Self-organized critical behavior and marginality in Ising spin glasses

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Self-organized critical behavior and marginality in Ising spin glasses

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Abstract.

We have studied numerically the states reached in a quench from various temperatures in the one-dimensional fully-connected Kotliar, Anderson and Stein Ising spin glass model. This is a model where there are long-range interactions between the spins which falls off as a power σ of their separation. We have made a detailed study in particular of the energies of the states reached in a quench from infinite temperature and their overlaps, including the spin glass susceptibility. In the regime where $\sigma \leq 1/2$, where the model is similar to the Sherrington-Kirkpatrick model, we find that the spin glass susceptibility diverges logarithmically with increasing N, the number of spins in the system, whereas for $\sigma > 1/2$ it remains finite. We attribute the behavior for $\sigma \leq 1/2$ to *self-organized critical behavior*, where the system after the quench is close to the transition between states which have trivial overlaps and those with the non-trivial overlaps associated with replica symmetry breaking. We have also found by studying the distribution of local fields that the states reached in the quench have marginal stability but only when $\sigma \leq 1/2$.

Main message: Threefold

Quenched states in the 1-d long-range power-law Ising spin glass

- Nature of final state determined by initial temperature T. If $T > T_c$, final state has trivial overlap $P(q) = \delta(q)$. If $T < T_c$, its form after quench resembles that of initial state.

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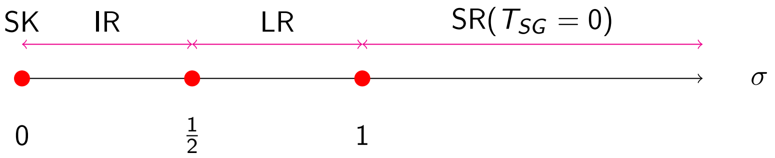
- Distribution of local fields $p(h)$ in final state consistent with *marginal* stability in the mean-field regime ($\sigma \leq 1/2$). Quenched state not marginal outside MF regime.
- *Static* features of SOC: spin glass susceptibility χ_{SG} via overlaps of quenched states obtained from different initial states. χ_{SG} diverges logarithmically with number of spins N, in the MF regime $\sigma \leq 1/2$. When $\sigma > 1/2$, no divergence in χ_{SG} indicating that the quenched state not critical.

Hamiltonian

The Kotliar, Anderson and Stein (KAS) Hamiltonian is

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j,$$

where the Ising spins S_i ($i = 1, 2, \cdots, N$), taking values ± 1, are arranged in a circle of perimeter N. The geometric distance between sites i and j is $r_{ij} = \frac{N}{\pi} \sin\left[\frac{\pi}{N}(i-j)\right]$, the length of the chord between the sites i, j. The interactions J_{ij} are long-ranged and depend on the distance r_{ij} as $J_{ij} = c(\sigma, N)\varepsilon_{ij}/r_{ij}^{\sigma}$, where ε_{ij} is a Gaussian random variable of mean zero and unit variance.


SK
IR
LR
SR($T_{SG} = 0$)
σ
0
$\frac{1}{2}$
1

Metastability. Quenching procedure.

At zero temperature, metastable states are generated by repeatedly aligning every spin along the local field direction:

$$S_i^0 = \frac{h_i}{|h_i|},$$

where the local fields are defined as

$$h_i = \sum_j J_{ij} S_j.$$

Parisi overlap.

Consider the overlap between two minima A and B defined as

$$q \equiv \frac{1}{N} \sum_i S_i^A S_i^B.$$

Distribution of overlaps

Theorem of Newman and Stein (1999)

In a quench from a random initial state, the final $P(q)$ *should be trivial*.

Our finding

For a quench which starts at any temperature $T > T_c$, the resulting $P(q)$ is trivial, in that it reduces in the large N limit to $P(q) = \delta(q)$. However, for quenches which start from a temperature $T < T_c$ a non-trivial $P(q)$ is found.

Overlap distribution

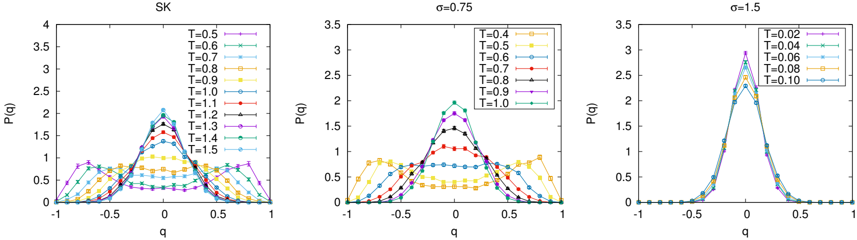


Figure: A range of temperatures T is shown for three values of $\sigma = 0, 0.75, 1.50$. The SK limit has a phase transition at $T_c = 1$, while at $\sigma = 0.75$, $T_c = 0.62$. No phase transition exists for $\sigma = 1.5$. Whether the final state displays a non-trivial $P(q)$ depends on whether the initial temperature T is less than T_c or not. For $\sigma = 1.5$, a trivial $P(q)$, that is one approaching $\delta(q)$ as $N \to \infty$, is obtained even for quenches from very low temperatures.

Marginality and the distribution of local fields $p(h)$

The magnitude of h_i after the quench is given by

$$h_i = S_i \sum_j J_{ij} S_j,$$

where S_i are the spins at the end of the quench.

For $\sigma < 1/2$, it is expected that $p(h) = h/H^2$ at small fields in the thermodynamic limit.

For $\sigma > 1/2$, it is expected that marginality requires $p(h) = \frac{1}{H}\left(\frac{h}{H}\right)^{(1/\sigma)-1}$.

Finite-size scaling of the distribution of fields (SK)

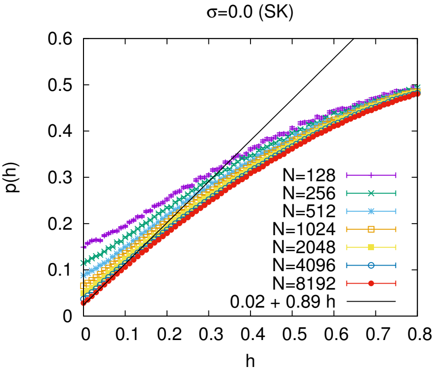


Figure: A plot of $p(h)$ for the SK model after a quench from a random initial state, for $\sigma = 0$ i.e. the SK model, for $h < 0.8$ for a range of N values from 128 to 8192. In the large N limit, $p(h)$ is linear at small h. Finite size effects cause $p(0)$ to be finite but this intercept on the y axis decreases as $1/\sqrt{N}$.

Anderson agrument

In the state reached in the quench, relabel the sites in order of their increasing local field h_i and consider the first n of these sites, where $1 \ll n \ll N$. Suppose one flips all n of the spins at these low-field sites: the consequent energy change is

$$\Delta E = 2\sum_{i=1}^{n} h_i - 2\sum_{i=1}^{n}\sum_{j=1}^{n} J_{ij}S_iS_j.$$

For $\sigma < 1/2 \quad \left[p(h) = h/H^2\right]$,

$$\Delta E = 2n\left(\frac{n}{N}\right)^{1/2}\left[\frac{2\sqrt{2}}{3}H - 1\right],$$

so the quenched state would be just marginal (i.e. has $\Delta E = 0$) if

$$H = \frac{3}{2\sqrt{2}}.$$

For $\sigma > 1/2 \quad \left[p(h) = \frac{1}{H}\left(\frac{h}{H}\right)^{(1/\sigma)-1}\right]$,

$$\Delta E = 2n\left(\frac{n}{N}\right)^{\sigma}\left[\frac{1}{(\sigma+1)\sigma^{\sigma}}H - 1\right].$$

Thus if the system is just marginal

$$H = (\sigma+1)\sigma^{\sigma}.$$

Marginality in the $\sigma \leq 0.5$ regime

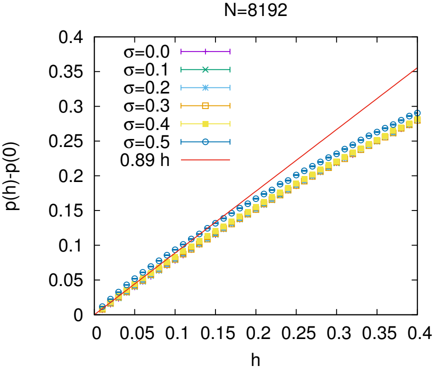


Figure: Plot of $p(h) - p(0)$ versus h for values of σ, $0.0, 0.1, 0.2, 0.3, 0.4$, and 0.5 for $N = 8192$ after a quench from a random initial state. The subtraction of $p(0)$ is done to reduce finite size effects. The red line is a line of slope 0.89 which is the value expected if the quenched states are just *marginal*.

Lack of marginality for $\sigma > 0.5$

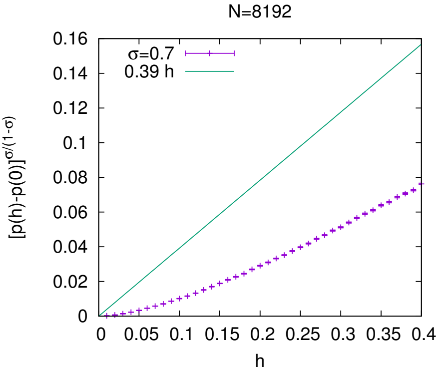


Figure: $(p(h) - p(0))^{\sigma/(1-\sigma)}$ plotted versus h for a quench from infinite temperature for a system of $N = 8192$ spins at $\sigma = 0.7$. The green line corresponding to a straight line of the form $0.39h$ is what would follow from the marginality expectation with H determined like shown earlier. The agreement is very poor implying that marginality in the sense of the Anderson argument is not present.

Lack of marginality for $\sigma > 0.5$: Alternate evidence

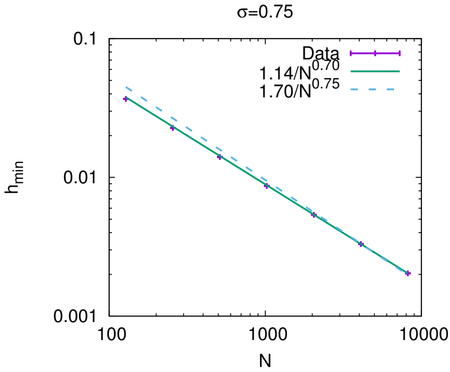


Figure: Size dependence of the magnitude of the smallest local field for the special case of $\sigma = 0.75$. A fit to the form b/N^c fixing $c = \sigma = 0.75$ as per the expectation from marginality, is roughly consistent with the data from the largest system sizes, but a value for c of 0.70 fits the data better over the entire set of N values studied.

Spin Glass Susceptibility

We have made a finite size scaling study of the spin-glass susceptibility χ_{SG}:

$$\chi_{SG} = \frac{1}{N} \sum_{i,j} \left[\langle S_i S_j \rangle^2 \right]_{av}, \quad (1)$$

where the angular brackets represent an average over the metastable minima for a given sample of disorder. The minima in this case were obtained from a random initial state, so that we are studying the case where $P(q)$ is trivial.

In the regime $0 \leq \sigma \leq 1/2$, χ_{SG} diverges as $\ln(N)$, whereas for the region $\sigma > 1/2$, χ_{SG} saturates to a finite value at large N, the form of this dependence being well fitted by $\chi_{SG} = a - b/N^{2\sigma-1}$.

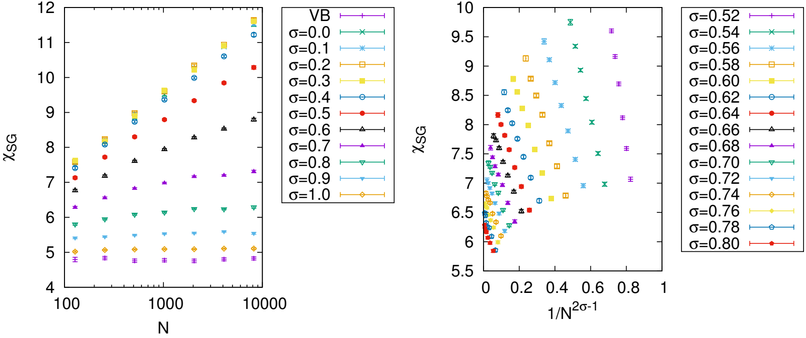


Figure: The N-dependence of χ_{SG} for a variety of σ. For $\sigma \leq 0.5$, χ_{SG} diverges as a logarithm of the system-size. The data points for $\sigma = 0.0, 0.1, 0.2, 0.3$ are so close as to be barely distinguishable. For $\sigma > 0.5$, there is a clear tendency for χ_{SG} to saturate at the largest sizes we are able to study: we are able to find very good fits to the saturating functional form $\chi_{SG} = a - b/N^{2\sigma-1}$.

Energy

There is an energy E_c in the large N limit which separates minima which are just at the brink of having a non-trivial form for $P(q)$ from those at higher energy which have trivial overlaps. E_c marks the transition to a state with broken replica symmetry.

E_c is the analogue of the transition temperature T_c in studies of the thermal spin glass susceptibility as the spin glass susceptibility diverges for $\sigma \leq 1/2$ as $\chi_{SG} \sim 1/\tau$, the usual mean-field form, where $\tau = (1 - T_c/T)$. Our quenches take us close to E_c but miss by an amount of $\mathrm{O}(1/\ln N)$ due to finite size effects; our analogue of τ is $\sim 1/\ln N$, so $\chi_{SG} \sim \ln N$.

Energy

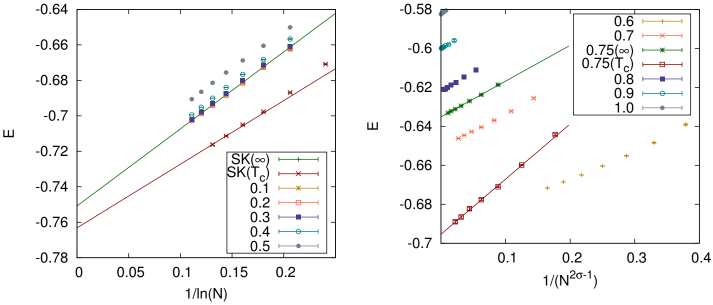


Figure: Energy per spin E as a function of system size N after quenches, for $\sigma \leq 0.5$ in the left panel and $\sigma > 0.5$ in the right panel. For $0 \leq \sigma \leq 0.5$, the energy saturates to a characteristic energy E_c as $1/\ln N$ for both the quench from infinite temperature and from T_c. For $\sigma > 0.5$, the energy of quenches fits well to the form $c + d/N^{2\sigma-1}$.

Conclusions and Outlook

- In the KAS model there is a connection between SOC behaviour and marginality. Because the states reached in the quenches are marginal when $\sigma \leq 1/2$, they are near the energy at which the states are becoming critical.
- It is striking that for the KAS model, the transition which is self-organized can be identified; it is the transition to states with correlations between them due to the onset of broken replica symmetry.
- Would be interesting to know if in other systems which are thought to be marginal, there is a similar connection to SOC.

Thank You