A case against conventional universality

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conventional universality

critical exponents depend on $d = 2, 3, \ldots$ and $n = 1, 2, 3, \ldots$ but independent of other details

e.g. we believe critical behavior of Ising model to be the same on honeycomb ($z = 3$), square ($z = 4$), or triangular ($z = 6$) lattice

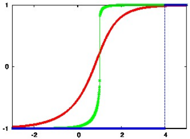
there is no rigorous proof of this but RG offers an intuitively appealing explation based on $\xi \to \infty$ at the critical point.

before RG, nonuniversal quantities like T_c were also studied for their dependence on z but this declined as the idea of universality gained currency.

for example, in an exactly solved model of electron localization by disorder on a Bethe lattice (D J Thouless, Physics Reports, 1974), the equations suggest a nontrivial dependence on z, but these studies were apparently not pursued further in the wake of RG ideas; subsequent and current studies focus on $z = 3$ only.

universality of critical hysteresis?

$H = -J \sum_{i,j} s_i s_j - \sum_i h_i s_i - h \sum_i s_i$
$s_i = \pm 1$; $h_i \Rightarrow N(0, \sigma^2)$
start with $s_i = -1$; use $T = 0$ Glauber dynamics to obtain
$m(h) = \frac{1}{N} \sum_i s_i$; it shows a discontinuity if $\sigma < \sigma_c$.



For $\sigma < \sigma_c$, system stays in an ordered metastable state upto $h < h_c$. At $h = h_c$ it rearranges itself into another ordered state via an infinite avalanche. Infinite avalanche plays the same role as correlation length $\xi \to \infty$ at equilibrium critical point. $\{\sigma_c, h_c\}$ is a non-equilibrium critical point.

surprising dependence of phase transition on coordination number of lattice

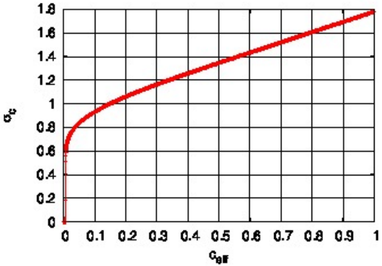
- no phase transition on a Bethe lattice if $z \leq 3$, but phase transition if $z > 4$.
- on periodic lattices, no phase transition if $z \leq 3$ irrespective of d.
- on a mixed Bethe lettice with $z = 3$ and $z = 4$ only, phase transition occurs for arbitrarily small fraction of $z = 4$ sites.
- on a randomly diluted $z = 4$ Bethe lattice, phase transition occurs if the fraction of $z4$ sites is greater than 0.5575.
- why???
- these issues were resolved only when we changed the mindset from "universal" to "local"

when and why does an avalanche diverge ?

Surprisingly the answer does not depend on the dimensionality of space in which the path of avalanche is embedded, neither does it depend on the average coordination number of sites on the path. It requires the presence of a nonzero fraction of $z \geq 4$ sites on a spanning path !

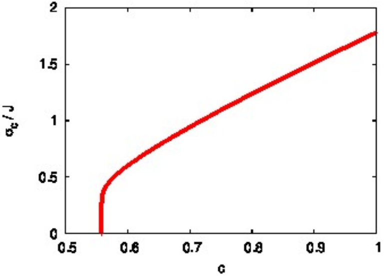
- a diverging avalanche obviously requires a spanning path across the system.
- a spanning path is guaranteed on a lattice with 100% occupancy but not on a randomly diluted lattice.
- if there is diverging avalanche, it must be there as $\sigma \to 0$.
- analysis on a Bethe lattice shows that a diverging avalanche in the limit $\sigma \to 0$ requires that there must be a nonzero fraction of $z \geq 4$ sites on the spanning path.
- on a $z = 4$ Bethe lattice with a fraction c of sites occupied, the minimum value of c for diverging avalanche is given by, $c_{min} = 2^{1/3}/(1 + 2^{1/3}) \approx 0.5575$.

σ_c vs. fraction of z_4 sites on $z_3 + z_4$ lattice



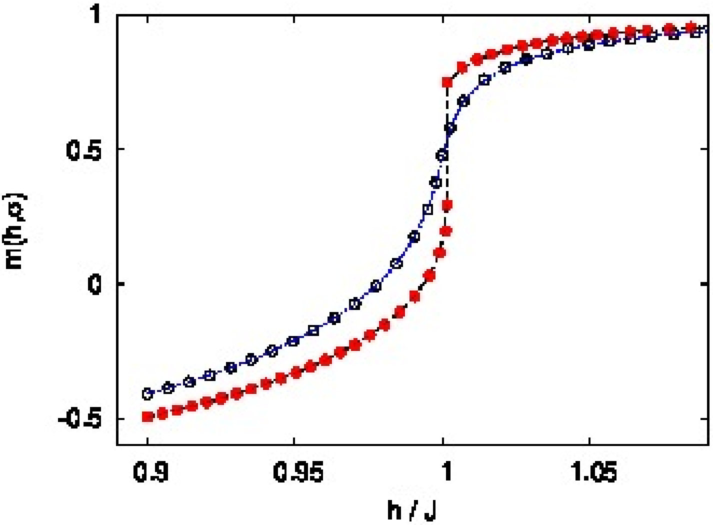
criteria for existence of $\sigma_c > 0$ is same as that of infinite avalance.

σ_c vs. fraction of z_4 sites on dilute z_4 lattice

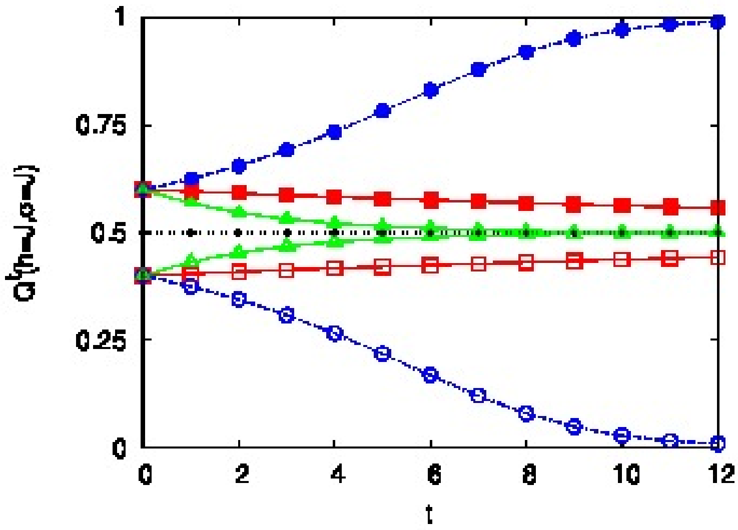


the rise of σ_c at c=0.557 is almost vertical !

sharpness of transition at $c = 0.557$



instability of the fixed point $Q^* = 0.5$ for $z = 4$



violation of universality on periodic lattices

extensive numerical study shows that the criticl exponents, i.e. the universality class of the model on triangular lattice is different from the one on the square lattice.
Ref: S Janicevic, S Mijatovic, and D Spasovic, Phys Rev E95, 042131 (2017).