Hydrodynamics, density fluctuation and universality in conserved stochastic sandpiles

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Reference

- *Hydrodynamics, density fluctuations and universality in conserved Manna sandpiles*, Sayani Chatterjee, Arghya Das, and Punyabrata Pradhan, *arXiv: 1711.03793v1*.

Introduction

Deriving hydrodynamics from microscopic dynamics is a fundamental problem in statistical physics.

Hydrodynamics

Why? Water flows, sand dune forms, ...

Evolution of observables (slow d.o.f) at large sptio–temporal scales

Density fluctuations

Irregular microscopic dynamics

Sandpiles (30 years)

Self–organized Criticality

Earthquake, river network, mountain ranges ...

“Sandpiles”

- Analogy to real sandpiles and self-organized criticality (SOC) *[Bak, Tang, and Wiesenfeld, Phys. Rev. Lett. (1987)]*
- Fixed-energy, or conserved-mass, sandpiles *[Vespignani, Dickman, Munoz, and Zapperi, Phys. Rev. Lett. (1998)]*

Studies in past decades

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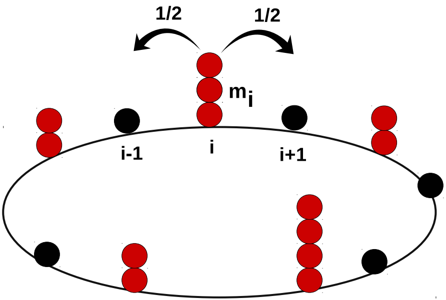
Where do we stand?

- Particle-transport and density-fluctuations, though at the heart of the problem, are far less studied and lack general theoretical understanding.

Conserved Stochastic Sandpiles: Manna Sandpiles



- Threshold-activated lattice gases
- Total mass $M = \sum_{i=0}^{L}$ conserved
- An active-absorbing phase transition upon tuning density $\rho = M/L$: active site density $a(\rho) = N_a/L \neq 0$ for $\rho > \rho_c$; $a(\rho) = 0$ otherwise.

Introduction of biased Manna sandpile

- Continuous-time dynamics (random sequential update):

$$m_i(t+dt) = \begin{cases} m_i(t) - 2 & \text{prob. } \hat{a}_i(c^F_{i,0} + c^F_{i,+} + c^F_{i,-})dt, \\ m_i(t) + 1 & \text{prob. } \hat{a}_{i-1} c^F_{i-1,0} dt, \\ m_i(t) + 1 & \text{prob. } \hat{a}_{i+1} c^F_{i+1,0} dt, \\ m_i(t) + 2 & \text{prob. } \hat{a}_{i-1} c^F_{i-1,+} dt, \\ m_i(t) + 2 & \text{prob. } \hat{a}_{i+1} c^F_{i+1,-} dt, \\ m_i(t) & \text{prob. } [1 - \Sigma dt], \end{cases}$$

- Small biasing force and modified rates $c^F_{i,0} = 1/2$, $c^F_{i,+} = (1+F)/4$, and $c^F_{i,-} = (1-F)/4$. *[Bertini, De Sole, Gabrielli, Jona-Lasinio, and Landim, Rev. Mod. Phys. (2015)]*

- Time evolution of local density

$$\frac{\partial \rho_i}{\partial t} = (a_{i-1} - 2a_i + a_{i+1}) + F\frac{a_{i-1} - a_{i+1}}{2}.$$

- Exact, but *not* closed!

Question of universality: A long debated issue!

- Phenomenological field theory for conserved directed percolation (CDP) universality,

$$\frac{\partial \rho(x,t)}{\partial t} = D_1 \frac{\partial^2 a(x,t)}{\partial x^2}$$
$$\frac{\partial a(x,t)}{\partial t} = D_2 \frac{\partial^2 a(x,t)}{\partial x^2} + b_1 a(x,t) - b_2 a^2(x,t) + \gamma a(x,t)\rho(x,t) + \text{noise term.}$$

- Since long, it's believed to represent Manna sandpile.
- However, there is no rigorous argument yet in its favour.
- In fact, some believe Manna sandpiles belong to a different universality of directed percolation (DP).

She is mysterious, and elusive so far.

Assumption of local steady state

- Activity field is *slave* to density field

$$a_i(t) = a[\rho_i(t)].$$

Continuity equation

- We get a closed time-evolution equation of density field, which becomes, in continuum limit,

$$\frac{\partial \rho(x,t)}{\partial t} = \frac{\partial^2 a(\rho)}{\partial x^2} - F\frac{\partial a(\rho)}{\partial x} \equiv -\frac{\partial J(\rho)}{\partial x}.$$

- Local current $J(\rho(x)) = J_{diff} + J_{drift}$ has two contributions:

$J_{diff} \equiv -D(\rho)\frac{\partial \rho}{\partial x}$ and $J_{drift} \equiv \chi(\rho)F.$

- *Density-dependent* bulk-diffusion coefficient and conductivity:

$D(\rho) = \frac{da}{d\rho}$ and $\chi(\rho) = a(\rho)$, respectively.

- Hydrodynamical evolution involves only a single variable.

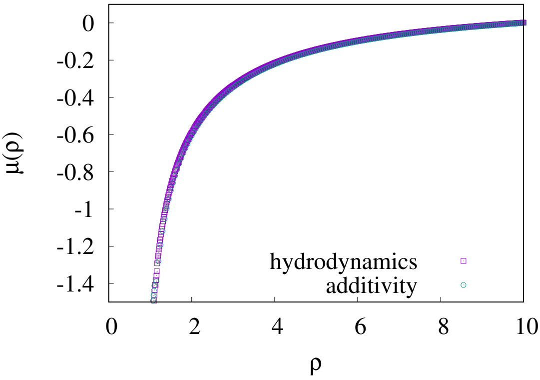
Verification of hydrodynamics

- Macroscopic fluctuation theory *[Bertini et. al., Phys. Rev. Lett. (2001)]* predicts an Einstein relation,

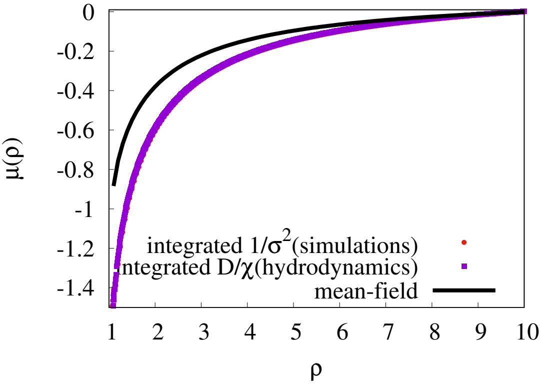
$$\lim_{v\to\infty} \frac{\langle m^2\rangle - \langle m\rangle^2}{v} \equiv \sigma^2(\rho) = \frac{\chi(\rho)}{D(\rho)}.$$

- We verify integrated form of Einstein relation,

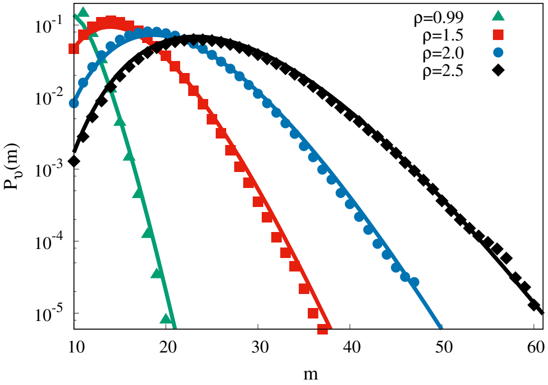
$$\mu(\rho) = \int_{\rho_0}^{\rho} \frac{1}{\sigma^2(\rho)} d\rho = \int_{\rho_0}^{\rho} \frac{D(\rho)}{\chi(\rho)} d\rho.$$



Comparison with mean-field theory



Manna sandpiles: Subsystem mass distribution $P_v(m)$



- $P_v(m) \simeq \text{const.} \frac{e^{-vh(m/v)}}{m}.$

Scaling relations

- $\frac{d\rho}{d\mu} = \sigma^2(\rho) \sim (\rho - \rho_c)^\delta$ with $\delta = 1$.
- $\tau_r \sim \xi^2 / D \Rightarrow z = 2 + \frac{\beta - 1}{\nu_\perp}$.

Summary

She is simple and beautiful!

- A remarkable hydrodynamic structure
- An equilibrium-like Einstein relation
- Two scaling relations
- Manna sandpiles do not belong to DP universality.

Thank You.