Droplets in isotropic turbulence

Deformation and breakup statistics

Samriddhi Sankar Ray

with D. Vincenzi

International Centre for Theoretical Sciences
Tata Institute of Fundamental Research
Bangalore, India

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ssray@icts.res.in

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Bangalore

Ray and Vincenzi, arXiv: 1712:07885 (2017)

Introduction

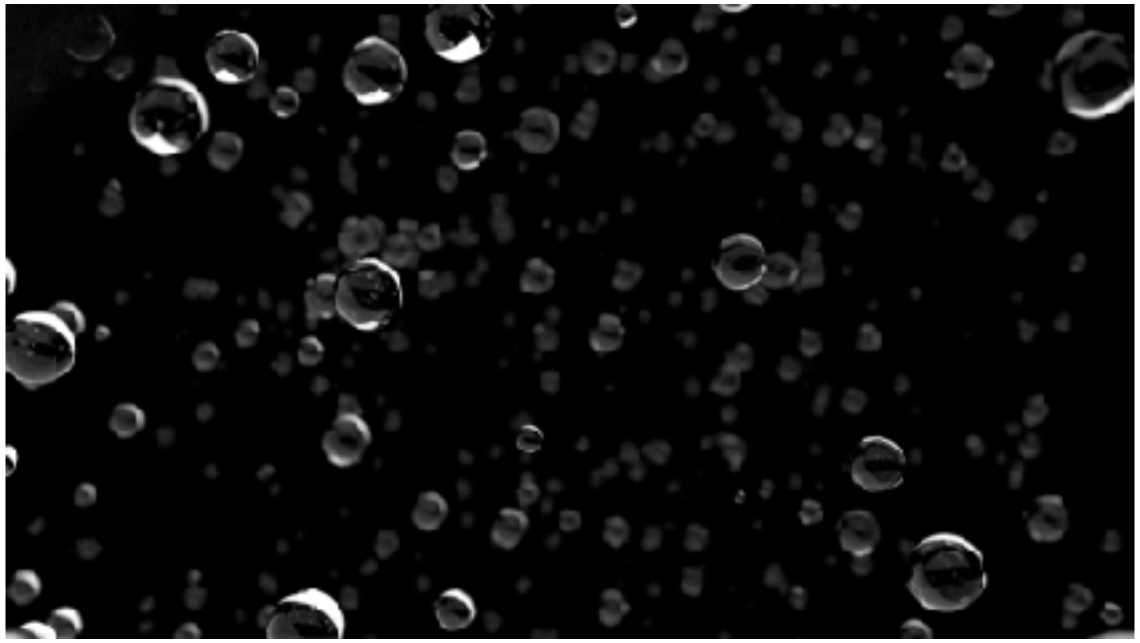
Preamble

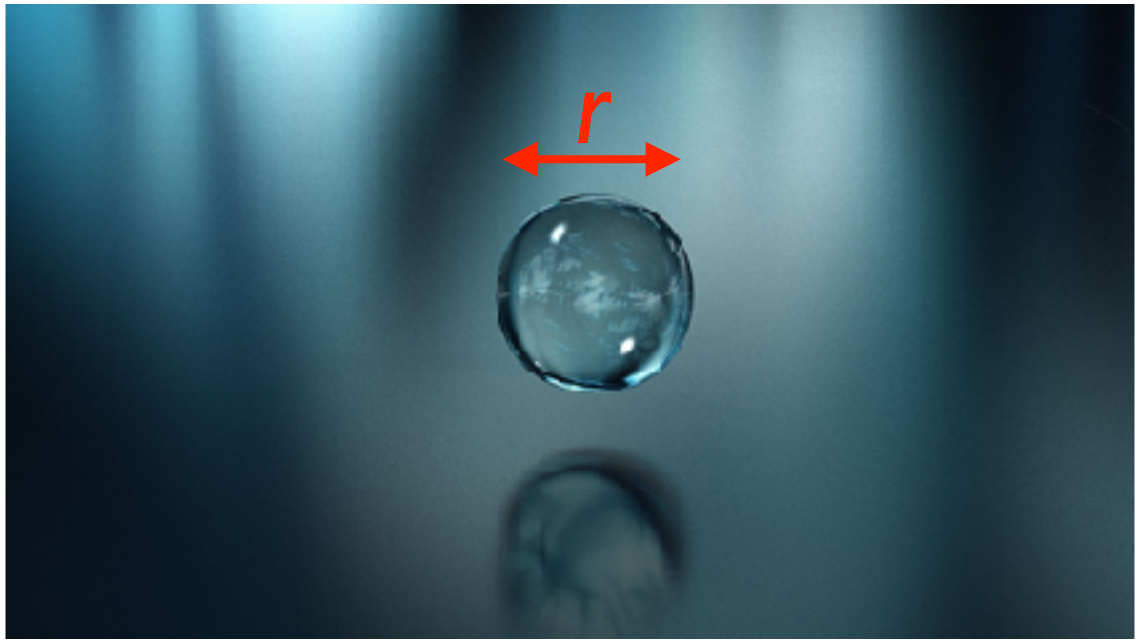
Dispersion, deformation and break-up of *immiscible* drops is an important problem.

Emulsion processes require detailed understanding of single droplet dynamics.

<u>Goal:</u> Elucidate and investigate deformation and break-up of *small* droplets.

<u>Approach:</u> Direct numerical simulations of a model *simple* enough to allow some analytical calculations.





$$\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r})-\boldsymbol{u}(\boldsymbol{x}) \approx \nabla \boldsymbol{u} \cdot \boldsymbol{r}$$

Walstra, *Chem. Eng. Sci.* (1993)
Windhab, *et al.*, *Chem. Eng. Sci.* (2005)

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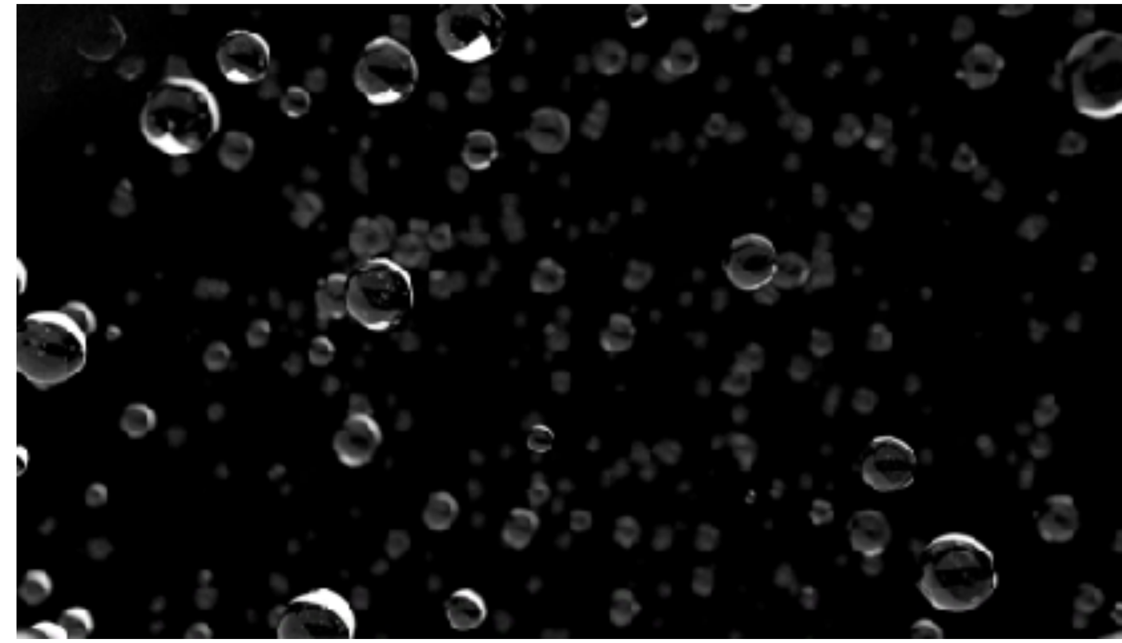
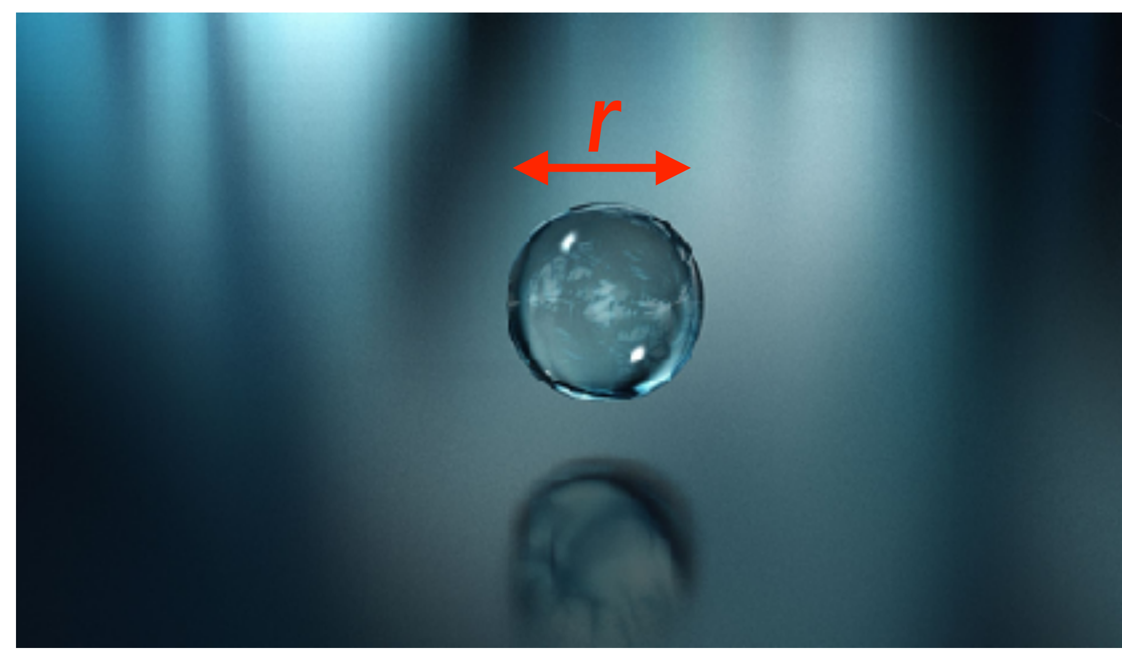


Control Parameters

Initial shape.
Capillary number.
Viscosity ratio.



$$u(x+r) - u(x) \approx \nabla u \cdot r$$

Maffetone and Minale, *J. Non-Newtonian Fluid Mech.* (1998)
Biferale, Meneveau, and Verzicco, *J. Fluid Mech.* (2014)

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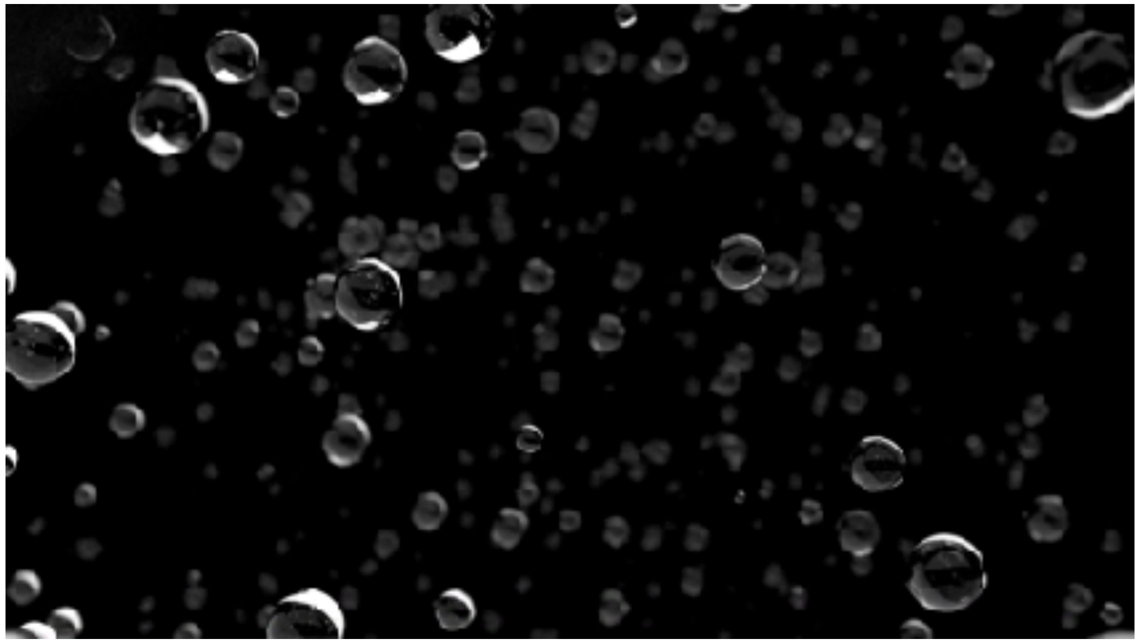
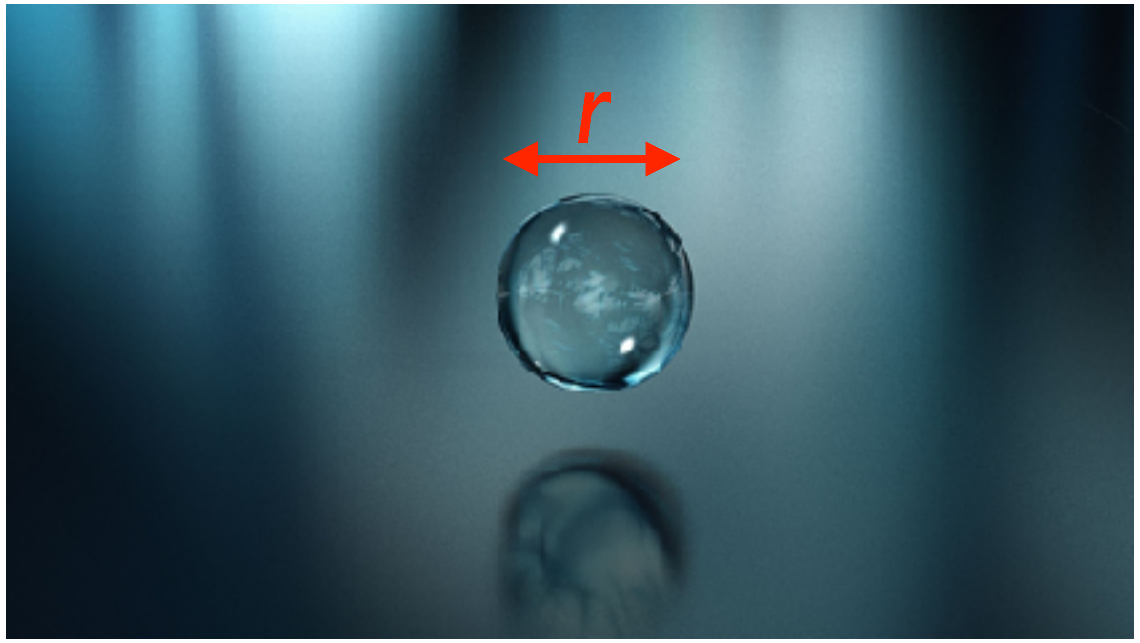
Caveat

Secondary break-ups not considered.
Break-up criterion *arbitrary*.
Ignore droplet inertia: Lagrangian simulations.





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Equations of Motion

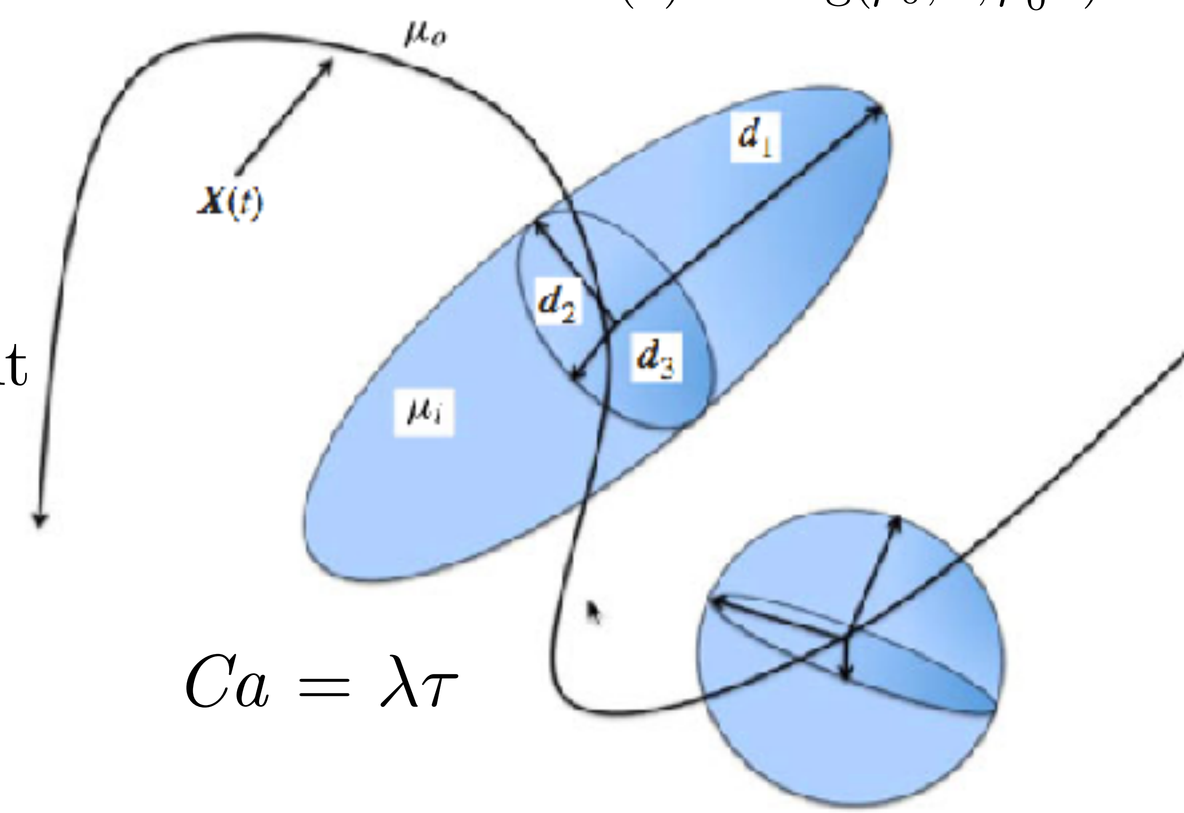
The droplet

Shape and orientation are determined by a second-order symmetric positive-definite tensor whose eigenvectors are the semi-axes of the drop and eigenvalues the squared-lengths of those axes.

Inertia tensor: $$\mathrm{M}^{ij} \equiv \int_{\mathcal{V}} (x^i - x^i_{\mathrm{cm}})(x^j - x^j_{\mathrm{cm}})\,\mathrm{d}\boldsymbol{x}$$

$$\boldsymbol{M}(0) = \mathrm{diag}(\rho_0, 1, \rho_0^{-1})$$

$$\dot{\boldsymbol{M}} = \boldsymbol{G}\boldsymbol{M} + \boldsymbol{M}\boldsymbol{G}^{\top} - \frac{f_1(\mu)}{\tau}[\boldsymbol{M} - g(\boldsymbol{M})\boldsymbol{I}]$$



$\boldsymbol{G} = f_2(\mu)\boldsymbol{S} + \Omega$ is an effective velocity gradient

$$\Omega = [\nabla \boldsymbol{u} - (\nabla \boldsymbol{u})^{\top}]/2 \qquad \boldsymbol{S} = [\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\top}]/2$$

$$f_1(\mu) = \frac{40(\mu+1)}{(2\mu+3)(19\mu+16)}, \quad f_2(\mu) = \frac{5}{2\mu+3}$$

$$Ca = \lambda\tau$$

Break-up Criterion: Ratio of the largest to the smallest eigenvalues greater than some threshold value.

Maffetone and Minale, *J. Non-Newtonian Fluid Mech.* (1998)

Equations of Motion

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The droplet

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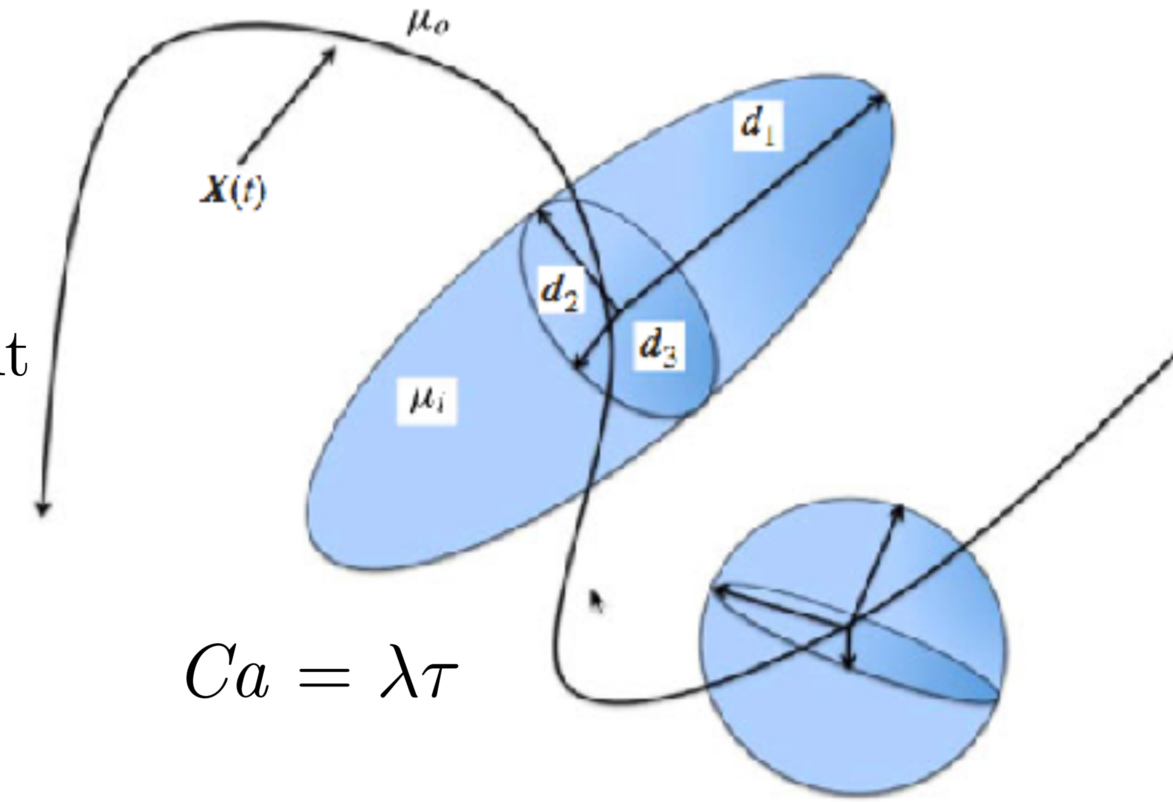

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The Fluid

Analytical Approach

Assume velocity gradients are Gaussian, delta-correlated in time to show the probability distribution function of the sizes satisfy a Fokker-Planck equation.

Maffetone and Minale, *J. Non-Newtonian Fluid Mech.* (1998)
Biferale, Meneveau, and Verzicco, *J. Fluid Mech.* (2014)
Spandan, Lohse, and Verzicco, *J. Fluid Mech.* (2016)
Olbricht, Rallison, and Leal, *J. Non-Newtonian Fluid Mech.* (1982)
Brunk and Koch, *Phys. Fluids* (1997)
Musacchio and Vincenzi, *J. Fluid Mech.* (2011)

Equations of Motion

The droplet

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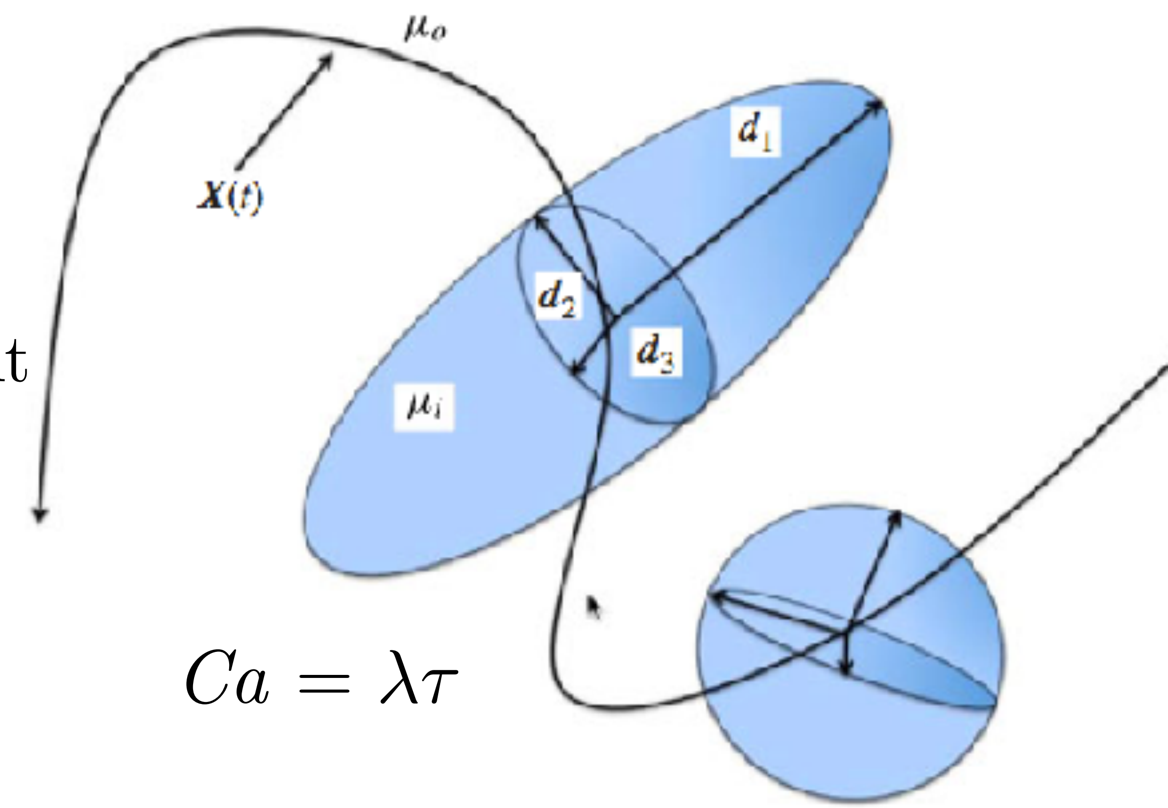
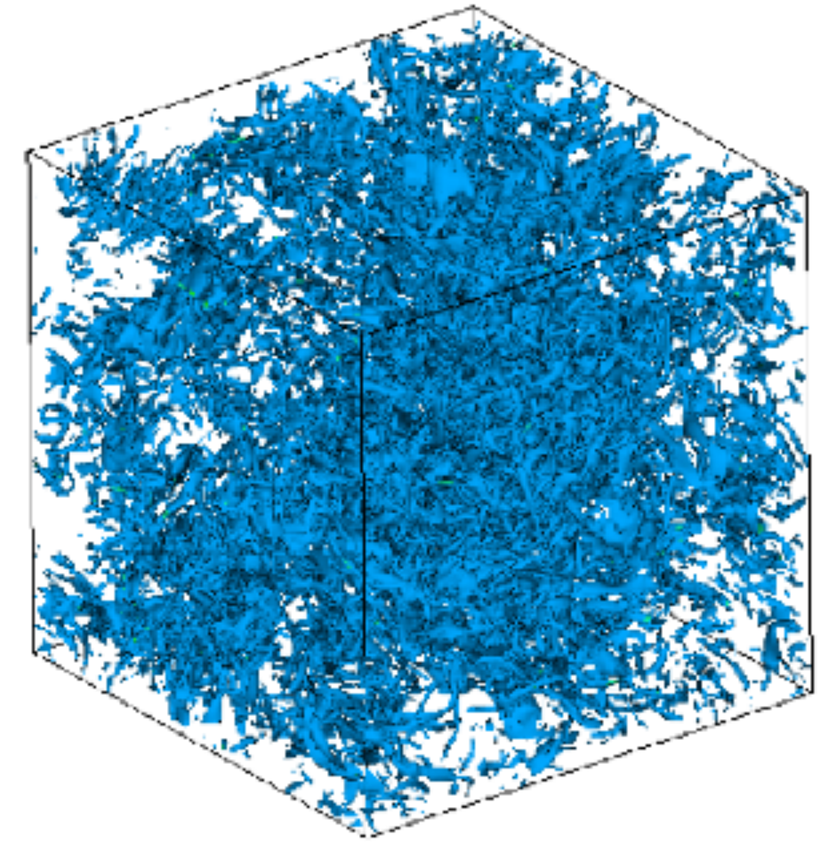


$$Ca = \lambda\tau$$

The Fluid

Direct Numerical Simulations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nu \Delta \boldsymbol{u} + \boldsymbol{F}$$



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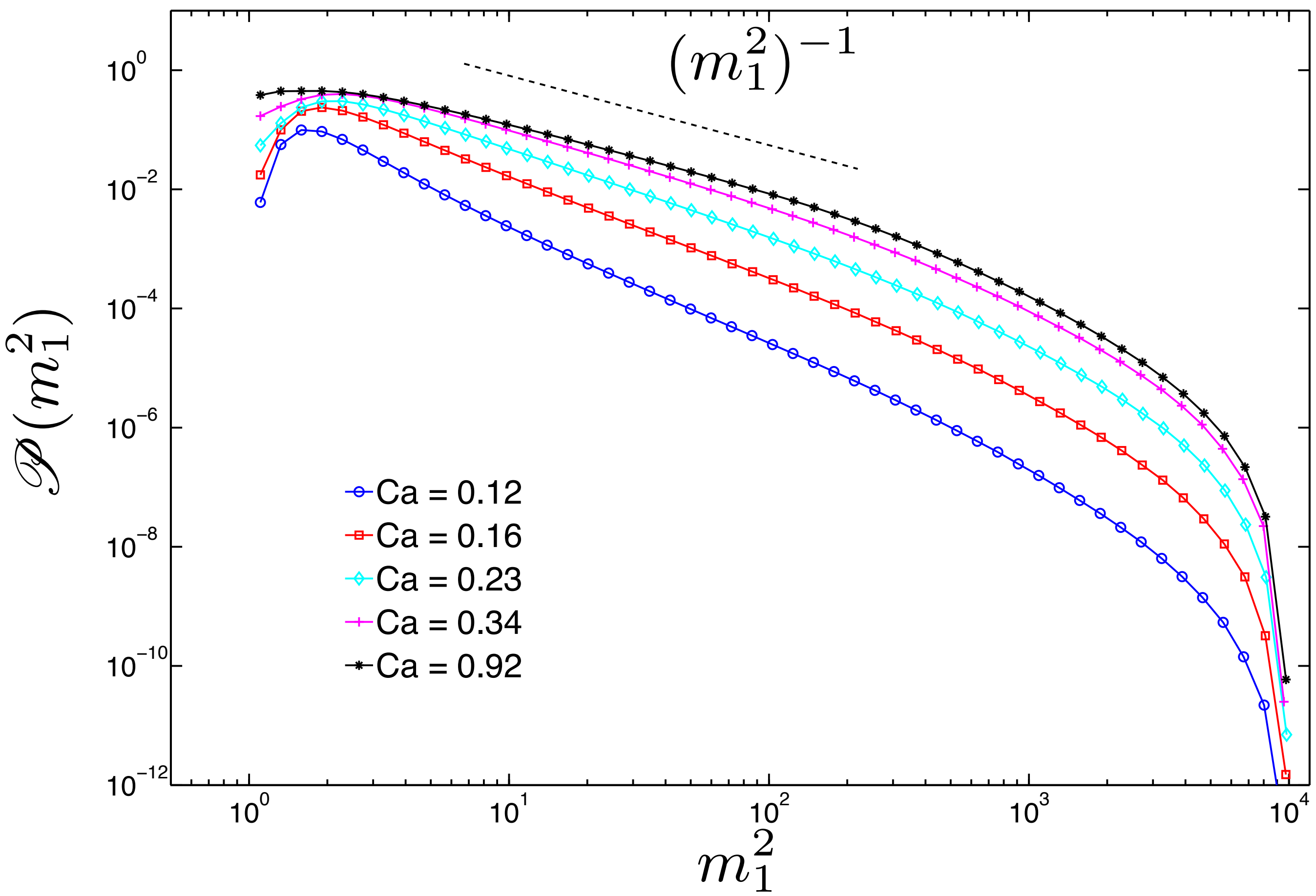
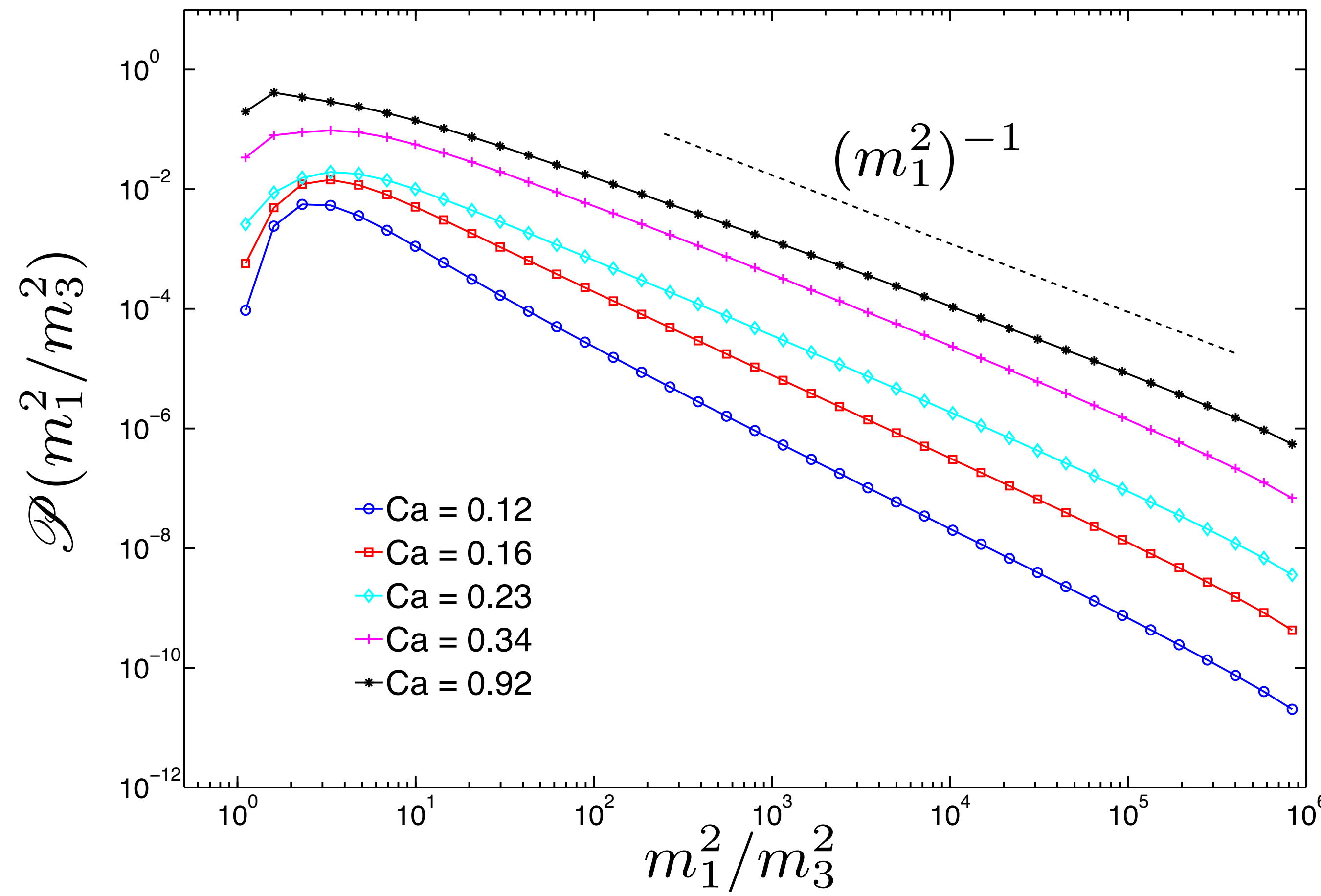
Results: Size distribution

Time-integrated





Prediction

$$\mathrm{Ca} < 1/2\gamma(\mu)$$

$$\mathcal{P}(r, r_0) \sim \begin{cases} r^{-\alpha} & (r_{\mathrm{eq}} \ll r \ll r_0) \\ r^{-\alpha} & (r_0 \ll r \ll \ell) \end{cases}$$

$$\mathrm{Ca} > 1/2\gamma(\mu)$$

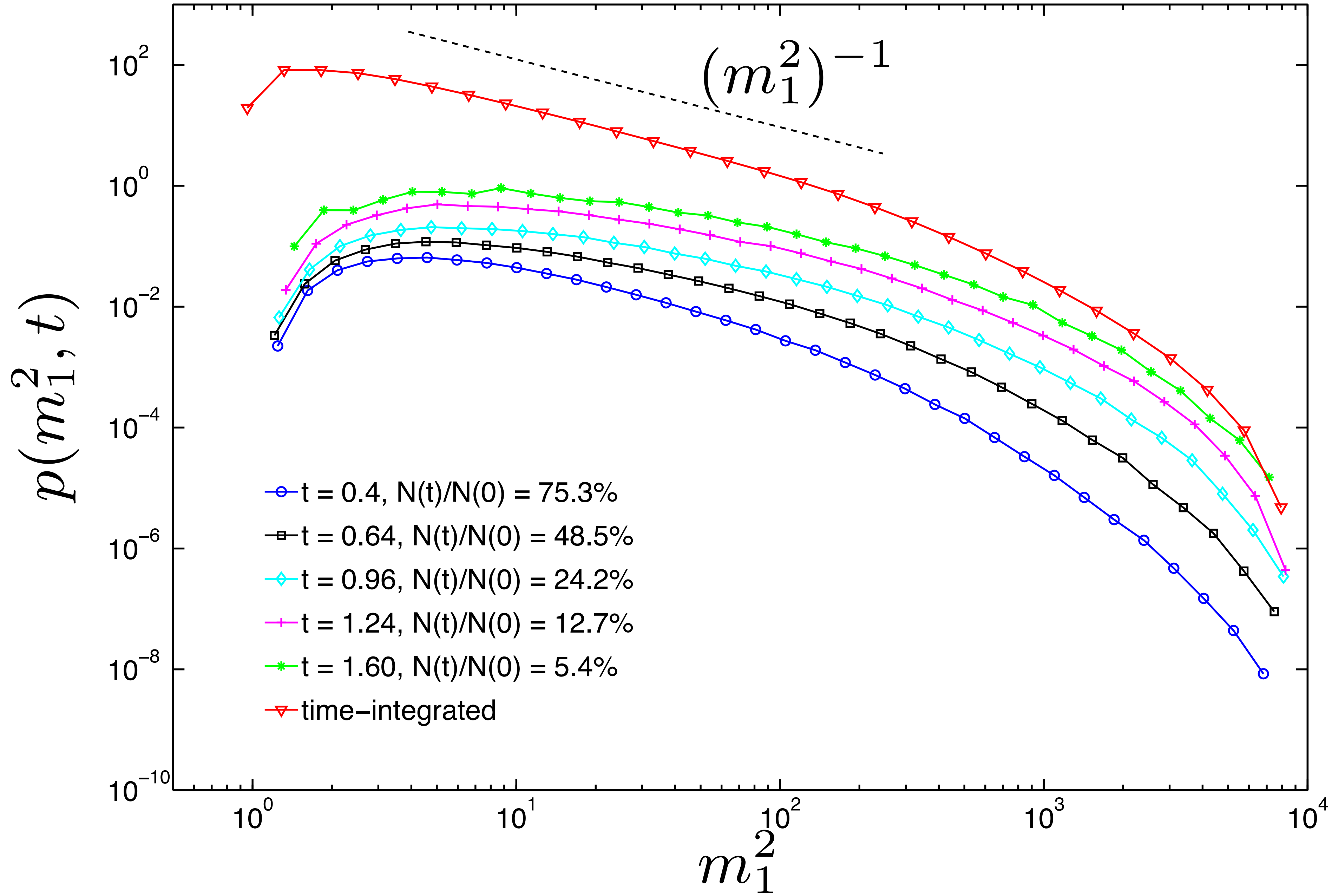
$$\mathcal{P}(r, r_0) \sim \begin{cases} r^{-\alpha} & (r_{\mathrm{eq}} \ll r \ll r_0) \\ r^{-1} & (r_0 \ll r \ll \ell) \end{cases}$$

$$\alpha = d - 1 - d/2\gamma(\mu)\mathrm{Ca}$$

Ray and Vincenzi, arXiv: 1712:07885 (2017)

Results: Size distribution

Time-dependent



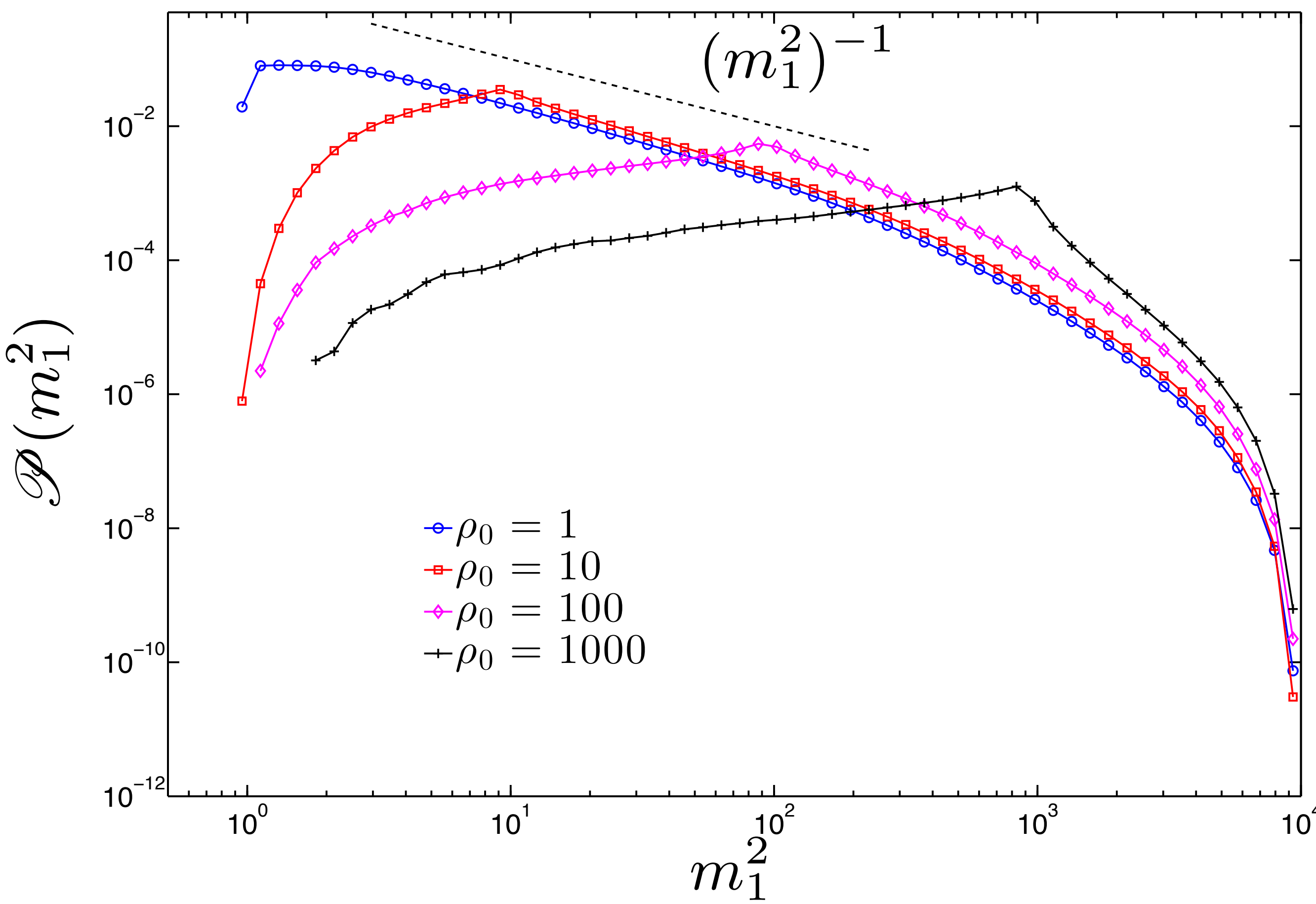
Prediction

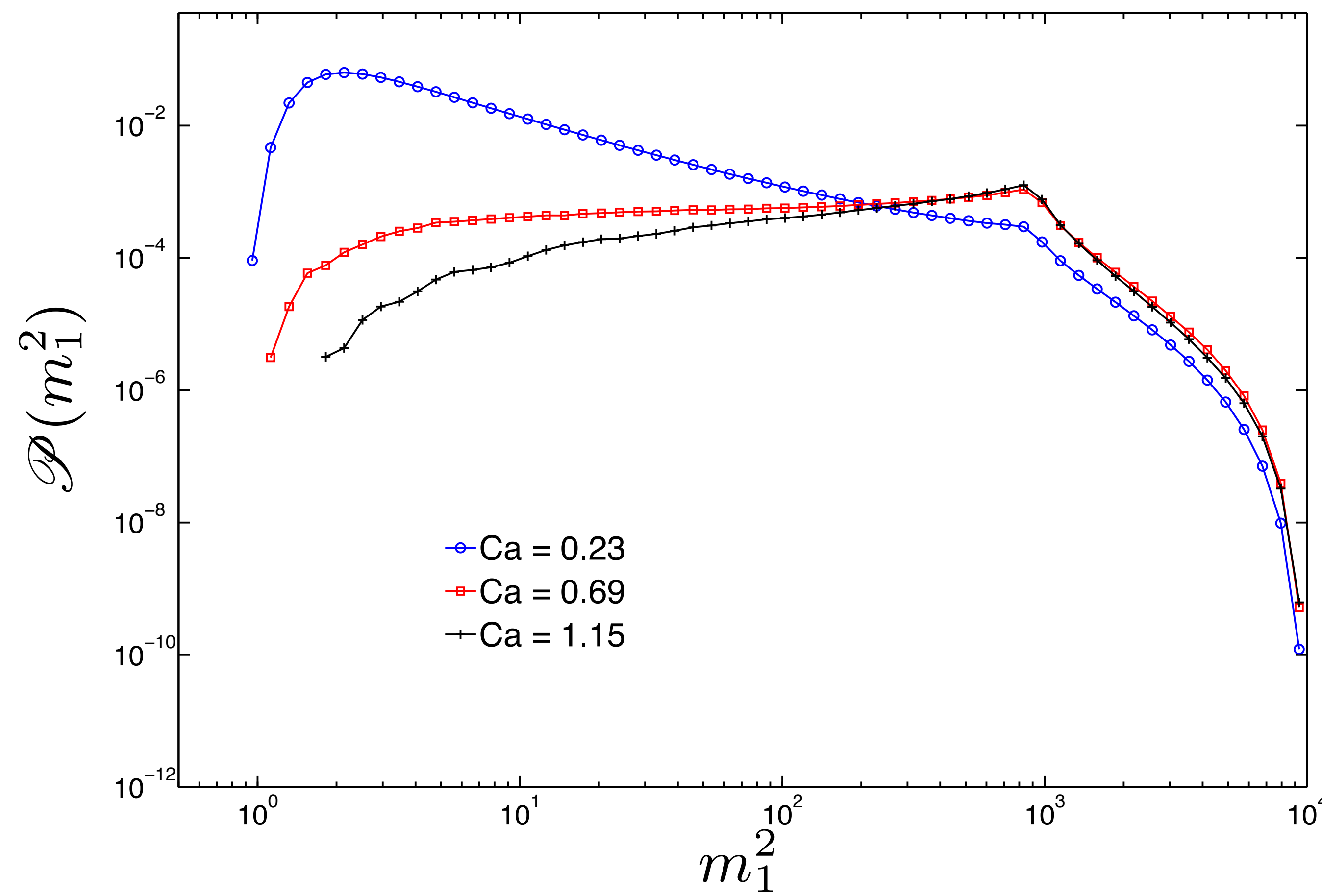
$$p(r,t) \sim \mathrm{e}^{-\nu_1 t} f_{\nu_1}(r) \quad \text{as } t \to \infty$$

Ray and Vincenzi, arXiv: 1712:07885 (2017)

Results: Size distribution

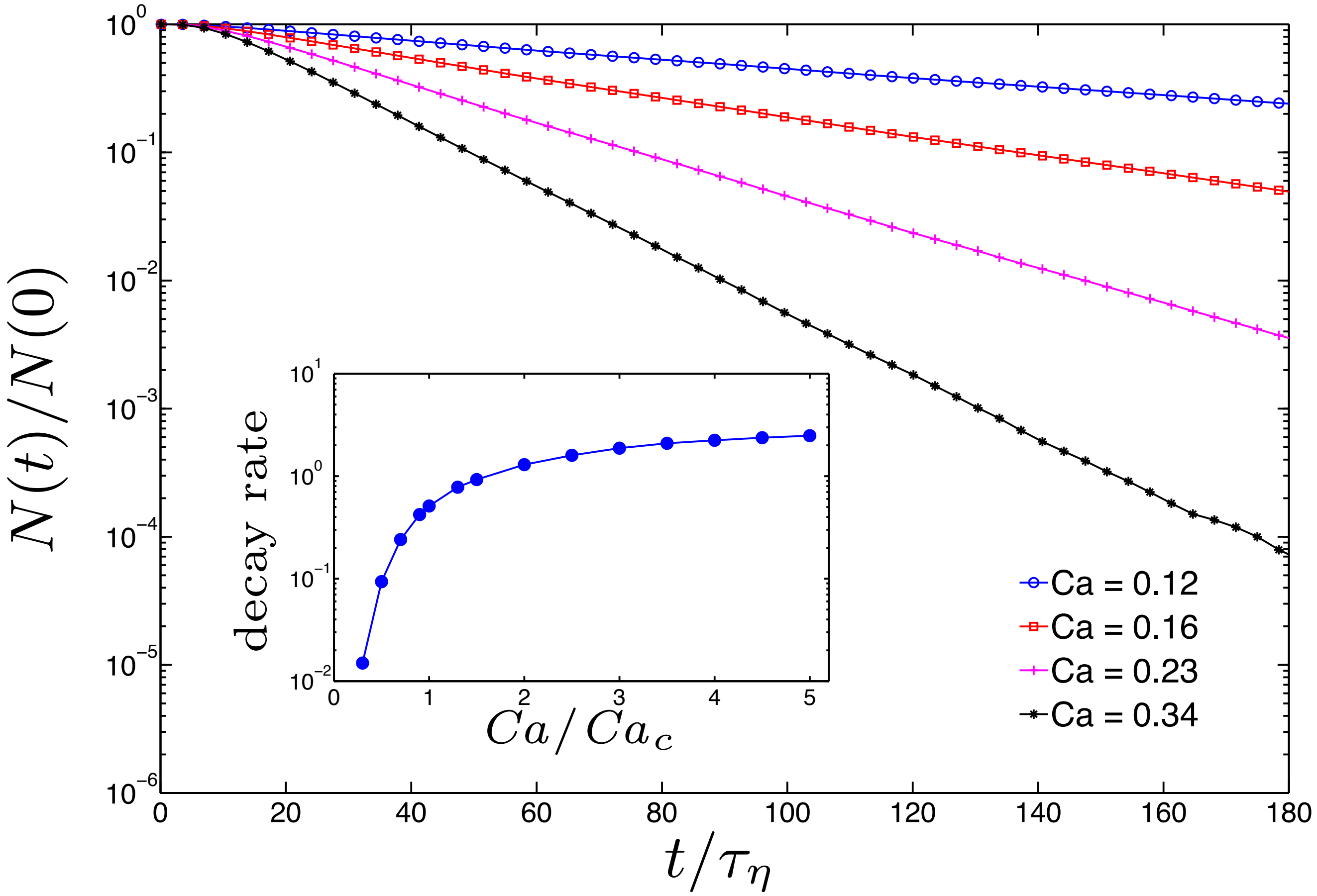
Dependence on initial distribution and viscosity ratios

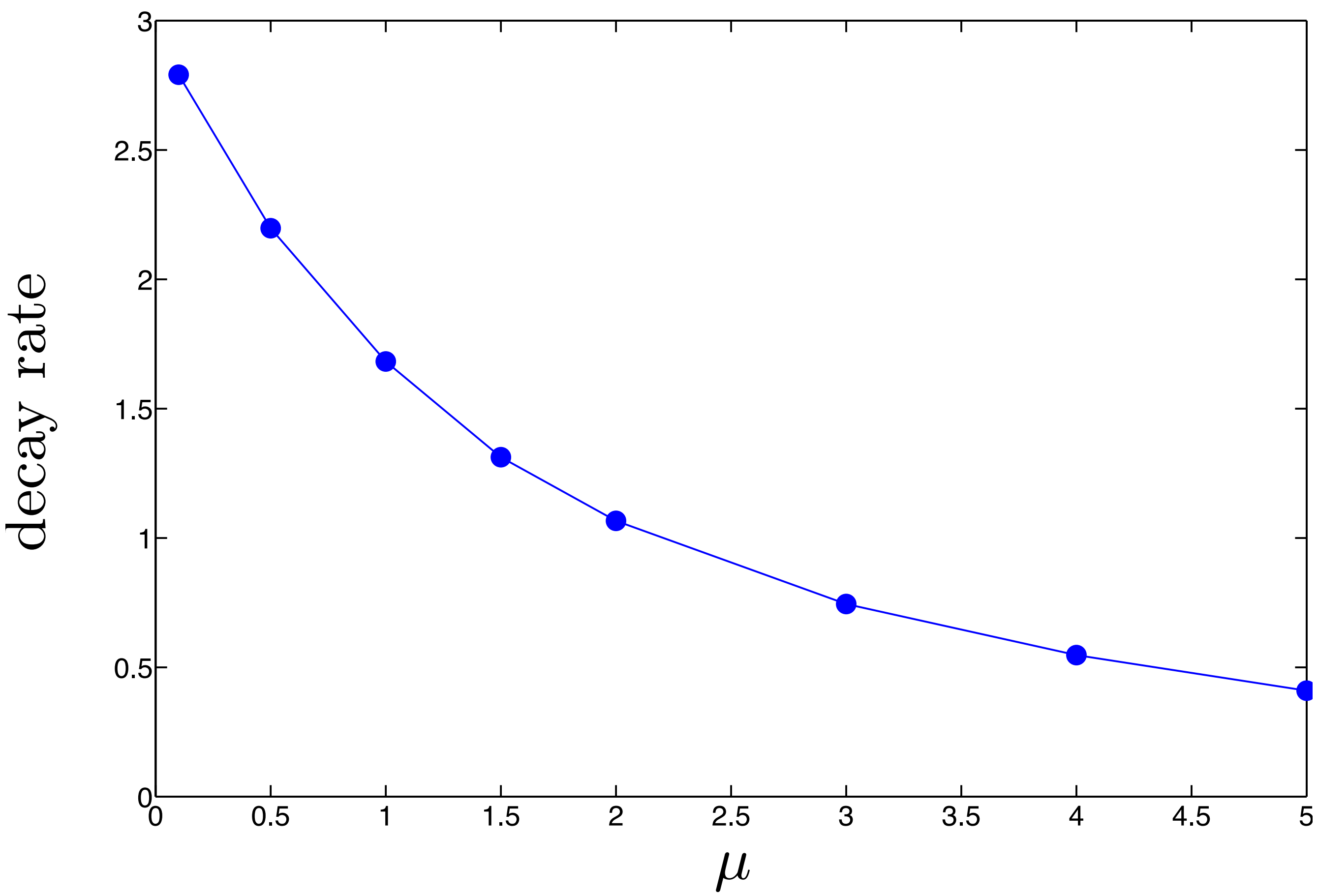




Ray and Vincenzi, arXiv: 1712:07885 (2017)

Results: Fraction of droplets which survive





Prediction

$$N(t)/N(0) \equiv \int_0^{\ell} p(r,t)\mathrm{d}r \sim \mathrm{e}^{-\nu_1 t} \quad \text{as } t \to \infty$$

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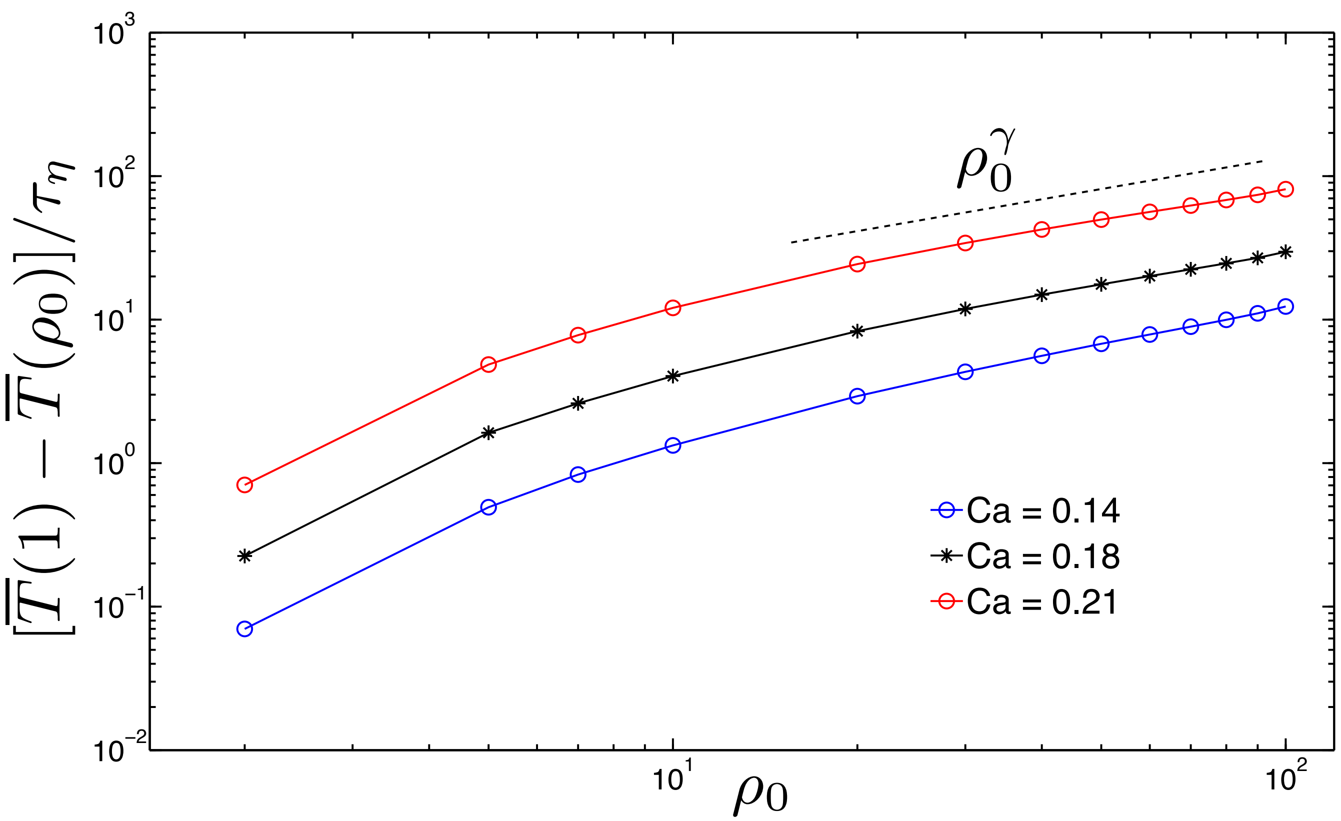
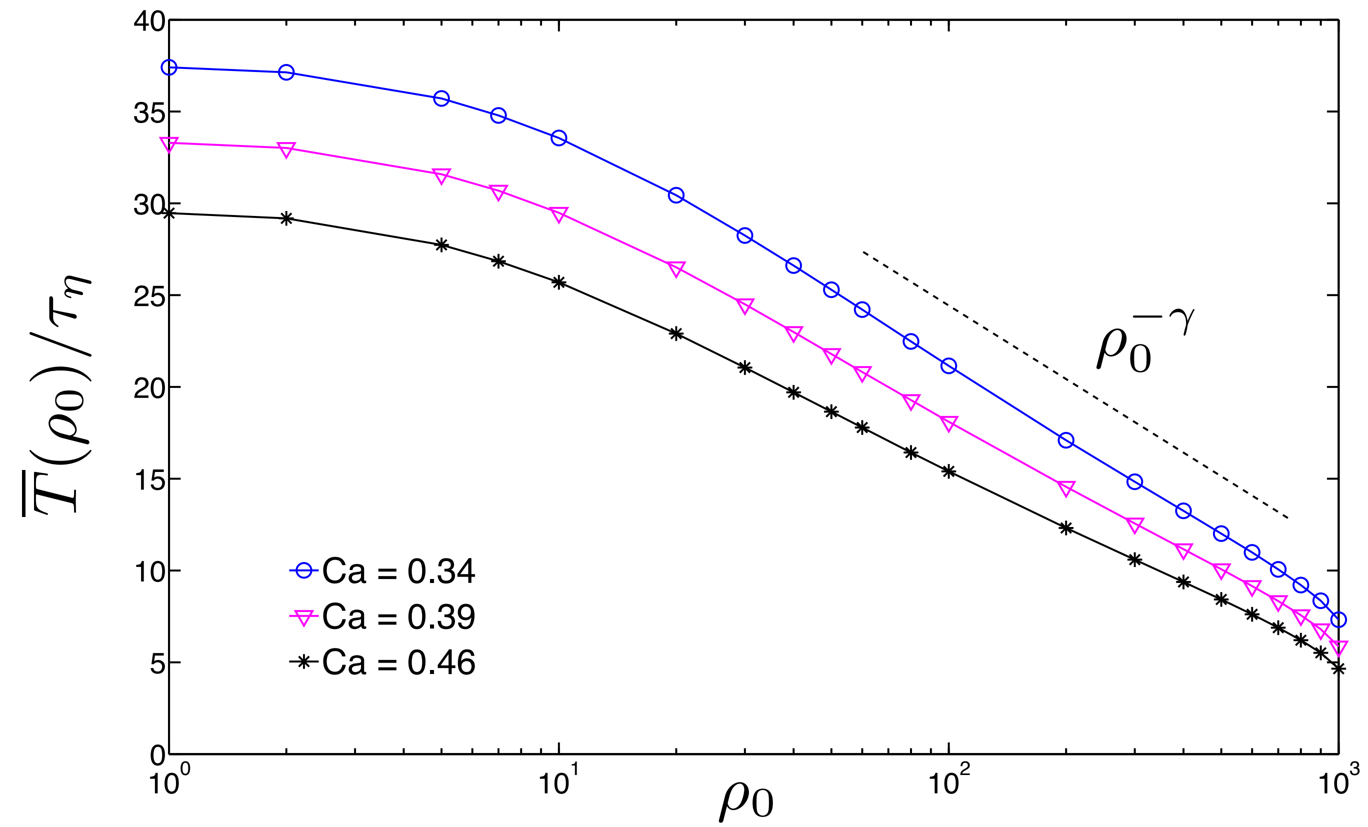
Results: Mean life time





Prediction

$$\overline{T}(r_0) \sim \begin{cases} \left(\dfrac{\ell}{r_{\mathrm{eq}}}\right)^{\beta-1} - \left(\dfrac{r_0}{r_{\mathrm{eq}}}\right)^{\beta-1} & \text{if } Ca < Ca_c \\ \ln\left(\dfrac{\ell}{r_0}\right) & \text{if } Ca > Ca_c \end{cases}$$

Ray and Vincenzi, arXiv: 1712:07885 (2017)

Conclusions and Perspective

- The probability distribution function of droplet sizes show a power-law behaviour.
  - For small Capillary numbers, a decreasing exponent.
  - For large Capillary numbers, it scales as r^{-1} for $r > r_0$ and a different, Ca-dependent exponent for $r < r_0$.
- The number of surviving drops decay exponentially.
  - The decay rate increases rapidly as $Ca \to Ca_c$.
  - The decay rate decreases as a function of μ.
- The mean life-time of a drop show different scaling laws
  - Power-law behaviour for $Ca < Ca_c$.
  - Logarithmic decrease for $Ca > Ca_c$.
- Extension of this approach beyond droplets to more extended objects such as dumbbells.