Non-equilibrium dynamics in quasiperiodic lattices

Uma Divakaran

February 17, 2018

Indian Institute of Technology Palakkad
@ISPCM-2018

Quasiperiodic Lattices

1. When a given term in the Hamiltonian has a spatial dependence of the form $\cos(2\pi\beta j + \phi)$ at lattice site j, with β being an irrational number

Example-1: Aubry-André Model

S. Aubry and G. Andre, Ann, Israel Phys (1980), & P. G. Harper, Proc. Phys. Soc. A (1955)

spin-1/2 XX chain in presence of position dependent transverse field

$$H = -\frac{J}{4}\sum_{n=1}^{L}(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) - \sum_n h_n\sigma_n^z$$

Choose a quasiperiodic lattice given by

$$h_n = h\cos(2\pi\beta n)$$

If β is an irrational number $\rightarrow$ delocalization-localization transition at $h = J$ $\rightarrow$ known as Aubry-André model

All states for $h < J$ are delocalized and localized for $h > J$

Quasiperiodic potential has spacial ordering which is intermediate between disorder and periodic

AA model

- Eigenstates in the localized phase are exponentially localized

$$|\psi(x - x_0)|^2 \sim \exp(-(x - x_0)/\xi)$$

- The localization length ξ is a function of h and given by

$$\xi \sim \frac{1}{ln(h/J)}$$

Non-equilibrium dynamics of Aubry André model G. Roosz, U. Divakaran, H. Rieger, F. Igloi, Phys. Rev. B (2014)

Set $h_n = h\cos(2\pi\beta n)$

Sudden quench: $h = 0$ **to** $h = h_0$

- Start from ground state of $h = 0$ (say $|\psi\rangle^0$ with $H = H_0$) and quench to $h = h_0$ ($H = H$)
- Study evolution of entanglement entropy S of a subsystem A with l sites
- $S_l = Tr_l[\rho_l(t)\ln\rho_l(t)]$ where $\rho_l(t) = Tr_{n>l}|\psi(t)\rangle\langle\psi(t)|$

Comparison of Results for homogeneous and random case

Homogeneous case
(Known Result)

$$
\begin{aligned}
S_l(t) &\sim t \;\; \text{for t} < \text{l/v}_{\text{max}} \\
&\sim S_l \;\; \text{for t} > \text{l/v}_{\text{max}}
\end{aligned} \tag{1}
$$

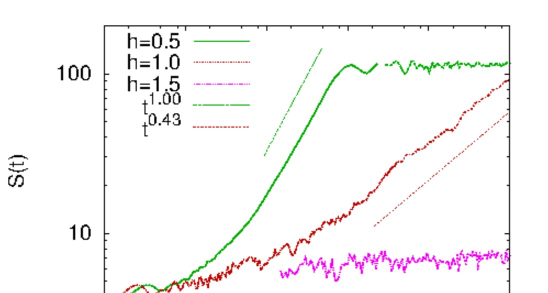
Random case
(Known Result)

- $S_l(t)$ saturates to a finite value due to localized excitations in non-critical phase
- ultra slow dynamics at the critical point $S_l(t) \sim \ln \ln t$

Sudden quench results in quasiperiodic lattice: $h_0 = 0 \to h$

Numerical results using free Fermion technique

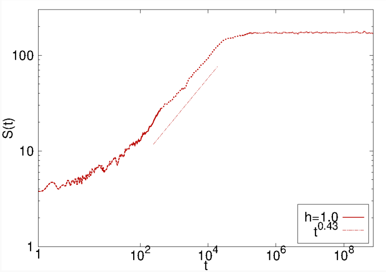
- Quench to extended phase $(0 < h < 1)$ $S_l(t) \sim \alpha(h)t$ and the saturation value is $\tilde{S}_l \sim l$
  similar to the homogeneous system
- Quench to localized phase $(h > 1)$ $S_l(t)$ saturates to l-independent value $\tilde{S}(h)$
  similar to random system



Quench results

- Quench to critical point

$$S_l(t) \sim t^{\sigma}$$



- → **Neither homogeneous nor random**

  Numerical value of $\sigma = 0.43$

  Can we understand this exponent σ??

- These results can be explained using quasiparticles propagating/diffusing in the lattice

Quasiparticle interpretation of $S(t)$ in homogeneous case

Checked quantitatively for transverse Ising model and XY model in transverse field

Calabrese, Cardy, J.Stat (2005), F. Igloi and H. Rieger, Phys. Rev. Lett (2011)

- Initial state $|\psi^0\rangle$ is an excited state with respect to final quenched Hamiltonian
- $|\psi^0\rangle =$ source of quasiparticle excitations
- Pair of quasiparticles produced randomly at sites
- The quasiparticle pair with one at subsystem A and the other in B at time $t \rightarrow$ contribute to the entanglement between A and B
- The analysis of spreading of wavepacket verifies the obtained results

Example 2: Transverse Ising model with a quasiperiodic transverse field

A. Chandran and C. R. Laumann, Phys. Rev. X (2017)

$$h_j = h + A_h \cos(2\pi\beta j + \phi)$$

- Can not be exactly solved
- Rich phase diagram consisting of low-lying delocalized states and localized excited states depending upon the parameters of the Hamiltonian
- How is this going to affect the non-equilibrium dyanmics??

Conclusions

- Sudden quench dynamics of Aubry André model shows clear characteristics of localized and delocalized phases
- Studied also by evolution of entanglement entropy
- Rich non-equilibrium dynamics in the quasiperiodic Ising model still to be fully understood