Universality properties of steady driven coagulation with collisional fragmentation

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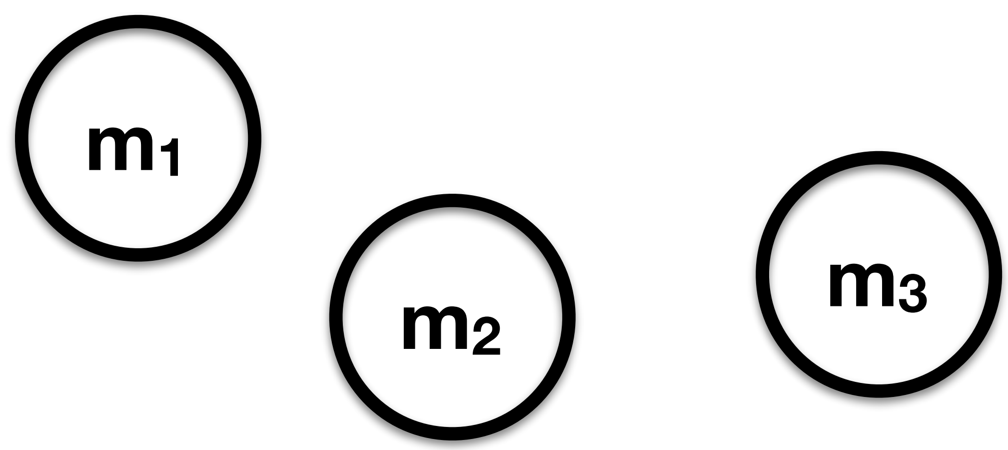
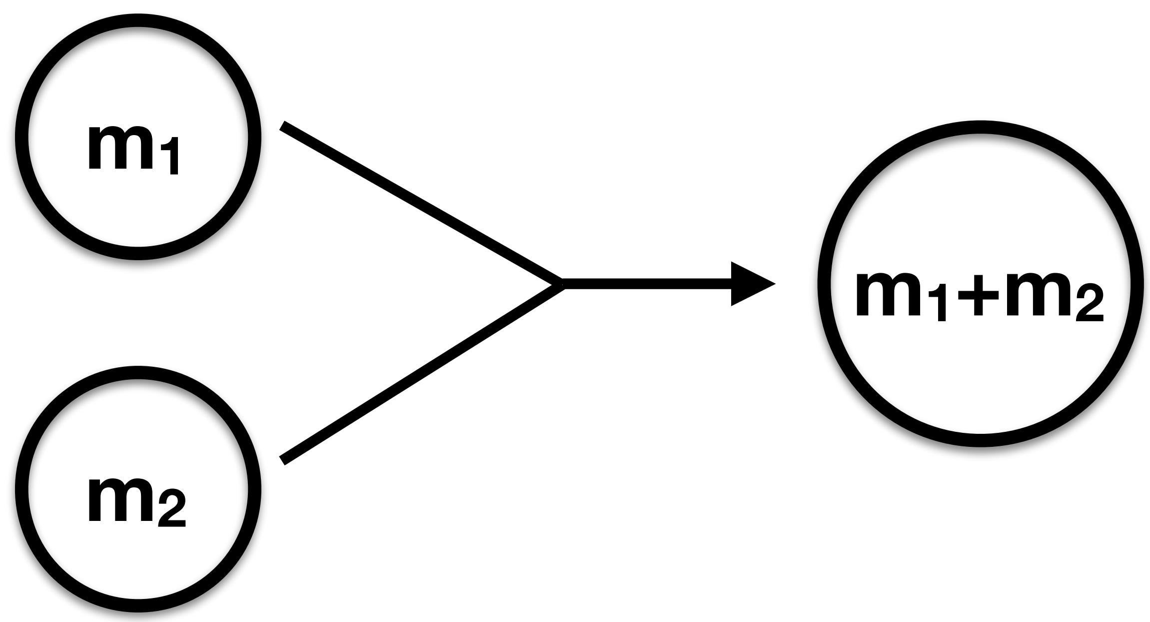
arXiv:1710.01875

The model



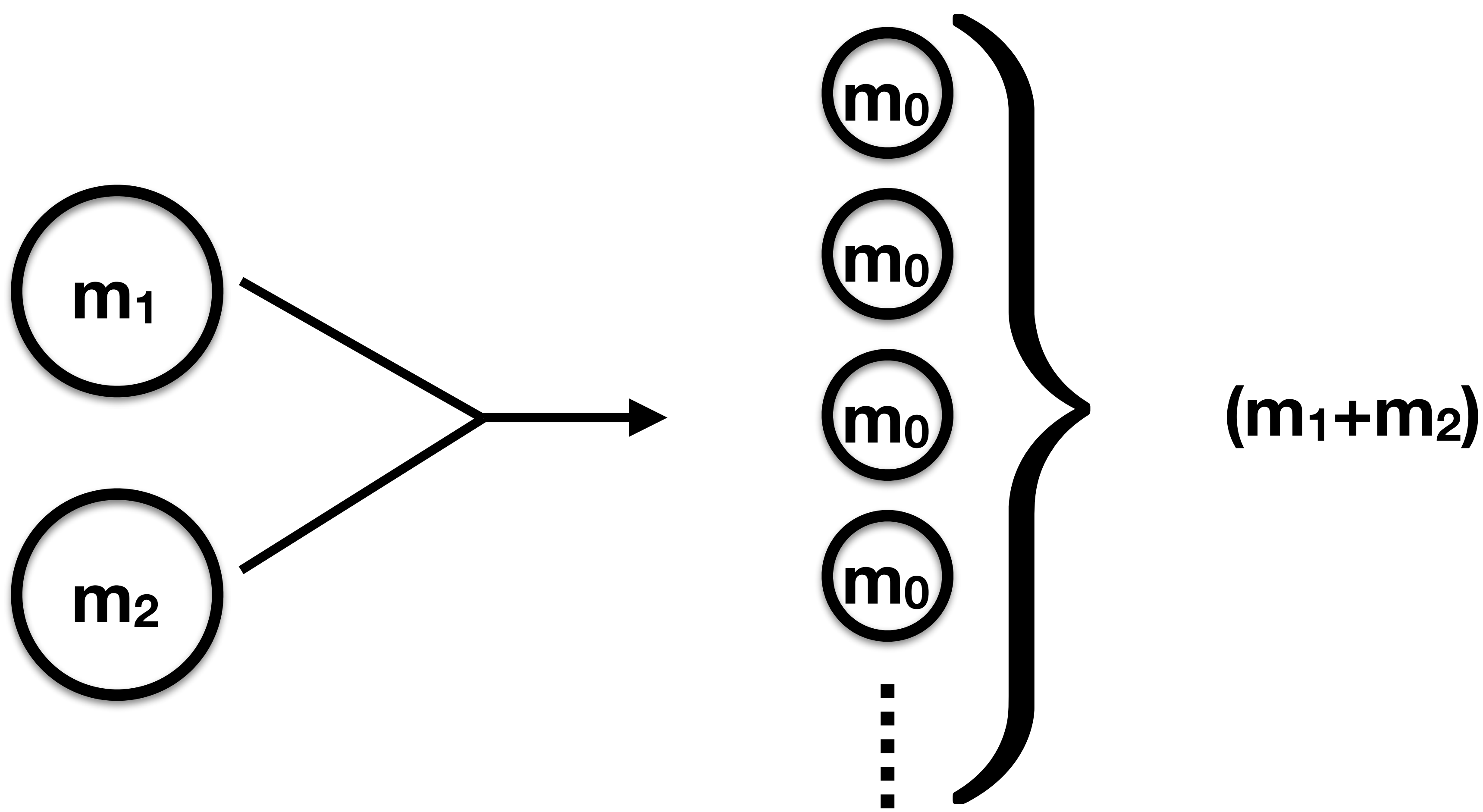
Mass measured in terms of smallest mass m_0 such that mass is an integer

Coagulation



Coagulation conserves mass but decreases number of particles

Collisional fragmentation



$m_1+m_2 \rightarrow (m_1+m_2)$ particles of mass m_0

Collision kernel $K(m_1, m_2)$: Example

- **Ballistic transport:** $$K(m_1, m_2) \propto (R_1 + R_2)^{d-1}\sqrt{v_1^2 + v_2^2}$$ $$= (m_1^{1/d} + m_2^{1/d})^{d-1}\sqrt{\frac{1}{m_1} + \frac{1}{m_2}}$$

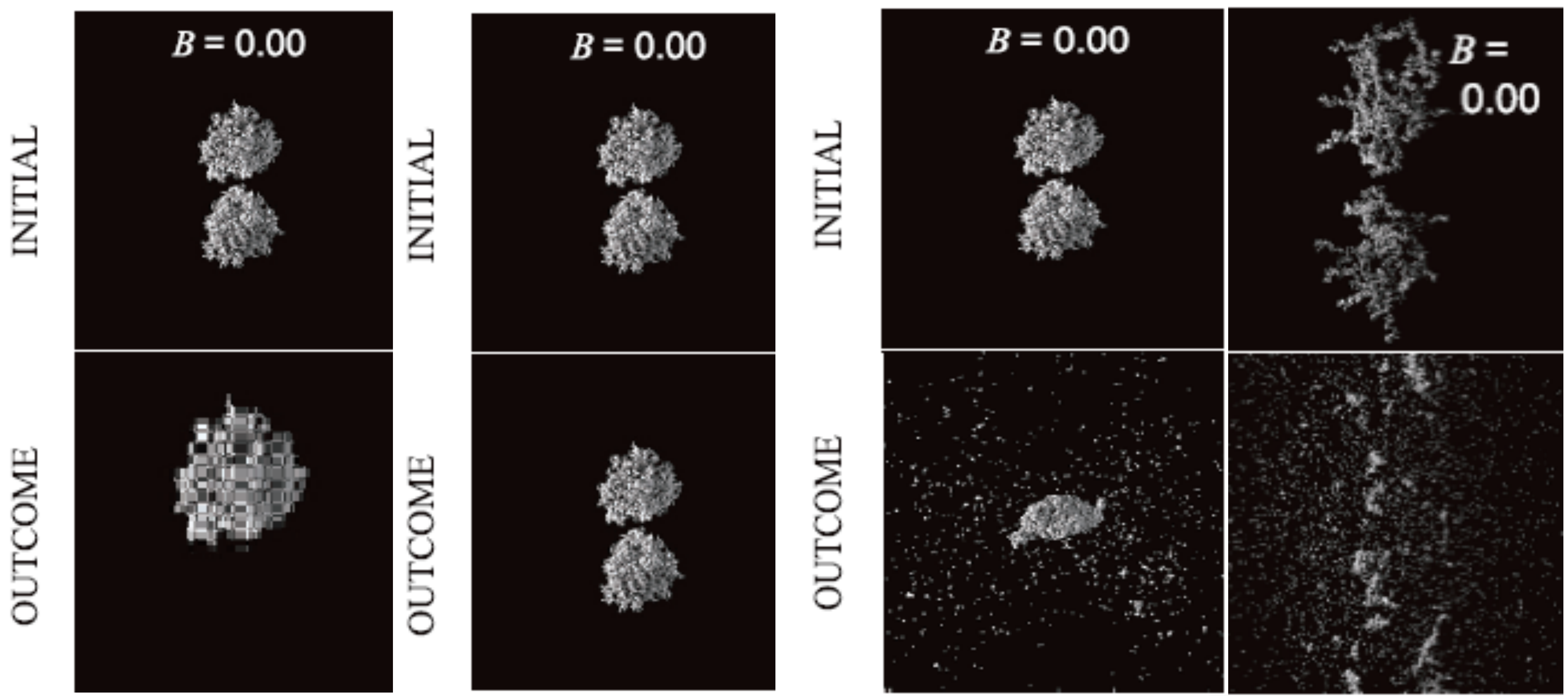
- **Homogeneous:** $$K(hm_1, hm_2) = h^{\frac{d-1}{d} - \frac{1}{2}} K(m_1, m_2)$$

- **$m_1 \gg m_2$:** $$K(m_1, m_2) \sim \frac{m_1^{\frac{d-1}{d}}}{\sqrt{m_2}} \sim m_1^{\nu} m_2^{\mu}$$

Collision kernel K(m_1,m_2)

- **Will consider kernel:** $$K(m_1, m_2) = \frac{1}{2}(m_1^{\mu} m_2^{\nu} + m_1^{\nu} m_2^{\mu})$$
- **Homogeneity exponent: $\beta=\mu+\nu$**
- **Locality exponent: $\theta=|\nu-\mu|$**
- **On collision**
  - **Coagulate** **Prob=1/(1+λ)**
  - **Fragment** **Prob=λ/(1+λ)**
- **Question: What is the steady state mass distribution?**
- **Model parameters: β, θ, λ**

Example

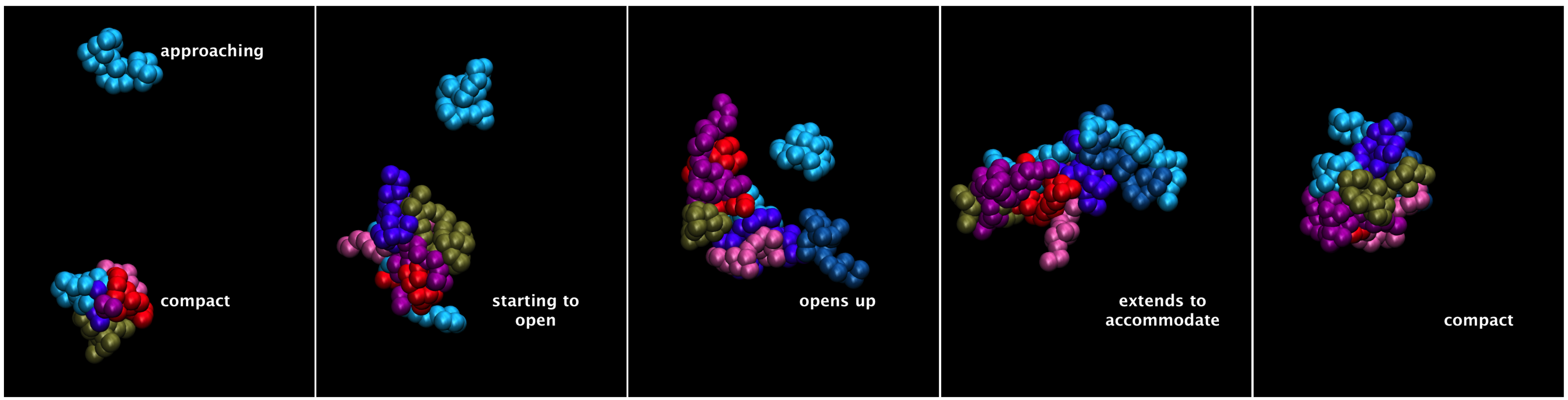


Small V_{imp}　***Medium V_{imp}***　***Large V_{imp}***

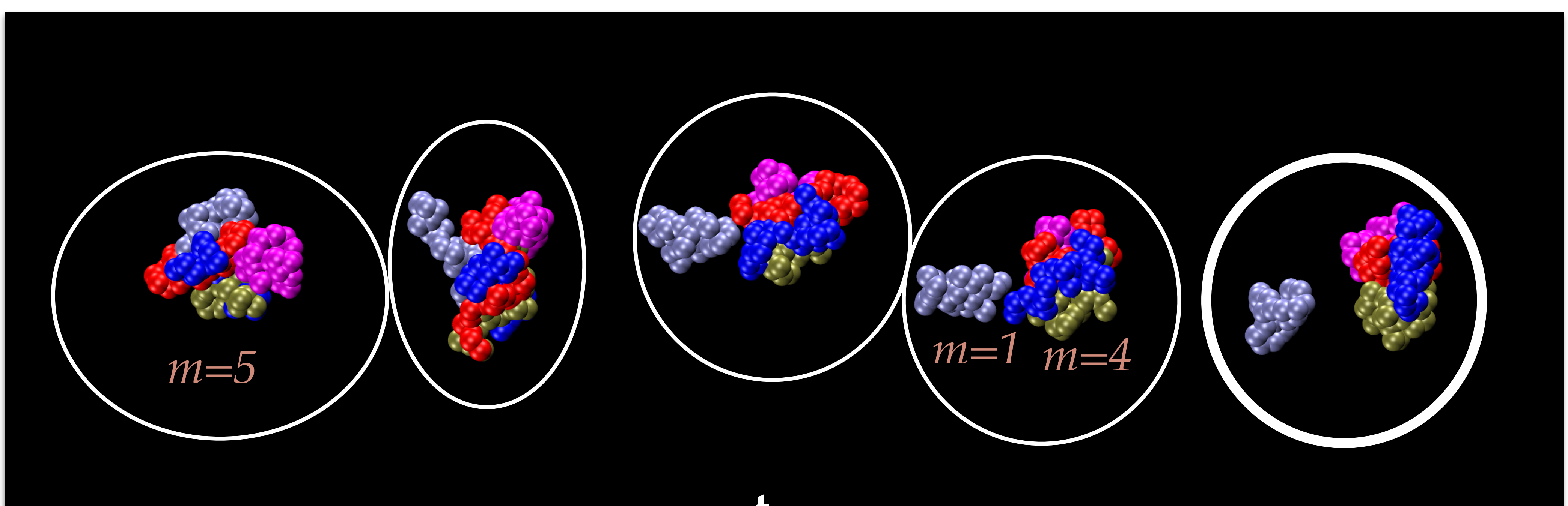
K. Wada, Astrophys. J. (2009)

Example

Aggregation

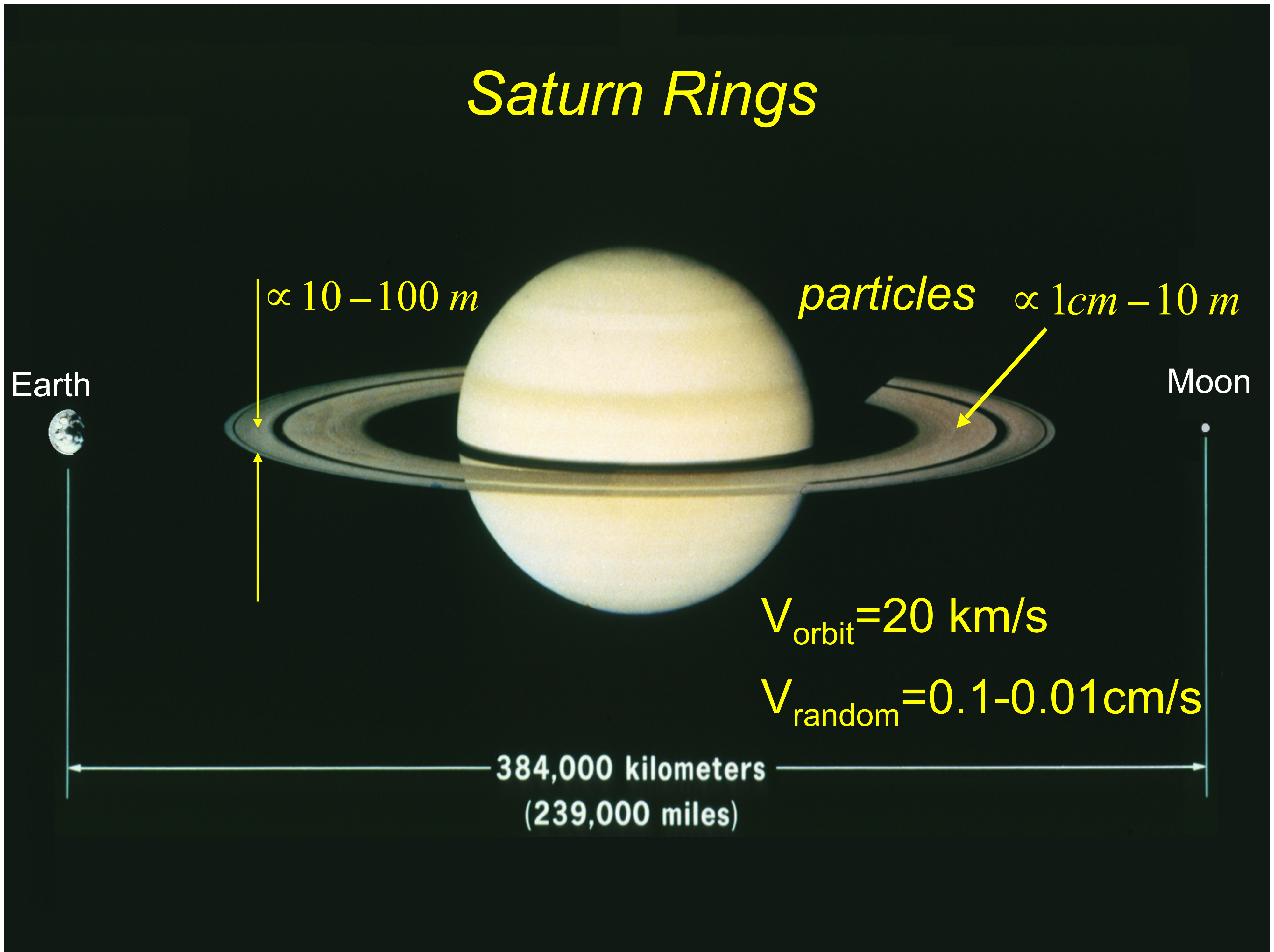


Fragmentation



Tom et al,
J. Chem. Phys. (2017)

Primary motivation: Rings of Saturns



Assumptions

- **Mean field treatment: assume well mixed system so that spatial inhomogeneities may be ignored**
- **Smoluchowski rate equation**

$$
\begin{aligned}
\frac{dN(m,t)}{dt} =& \frac{1}{2}\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty} N(m_1,t)N(m_2,t)K(m_1,m_2)\delta(m_1+m_2-m) \\
&- (1+\lambda)\sum_{m_1=1}^{\infty} N(m_1,t)N(m,t)K(m_1,m) \\
&+ \frac{\lambda}{2}\delta_{m,1}\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty} N(m_1,t)N(m_2,t)K(m_1,m_2)(m_1+m_2).
\end{aligned}
$$

Approximate solution

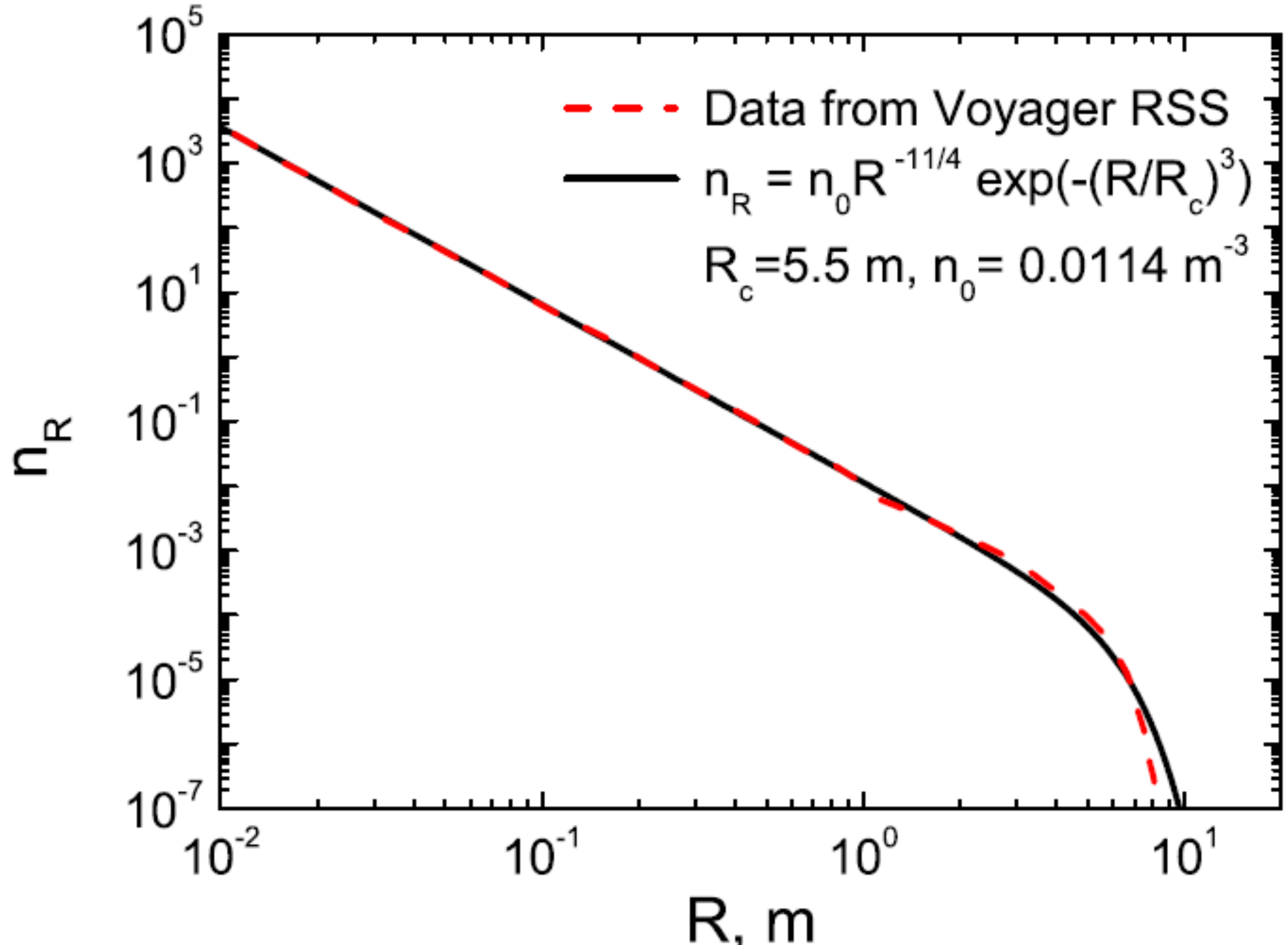
- **Ballistic transport:** $K(m_1, m_2) = (m_1^{1/d} + m_2^{1/d})^{d-1}\sqrt{\frac{1}{m_1} + \frac{1}{m_2}}$
- **β:** $K(hm_1, hm_2) = h^{\frac{d-1}{d} - \frac{1}{2}} K(m_1, m_2)$
- **$m_1 \gg m_2$:** $K(m_1, m_2) \sim \frac{m_1^{\frac{d-1}{d}}}{\sqrt{m_2}} \sim m_1^{\nu} m_2^{\mu}$

d	μ	ν	β	θ
3	-1/2	2/3	1/6	7/6

- **Difficult to solve with θ=7/6**
- **Instead solve with θ=0 keeping β=1/6 the same**

Brilliantov et al, PNAS (2015)

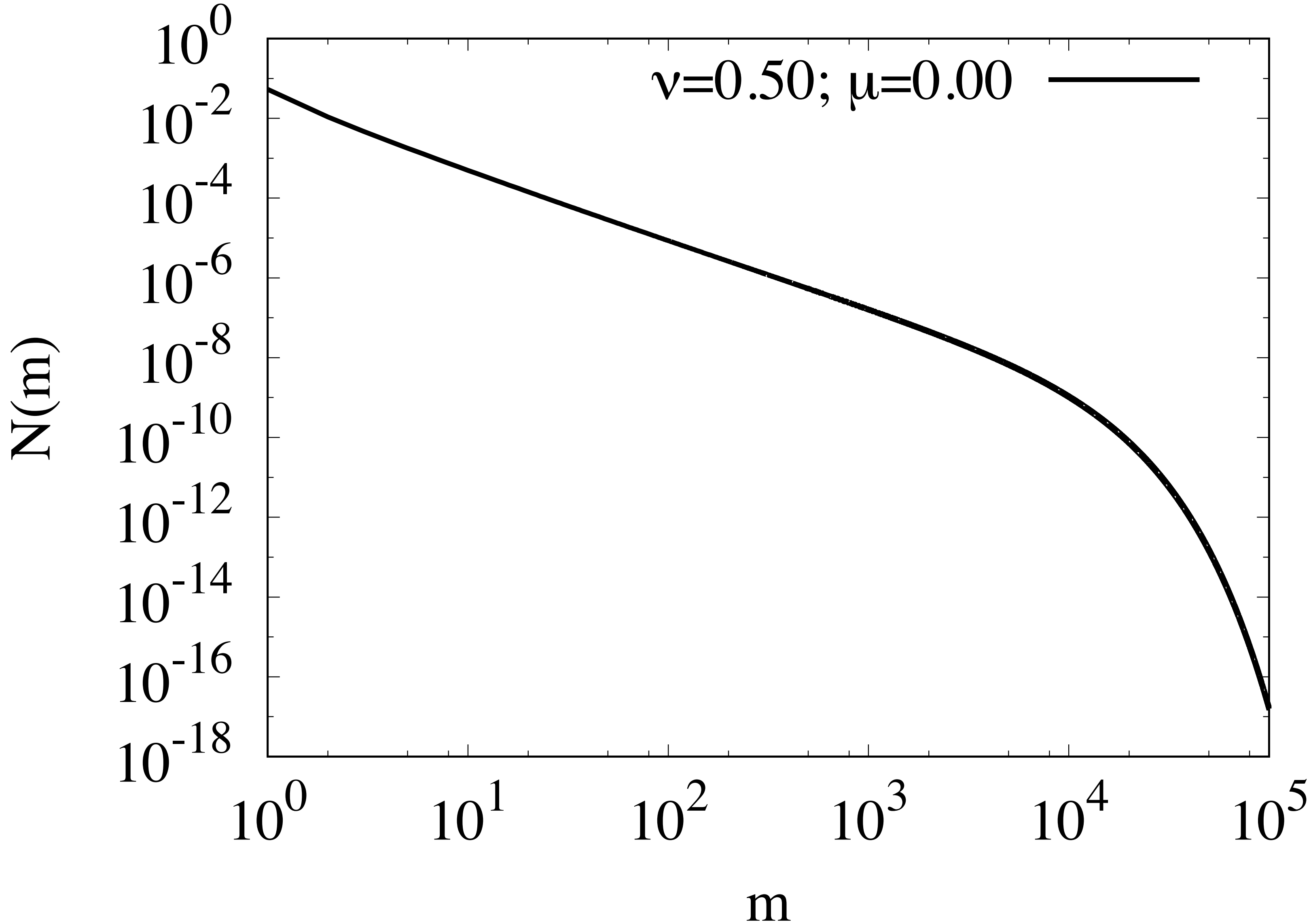
Rings of Saturn



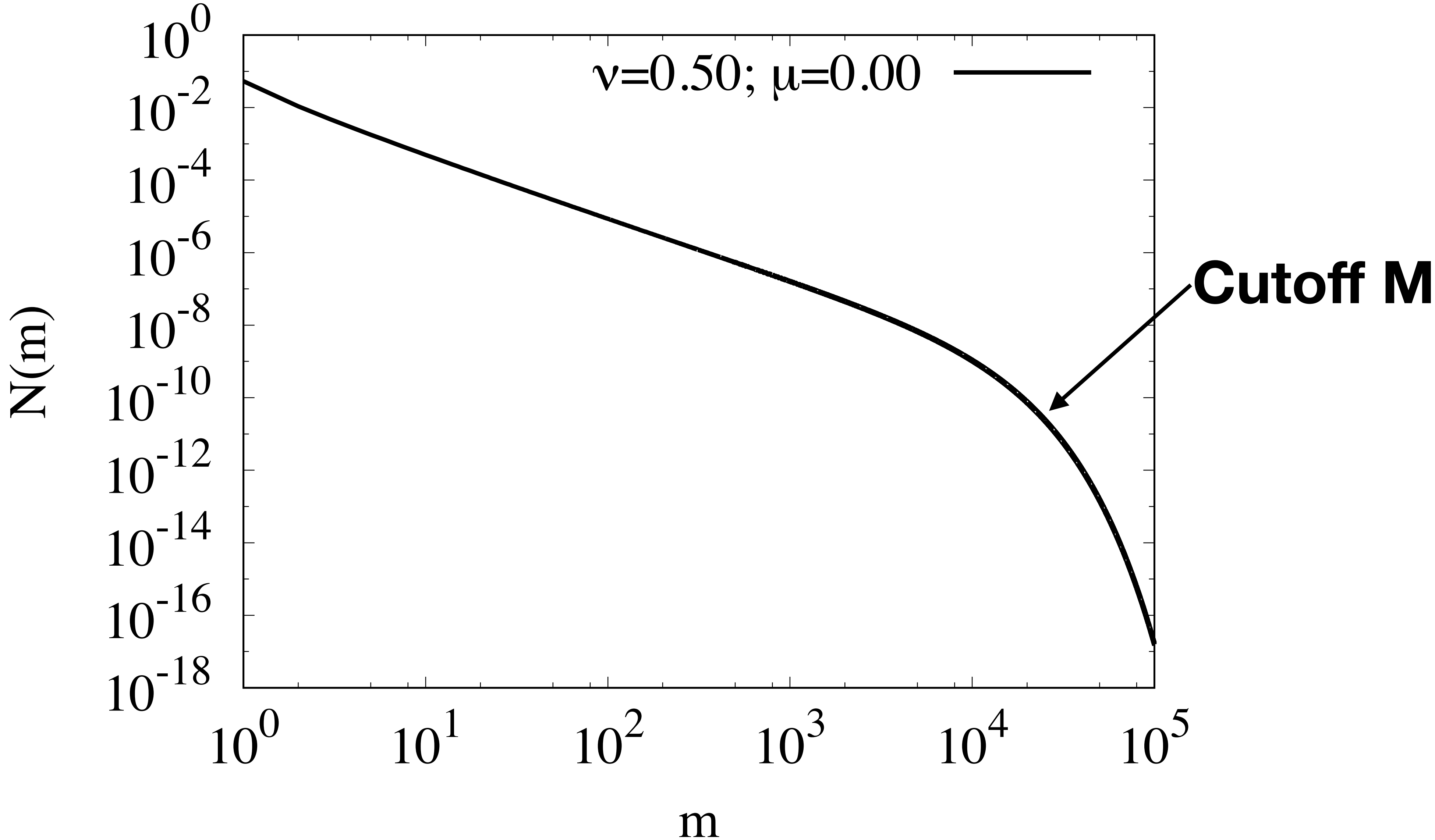
Brilliantov et al, PNAS (2015)

Excellent fit.
λ is a fitting parameter

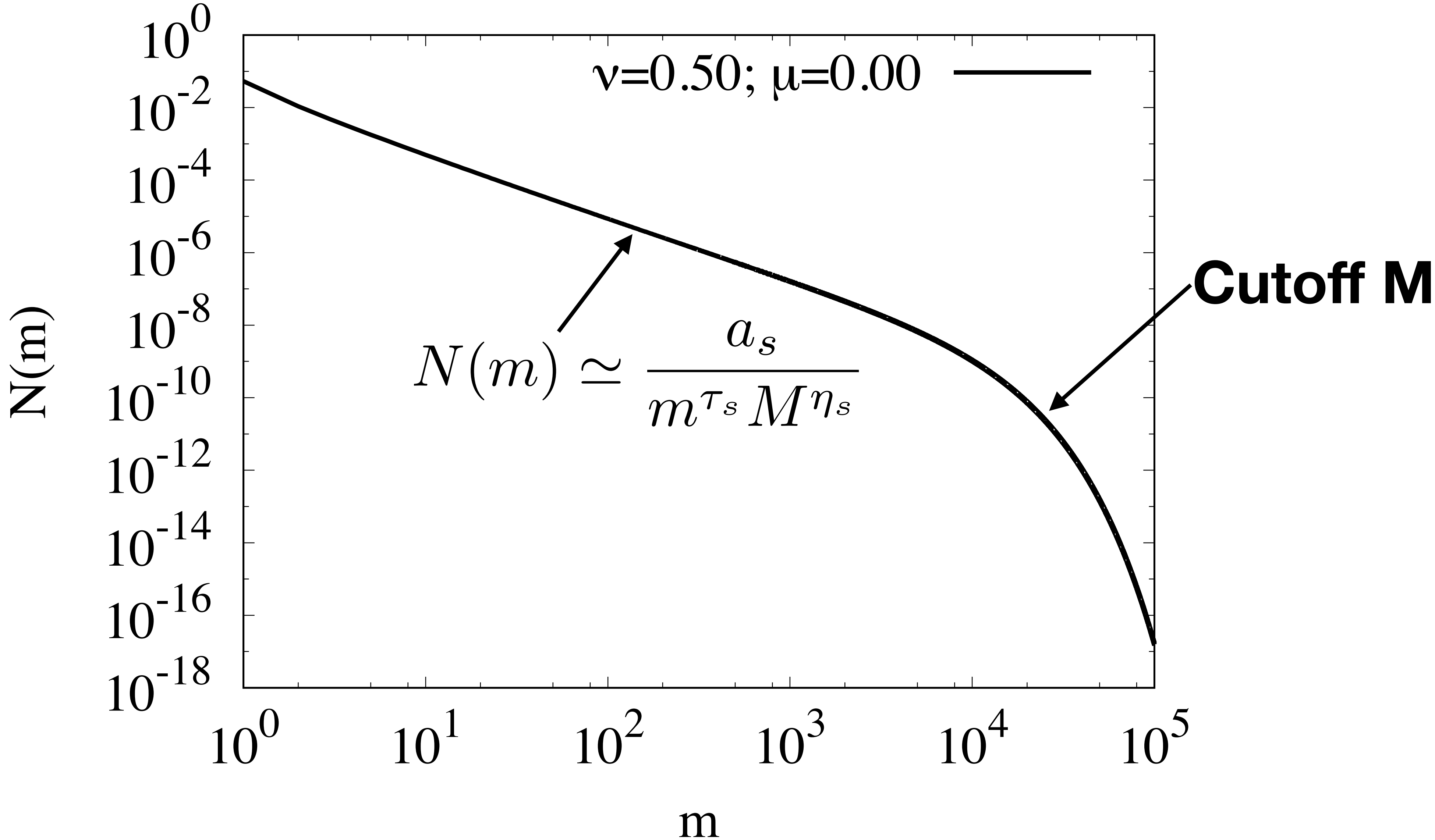
Characterising the mass distribution



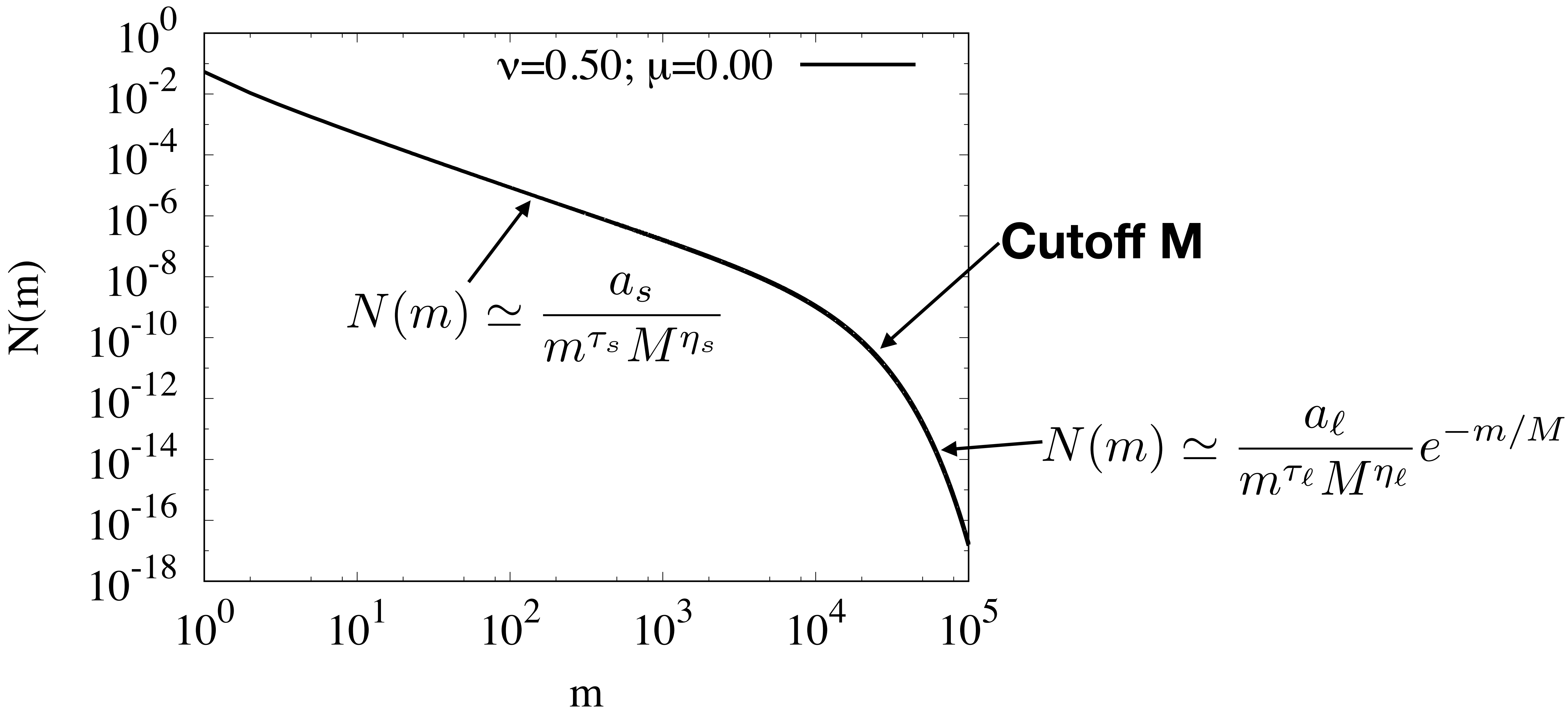
Characterising the mass distribution



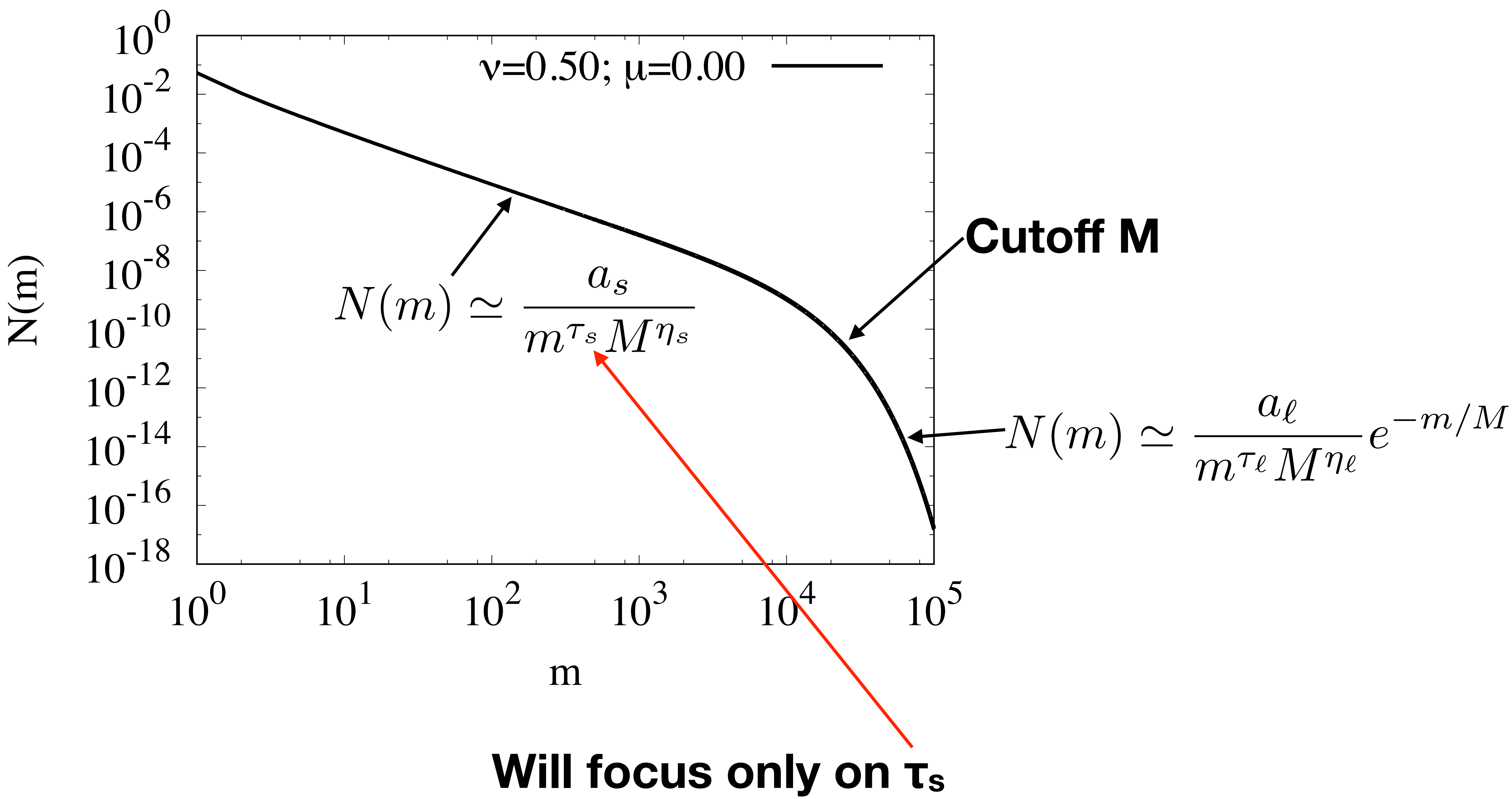
Characterising the mass distribution



Characterising the mass distribution



Characterising the mass distribution

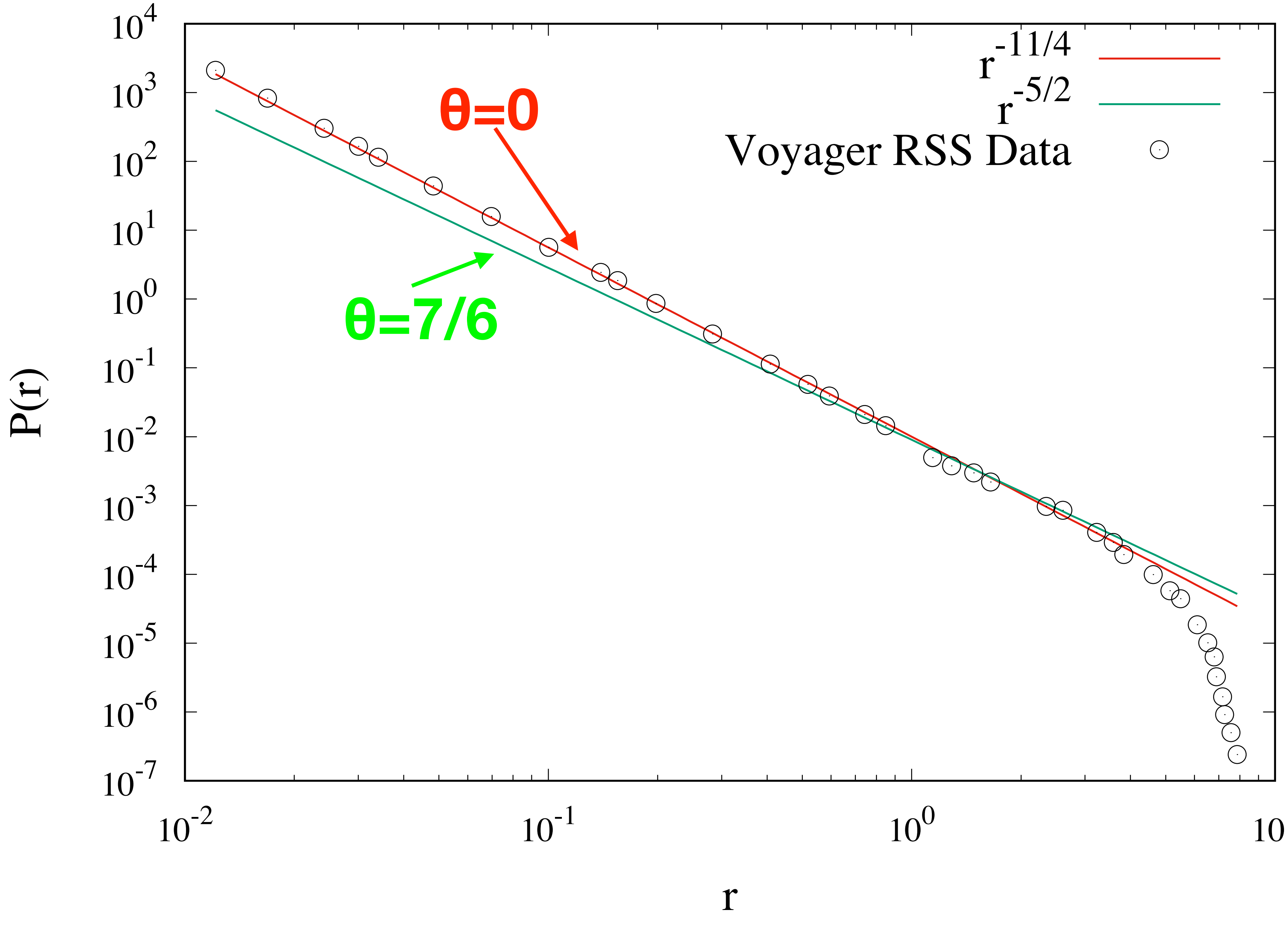


Result

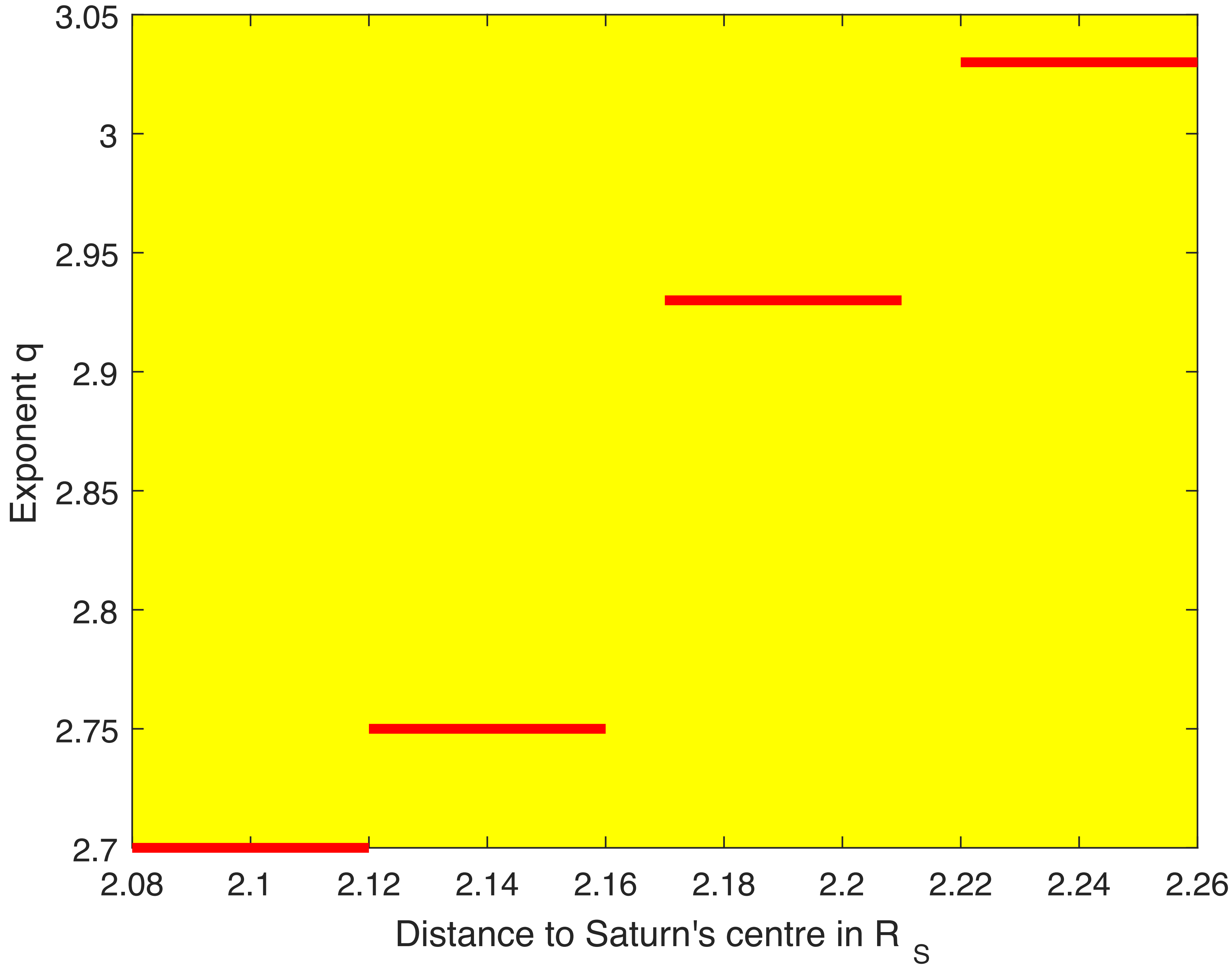
θ	τ_s
[0,1)	$(3+\beta)/2$
(1,2)	$(4-\theta+\beta)/2$
(2,∞)	$(\theta+\beta)/2$

← **θ=7/6**

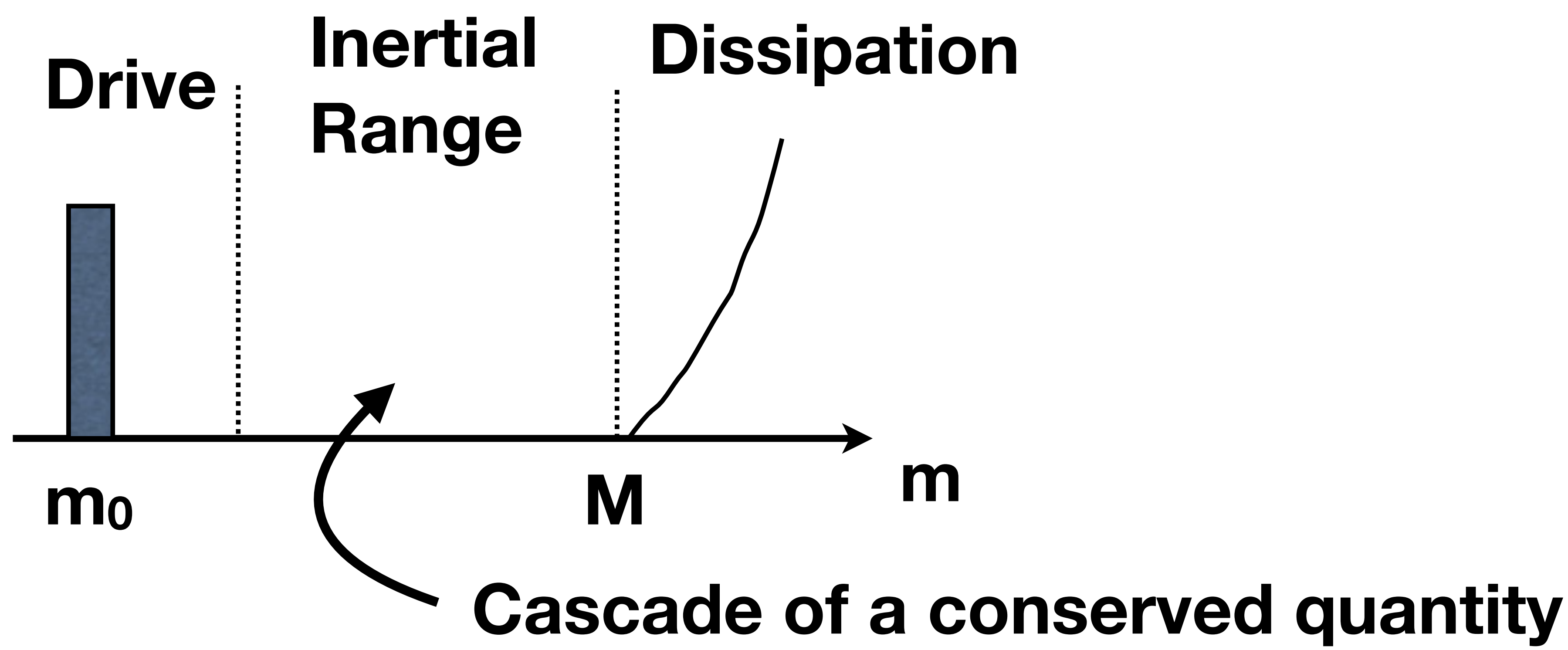
Discussion: rings of Saturn



Discussion: rings of Saturn



Discussion: driven dissipative system



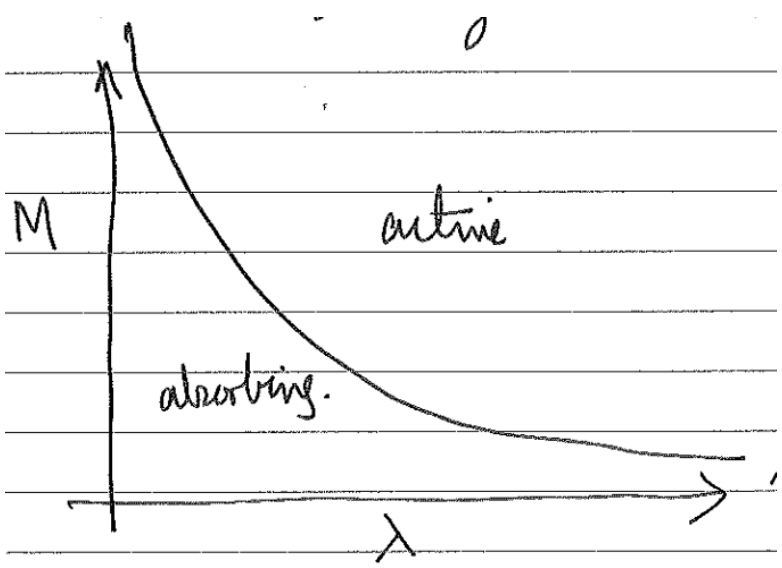
$$J(m) \sim m^3 m^\beta N^2(m)$$

$$N(m) \sim m^{-\frac{3+\beta}{2}}$$

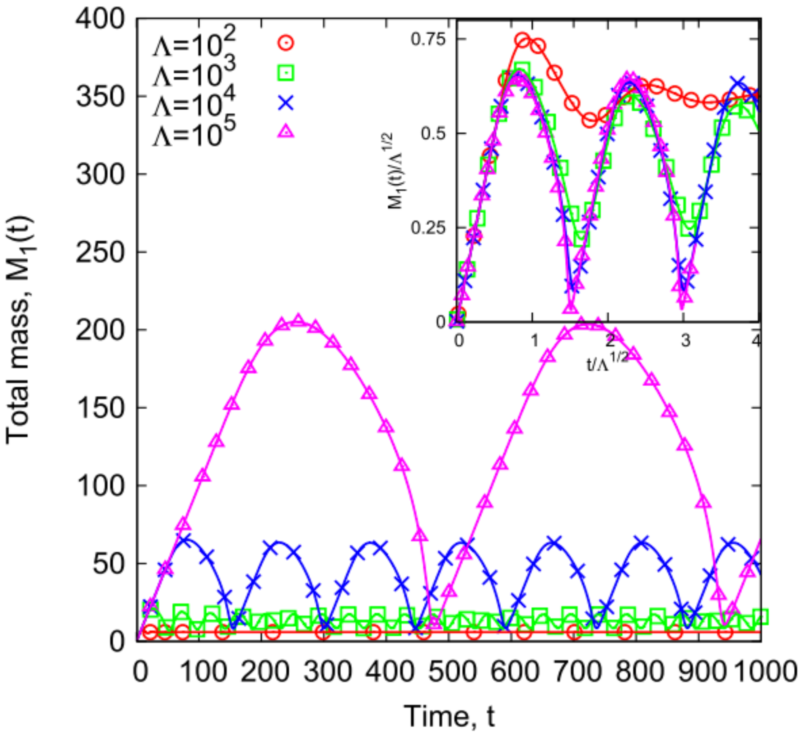
θ	τ_s
[0,1)	(3+β)/2
(1,2)	(4-θ+β)/2
(2,∞)	(θ+β)/2

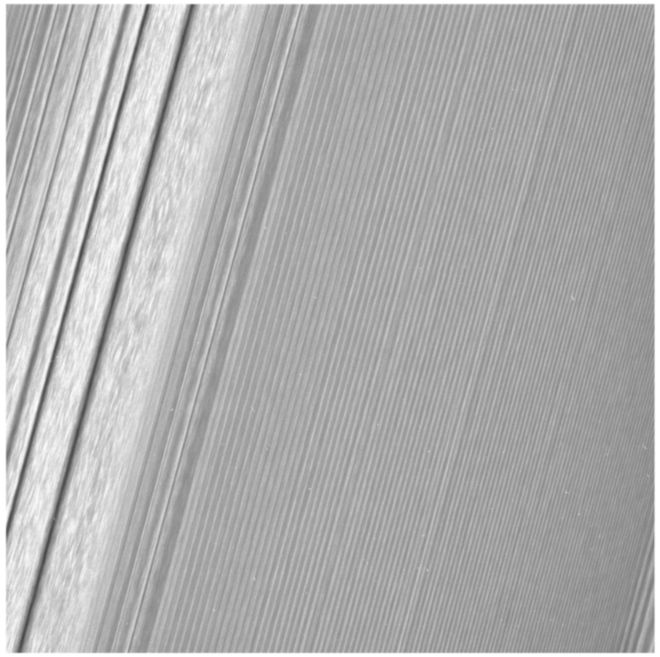
Outlook

- **Stability of solution?**
- **Non locality leads to instability and onset of oscillations**
- **Effect of space: oscillations in space?**
- **Effect of stochasticity?**



Connaughton et al, PRL (2012)





Jet propulsion lab, caltech