Active Bath Heat Engine

Rahul Marathe

Department of Physics
IIT, Delhi

Arnab Saha
Department of Physics
Pune University

Priyo Pal, Arun Jayannavar
IOP, Bhubaneshwar

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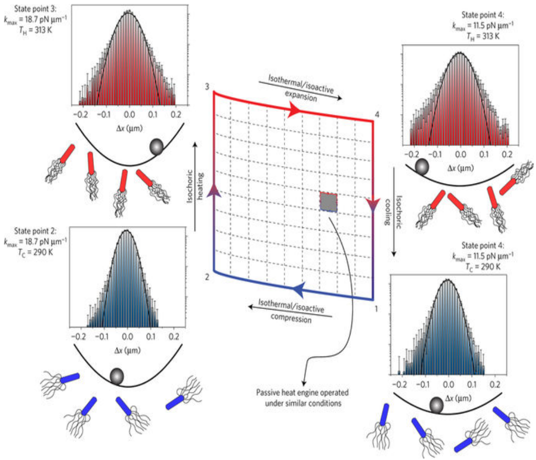
Background:

- Carnot, Stirling Engine like protocols on Brownian particle (theory + experiments)
- Typically consists of two isotherms and two adiabats.
  - Two temperatures involved.
- Expansion/Compression steps are performed using an Optical Trap.
- Interests:- Finite time efficiency, non-equilibrium behavior, work, heat distributions.

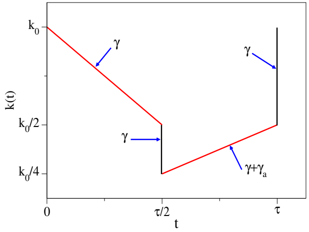
Motivation: Experiments with Bacteria (Active Particles)

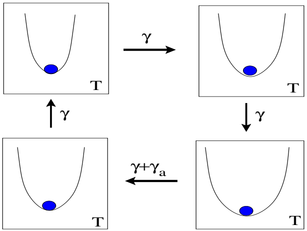


S. Krishnamurthy et al. Nature Physics **12**, 1134 (2016)

Model:

- Single Underdamped Brownian particle in Harmonic Trap
- Single Temperature, dissipation changes $\rightarrow$ Active bath





Equations:

$$m\dot{v}(t) = -\gamma\ v(t) - k_0 \left(1 - \frac{t}{\tau}\right)\ x(t) + \sqrt{2m\gamma k_B T}\ \xi(t), \quad 0 \le t \le \frac{\tau}{2}$$

$$m\dot{v}(t) = -(\gamma + \gamma_a)\ v(t) - k_0 \frac{t}{2\tau}\ x(t) + \sqrt{2m\gamma k_B T}\ \xi(t), \quad \frac{\tau}{2} < t \le \tau$$

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = \delta(t - t')$$

- τ finite $\rightarrow$ non-quasistatic, non-equilibrium.

Evolution equations for $\sigma_x = \langle x^2 \rangle$, $\sigma_v = \langle v^2 \rangle$

$$
\begin{aligned}
\frac{d\sigma_v}{dt} &= -2\gamma\sigma_v - k_1(t)\frac{d\sigma_x}{dt} + D, \\
\frac{d^2\sigma_x}{dt^2} &= 2\sigma_v - \gamma\frac{d\sigma_x}{dt} - 2k_1(t)\sigma_x, \\
\frac{d\sigma_v}{dt} &= -2(\gamma+\gamma_a)\sigma_v - k_2(t)\frac{d\sigma_x}{dt} + D, \\
\frac{d^2\sigma_x}{dt^2} &= 2\sigma_v - (\gamma+\gamma_a)\frac{d\sigma_x}{dt} - 2k_2(t)\sigma_x
\end{aligned}
$$
[1],

where, $k_1(t) = k_0\left(1 - \frac{t}{\tau}\right)$, $k_2(t) = k_0\frac{t}{2\tau}$, $D = \frac{2\gamma k_B T}{m}$
Can be solved numerically,
Analytical results $\tau \to \infty$, quasi-static limit..

[1] Dominic Arold et al., arXiv:1707.06441

Definitions of work and heat

- Work done in isothermal/adiabatic expansion and compression steps:

$$\langle W_1\rangle^{iso} = \frac{1}{2}\int_0^{\tau/2} \dot{k}_1(t)\sigma_x(t)\ dt,\quad \langle W_2\rangle^{iso} = \frac{1}{2}\int_{\tau/2}^{\tau} \dot{k}_2(t)\sigma_x(t)\ dt$$

$$\langle W_1\rangle^{adb} = U(\frac{\tau^+}{2}) - U(\frac{\tau^-}{2}),\quad \langle W_2\rangle^{adb} = U(0) - U(\tau^-),$$

change in internal energy at the steps.

- Heat in isothermal steps:

$$\langle Q_1\rangle^{iso} = \int_0^{\tau/2} \langle(-\gamma v + \sqrt{D}\xi(t))v\rangle\ dt = -\gamma\int_0^{\tau/2}\sigma_v\ dt + \frac{D\tau}{4}$$

$$\langle Q_2\rangle^{iso} = \int_{\tau/2}^{\tau} \langle(-(\gamma+\gamma_a)v+\sqrt{D}\xi(t))v\rangle\ dt = -(\gamma+\gamma_a)\int_{\tau/2}^{\tau}\sigma_v\ dt+\frac{D\tau}{4}$$

*Heats in adiabatic steps are zero

Quasistatic limit: $\tau \to \infty$

In this limit one can match the orders of $1/\tau$ in equations for σ_x, σ_v.

- Isothermal expansion:

$$\sigma_v = \frac{D}{2\gamma}$$

$$\sigma_x = \frac{D}{2\gamma k_0 \left(1 - \frac{t}{\tau}\right)}$$

- Isothermal compression:

$$\sigma_v = \frac{D}{2(\gamma + \gamma_a)}$$

$$\sigma_x = \frac{D}{(\gamma + \gamma_a) k_0 \frac{t}{\tau}}$$

Quasistatic limit: Work, Heat

Work:

$$\langle W_1\rangle^{iso} = \frac{-k_B T}{2}\ \ln(2), \quad \langle W_2\rangle^{iso} = \frac{k_B T}{2}\left(\frac{\gamma}{\gamma+\gamma_a}\right)\ln(2),$$

$$\langle W_1\rangle^{adb} = \frac{-k_B T}{4}, \quad \langle W_2\rangle^{adb} = \frac{k_B T}{2}\left(\frac{\gamma}{\gamma+\gamma_a}\right)$$

$$\langle W\rangle = \langle W_1\rangle^{iso} + \langle W_2\rangle^{iso} + \langle W_1\rangle^{adb} + \langle W_2\rangle^{adb}$$

Heat: Use first law:

$$\langle Q_1\rangle^{iso} = U(\tau/2) - U(0) - \langle W_1\rangle^{iso} = k_B T - \frac{3}{2}\frac{\gamma k_B T}{\gamma+\gamma_a} + \frac{k_B T}{2}\ \ln(2)$$

$$\langle Q_2\rangle^{iso} = U(\tau) - U(\tau/2) - \langle W_2\rangle^{iso} = -\frac{3k_B T}{4} + \frac{\gamma k_B T}{\gamma+\gamma_a} - \frac{k_B T}{2}\ \frac{\gamma}{\gamma+\gamma_a}\ \ln(2)$$

Note:

$$\langle Q_1\rangle^{iso} + \langle Q_2\rangle^{iso} + \langle W\rangle = 0$$

Quasistatic limit: Total Work, Efficiency

Total Work:

$$\langle W \rangle = \frac{k_B T}{2(\gamma + \gamma_a)} \left[\left(\frac{\gamma - \gamma_a}{2} \right) - \gamma_a \ln(2) \right]$$

$\langle W \rangle < 0$ if:

$$\gamma < \gamma_a (1 + 2 \ln(2))$$

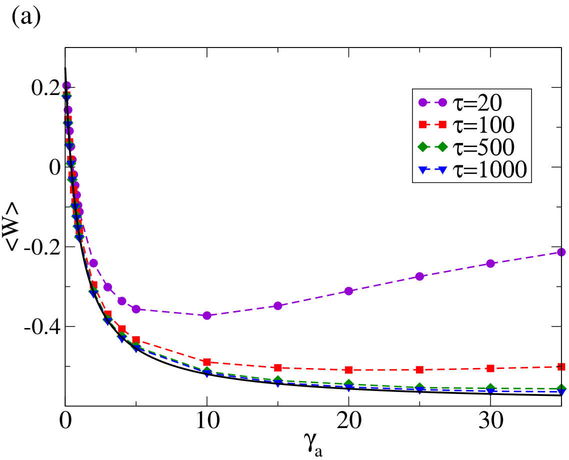
Efficiency:

$$\eta = \frac{\left| \langle Q_1 \rangle^{iso} + \langle Q_2 \rangle^{iso} \right|}{\langle Q_1 \rangle^{iso}} = \frac{|\langle W \rangle|}{\langle Q_1 \rangle^{iso}} = \frac{\left| \left(\frac{\gamma - \gamma_a}{2} \right) - \gamma_a \ln(2) \right|}{(2\gamma_a - \gamma) + (\gamma + \gamma_a) \ln(2)}$$

$\gamma \sim 1$, and in the limit $\gamma_a >> \gamma$:

$$\eta = \frac{1 + 2 \ln(2)}{4 + 2 \ln(2)} \sim 0.44$$

Simulation Results:



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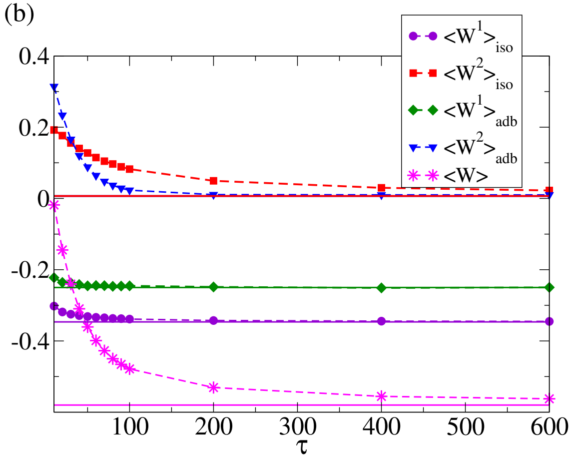

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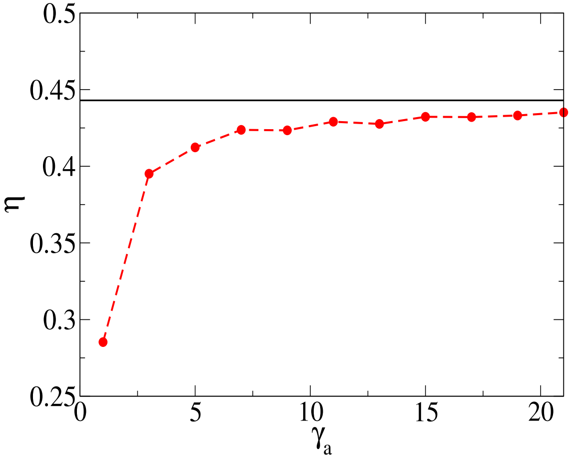

Other Aspects:

- Probability distributions of work, heat, efficiency.
- Optimal protocol, Efficiency at maximum power.
- Correlated noise.
- Looking at actual active Langevin dynamics e.g. velocity dependent potentials.

Thank You