

# Permanent spin currents in cavity-qubit systems

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PRB 97, 064506 (2018)

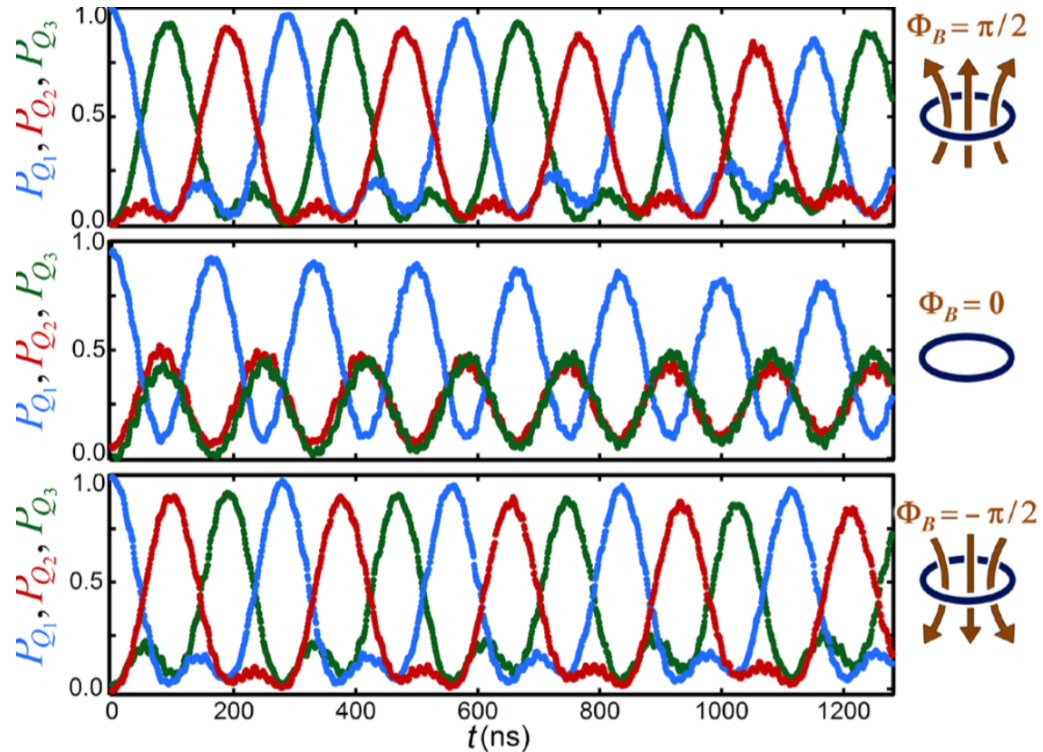
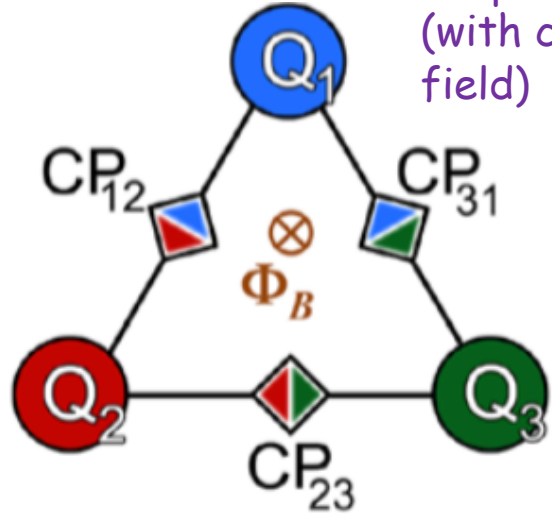
Collaboration:

C. Aron (ENS Paris / Belgium)  
S. Hein (Technische Univ, Berlin)  
E. Kapit (Tulane, USA)

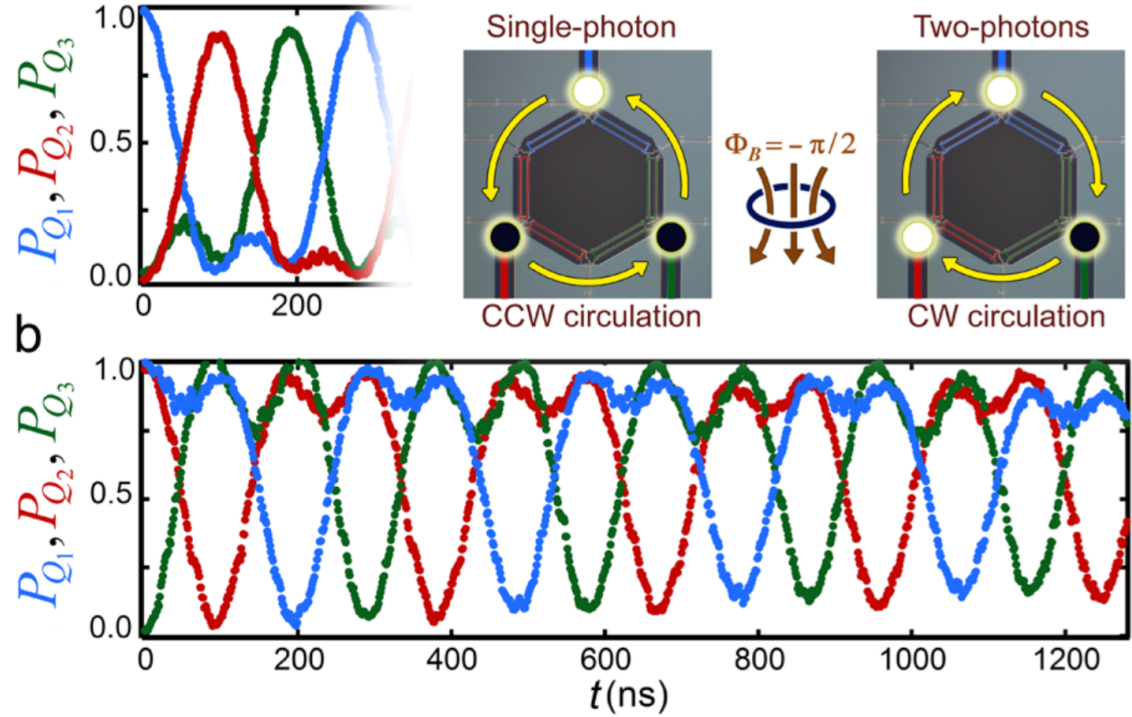
Also, PRL 116, 240503 (2016)  
PRX 6, 011032 (2016)



Coupled two level systems  
(with complex coupling, artificial gauge field)



Google Inc / Santa Barbara Experiment



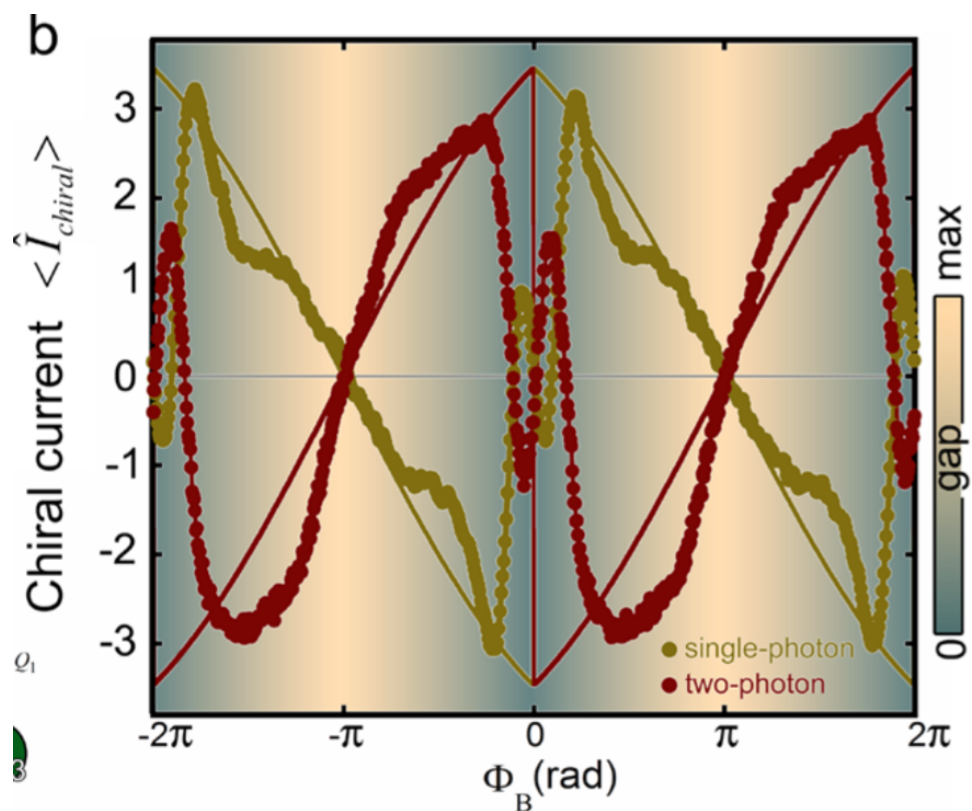
$$H(t) = \hbar \sum_{j=1}^3 \omega_j (\hat{n}_j + 1/2) + \hbar \sum_{j,k} g_{jk}(t) (a_j^\dagger a_k + a_j a_k^\dagger) + H_{\text{int}}$$

J. Martinis (Google Inc / Santa Barbara)

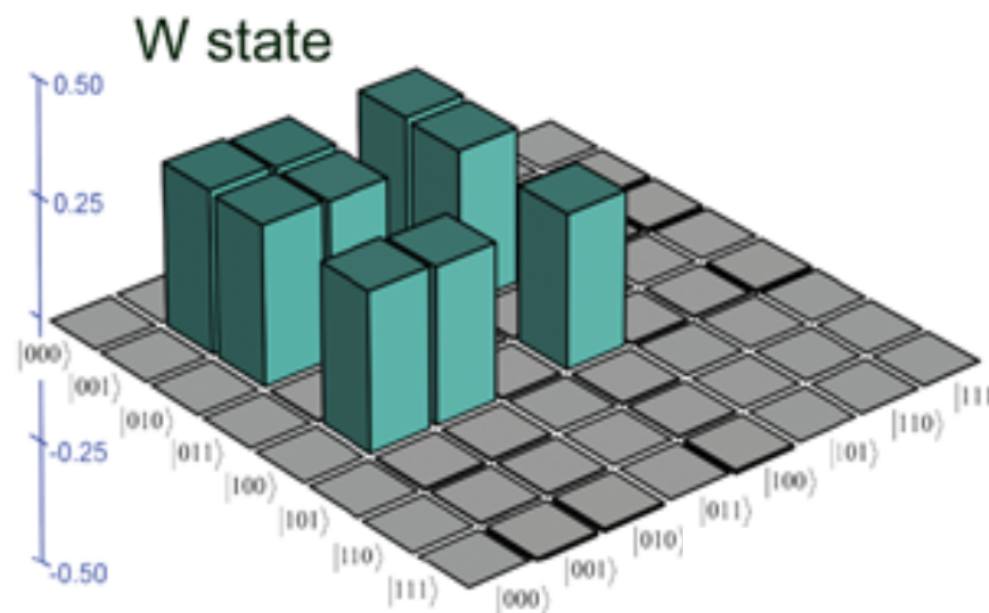
*P. Roushan et al,*

*Nature Physics* **13**, 146–151 (2017)

## Chiral currents



## Creating W-states



*P. Roushan et al Nature Physics 13, 146–151 (2017)*

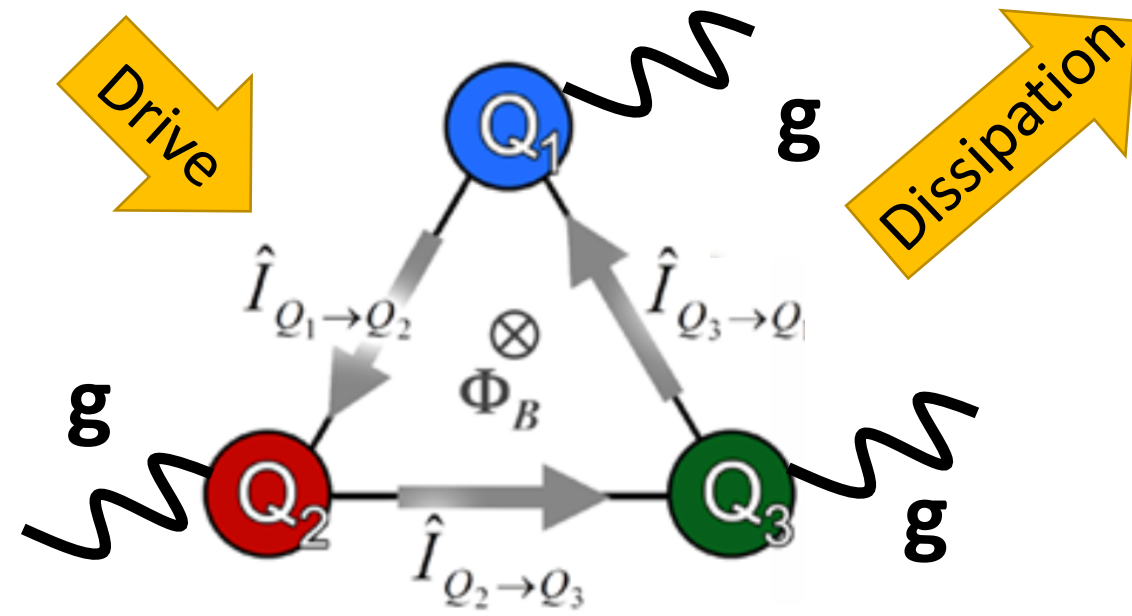
Quantum State Tomography

However, currents decay and are not persistent.

Systems are subject to inevitable environmental effects

# Designing a protocol to create persistent currents

- striking a delicate balance between drive and dissipation to activate, specific entangled states with high fidelity, which are capable of carrying current
- power of quantum bath engineering approaches to realize highly non-trivial non-equilibrium steady states in Open Quantum Systems.



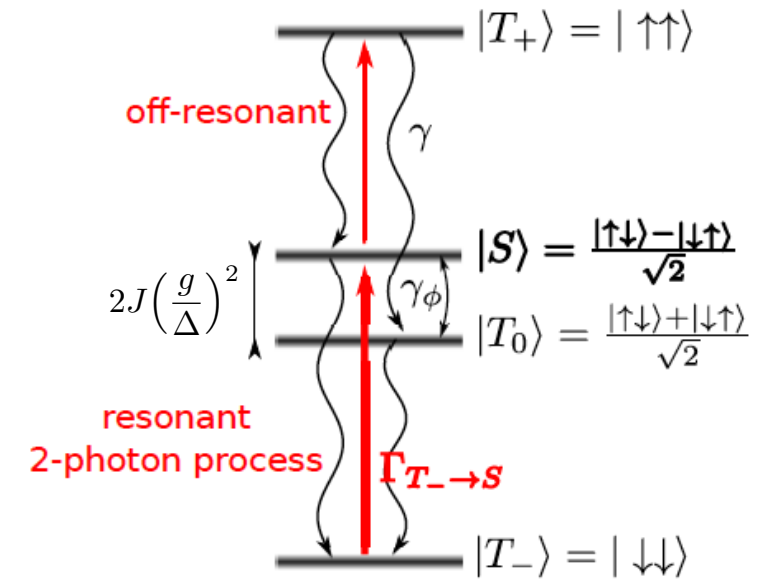
- Sources of dissipation are qubit dissipation/dephasing and cavity decay
- Qubits are coupled with complex hopping
- Each Qubit is coupled to a respective photon mode ( $g$ )
- Photons are driven by a classical microwave drive

Hamiltonians with complex coupling realized

*P. Roushan et al Nature Physics (2017)*

Many fascinating variations possible

*E. Kapit, PRA (2015)*



(Experiment realized @Berkeley, PRL 2016)

Protocols to strike a delicate balance between drive and dissipation (especially in context of entanglement)

Aron, **Kulkarni**, Tureci (PRX 2016)

Schwartz, Martin, Flurin, Aron, **Kulkarni**, Tureci, Siddiqi (PRL 2016)

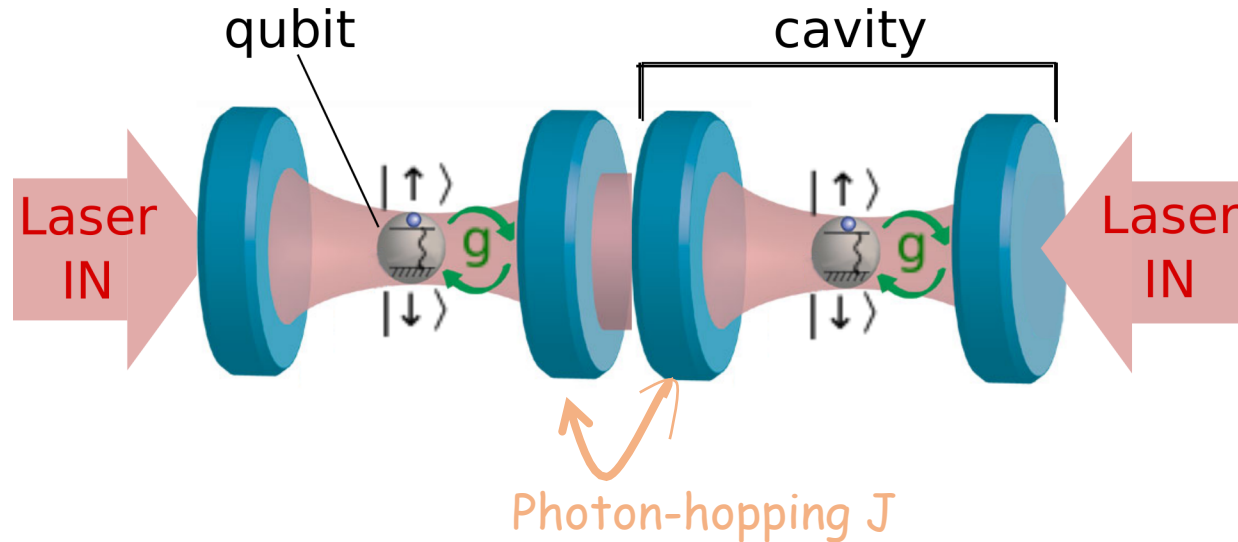
Aron, **Kulkarni**, Tureci (PRA 2014)

*We try to unite these two ideas*

# Engineering a quantum device and the environment

Quantum Nanoelectronics Laboratory, Berkeley

Schwartz, Martin, Flurin, Aron, **Kulkarni**, Tureci, Siddiqi (PRL 2016)



## Driven unitary evolution

- two-level systems  $\omega_q \frac{\sigma_i^z}{2}$ ,  $\omega_q \sim 10$  GHz
- far-detuned cavities  $\omega_c \neq \omega_q$
- light-matter coupling  $g \sim 0.1$  GHz
- inter-cavity coupling  $J \sim 0.1$  GHz
- microwave drives:  $\epsilon_d \cos(\omega_d t)(a_i + a_i^\dagger)$

## Dissipative processes

- cavity decay  $\kappa \sim 10^{-3}$  GHz
- qubit decay  $\gamma \sim 10^{-4}$  GHz
- qubit dephasing  $\gamma_\phi \sim 10^{-5}$  GHz

## Jaynes-Cummings Array to XY Spin chain coupled to photons

$$H_{\sigma} = \sum_i \omega_q \frac{\sigma_i^z}{2}, \quad H_{\sigma a} = g \sum_i [a_i^{\dagger} \sigma_i^{-} + \text{H.c.}]$$

$$H_a(t) = \sum_i [\omega_c a_i^{\dagger} a_i - J(a_i^{\dagger} a_{i+1} + \text{H.c.}) \\ + 2\epsilon_i^d \cos(\omega_d t + \Phi_i)(a_i + a_i^{\dagger})].$$

RWA+ Schrieffer-Wolff Transformation

$$H_{\sigma} = \sum_i h_i \cdot \frac{\boldsymbol{\sigma}_i}{2} - \frac{J}{2} \left( \frac{g}{\Delta} \right)^2 [\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y],$$

$$H_{\sigma a} = \sum_i \left( \frac{g}{\Delta} \right)^2 \sigma_i^z (\Delta a_i^{\dagger} a_i + \epsilon_i^d a_i^{\dagger} + \epsilon_i^{d*} a_i).$$

# Baths/Noise/Imperfections

$$H_{total} = H_{system} + H_{bath}$$

$$H_{\sigma,b} = \eta \sum_{i,n} \sigma_i^x (b_{in} + b_{in}^\dagger), \quad H_b = \sum_{i,n} \omega_n b_{in}^\dagger b_{in} \quad \text{Qubits coupled to Bath}$$

$$H_{a,\tilde{b}} = \tilde{\kappa} \sum_{i,n} (a_i^\dagger \tilde{b}_{in} + \tilde{b}_{in}^\dagger a_i), \quad H_{\tilde{b}} = \sum_{i,n} \tilde{\omega}_n \tilde{b}_{in}^\dagger \tilde{b}_{in} \quad \text{Cavity (photons) coupled to bath}$$

$$H_\sigma = \sum_i h_i \cdot \frac{\boldsymbol{\sigma}_i}{2} - \frac{J}{2} \left( \frac{g}{\Delta} \right)^2 [\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y],$$

$$H_{\sigma a} = \sum_i \left( \frac{g}{\Delta} \right)^2 \sigma_i^z (\Delta a_i^\dagger a_i + \epsilon_i^d a_i^\dagger + \epsilon_i^{d*} a_i).$$

System Hamiltonian

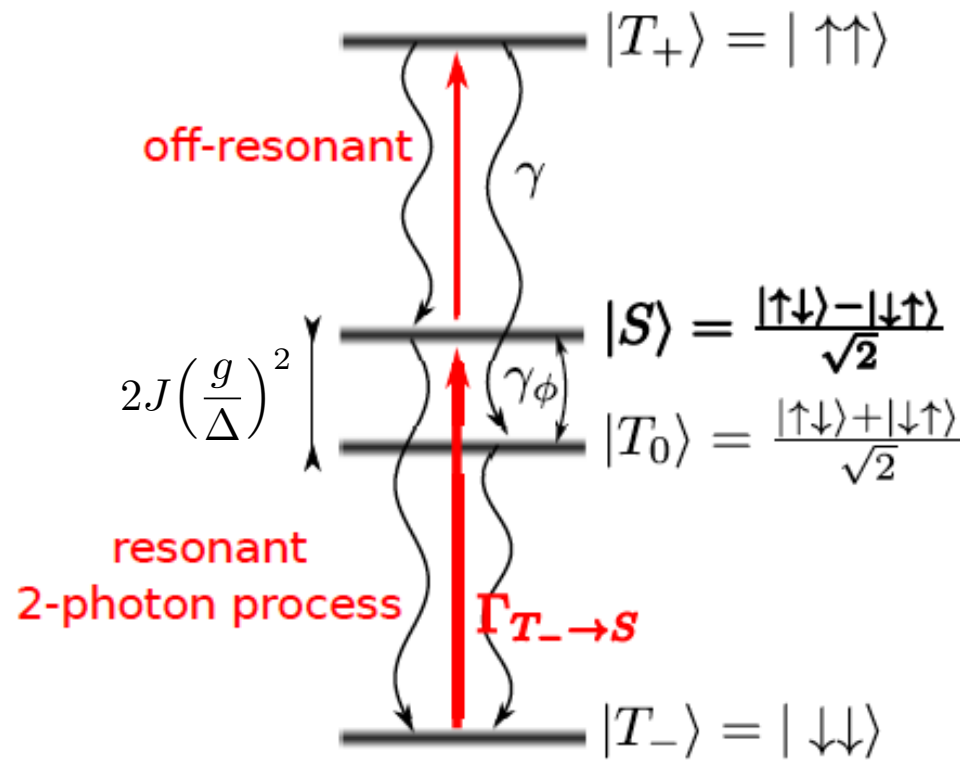


Aron, **Kulkarni**, Tureci (PRA, 2014)

Aron, **Kulkarni**, Tureci (PRX 2016)

Nonequilibrium rate from  $|T_-\rangle$  to  $|S\rangle$

$$\Gamma_{T_-\rightarrow S} \propto 2\pi(g^3\epsilon_d^2)^2\rho_-(\omega_d + E_{T_-} - E_S)$$



Transverse-field isotropic XY-model

$$H_\sigma = \sum_{i=1}^2 \mathbf{h} \cdot \frac{\boldsymbol{\sigma}_i}{2} - \frac{J}{2} \left(\frac{g}{\Delta}\right)^2 [\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y]$$

$$h^x \equiv g\epsilon_d, h^y \equiv 0, h^z \equiv \omega_q - \omega_d$$

Weakly-coupled photon-fluctuation bath

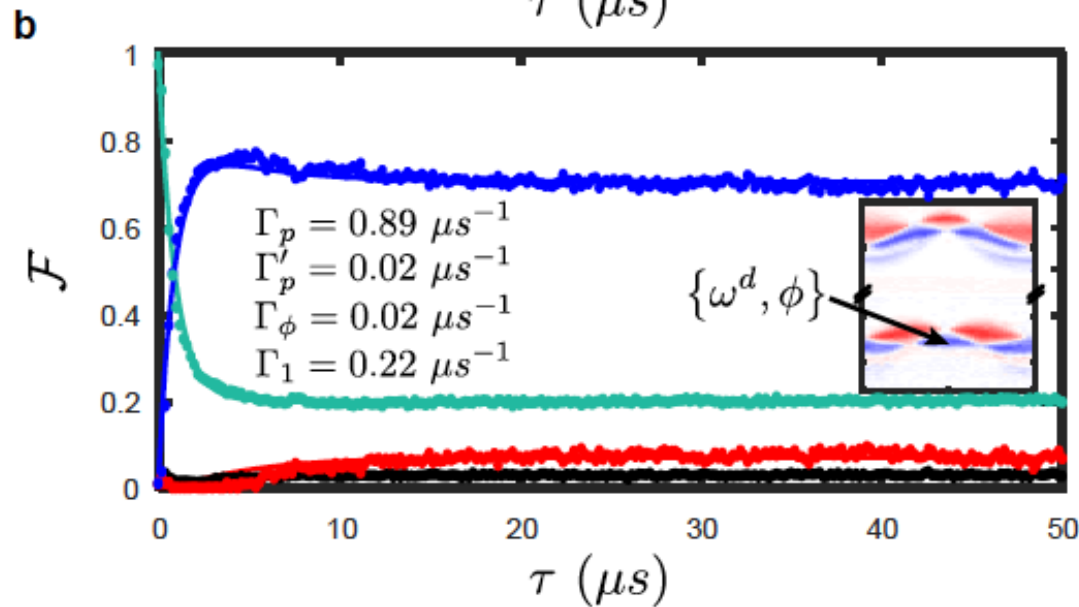
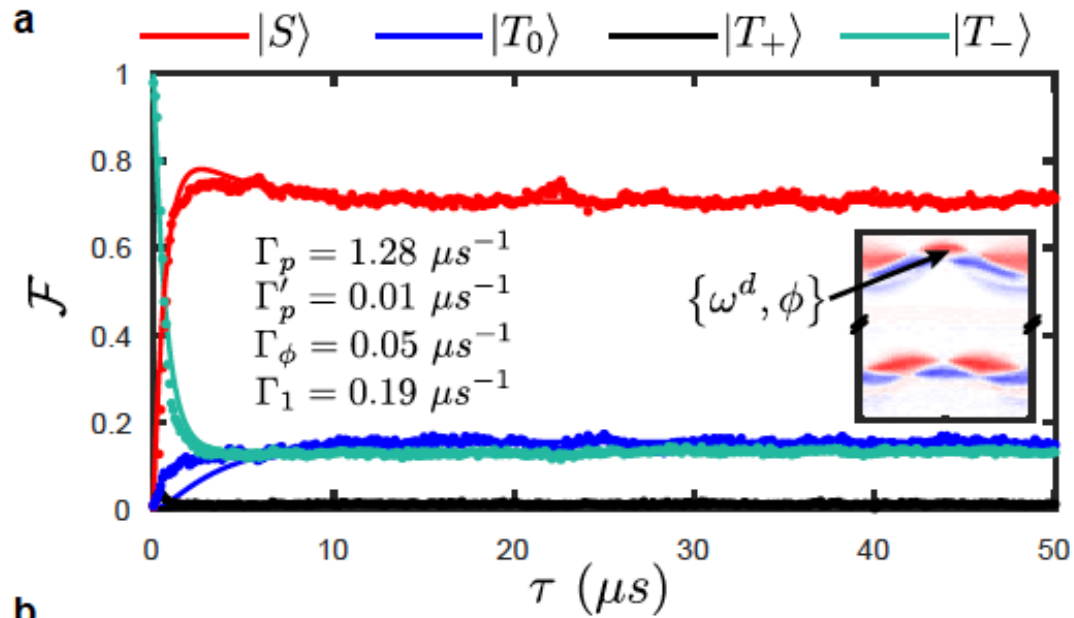
$$H_{\sigma d} = \# \left(\frac{g}{\Delta}\right)^2 \epsilon_d \sum_{i=1}^2 \sigma_i^z (d_i + d_i^\dagger)$$

$$H_d = \sum_{i=1}^2 (\omega_c - \omega_d) d_i^\dagger d_i - J(d_1^\dagger d_2 + \text{h.c.})$$

# Experimental Results: Time Dynamics (time evolution of entanglement)

Quantum State Tomography  
@Berkeley

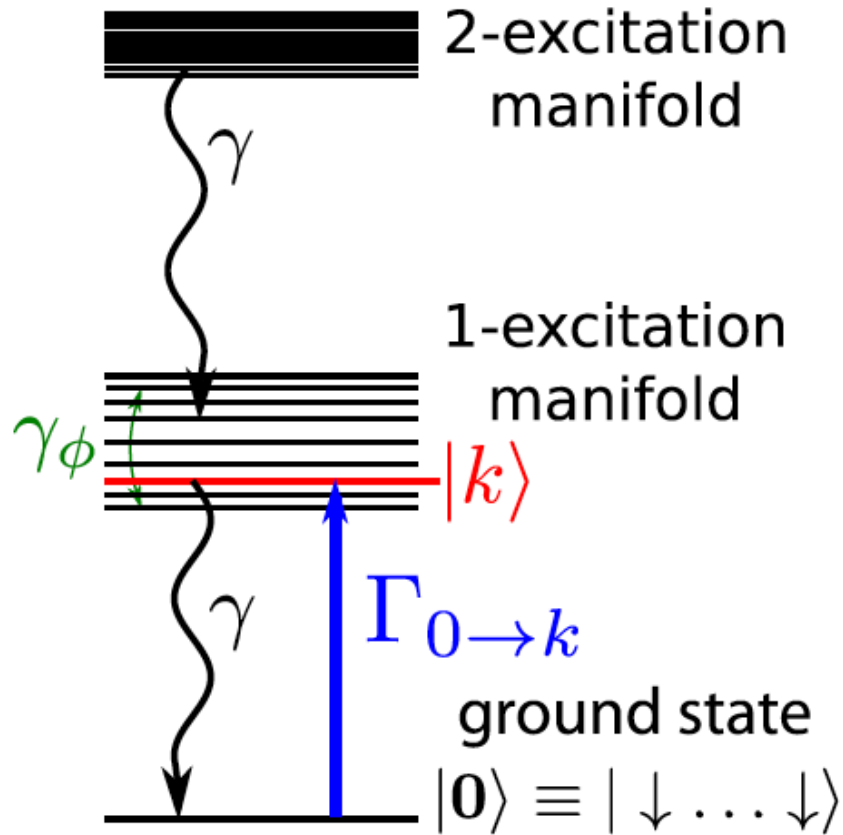
Agreement between  
experiment and theory !



Schwartz, Martin, Flurin, Aron,  
Kulkarni, Tureci, Siddiqi (PRL 2016)

# Creating a target generalized-W state

## Target state



$$|k\rangle \equiv \sum_j e^{ikj} |j\rangle$$

$$|j\rangle = |\downarrow_0 \dots \downarrow_{j-1} \uparrow_j \downarrow_{j+1} \dots \downarrow_{N-1}\rangle$$

### Single magnon excitations

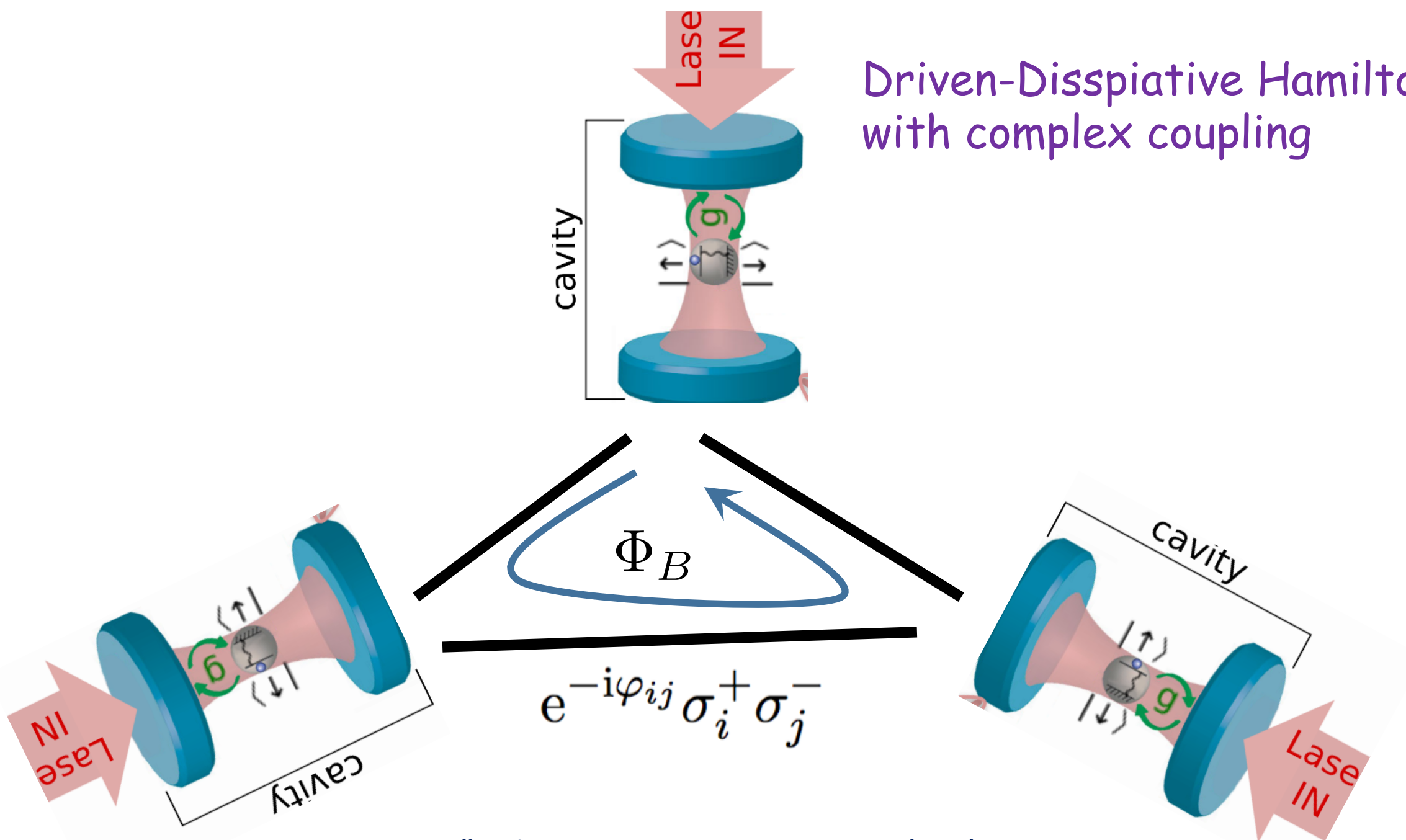
For e.g.,  $N=2$  and  $k=0$ , target state is

$$|T_0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

For e.g.,  $N=3$  and  $k=0$ , target state is

$$W = \frac{1}{\sqrt{3}} (|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle)$$

# Driven-Dissipative Hamiltonian with complex coupling



$$H_\sigma = \sum_i (\omega_q - \omega_d) \frac{\sigma_i^z}{2} - \sum_{\langle ij \rangle} J_0 \left[ e^{-i\varphi_{ij}} \sigma_i^+ \sigma_j^- + \text{h.c.} \right]$$

$$H_{\sigma a} = \sum_i g \left[ a_i^\dagger \sigma_i^- + \text{h.c.} \right],$$

$$H_a = \sum_k (\omega_c - \omega_d) a_k^\dagger a_k + (\epsilon_k^d a_k^\dagger + \text{h.c.}).$$



SW transformation

$$H_\sigma = \sum_i \mathbf{h}_i \cdot \frac{\boldsymbol{\sigma}_i}{2} - \sum_i J_0 \left[ e^{i\phi} \sigma_i^+ \sigma_{i+1}^- + \text{h.c.} \right],$$

$$H_{\sigma a} = \sum_i \left( \frac{g}{\Delta} \right)^2 \sigma_i^z \left( \Delta a_i^\dagger a_i + \frac{1}{2} \epsilon_i^d a_i^\dagger + \frac{1}{2} \epsilon_i^{d*} a_i \right)$$

$$H_a = \sum_k (\omega_k - \omega_d) a_k^\dagger a_k + (\epsilon_k^d a_k^\dagger + \text{h.c.})$$

$$H_d = \sum_k (\omega_c - \omega_d) d_k^\dagger d_k \quad H_\sigma = \sum_k E_k |k\rangle \langle k| + \left(\frac{g}{\Delta}\right) (\epsilon_k^d |k\rangle \langle \mathbf{0}| + \text{h.c.})$$

$$H_{\sigma d} = \left(\frac{g}{\Delta}\right)^2 \sum_i \sigma_i^z [(\Delta \bar{a}_i + \frac{1}{2} \epsilon_i^d) d_i^\dagger + \text{h.c.}]$$



$$\begin{aligned} \partial_t \rho_\sigma^{\text{NESS}} = 0 = & -i [H_\sigma, \rho_\sigma^{\text{NESS}}] + \sum_k \Gamma_{\mathbf{0} \rightarrow k} \mathcal{D}[|\tilde{k}\rangle \langle \tilde{\mathbf{0}}|] \rho_\sigma^{\text{NESS}} \\ & + \gamma \sum_k \mathcal{D}[|\tilde{\mathbf{0}}\rangle \langle \tilde{k}|] \rho_\sigma^{\text{NESS}} + \frac{2\gamma\phi}{N} \sum_{kq} \mathcal{D}[|\tilde{q}\rangle \langle \tilde{k}|] \rho_\sigma^{\text{NESS}}. \end{aligned}$$

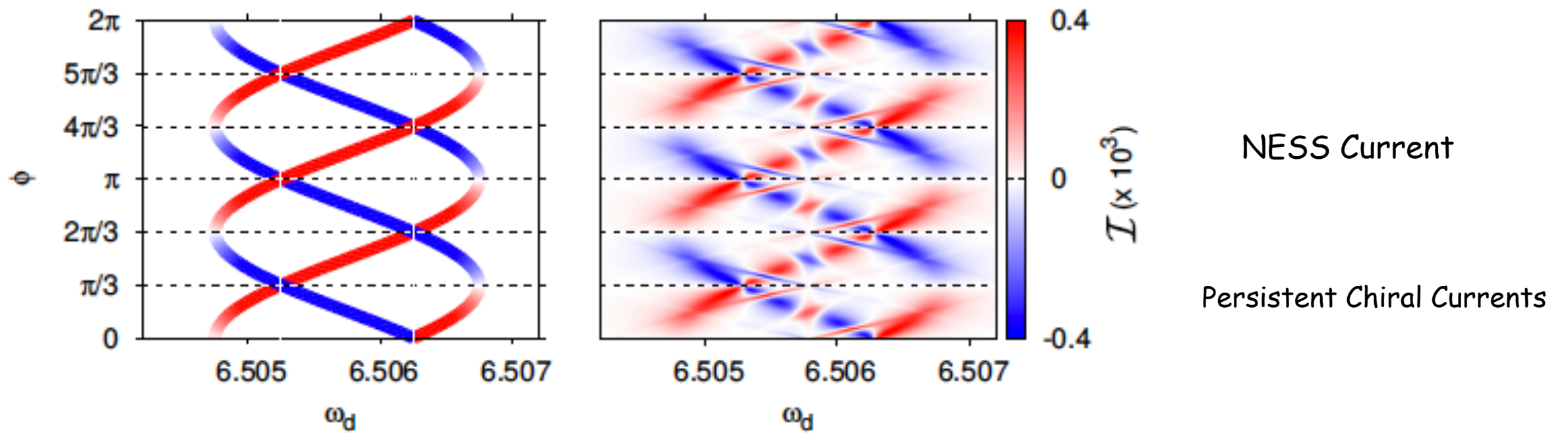
$$\Gamma_{\mathbf{0} \rightarrow k} = 2\pi \sum_q \Lambda_{kq}^2 \rho(\omega_d + \tilde{E}_0 - \tilde{E}_k)$$

$$\omega_d = \frac{\omega_c + \omega_q + \delta\omega_q}{2} - J_0 \cos(k + \phi) + \frac{1}{2} \left( \frac{g}{\Delta} \right)^2 \sum_{q \neq k} \frac{|\epsilon_q^d|^2}{\Delta_q}$$

Once we compute density matrix we can calculate current:

$$\mathcal{J}_i = -iJ_0 \left[ e^{i\phi} \sigma_i^+ \sigma_{i+1}^- - \text{H.c.} \right]$$

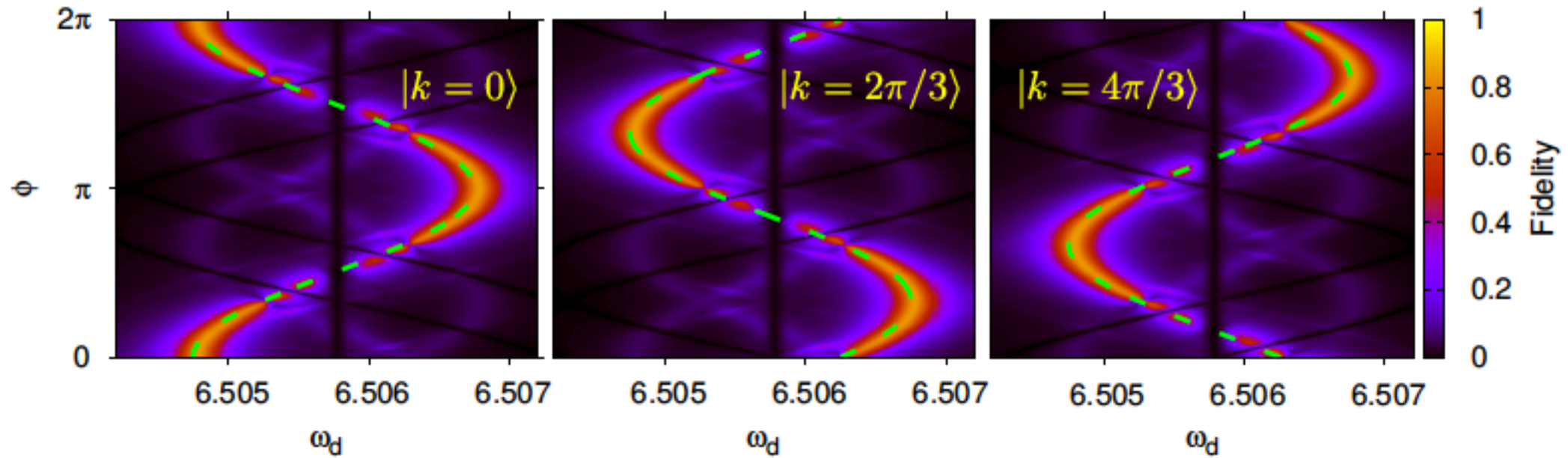
We also compute the fidelity to go to target eigenstates



$$\mathcal{I}(\omega_d, \phi) = \frac{2}{N} J_0 \sum_k \sin(k + \phi) n_k(\omega_d, \phi)$$

Kulkarni, Hein, Kapit, Aron, **PRB 97, 064506 (2018)**





Steady State Population (Fidelities)

$$\omega_d^{\text{opt}}(\phi) = \bar{\omega}_d - J_0 \cos(k + \phi) \quad \text{Optimal Drive}$$

# Conclusions

Kulkarni, Hein, Kapit, Aron, **PRB 97, 064506 (2018)**

- Persistent (infinitely long-lived) current
- Preparation of High fidelity states
- Entanglement Dynamics

## Outlook

- Extending to ring geometries (many qubits & cavities)
- Performing an exact brute force numerical computation (including spins and photons)
- Treat higher excitation manifolds