# Higher-dimensional Sachdev-Ye-Kitaev Non-Fermi Liquids at Lifshitz transitions

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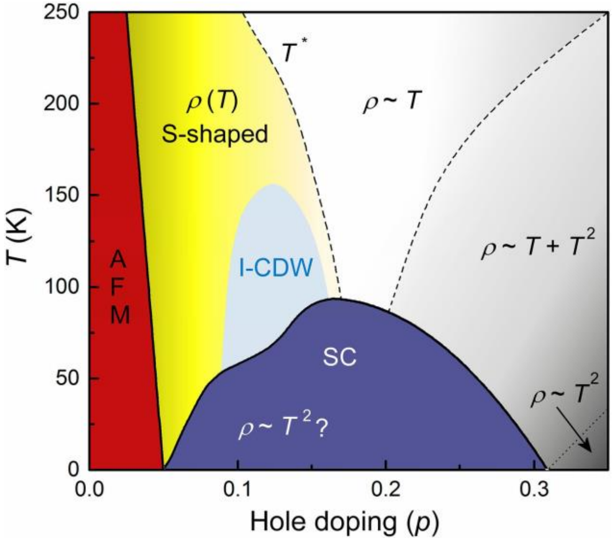
arXiv:1710.00842 (2017)

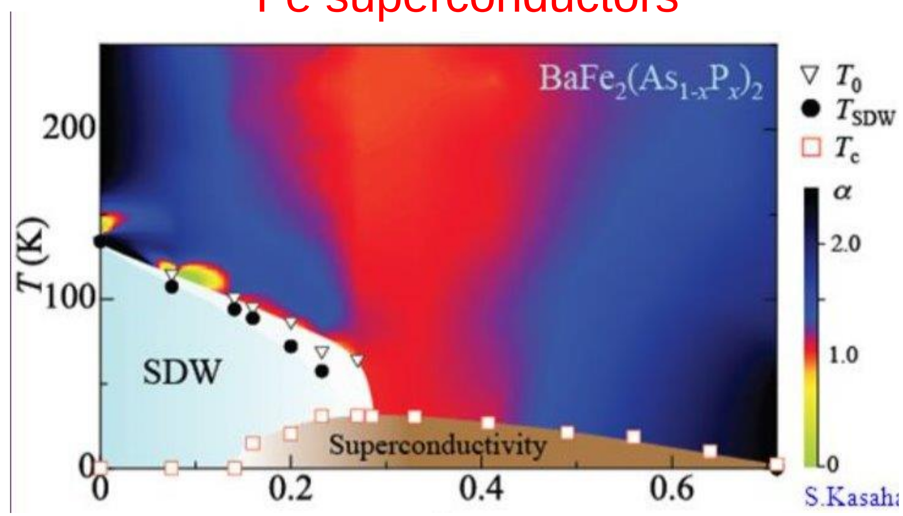
# How to describe transport in metals without quasiparticles: `Strange metals'

Cuprates




Proust et al. 2016

Fe superconductors




Kasahara et al. 2010

- Resistivity of metals, $\Delta\rho(T) \sim T^{\alpha}$
- Strange metals or non-Fermi liquids, $\alpha < 2$

Electronic self-energy,

$\Sigma(\omega) \sim \omega^{\theta} \gg \omega, \;\; \omega \to 0$

$\theta < 1$

No well defined quasiparticle

# 0-dimensional model of NFL: Sachdev-Ye-Kitaev (SYK) model

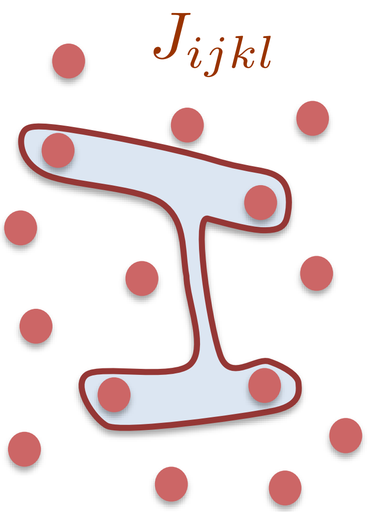
$$H_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_{i}^{N} c_i^\dagger c_i$$

$$P(J_{ijkl}) \sim e^{-\frac{|J_{ijkl}|^2}{J^2}}$$

- Solvable in strong coupling for large $N$

→ Non-Fermi liquid ground state

- Emergent conformal (time reparameterization) symmetry at low energy.




$N$ sites

Kitaev → Solvable model for holography

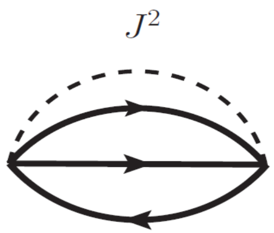
Sachdev & Ye, PRL (1993)
Kitaev, KITP (2015)
Sachdev, PRX (2015)

# SYK non-Fermi liquid phase

Half filling, $\mu = 0$

- Disordered averaged saddle point for $N \to \infty$

$$G^{-1}(\omega) = \cancel{\omega} - \Sigma(\omega)$$




$$\Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

→ Conformal (time reparameterization) symmetry

$$\tau = f(\sigma)$$

$$f'(\sigma) = \frac{\partial f}{\partial \sigma}$$

$$\tilde{G}(\sigma_1, \sigma_2) = [f'(\sigma_1) f'(\sigma_2)]^{\Delta}\, G(f(\sigma_1), f(\sigma_2))$$

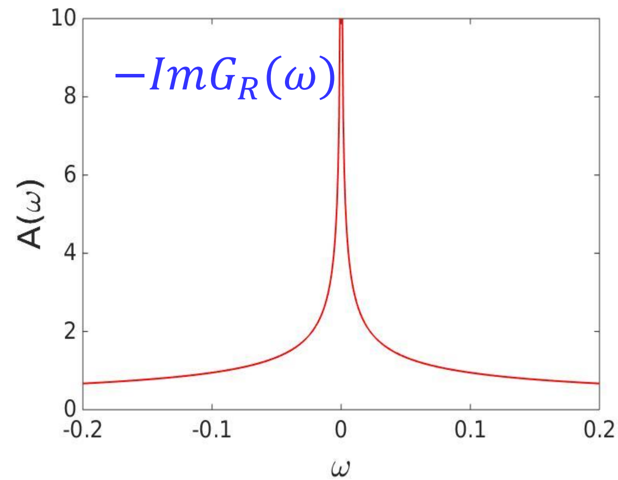
Scaling dimension, $\Delta = 1/4$

- Non Fermi liquid fixed point

$$G_R(\omega) = \Lambda e^{-\frac{i\pi}{4}} \sqrt{\omega} \sim \omega^{2\Delta - 1}$$

$$\Sigma_R(\omega) \sim -\Lambda^3 e^{\frac{i\pi}{4}} \sqrt{J\omega}$$

* Contrast with $\Sigma_{FL}(\omega) \sim \omega^2$

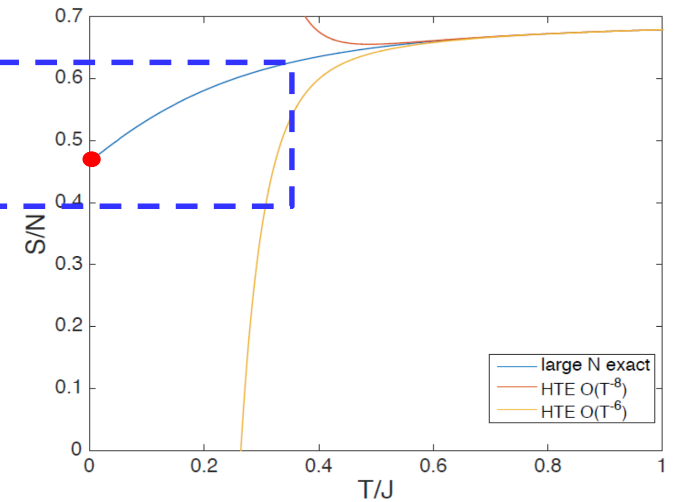



Divergent density of states (DOS)

# Residual entropy and quantum chaos

○ Extensive *T=0* residual entropy (for *T→0, N→∞*)

→ Black hole entropy (?)




Fu & Sachdev, PRB (2016)

○ Many-body quantum chaos

Out-of-time-order correlation (OTOC)

$$\langle c_i^\dagger(t) c_j(0) c_i^\dagger(t) c_j(0) \rangle \sim 1 - \left(\frac{\beta J}{N}\right) e^{\lambda_L t}$$

Kitaev, 2015

Lyapunov exponent or information scrambling rate

$$\lambda_L = 2\pi T$$

Upper bound to quantum chaos

Maldacena, Shenker & Stanford (2015)

Fastest scrambler in nature, like a black hole!

→ Dual to quantum gravity in $AdS_2$ (?)

# How to get a Fermi liquid out of SYK?

Very easy! add a quadratic term

$$\mathcal{H}_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{N}} \sum_{ij} t_{ij} c_i^\dagger c_j \qquad \overline{|t_{ij}|^2} = t^2$$

Solvable model of Fermi liquid for $N \to \infty$

- Constant DOS and Fermi-liquid self-energy for $t \neq 0$

→ Interaction irrelevant for $\omega \to 0$

$$G_R(\omega) \sim -i/t \qquad \Sigma_R(\omega) \sim \omega^2$$

Quadratic term is relevant. Always a Fermi liquid.

But, SYK model is 0-dimensional (infinite range), like a quantum dot!

→ Can not ask meaningful question about transport.
Or, gravity in higher dimension.

How to generalize SYK model to higher dimension?
→ Can we get new higher dimensional analogue of SYK non-Fermi liquid?

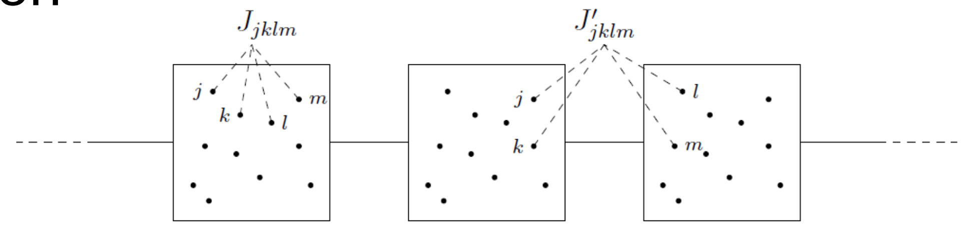
Not easy! Interaction effect for fermion becomes marginal or irrelevant in higher dimensions.

→ Hard to get a SYK non-Fermi liquid in higher dimension.

# Lattice and higher dimensional generalizations of SYK

- Lattice of 'SYK dots' with random inter dot interaction

Gu, Qi and Stanford (2017), .......




→ Diffusive transport in a metal without quasiparticles
Connection between transport and chaos

→ However, The NFL is same as the 0-dimensional SYK at the saddle point, no dispersive fermions.

- Lattice of 'SYK dots' with random or uniform inter dot hopping.

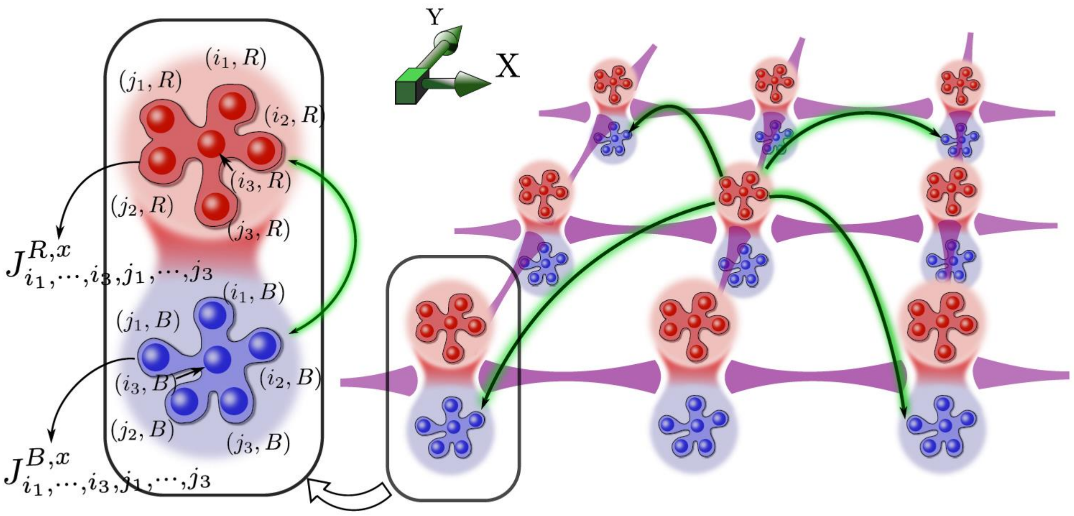
→ Fermi liquid phase at zero temperature, crossover to SYK-like behavior above some temperature.

- 1+1D 'low pass SYK model'

Berkooz et al. (2017)

→ New higher-dimensional NFL, but very phenomenological construction

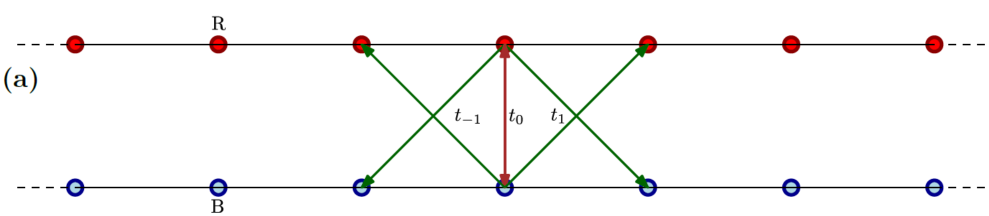
# A model for 'canonical' higher-dimensional SYK non-Fermi liquids




SYK dots with two colors (R and B) with uniform interdot hopping

- Particle-hole symmetric energy dispersion

$$\epsilon_{\pm}(k) = \pm|k|^p$$



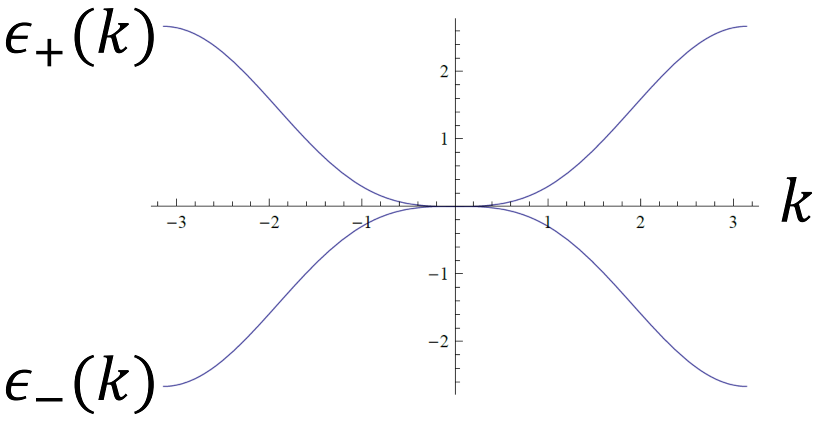

→ Appropriate choice of the hoppings $t_{-1}, t_0, t_1, \ldots$ → $p = 1,2,3, \ldots$

# Singular non-interacting density of states

$\epsilon_{\pm}(k) = \pm|k|^p$

Higher-order Band touching point, e.g. quadratic, cubic, ..

Appear at Lifshitz transitions, e.g. between topological band insulating phases




Heikkila, Volovik (2010), …

→ Diverging non-interacting density of states for $d < p$

$$g(\epsilon) \sim |\epsilon|^{-\gamma} \qquad \gamma = 1 - \frac{d}{p}$$

$$\int_{-\Lambda}^{\Lambda} g(\epsilon) d\epsilon = 1$$

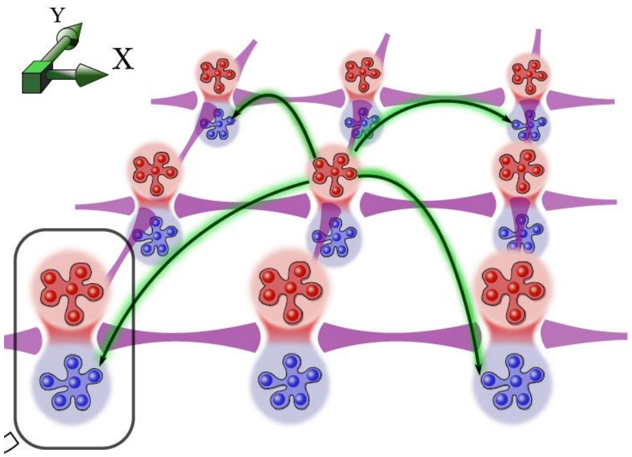
Normalizable DOS → $0 < \gamma < 1$

* $\gamma = 0$ → typical constant DOS

How does it help to realize higher-dimensional non-Fermi liquids?

Generalized SYK model for the dots: $SYK_{2q}$

$q$-body intra-dot SYK interaction




$$S = \int d\tau \left[ \sum_{ixa} \bar{c}_{ixa} \partial_\tau c_{ixa} + \sum_{ixx'a} t_{x-x'} \bar{c}_{ixa} c_{ix'a} + \sum_{i_1,\dots,i_q,j_1,\dots,j_q,xa} J_{i_1\dots i_q j_1 \dots i_q, x} \bar{c}_{i_1xa} \dots \bar{c}_{i_qxa} c_{j_1xa} \dots c_{j_qxa} \right]$$

Here $\overline{|J_{i_1\dots i_q j_1 \dots j_q, x}|^2} = 2J^2/qN^{2q-1}$

Large -$N$ Saddle point equations

$$G(\omega) = \frac{1}{L^d} \sum_{k,s=\pm} \frac{1}{\omega - \epsilon_{ks} - \Sigma(\omega)} = \int_{-\Lambda}^{\Lambda} d\epsilon \frac{g(\epsilon)}{\omega - \epsilon - \Sigma(\omega)}$$

$$\Sigma(\tau) = (-)^{q+1} J^2 G^q(\tau) G^{q-1}(-\tau)$$

- At zero temperature and low energies, $\omega, \Sigma(\omega) \ll \Lambda$

$$G(\omega) \sim \left(\omega - \Sigma(\omega)\right)^{-\gamma}$$

# Saddle point solutions: NFL and FL fixed points

$$G(\omega) \sim \left(\omega - \Sigma(\omega)\right)^{-\gamma}$$

- NFL fixed point, $\omega \ll \Sigma(\omega)$ $\quad G(\omega) \sim \Sigma^{-\gamma}(\omega)$ → $G_R(\omega) \sim \omega^{2\Delta-1}$

Fermion scaling dimension

$$\Delta = \frac{1+\gamma}{2(1-\gamma+2\gamma q)}$$
$$= \frac{1}{2} \qquad \gamma = 0$$ ← Non-interacting fermions
$$= \frac{1}{2q} \qquad \gamma = 1$$ ← 0-dimensional SYK

Dynamical exponent

$$z = \frac{p}{2(2q-1)\Delta - 1}$$

Distinct non-Fermi liquids from parent SYK!

NFL is self-consistent only for $\omega \ll \Sigma(\omega)$ as $\omega \to 0$ → Critical value of $\gamma$

$$\gamma \geq \gamma_c = \frac{2q-3}{2q-1}$$

- For $\gamma < \gamma_c$ → Perturbative fixed point , Lattice Fermi liquid (LFL)

$$G(\omega) \sim \omega^{-\gamma}$$

******* * ** * * 0-dim SYK fixed points

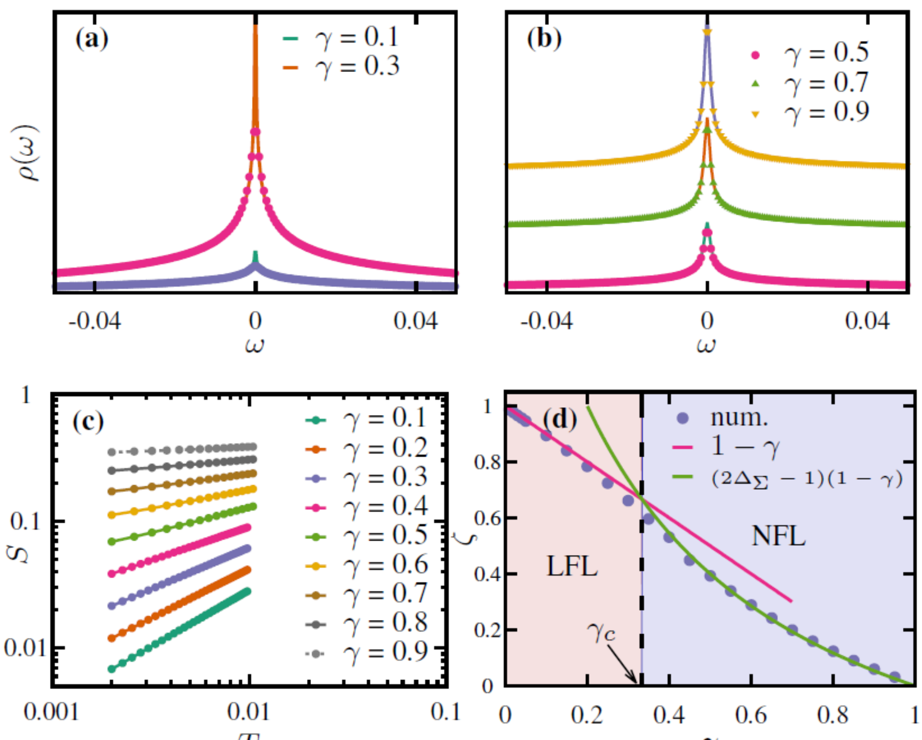
Line of distinct NFL fixed points

# Low-temperature entropy

$$S \sim T^{1-\gamma}$$

$$S \sim T^{\zeta} \qquad \zeta = \frac{2(q-1)(1-\gamma)}{1-\gamma+2\gamma q}$$

*Recovers the residual entropy only for $\gamma \to 1$




← Numerical solution of saddle-point equation

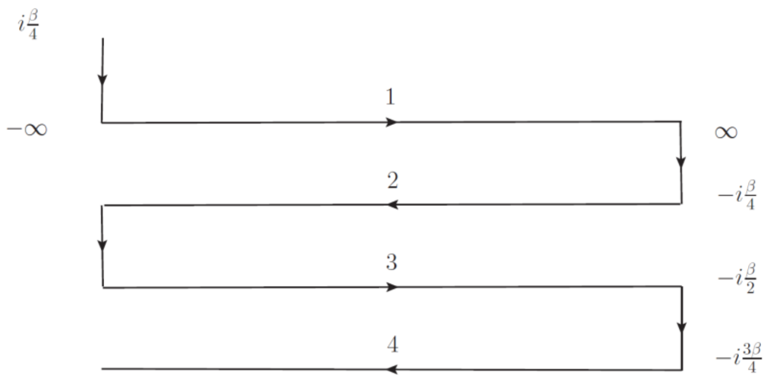
What happens to quantum chaos across the NFL-FL transition?

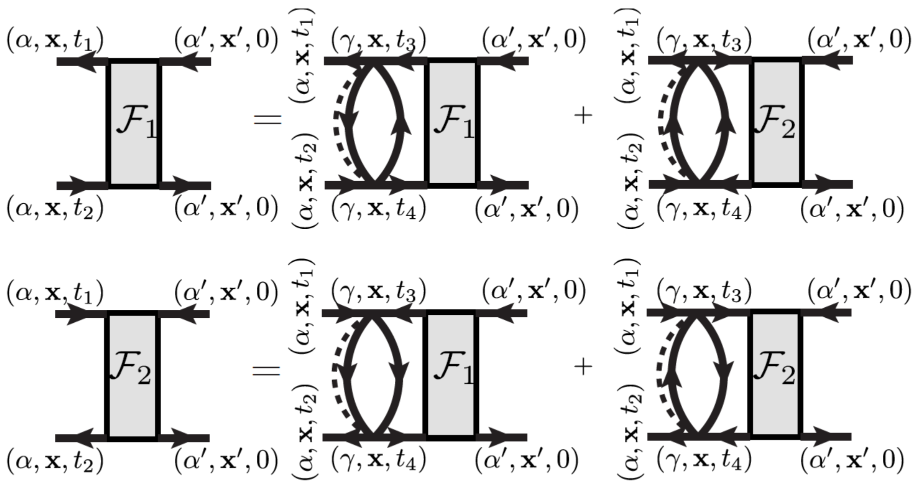
# Lyapunov exponent

## Out-of-time-order correlations (OTOC)

$$(1/N^2)\sum_{ij}\langle c^\dagger_{i\alpha x}(t)c^\dagger_{j\beta x'}(0)c_{i\alpha x}(t)c_{j\beta x'}(0)\rangle$$

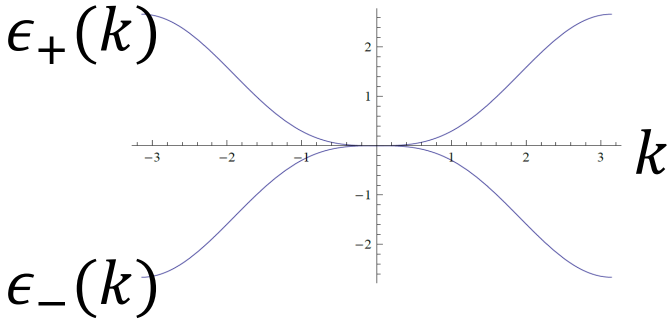
## Keldysh contour

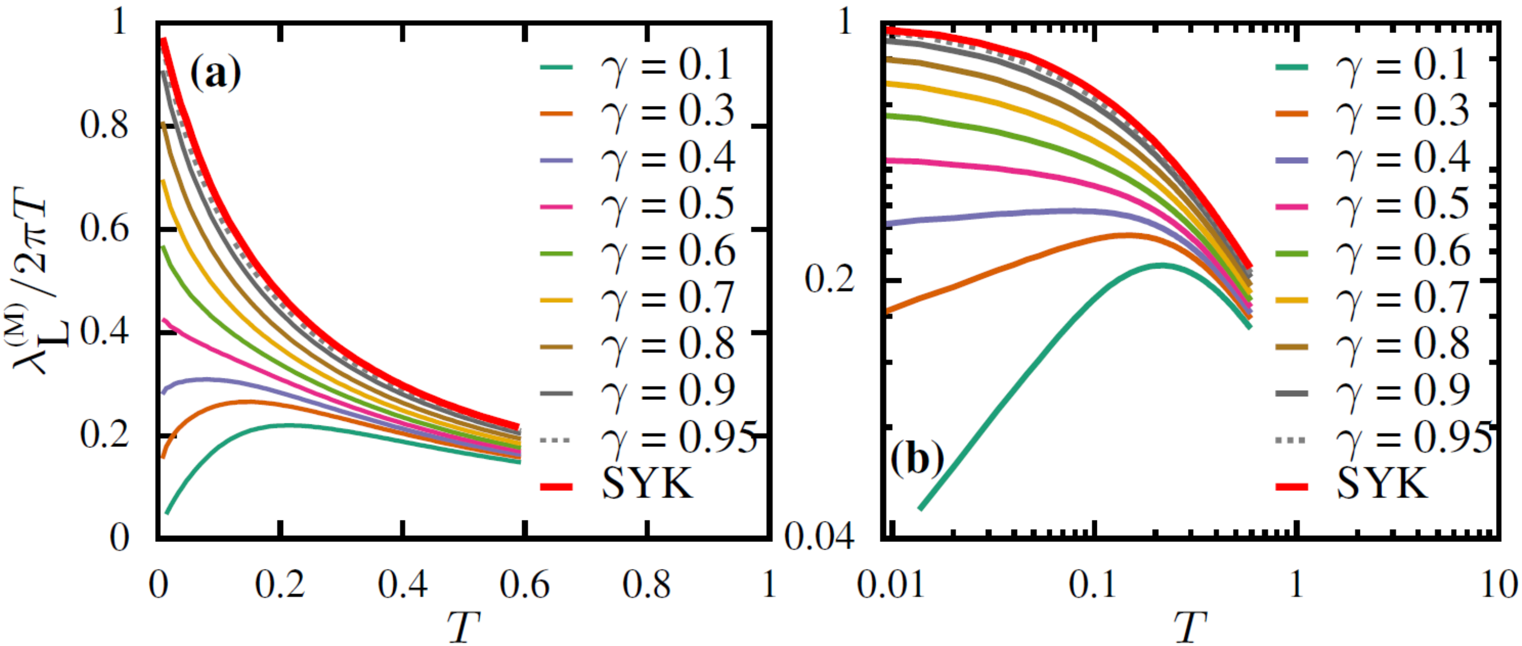


Fast ($\boldsymbol{q} = 0$) `intraband' scrambling mode

→ Maximum Lyapunov exponent $\lambda_L^{(M)}$




○ $\gamma > \gamma_c$, NFL phase → $\lambda_L^{(M)} = \alpha T$ , $\alpha \to 2\pi$ for $\gamma \to 1$

○ $\gamma < \gamma_c$, FL phase → $\lambda_L^{(M)} \sim T^\eta$ , $\eta > 1$