

# Distribution of Waiting Time in Traffic Congestion

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Collaborator : *Sitabhra Sinha (IMSc, Chennai)*

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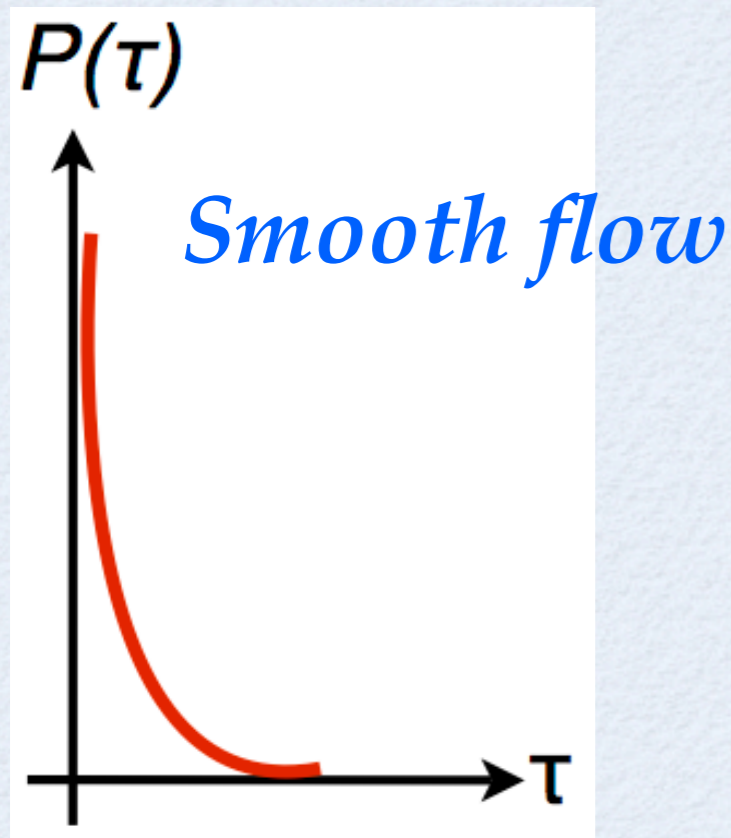
# WHAT IT IS ALL ABOUT

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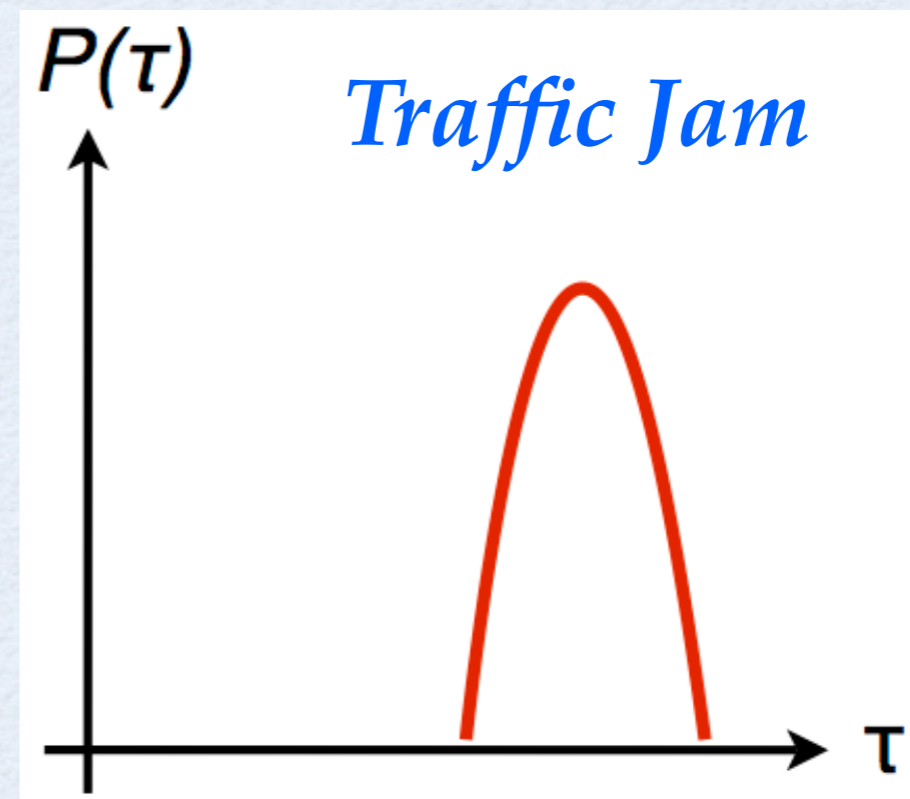
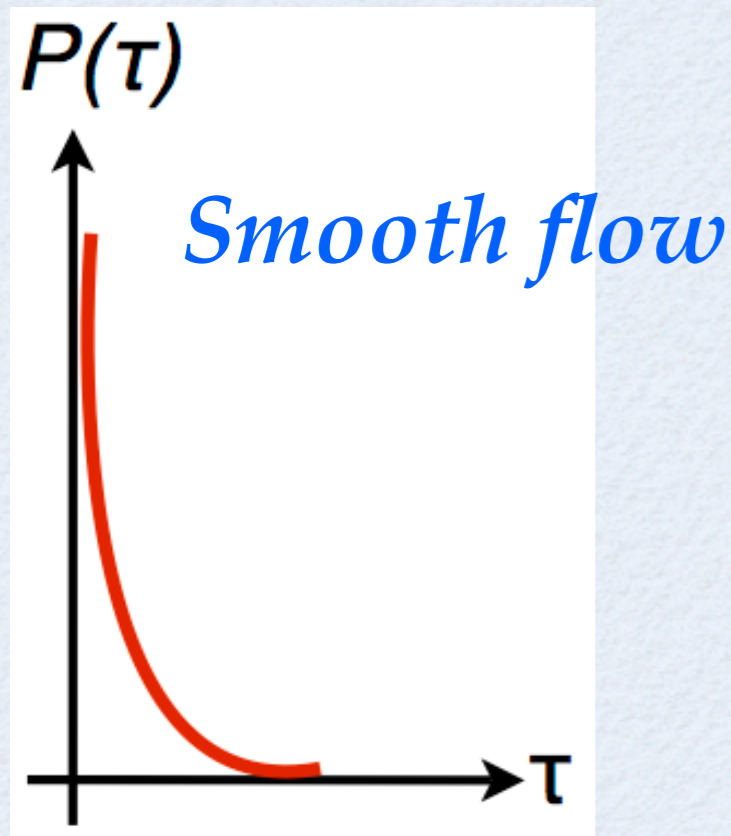
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# OUTLINE



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- Analysis of real-life data for traffic congestion
- Model explaining the results. Based on aggressiveness of the drivers at traffic signals

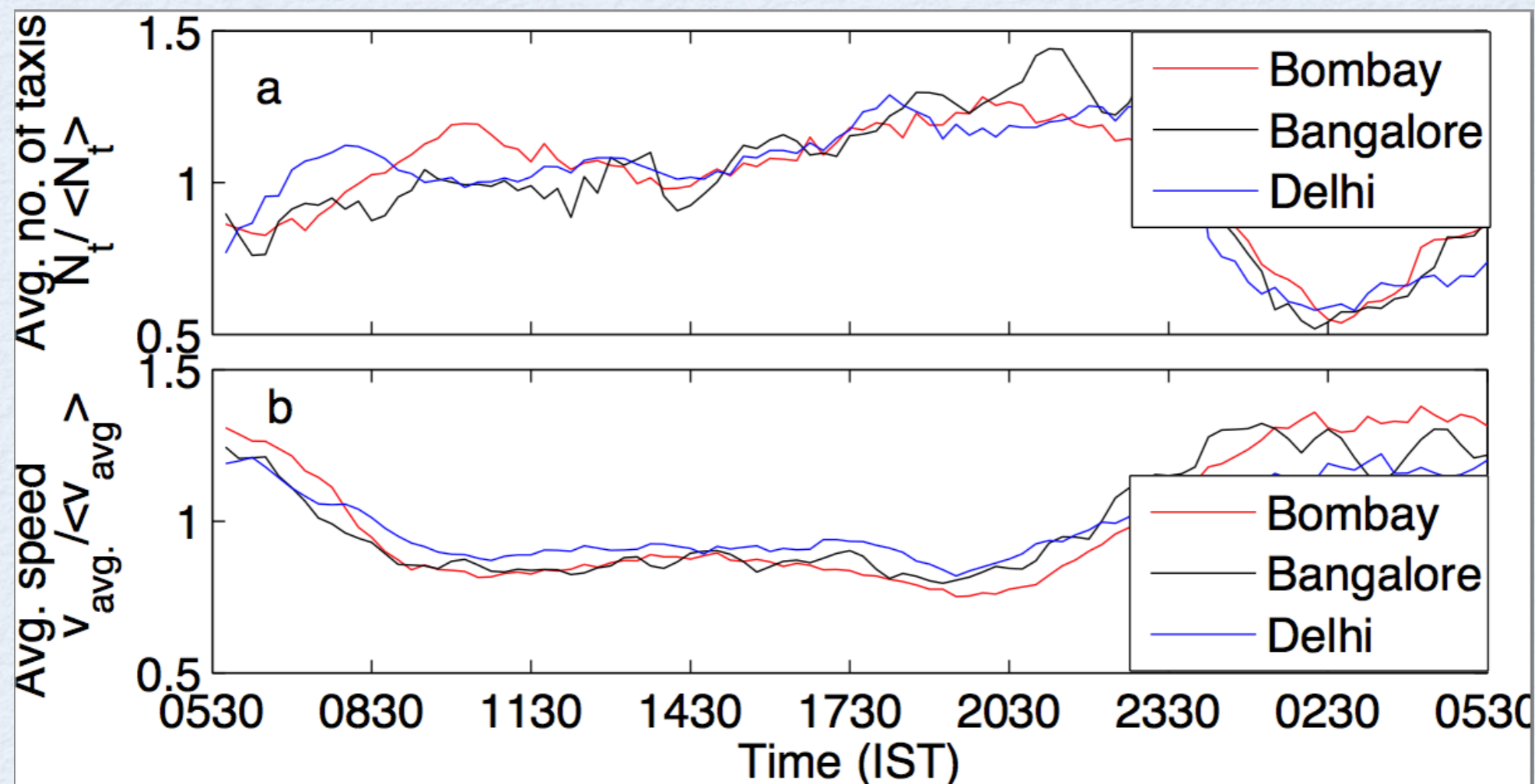


# AVAILABLE DATA

GPS data of ~ 1000 taxis in Delhi, Bengaluru, Mumbai, for several months in 2013- 2014

Collaboration with IIT Madras (Prof. K. Jagannathan)

For each vehicle : time, latitude, longitude, velocity





# CONGESTION TIME

When a car has speed  $< 10$  kmph for  $\tau$  secs, we call it a *congestion time* of  $\tau$  secs

Compute the statistics of this  $\tau$  for all cars, on all days to obtain the probability distribution function  $P(\tau)$



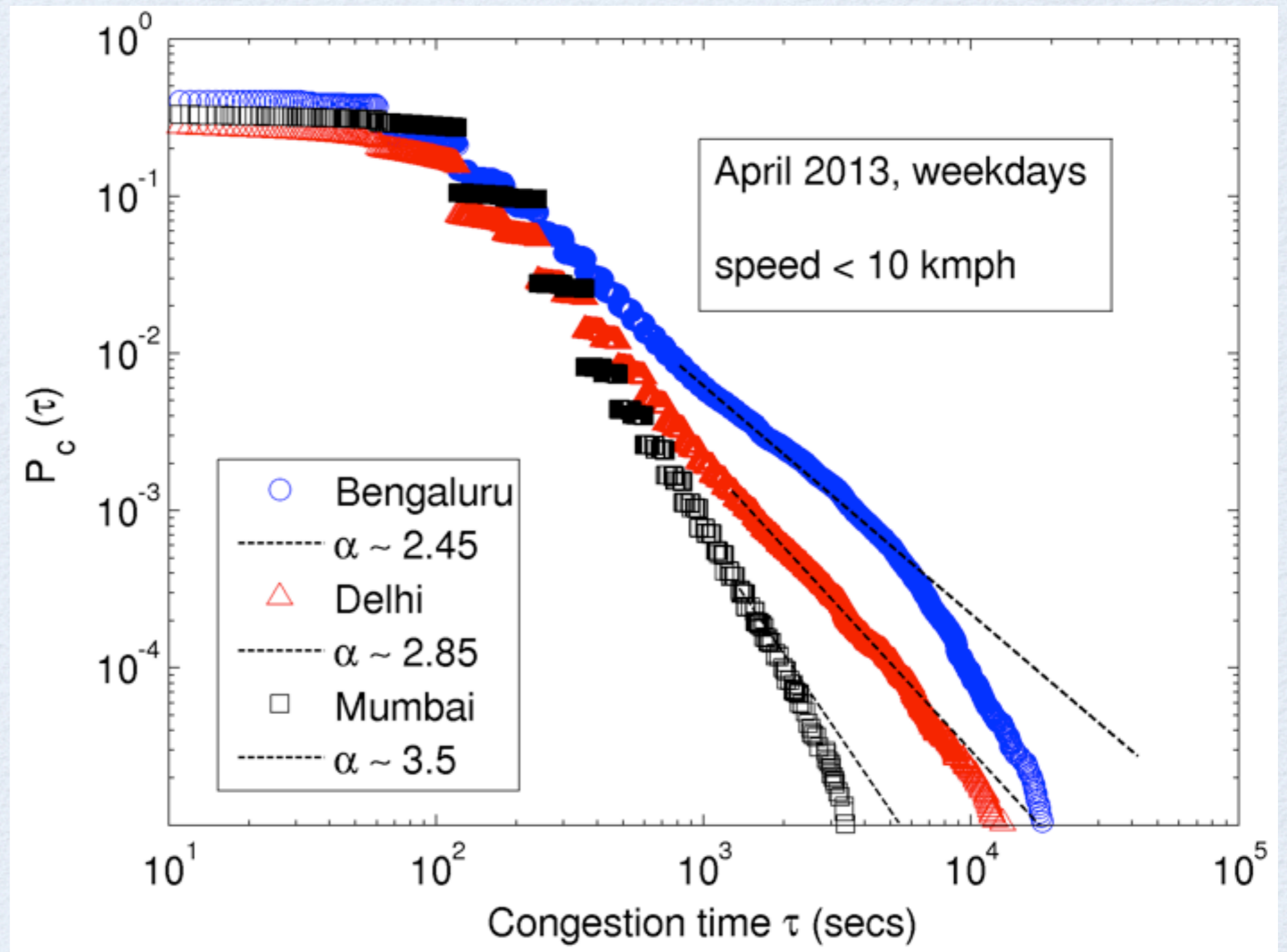
# RESULT FOR CONGESTION TIME

$$P(\tau) \sim \tau^{-\alpha}$$

scale free

Why  
power-law??

Microscopic  
model ??

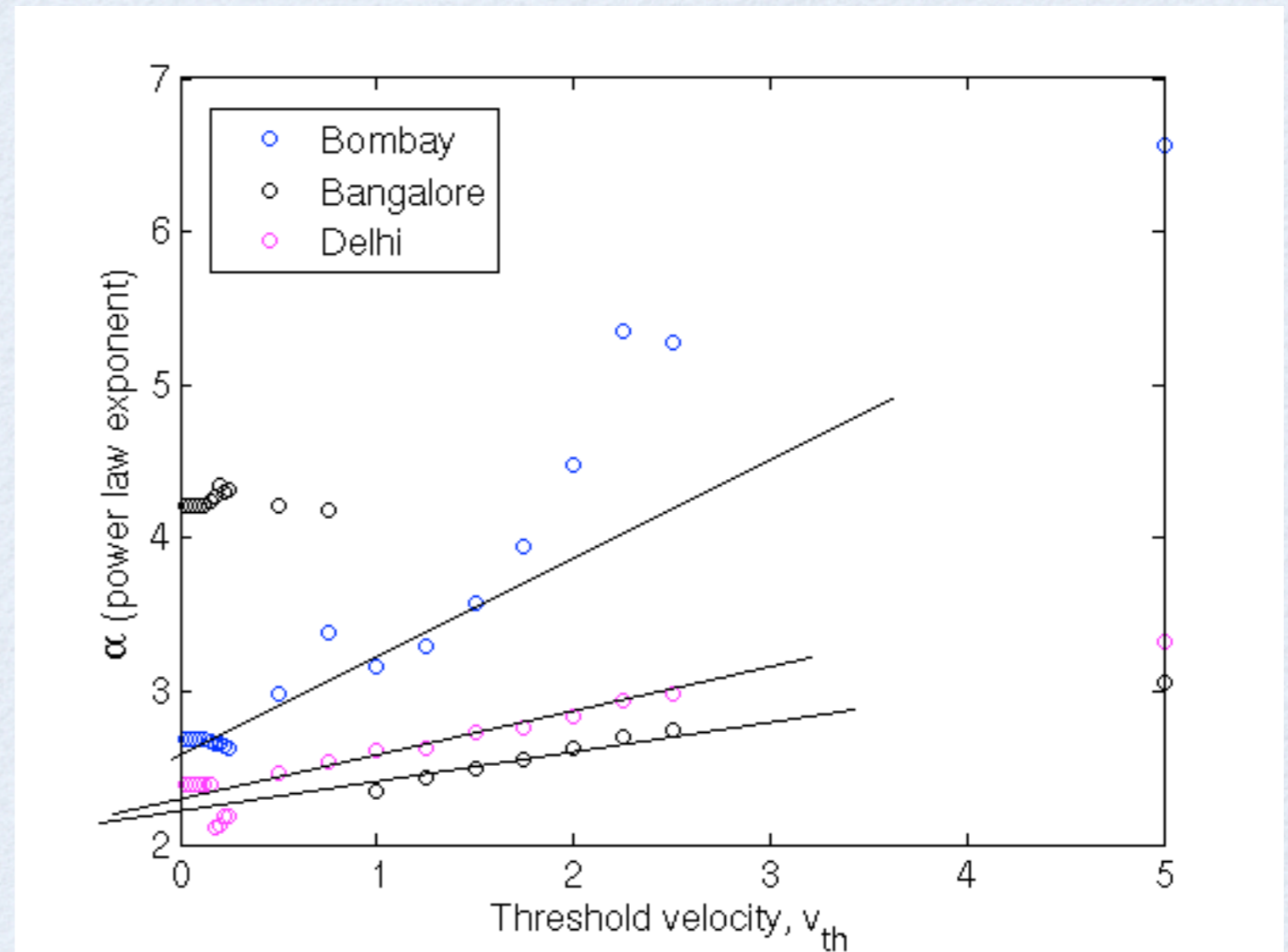




# VALUE OF EXPONENT

The value of exponent depends on the choice of the threshold velocity.

The limiting value of the exponent for small threshold velocity seems to be between 2 to 3.





# MORE DATA

Motorway in  
Cologne, Germany

Krause, Habel, Guhr,  
Schreckenberg (2017)

$$P(\tau) \sim \tau^{-\alpha} \quad \alpha=1.5$$

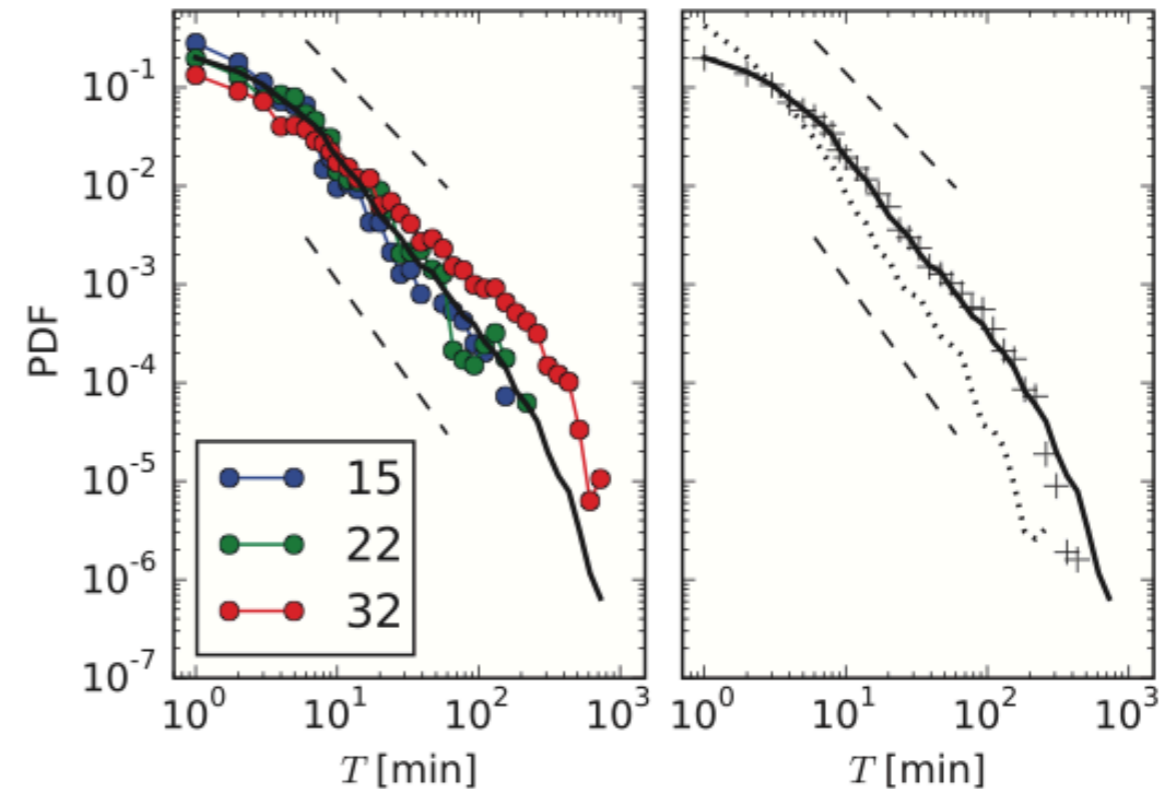


Fig. 3: (Colour online) Left: PDF of traffic congestion durations  $T$  for  $v_{\text{jam}} = 50$  km/h in double logarithmic plot. Symbols indicate different cross-sections, the black solid line is the average over cross-sections 3 to 33. For comparison, power laws  $T^{-\gamma}$  with exponent  $\gamma = 3/2$  (upper dashed line) and  $\gamma = 2$  (lower dashed line) are shown. Right: The average result (black solid line) changes only slightly for data reduced to the first three months (crosses), or for reduced  $v_{\text{jam}} = 20$  km/h (dotted line).



# MODEL

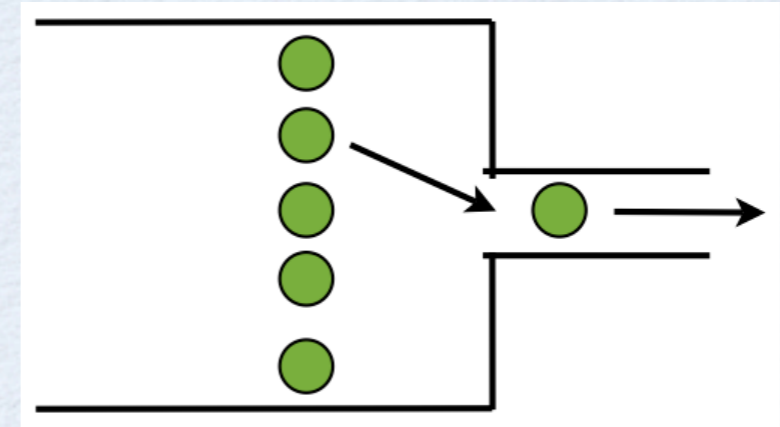


# MODEL

Consider a bottleneck.

Only one car can pass through.

The most *aggressive* driver will pass

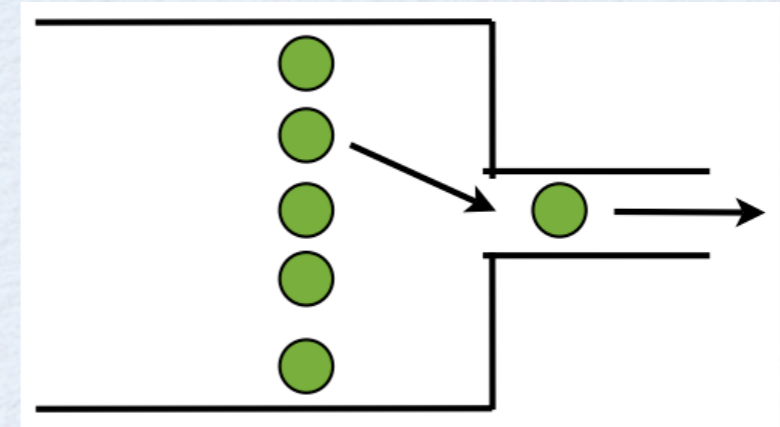




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**HYPOTHESIS:**

can be quantified by an attribute  $A$

Aggressiveness

depends only on  
(i) the intrinsic nature  
(ii) the duration of waiting



# MODEL



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## HYPOTHESIS:

Aggressiveness (for  $i$ -th person waiting for time  $\tau$ )

$$A_i = N_i \tau^\sigma$$

- ★  $N_i$  varies randomly from person to person  
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- ★  $\sigma$  is the same for all persons



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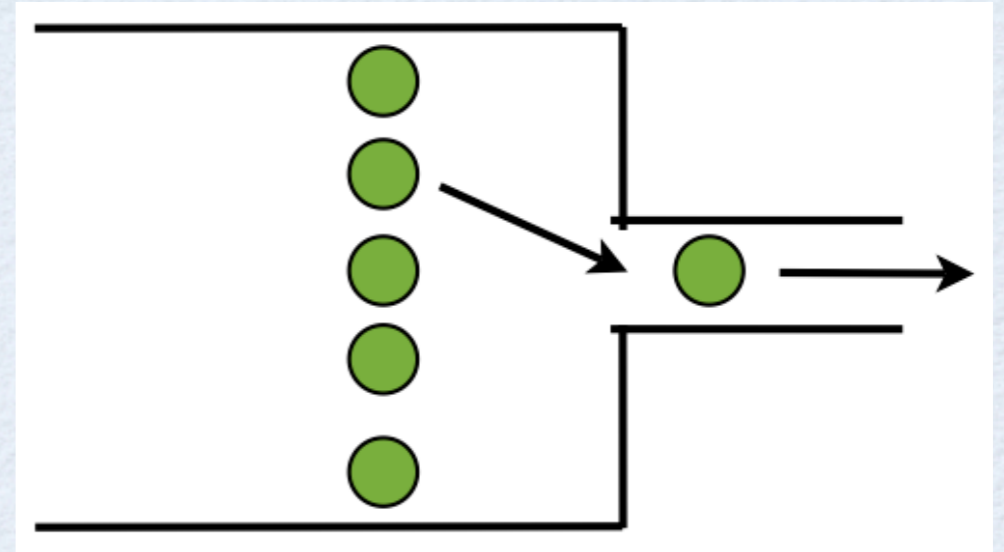
**This gives** (numerical, analytic)  $P(\tau) \sim \tau^{-\alpha}$   $\alpha = \sigma + 1$

$\sigma = 1$  to 2 in India &  $\sigma = 0.5$  in Germany

(probability of congestion time)



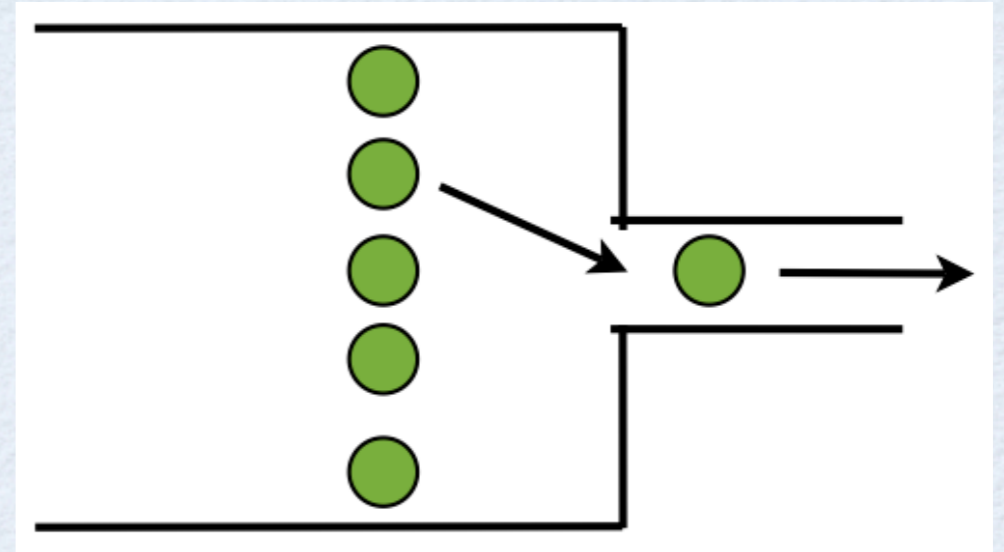
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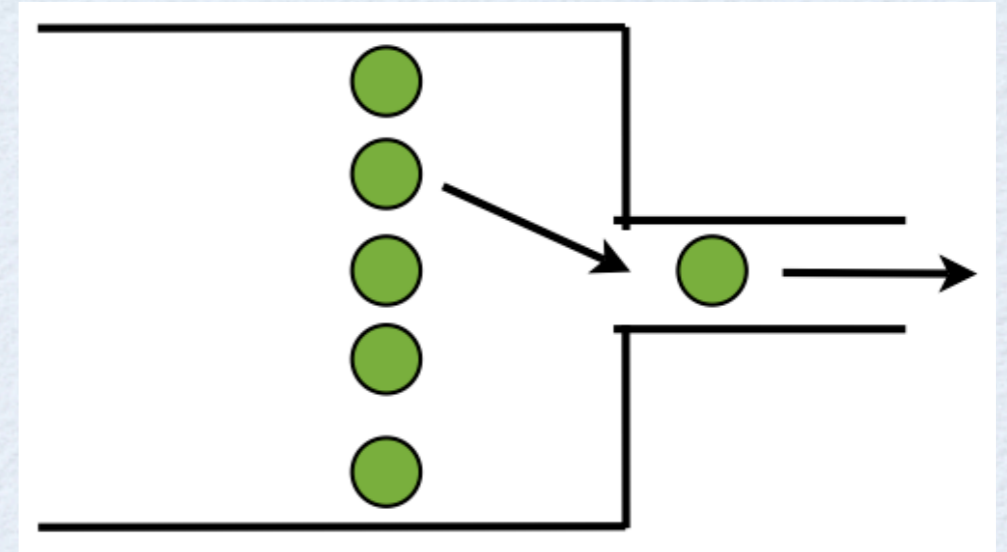
- Start at time  $t=0$  with a queue of  $L$  cars with  $N_1, N_2, \dots, N_L$  drawn from a uniform distribution in  $(0, 1)$ .





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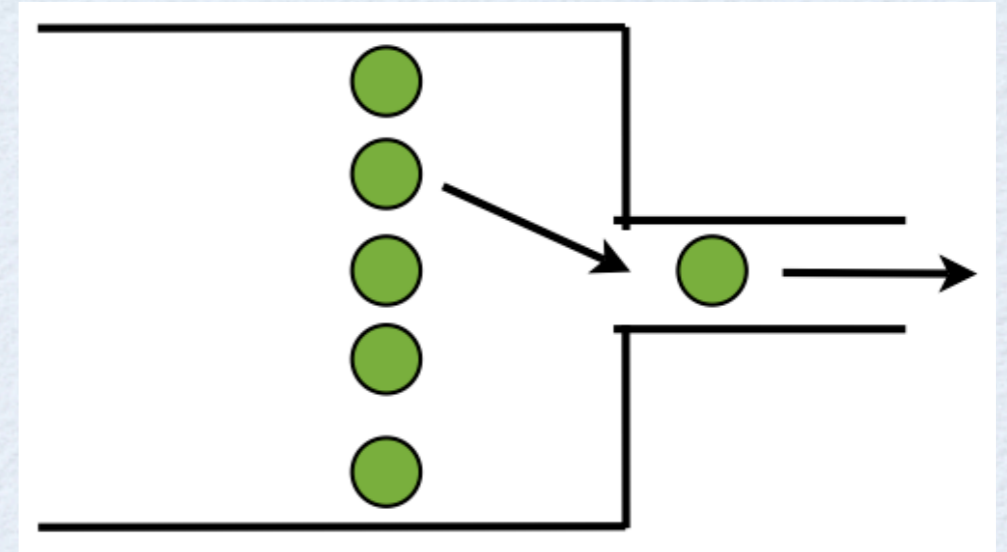


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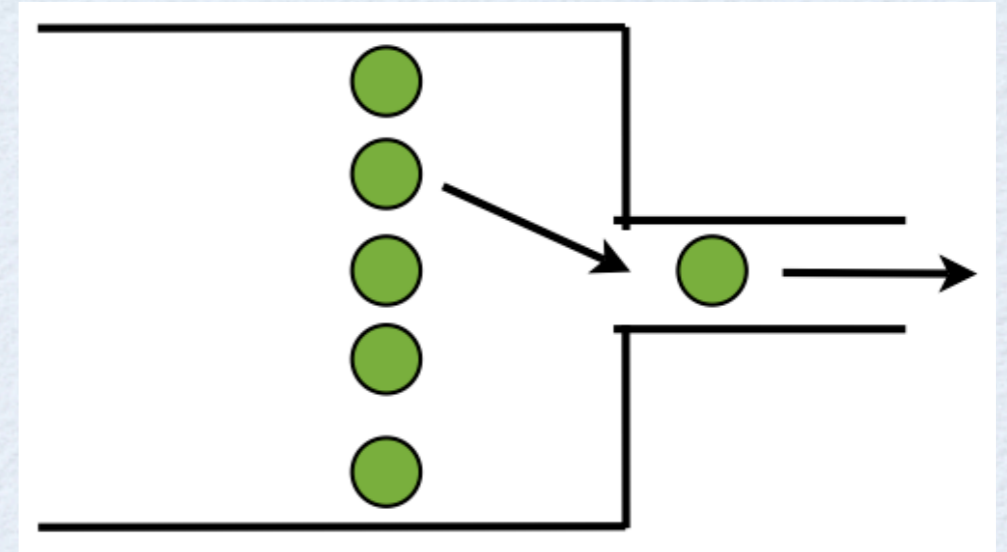


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Note : The number of cars queued up remains the same with time.



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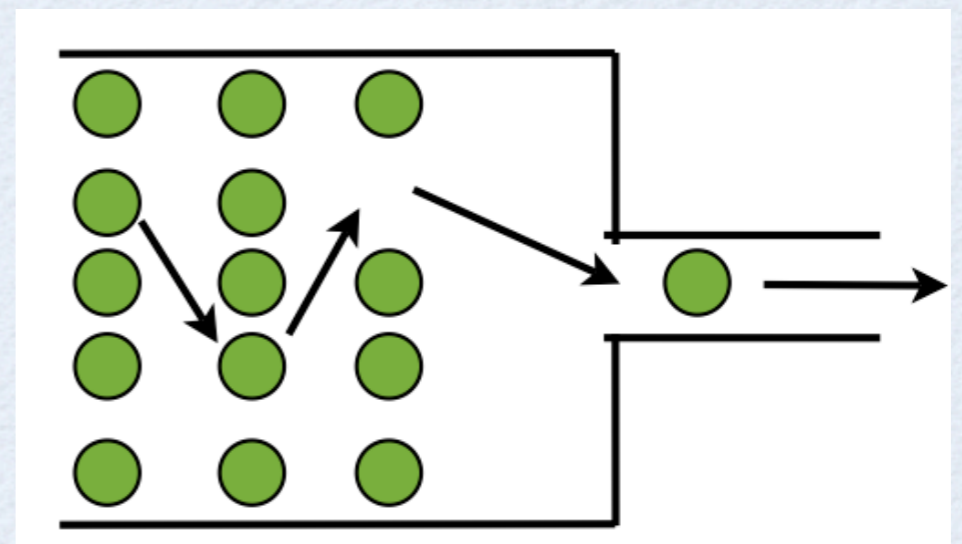
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- ★ Similar result, if the total number of cars vary randomly about a mean value.
- ★ A variant of the model with multiple layer also gives similar result

Replace removed car  
by the one with the largest  $A$   
in the row just behind





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★ In a traffic bottleneck, only the most *aggressive* driver passes through at a time.

★ The aggressiveness can be quantified as  $A_i = N_i \tau^\sigma$  where  $N_i \in (0,1)$  varies randomly from person to person,  $\sigma$  is the same for all, and  $\tau$  is the time for which it is stranded.



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**THIS GIVES :**  $P(\tau) \sim \tau^{-\alpha}$  with  $\alpha = \sigma + 1$  which agrees with available empirical data, with  $\sigma = 1$  to  $2$  in India &  $\sigma = 0.5$  in Germany. *Does  $\sigma$  depend on country?*



# CAUTION

Data for 3 cities in India (in 2013) and 1 city in Germany (in 2016) show that probability distribution of congestion time is  $P(\tau) \sim \tau^{-\alpha}$

Data for many other places need to be analysed.



*Thank you for your attention*