Negative mobility in interacting particle systems

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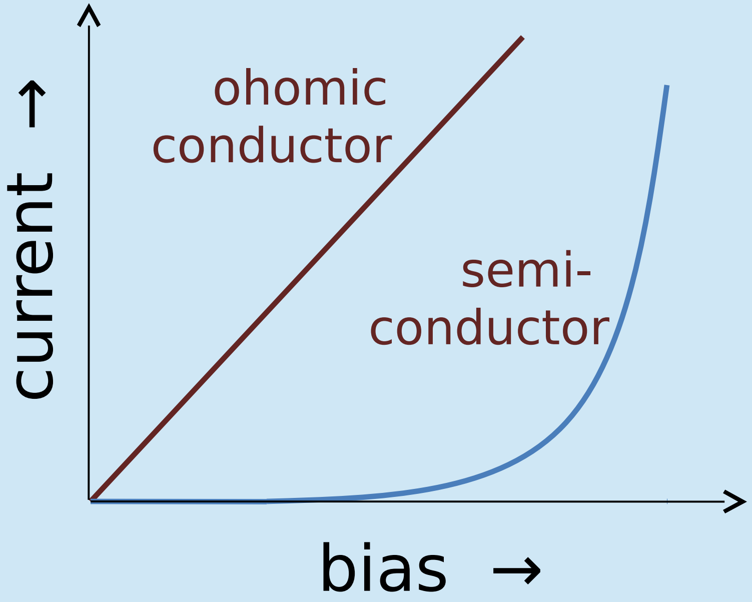
NEGATIVE RESPONSE

normally

current always increases with bias



negative response

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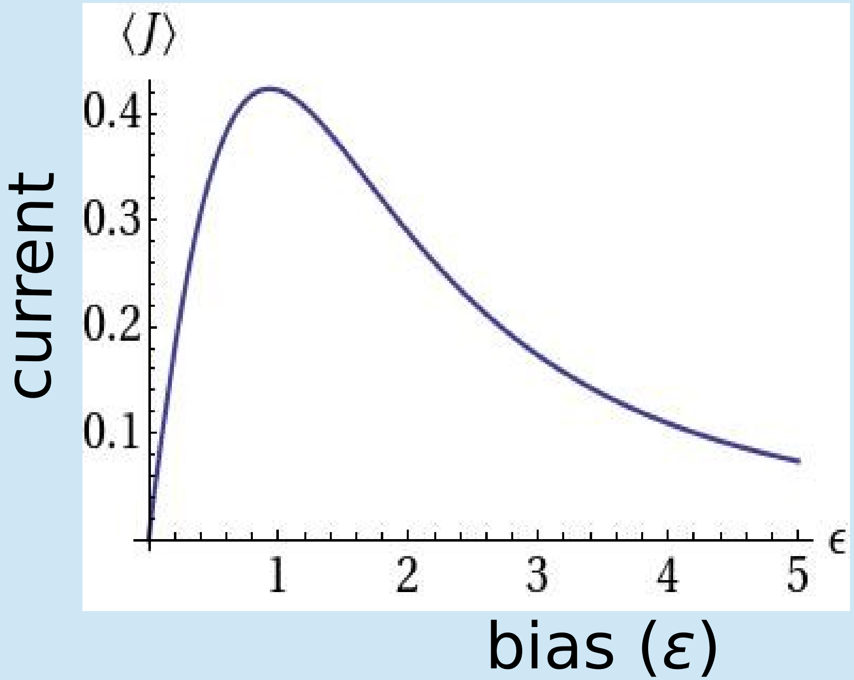

What I intend to discuss......

Mechamism of

NEGATIVE RESPONSE

in

NON-EQUILIBRIUM SYSTEMS

In equilibrium $<J> = 0$ but current fluctuates (*microscopic*)

$<J^2> = finite$

Like in a paramagnet $<M> = 0$ but $<M^2> = finite$

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Description : $Z(z) = \sum_{M=0}^{\infty} z^M Q_M$ $\quad z = e^{-h}$ $\quad <M> = z \frac{d}{dz} \ln Z$

(of para-magnet)

$$\frac{d}{dh} <M> = <M^2> - <M>^2 \equiv \chi$$

With a small field: $<M>_h = <M>_0 + h \frac{d}{dh} <M>_0 = h <M^2>_0$

Response would be propertional to the fluctuations at zero-filed.

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Response would be propertional to the fluctuations at zero-filed.

Same analogy

$$J(\epsilon) = J(0) + \epsilon\left(<J^2> - <J>^2 \right)_{\epsilon=0}$$
$$= \epsilon <J^2>_0$$

$$\frac{dJ}{d\epsilon} = <J^2> - <J>^2 \geq 0$$

Response is positive

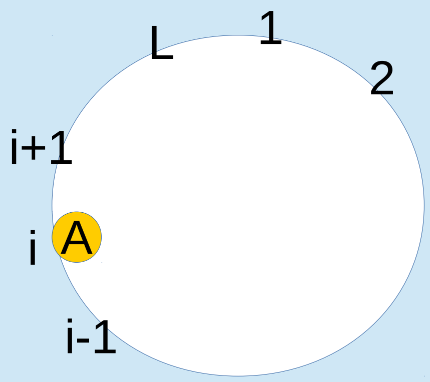
Direction of current = direction of fileld

Calculating Response:

Single random walker(A):

1-D periodic lattice, *L* sites

$$A0 \underset{q}{\overset{p}{\rightleftharpoons}} 0A$$



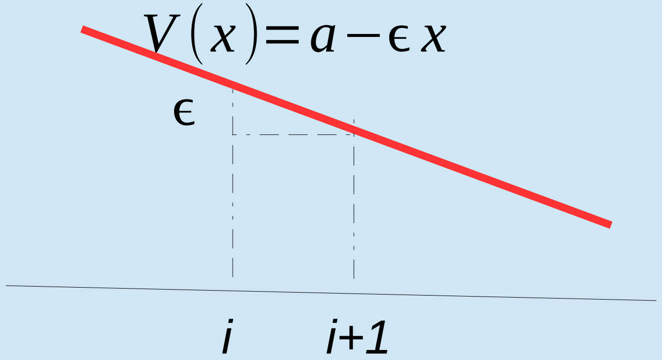
Current :

Number of right/left moves in time T : N_R, N_L

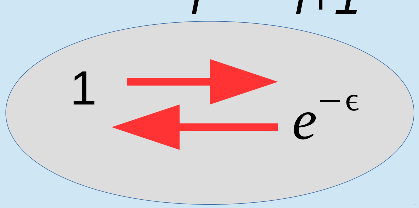
$$<J> \; = \lim_{T \to \infty} \frac{1}{T} \left(<N_R> - <N_L> \right) = p - q$$

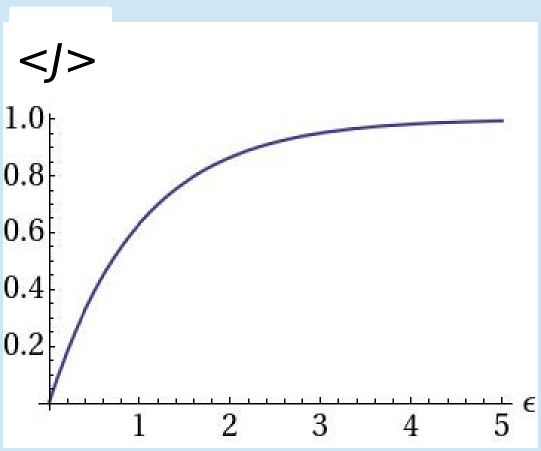
Bias:

$$V(x) = a - \epsilon x$$



MonteCarlo Dynamics :



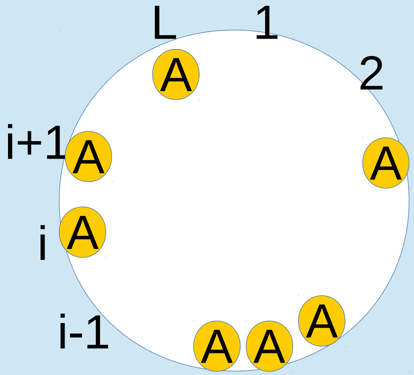


p=1, q=e^{-ε}, ε is bias — $p=1,\ q=e^{-\varepsilon}$, ε is bias

Many walker & exclusion :

$$A0 \underset{q}{\overset{p}{\rightleftharpoons}} 0A$$

$p=1,\ q=e^{-\varepsilon}$



$$<J> \;= p - q$$

$$\Rightarrow \quad \frac{dJ}{d\epsilon} \geq 0$$

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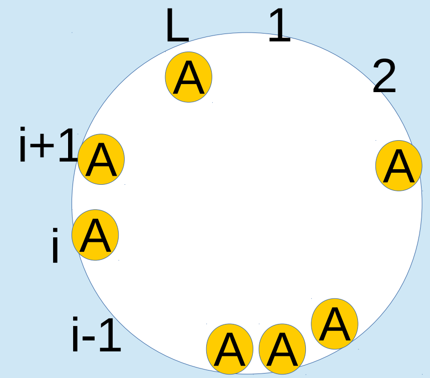
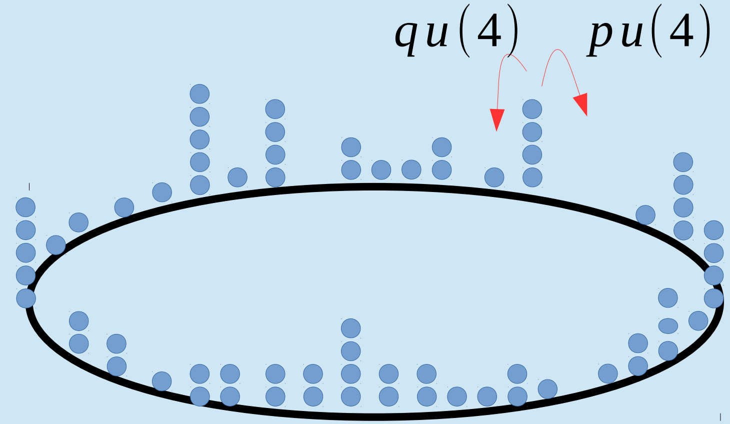


$$<J> \ = p - q$$

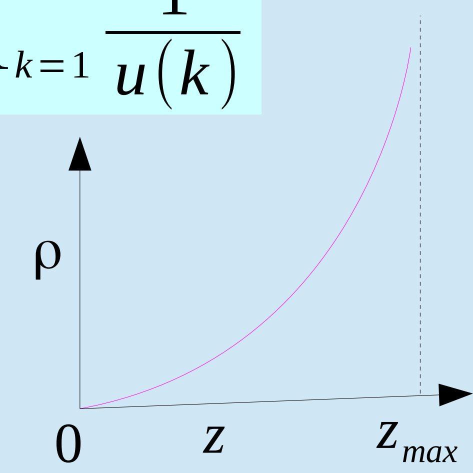
$$\Rightarrow \quad \frac{dJ}{d\epsilon} \geq 0$$

Many walker, no exclusion :



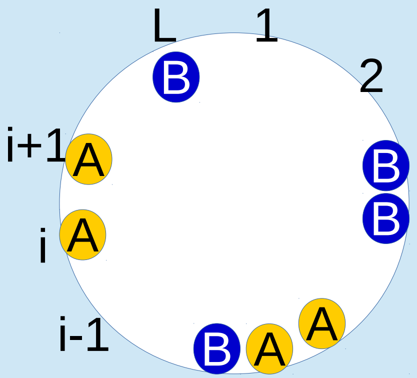
- Right/left hop rates: $u_R(n) = p\,u(n)$ $\quad u_L(n) = q\,u(n)$
- Zero Range Process (ZRP), exactly solvable.
- Steadystate weights: $P(\{n_i\}) \sim \prod_{i=1}^{L} f(n_i)$, where $f(m) = \prod_{k=1}^{m} \frac{1}{u(k)}$
- Current : $J = <u_R(n)> - <u_L(n)> \ = (p-q)\,z$

$$\Rightarrow \quad \frac{dJ}{d\epsilon} \geq 0$$



Two species exclusion process (TSEP) :

$$A0 \underset{q}{\overset{p}{\rightleftharpoons}} 0A \qquad B0 \underset{\psi}{\overset{\psi}{\rightleftharpoons}} 0B \qquad AB \underset{\alpha}{\overset{\alpha}{\rightleftharpoons}} BA$$



► $p=1,\ q=e^{-\varepsilon}$ Only ***A***s are biased to move towards right

For any fixed ψ, α ➡ $\frac{dJ_A}{d\epsilon} \geq 0$

Numerical results

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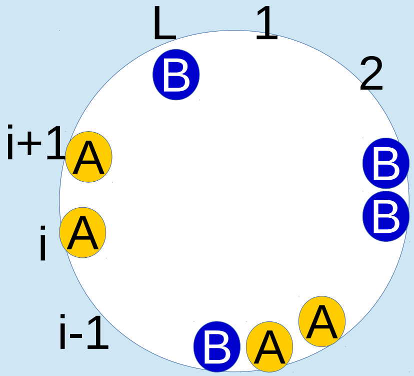

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Numerical results

- Additional field on ***B*** particles $B0 \underset{\psi}{\overset{\varphi}{\rightleftharpoons}} 0B$

For any fixed field $\log\left(\frac{\varphi}{\psi}\right)$ and α, ➡ $\frac{d J_A}{d \epsilon} \geq 0$

Numerical results

Additional interaction among A,B particles :

$$A0 \underset{q}{\overset{p}{\rightleftharpoons}} 0A$$

$$AB \underset{\alpha}{\overset{\alpha}{\rightleftharpoons}} BA$$

+

$$\tau B0 \overset{r(\tau)}{\rightarrow} \tau 0B$$

right move of B

$$0B\tau \overset{l(\tau)}{\rightarrow} B0\tau$$

left move of B

$$\tau = 0, A, B$$

We consider $r(0)=w, l(0)=1-w$; all other rates $1-w$

Only isolated **B** particles are biased.

(**A**s are anyway biased as $p=1, q=e^{-\epsilon}$)

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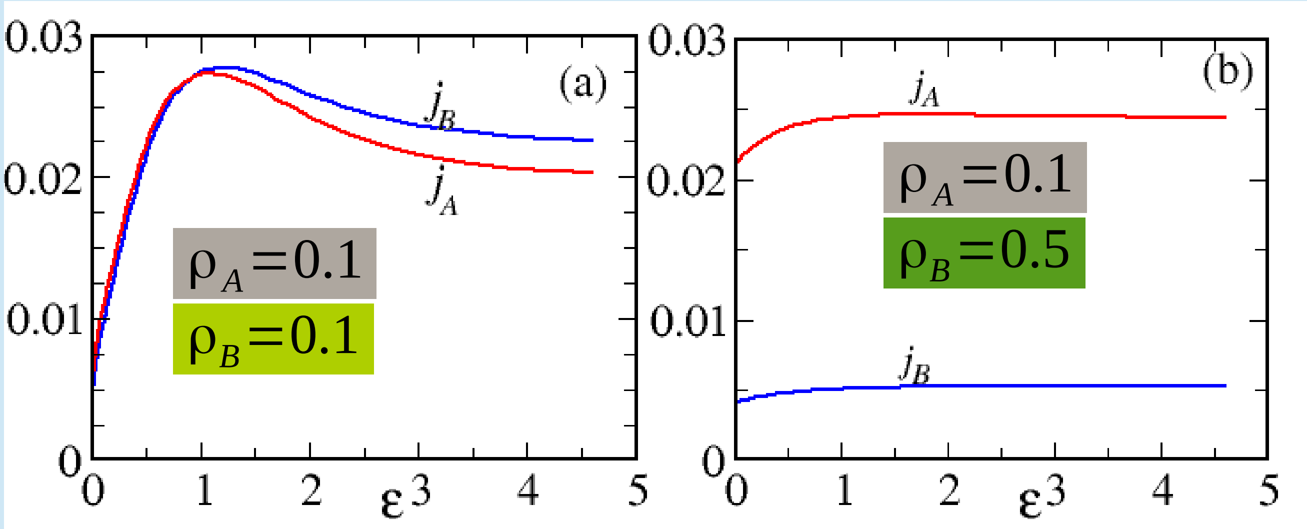
Currents:

Current of A, B particles

$\alpha=0.01$
$w=0.9$



B: fixed +ve bias (to right)
A: +ve bias (to right)

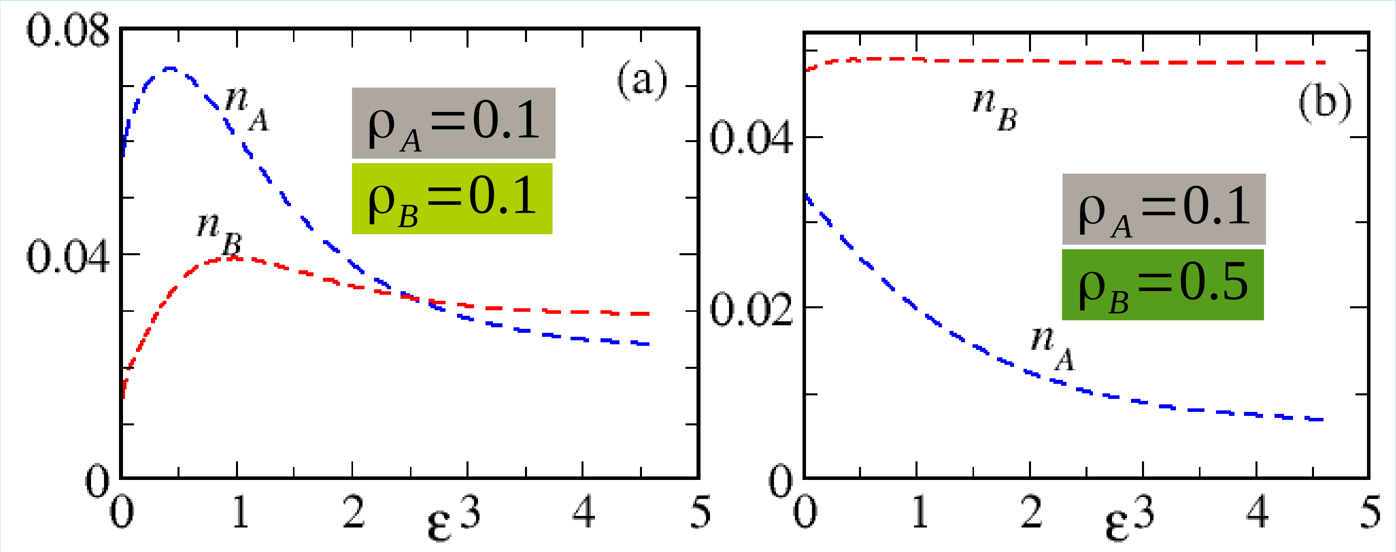
(a) Negative diffrential mobility/respose (NDM) for $j_A, j_B, \quad j=j_A+j_B$

Why does it exhibit NDM ?

Called ***traffic***

We measure other observables $n_{A,B} = \langle n_r^{A,B} + n_l^{A,B} \rangle$

along with $j_{A,B} = \langle n_r^{A,B} - n_l^{A,B} \rangle$



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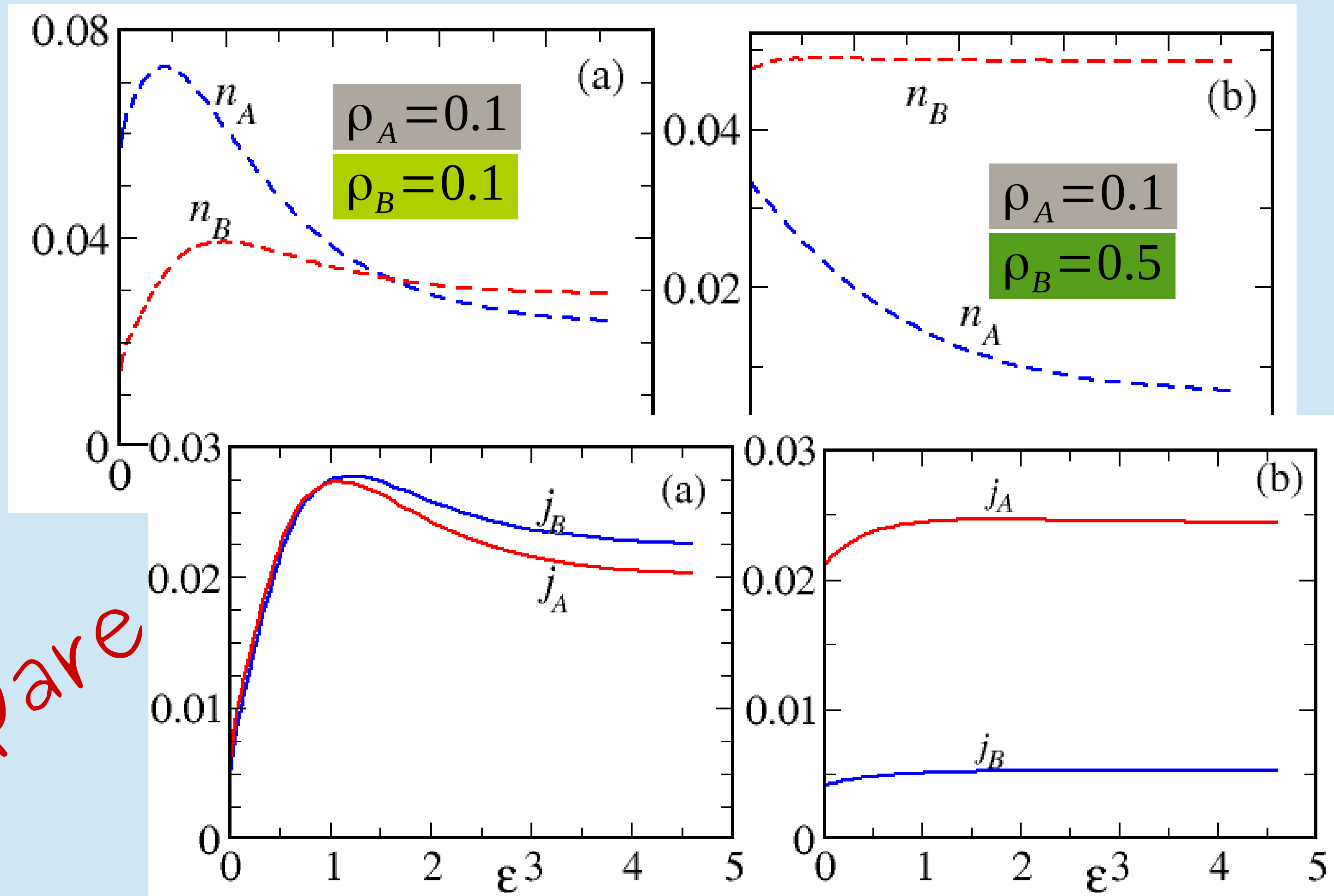
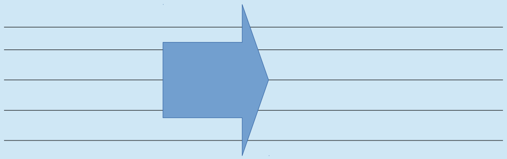


compare

Current of A particles (driven-sector) follows the traffic of B (not driven by ϵ)

Conjecture :

Possible mechanism for NDM in interacting systems:
particle current in a driven many particle system might show a non-monotonic behaviour if some modes, which are not driven by the external field, slow down with increased driving.



Interacting particle system has
many current-carrying modes
(here A, B)

What is necessary for NDM: traffic of driven degrees must slow down

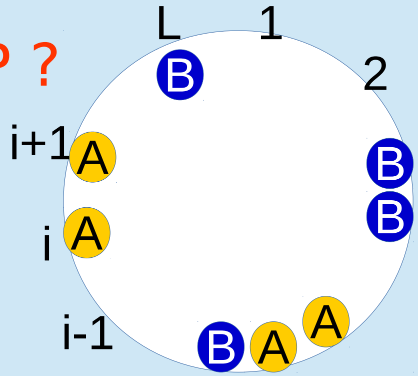
Mechanism :

Slow down of non-driven modes

Slows down traffic of driven modes due to interaction

NDM

What, if we explicitly slow down Bs in TSEP ?



$$A0 \underset{q}{\overset{p}{\rightleftharpoons}} 0A \qquad B0 \underset{\psi}{\overset{\psi}{\rightleftharpoons}} 0B \qquad AB \underset{\alpha}{\overset{\alpha}{\rightleftharpoons}} BA$$

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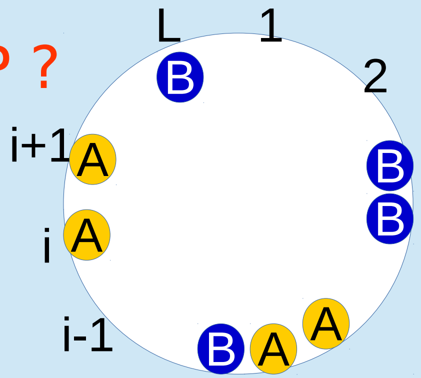

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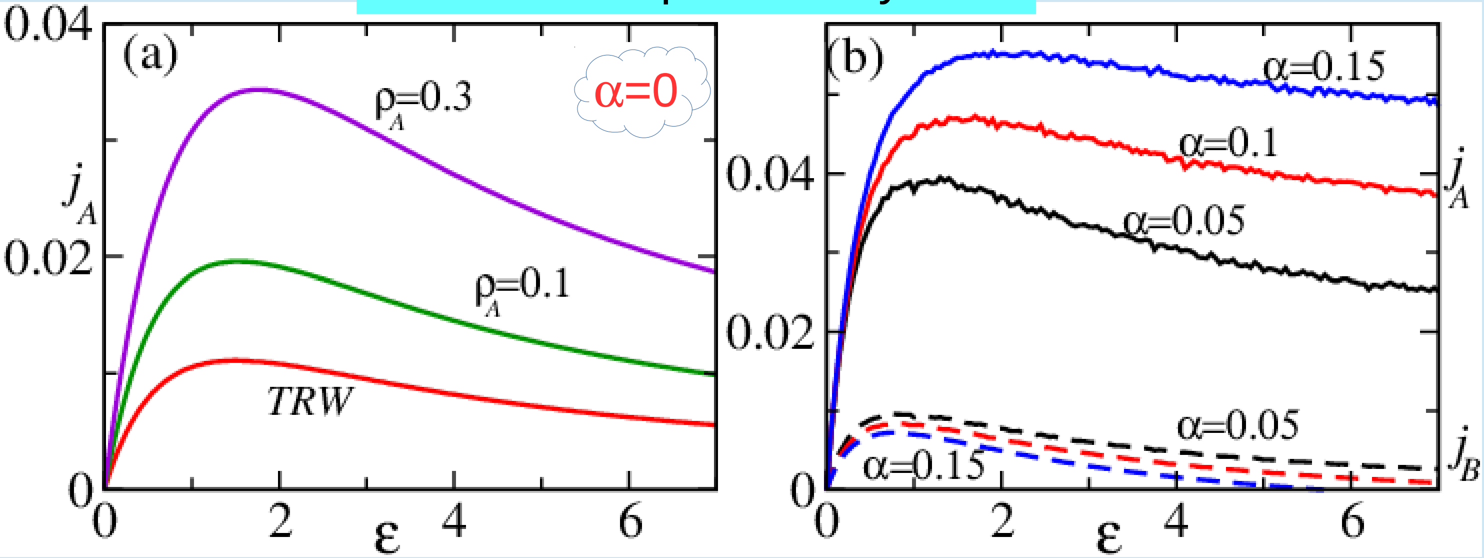
Current for equal-density case



Indeed TSEP now exhibits NDR

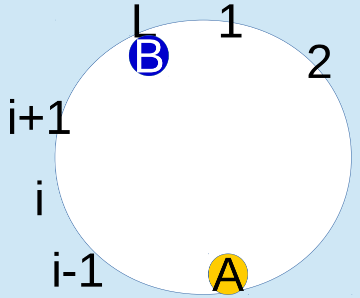
TSEP is exactly solvable for α=0

- **Just two walkers :**

(no exchange)

$$A0 \underset{q}{\overset{p}{\rightleftharpoons}} 0A \qquad B0 \underset{\psi}{\overset{\psi}{\rightleftharpoons}} 0B$$

$$s_i = A, B, 0$$



Exact solution using **Matrix Product Ansatz**

$$P(C) = P(\{s_i\}) = Tr\left(\prod_{i=1}^{L} X_i\right)$$

$$X_i = A\,\delta_{s_i,A} + B\,\delta_{s_i,B} + E\,\delta_{s_i,0}$$

A	***A***
B	***B***
vacancy	***E***

$$P(000A0\ldots B0\ldots) = Tr(EEEAE\ldots BE\ldots)$$

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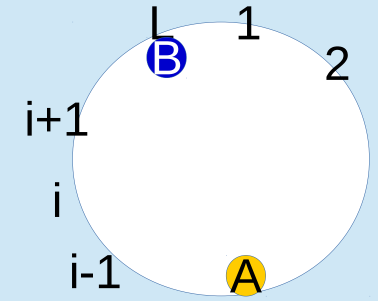

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Matrix Algebra :

$p\,A\,E - q\,E\,A = x_0\,A$
$\psi\,(B\,E - E\,B) = x_0\,B$
$A^2=0 \quad , \quad B^2=0$

Matrix representation :

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad E = \begin{pmatrix} r & 0 \\ 0 & 1 \end{pmatrix}$$

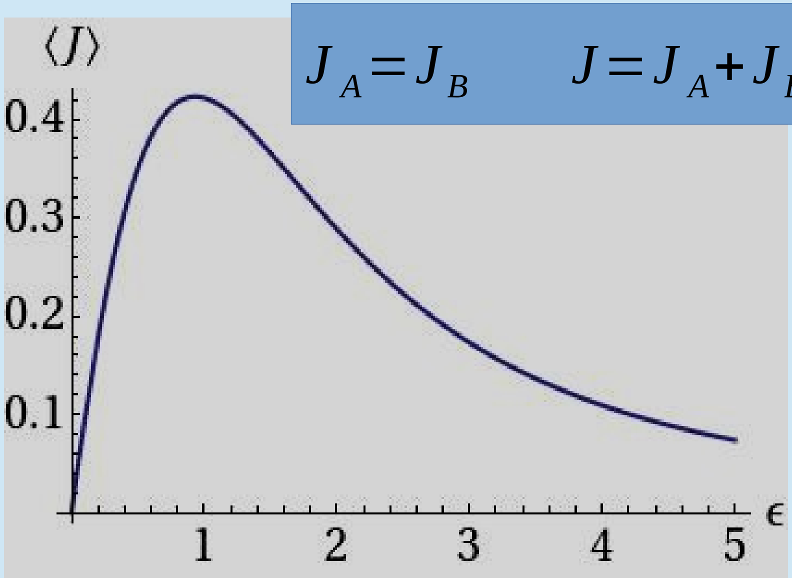
Here, $r = \dfrac{p+\psi}{q+\psi}$

• Average particle current :

$$<J> = \left(p<AE> - q <EA>\right) + \psi(<BE> - <EB>) = \frac{2(p-q)\psi}{p+\psi}$$

$$\langle J\rangle(\varepsilon) = \frac{2(1-e^{-\varepsilon})}{(2+\varepsilon^2)}$$

$$p=1,\ q=e^{-\varepsilon},\ \psi = \frac{1}{(1+\varepsilon^2)}$$



$J_A = J_B \qquad J = J_A + J_B$

- NDM for any ψ, as long as $\psi' < 0$
- **A,B are representative current carrying modes**

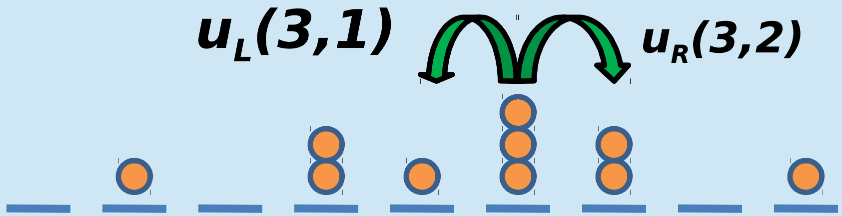
More examples: *Asymmetric misanthrope process*

1-D periodic lattice

$i = 1, \ldots, L$ sites

$n_i \geq 0$ particles

$\sum_{i=1}^{L} n_i = N$

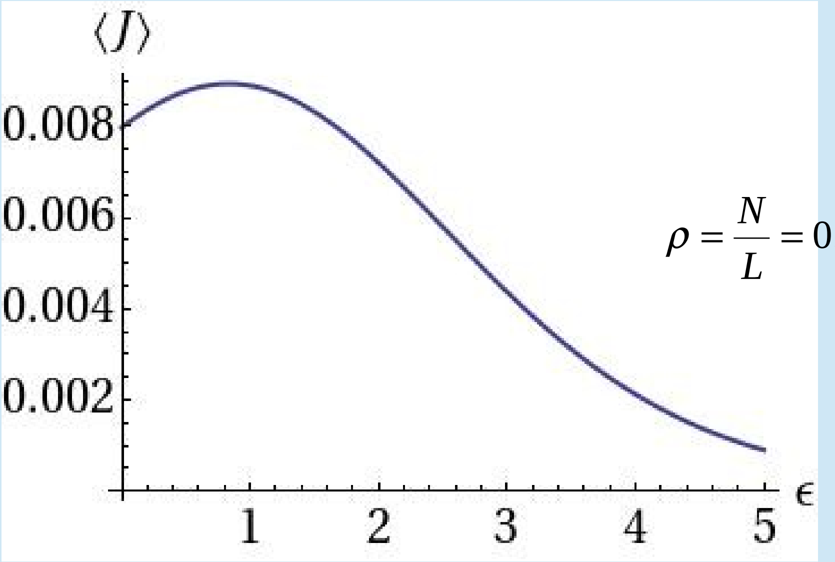


No hard core exclusion

Dynamics - right hop: $u_R(n_i, n_{i+1})$ left hop: $u_L(n_i, n_{i+1})$

$u_R(m,n)$	p_0 , $n=0$
	1, otherwise

$u_L(m,n)$	p_0 , $m=1$
	$e^{-\varepsilon}$, $m>1, n=0$
	½ , $m>1, n>0$



$\rho = \frac{N}{L} = 0.15$

But, what went wrong with the response formula ?

N_R, N_L fluctuates independently

Correspondingly, $J = N_R - N_L \quad S = N_R + N_L$ fluctuates, but J and S ***are correlated***

Correct response formula :

$$\frac{d}{d\epsilon} < J > = < J ; J > - const. < J ; S >$$

In equilibrium: $< J > = 0,$

Baiesi et. al., J. Stat. Phys 2009

$$\Rightarrow \frac{d}{d\epsilon} < J > = < J ; J > = < J^2 >$$

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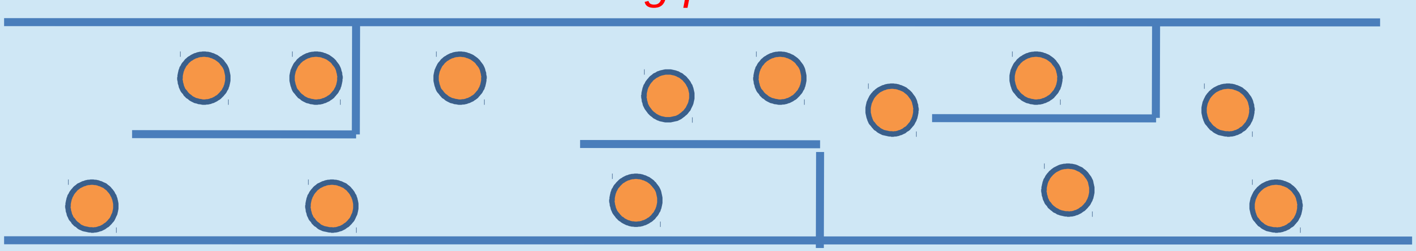
Baiesi et. al., J. Stat. Phys 2009

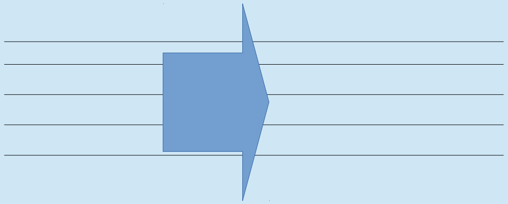
Non- equilibrium: *driven non-interacting particles*



Baerts et.al. , PRE 88, 052109 (2013)

Summary

What gives rise to negative response ?



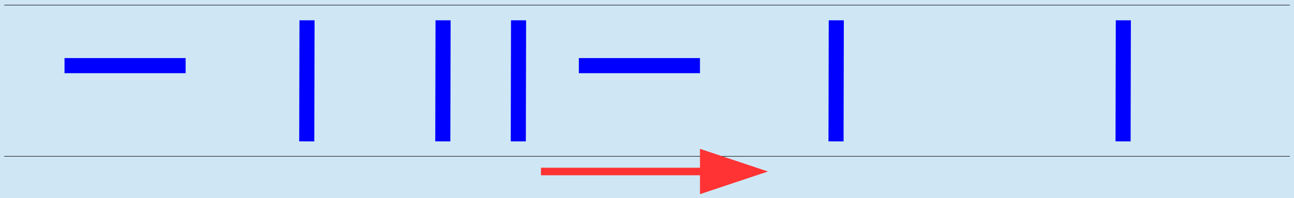
Interacting particle system has many current-carrying modes

Mechanism :

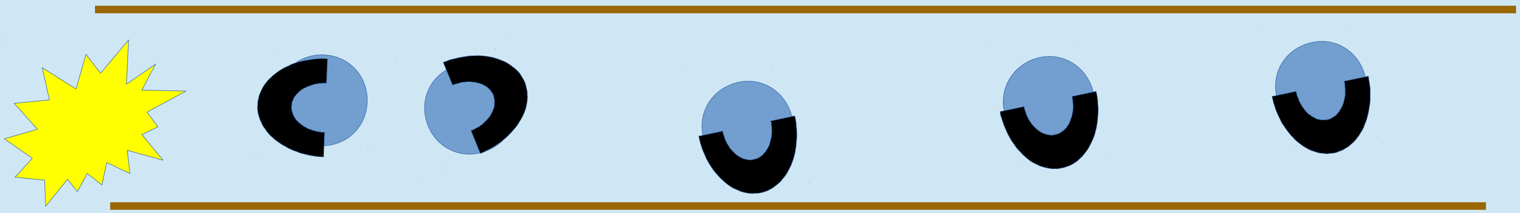
If external field "bias" someof the modes (dominatly) and slow-down other modes, *one expects negative response*

Experimrnts ?

(a)



(b)



Thanks to Amit, Urna

Zero range and finite range processes with asymmetric rate functions

A Chatterjee, PKM, JSTAT2017

MPA for interacting-particles systems without hardcore-constraints.

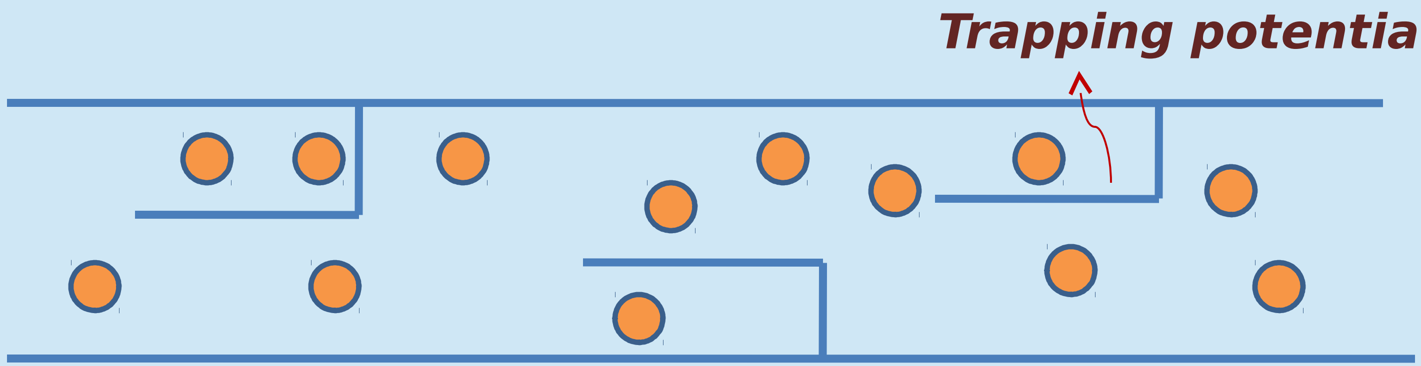
A Chatterjee, PKM, Jphys A 2017

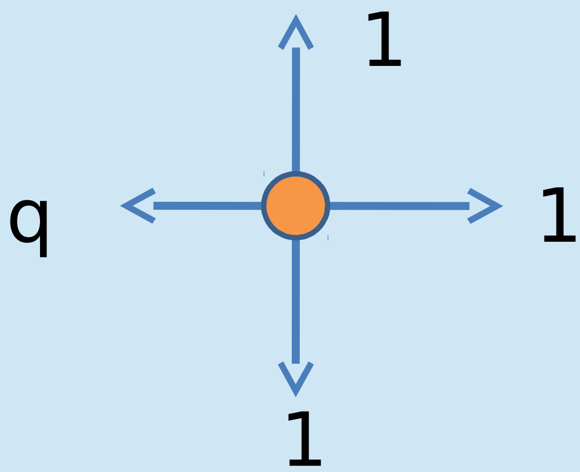
Mechanism of negative response in interacting-particles systems

A Chatterjee, U basu, PKM,
(*arXiv:1712.01236)*

Thank you.....

• **Negative Response** : for *non-interacting particles*





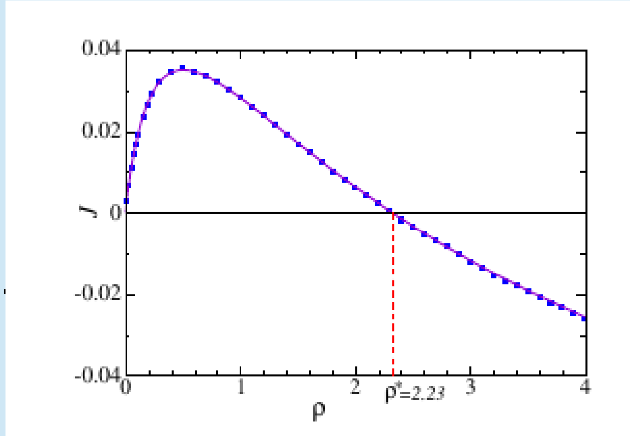
(a) q=1 *J=0*

(b) q=0.9 , only few particles get trapped , **positive response**

(c) q=0.05 , most of the particles get trapped, **negative response**

Baerts et.al. , PRE 88, 052109 (2013)

Many other examples



- Asymmetric misanthrope process (AMAP):
  Hop-rate depends on occupation of dep. and arv. sites.
  But the rate functions are different for R,L

- Asymmetric finite range process (AFRP):
  Hop-rate depends on occupation of dep. Site and its R- nearest neighbours.
  But the rate functions are different for R,L.

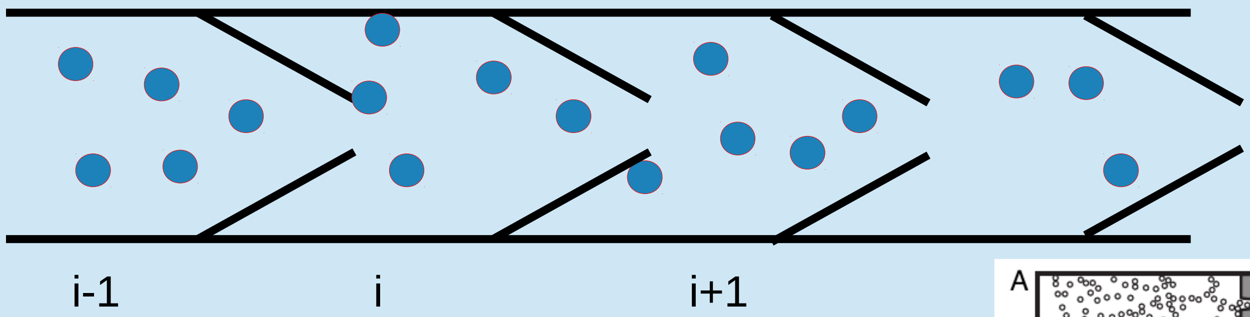
- Higher dimensions:
  AZRP and AMAP shows current reversal in higher dimension.

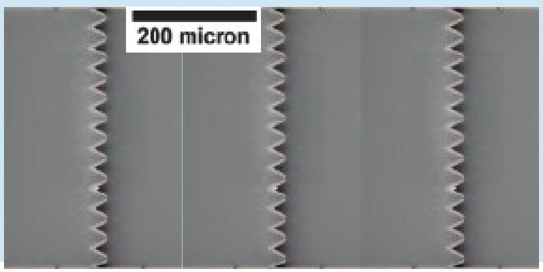
Requirement :

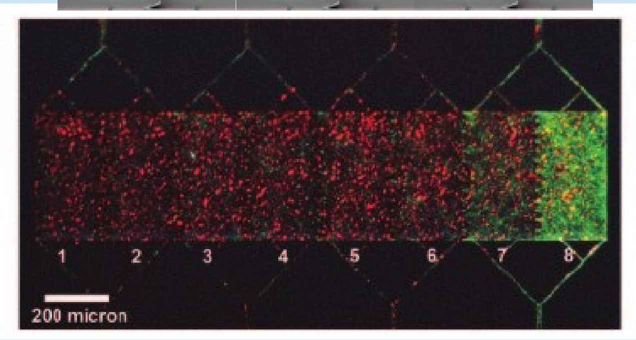
Interacting particle system where some internal degrees experience a different bias then the rest.

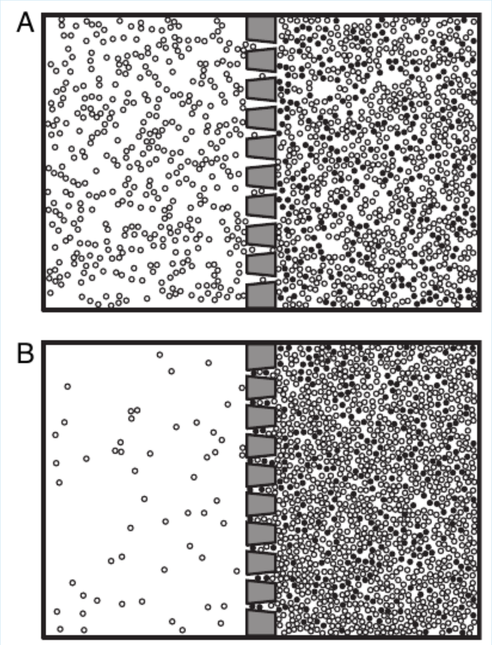
Up-down, active-inactive, isolated-crowded

Asymmetry from geometry









A Wall of Funnels Concentrates Swimming Bacteria,
P Galajda, J Keymer,PChaikin,and Robert Austin.

JOURNAL OF BACTERIOLOGY, Dec. 2007, p. 8704–8707

Geometry-induced asymmetric Diffusion, RS Shaw, N Packard, Swine.

9580–9584 | PNAS | June 5, 2007 | vol. 104 |