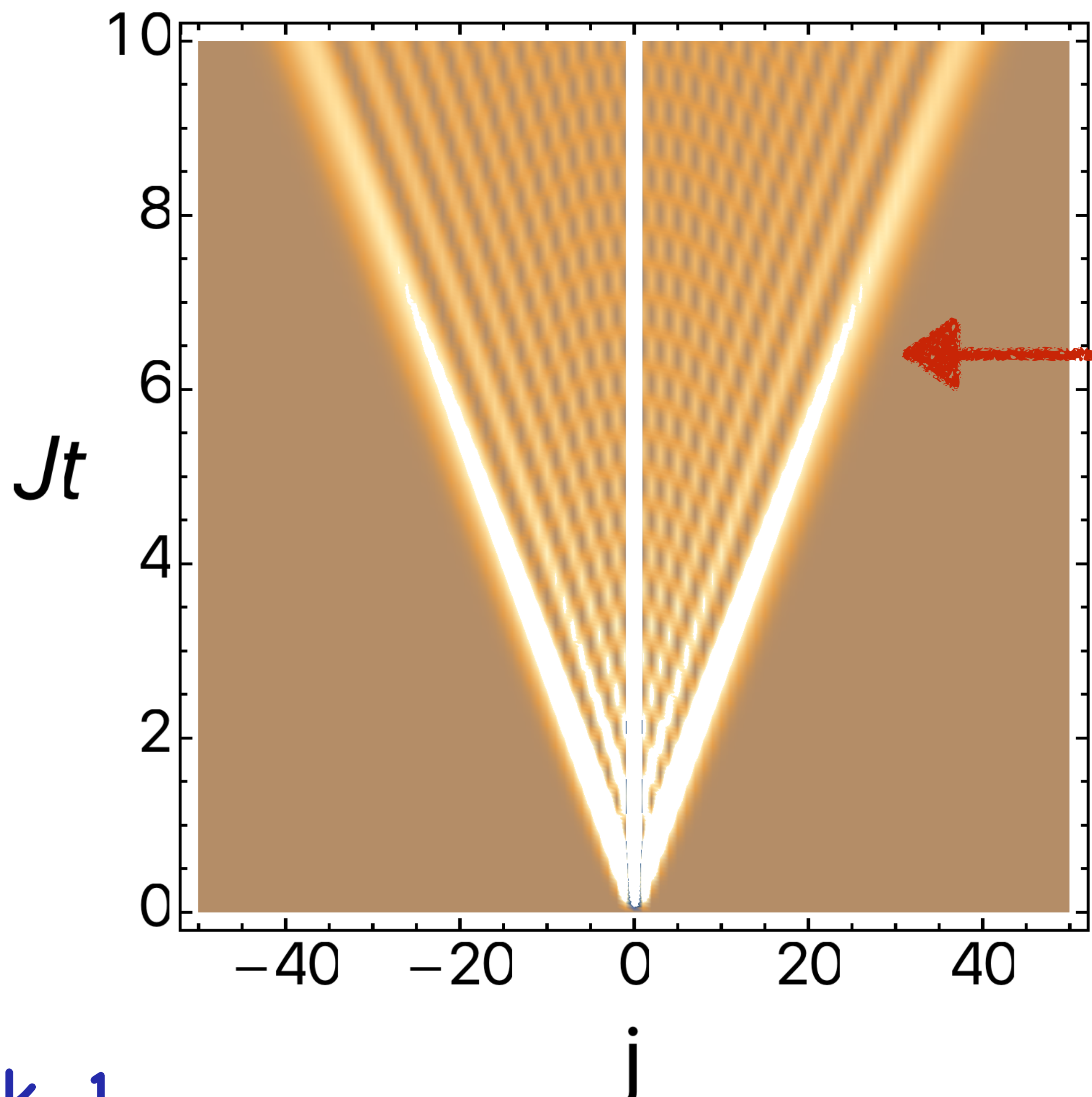
IV. Spreading of correlations: “Light-cone” effects

Connected density-density correlator in our example

$$\langle \Psi(0) | n_j(t) n_k(t) | \Psi(0) \rangle_c = \frac{\delta_{j,k}}{4} - \frac{1}{4} J^2_{|j-k|}(4Jt)$$



k=1

"light-cone" effect

connected correlators exponentially small for

$$t < \frac{|j-k|}{2\mathrm{v}_{\max}}$$

$$\mathrm{v}_{\max} = \max_p \frac{d\epsilon(p)}{dp} = 2J$$

Light-cone effects in connected correlators are general!

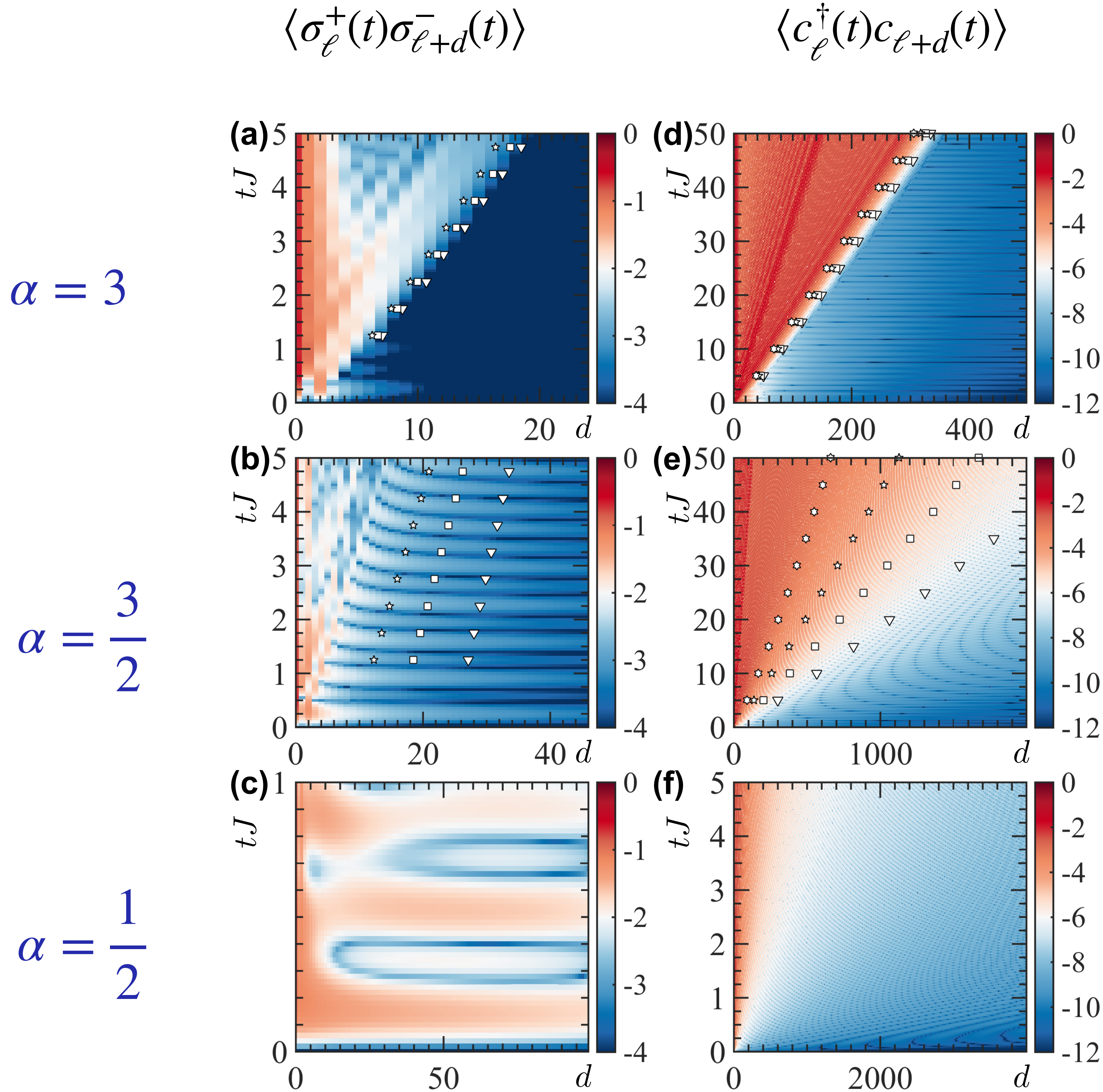
- If correlation length in initial state is finite the effect is very pronounced (as in our example).
- If correlation length in initial state is infinite the effect is (much) weaker.
- For long-range interactions the light-cone effect is different:

$$H_1 = \sum_{l \neq j}^{M} J_{l,j} \sigma_l^x \sigma_j^x + B \sum_j \sigma_l^z \qquad J_{l,j} = J \left| \frac{M}{\pi} \sin \left[\frac{\pi(l-j)}{M} \right] \right|^{-\alpha}$$

$$H_2 = \sum_{l \neq j}^{M} J_{l,j} c_l^\dagger c_j + \Delta \sum_j c_l^\dagger c_{l+1}^\dagger + \mathrm{h.c.}$$

Buyskikh et al '16

Quenches $\frac{B}{J} = \infty \to 1 \qquad \frac{\Delta}{J} = 10 \to 1$

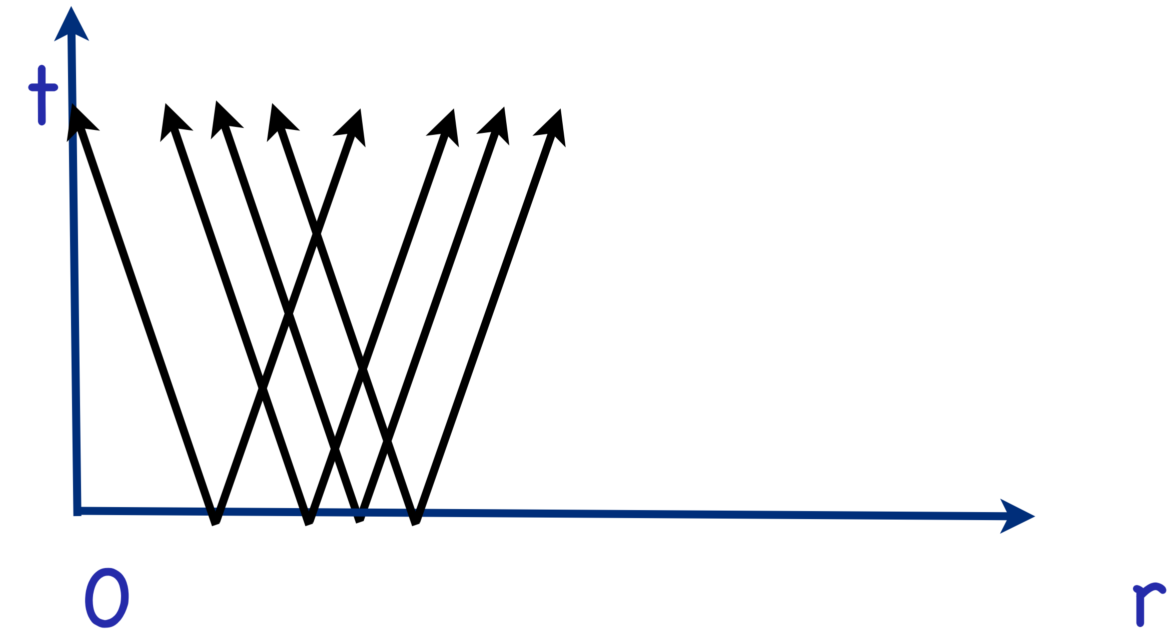


$\langle \sigma^+_\ell(t)\sigma^-_{\ell+d}(t)\rangle$
$\langle c^\dagger_\ell(t)c_{\ell+d}(t)\rangle$
$\alpha = 3$
$\alpha = \frac{3}{2}$
$\alpha = \frac{1}{2}$
(a)
(b)
(c)
(d)
(e)
(f)
tJ
d

"Quasiparticle" picture for light-cone effect

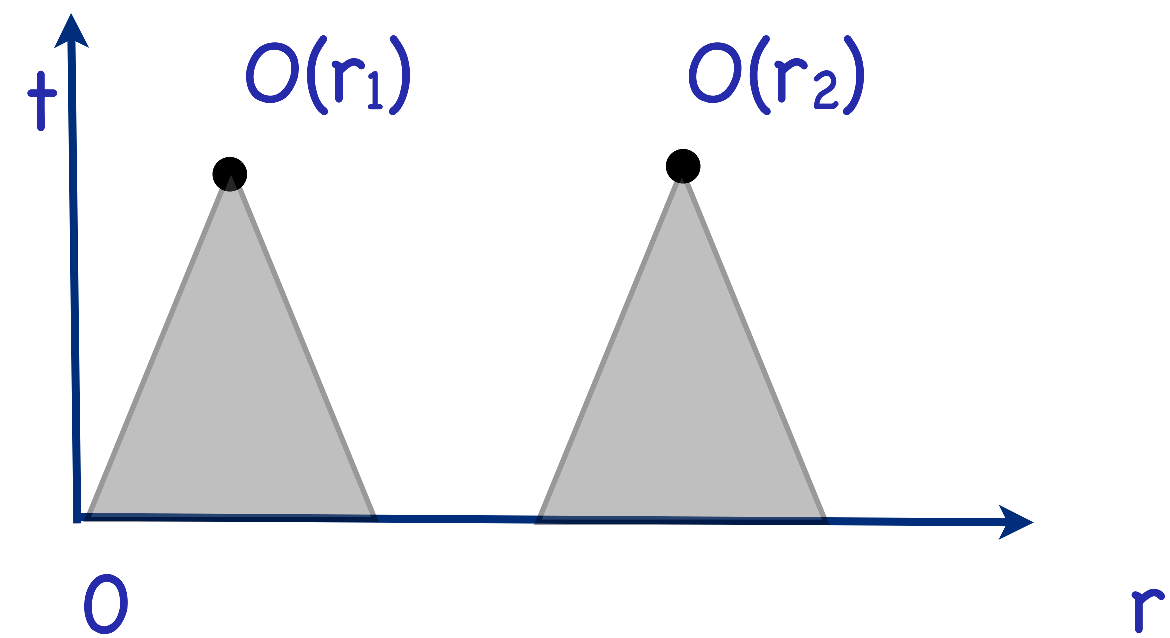
Calabrese & Cardy '05, '06

Quench creates quasiparticles at t=0, which start propagating with velocity $|v| \leq v_{max}$

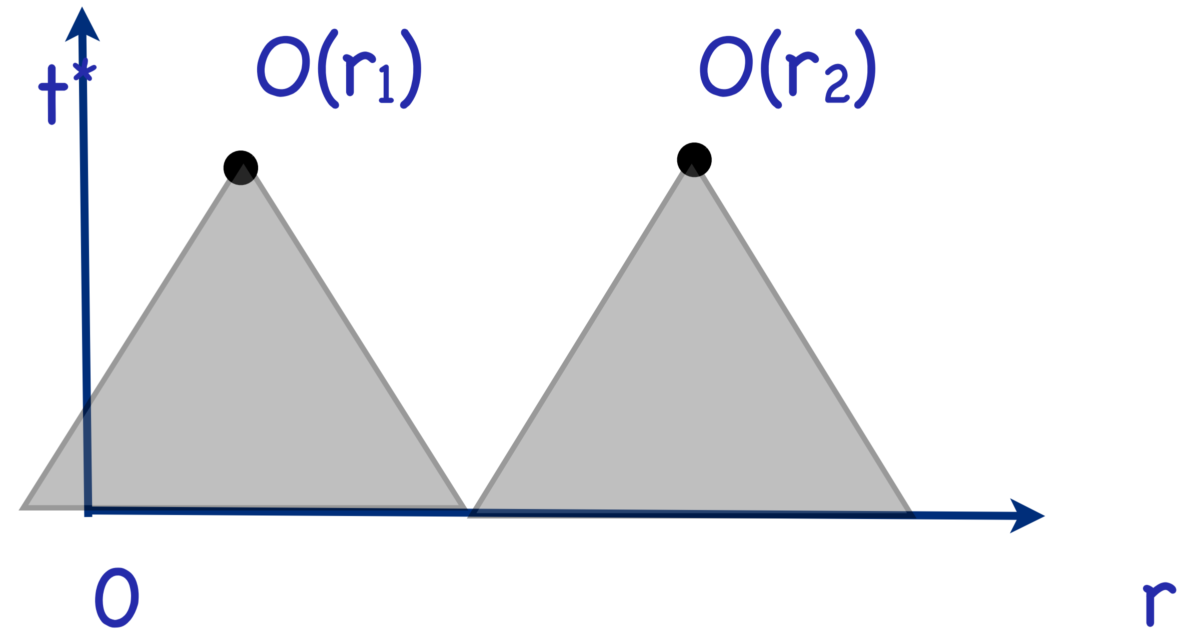


Operators at r_j get "hit" by quasiparticles from within the backwards light cone

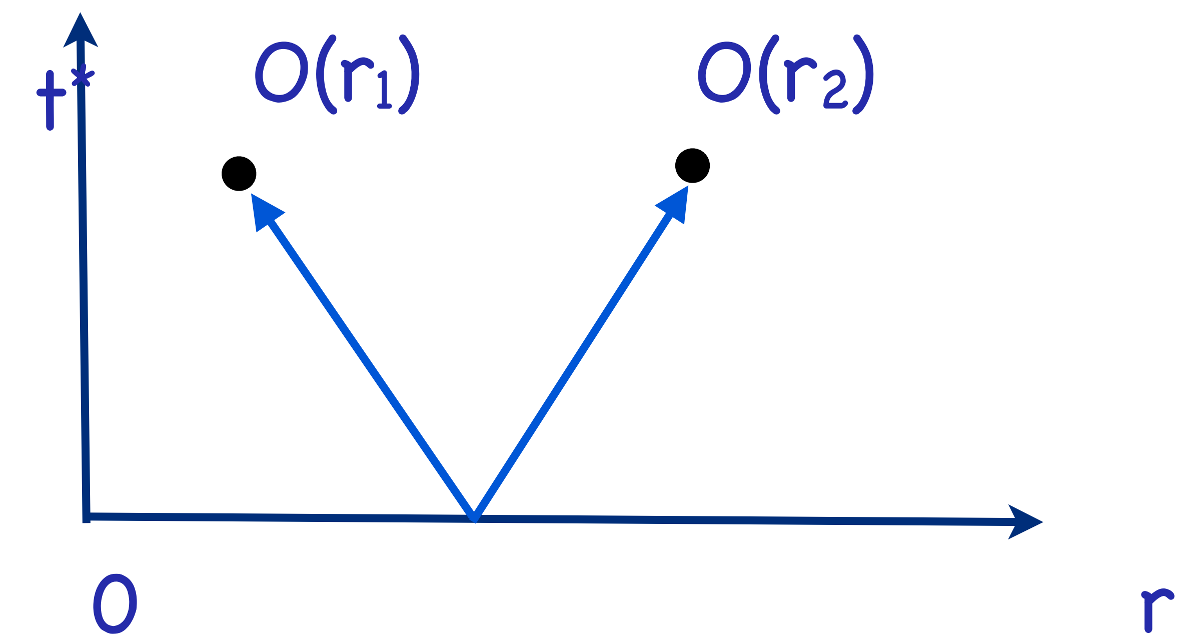
→ dephasing of 1-point fns



At $t^* = |r_2 - r_1|/(2v_{max})$ backwards light cones touch, and **connected correlations develop**

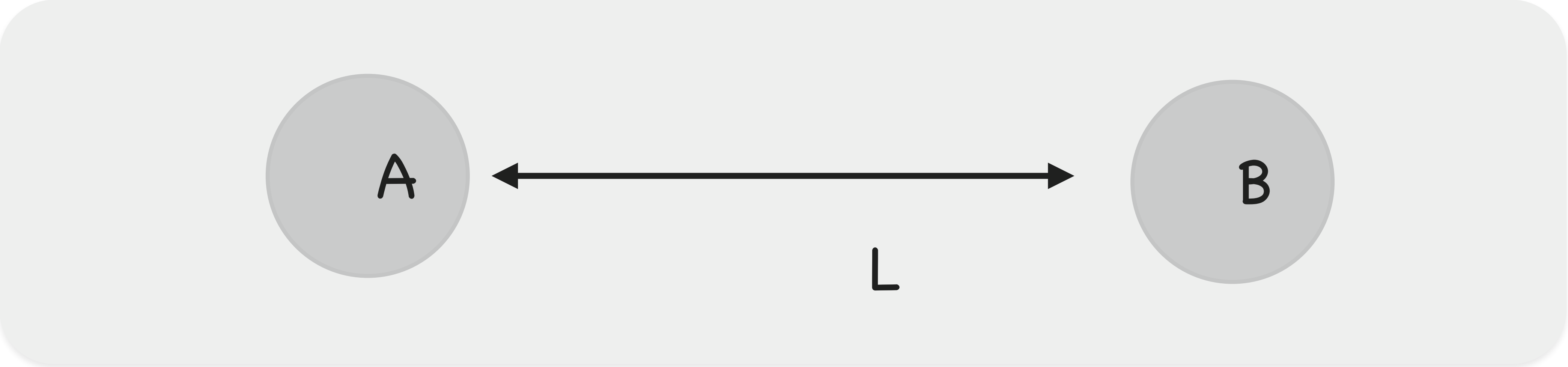


Correlations induced by **entangled quasiparticle pairs.**



Relation to Lieb-Robinson bounds

Consider some non-relativistic quantum spin system with **short-ranged** Hamiltonian H



Let $O_{A/B}$ be operators acting only in subsystem A/B and $O_A(t)=\exp(iHt)\, O_A \exp(-iHt)$. Then the following bound holds

$$\| \, [O_A(t), O_B(0)] \, \| \leq c N_{min} \|O_A\| \, \|O_B\| \, \exp\left(- \frac{L - v|t|}{\xi} \right),$$

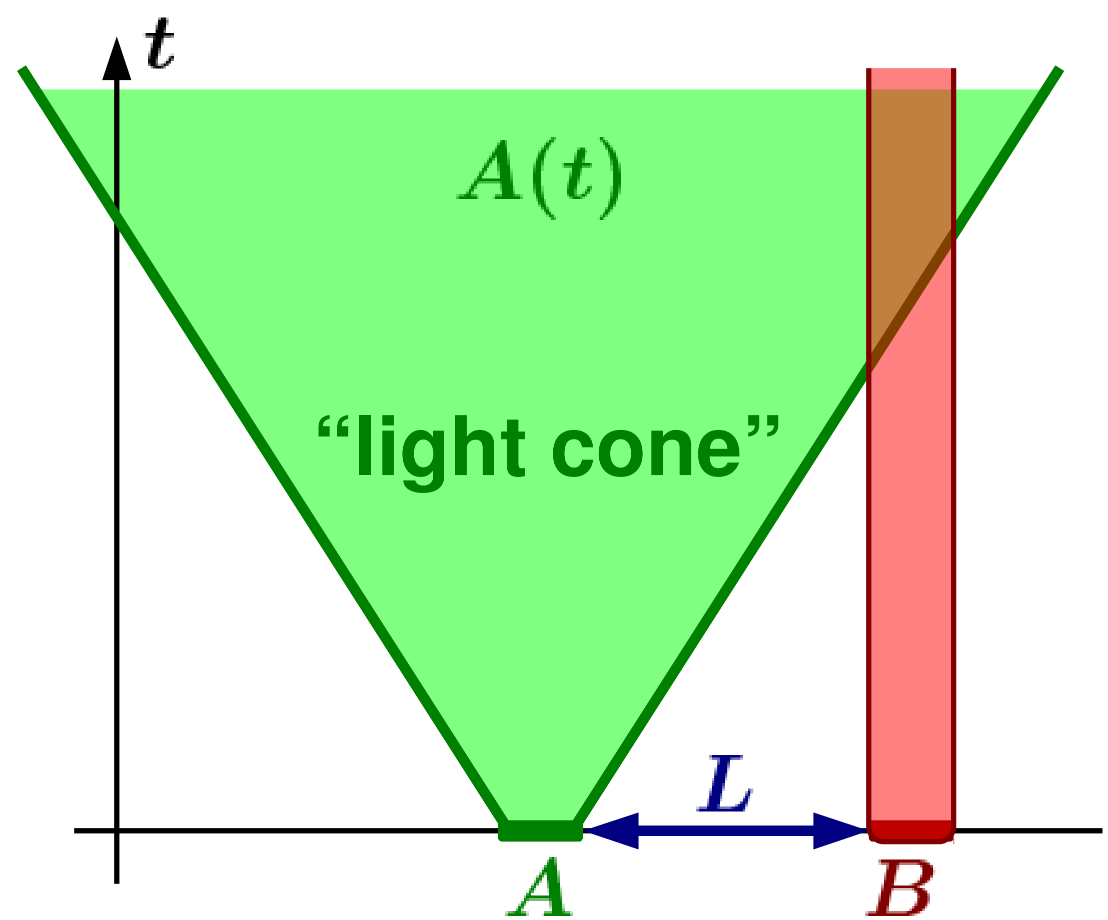
Lieb & Robinson '72

$$\| [O_A(t), O_B(0)] \| \leq c N_{min} \|O_A\| \, \|O_B\| \, \exp\left(-\frac{L - v|t|}{\xi}\right),$$

Lieb & Robinson '72

- RHS exponentially small until L≈vt → operators ≈ **commute**
- perturbation in B does not affect region A **significantly** until **at least** L/v **for some v.**

there **is** an exponentially small effect **immediately**, →"approximate causality"



Linear response theory:

$$H(t) = H_0 + \varepsilon V(t)$$

$$\langle \mathcal{O}_A(t) \rangle = \langle \mathcal{O}_A^{(0)}(t) \rangle - \frac{i\varepsilon}{\hbar} \int_0^t dt' \langle [\mathcal{O}_A(t), V(t')] \rangle + \mathcal{O}(\varepsilon^2)$$

$\langle \quad \rangle$ = expectation value wrt some density matrix

$$\mathcal{O}_A^{(0)}(t) = e^{iH_0 t} \mathcal{O}_A e^{-iH_0 t}$$

Take $V(t) = \mathcal{O}_B \delta(t) \Rightarrow$

$$\langle \mathcal{O}_A(t) \rangle = \langle \mathcal{O}_A^{(0)}(t) \rangle - \frac{i\varepsilon}{\hbar} \langle [\mathcal{O}_A(t), \mathcal{O}_B(0)] \rangle + \mathcal{O}(\varepsilon^2)$$

But

$$\begin{aligned}
|\langle [\mathcal{O}_A(t), \mathcal{O}_B(0)] \rangle| &= |\mathrm{Tr}\, \rho\, [\mathcal{O}_A(t), \mathcal{O}_B(0)]| \\
&= \Big| \sum_n p_n \langle n | [\mathcal{O}_A(t), \mathcal{O}_B(0)] | n \rangle \Big| \\
&\leq \sum_n p_n \, | \langle n | [\mathcal{O}_A(t), \mathcal{O}_B(0)] | n \rangle | \\
&\underset{\text{LR}}{\leq} \underbrace{\sum_n p_n}_{=1} \; c N_{min} \|\mathcal{O}_A\| \, \|\mathcal{O}_B\| \, e^{-\frac{L - vt}{\xi}}
\end{aligned}$$

$$\| [O_A(t), O_B(0)] \| \leq c N_{min} \|O_A\| \, \|O_B\| \, \exp\left(-\frac{L - v|t|}{\xi} \right),$$

Lieb & Robinson '72

exponentially decaying correlations in initial state

$$\langle \mathcal{O}_A(t) \mathcal{O}_B(t) \rangle_{\text{conn}} < \bar{c}(|A| + |B|) e^{-(L-2vt)/\xi}$$

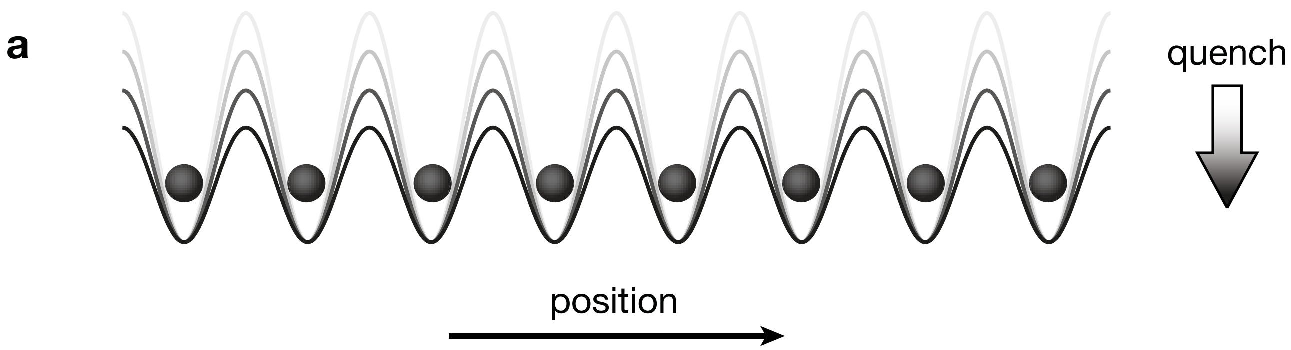
$\bar{c}, \xi, v$ are constants

Bravyi, Hastings & Verstraete '06

There is a kind of "speed limit" for "sizeable" connected correlations to emerge.

Cold atom Experiments

Cheneau et al '12



$$\hat{H} = \sum_j \Big\{ -J\,(\hat{a}_j^\dagger\,\hat{a}_{j+1} + \mathrm{h.\,c.}) + \frac{U}{2}\hat{n}_j(\hat{n}_j - 1)\Big\}\,,$$

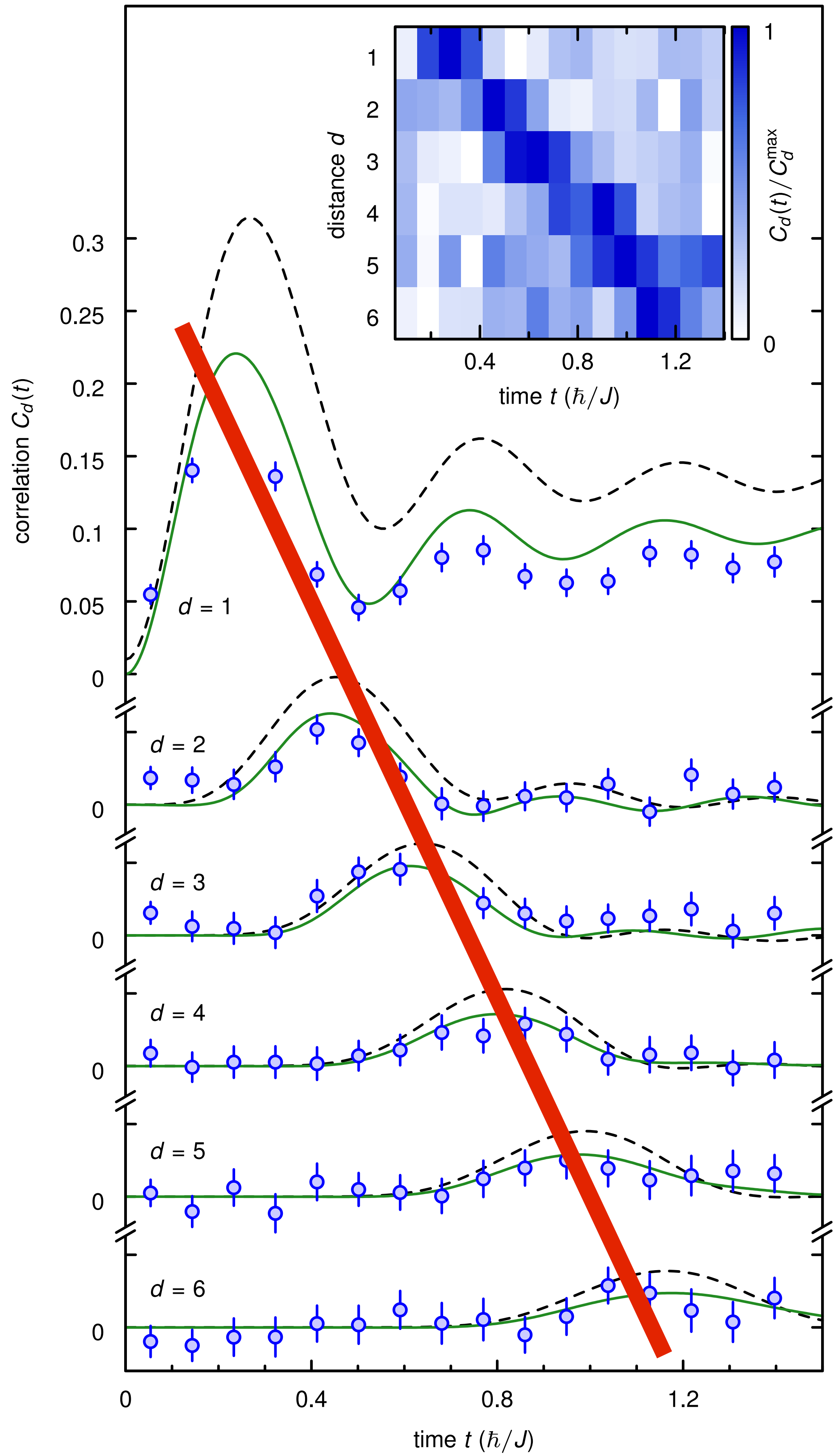
quench

$U_0/J=40 \rightarrow U/J=9$

occupation parity 2-point function

$$C_d(t) = \langle \hat{s}_j(t)\hat{s}_{j+d}(t)\rangle - \langle \hat{s}_j(t)\rangle\langle \hat{s}_{j+d}(t)\rangle\,,$$

$$\hat{s}_j(t) = \exp(i\pi[\hat{n}_j(t) - \bar{n}])$$



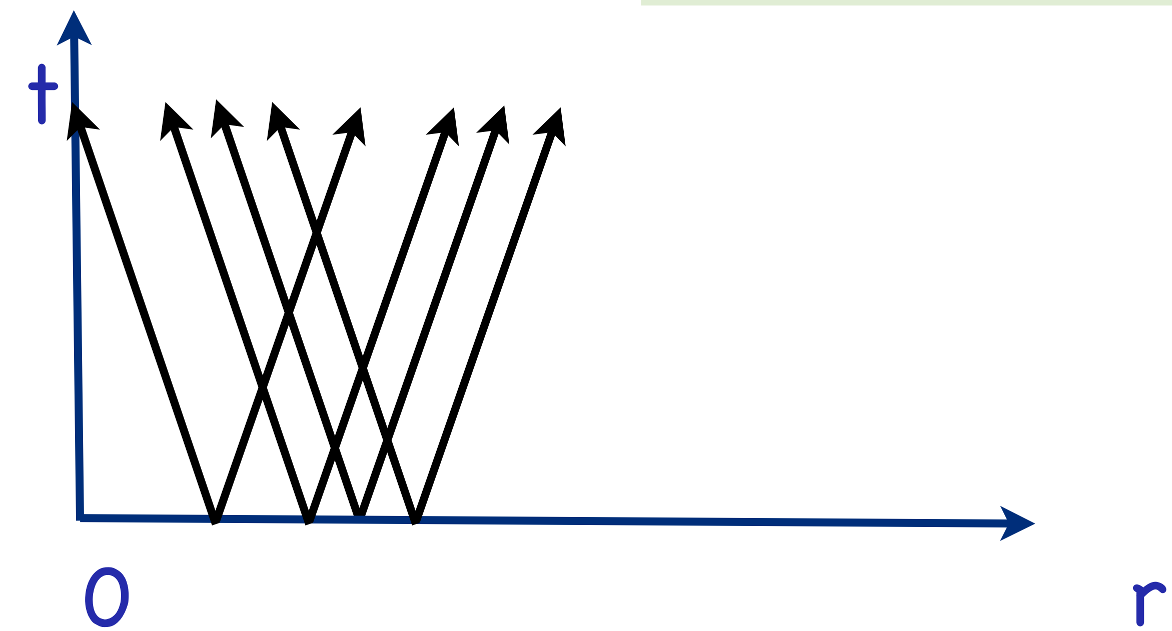
distance d
1
2
3
4
5
6
0.4
0.8
1.2
time t (ħ/J)
$C_d(t)/C_d^{max}$
1
0
correlation $C_d(t)$
0.3
0.25
0.2
0.15
0.1
0.05
0
d = 1
d = 2
d = 3
d = 4
d = 5
d = 6
0
0.4
0.8
1.2
time t (ħ/J)

"Light cone"

"Quasiparticle" picture for entanglement growth

Calabrese & Cardy '05, '06

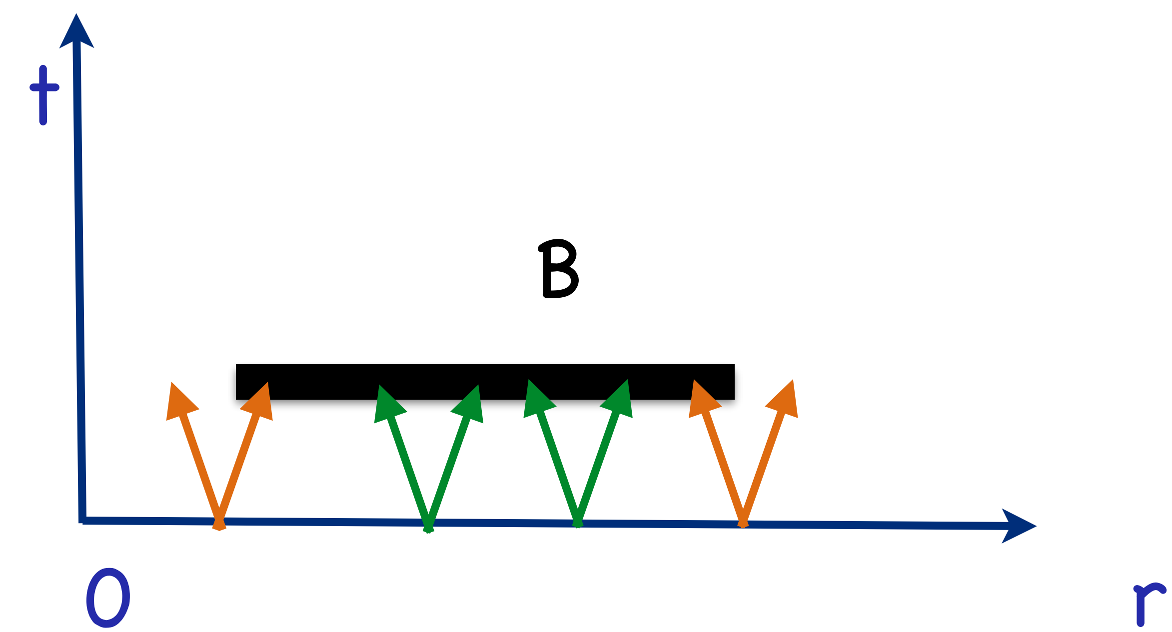
Quench creates quasiparticles at t=0, which start propagating with velocity $|v| \leq v_{max}$



Idea: entanglement spreads through propagation of entangled QP pairs with equal but opposite momenta

These induce entanglement of B with the outside world

These don't

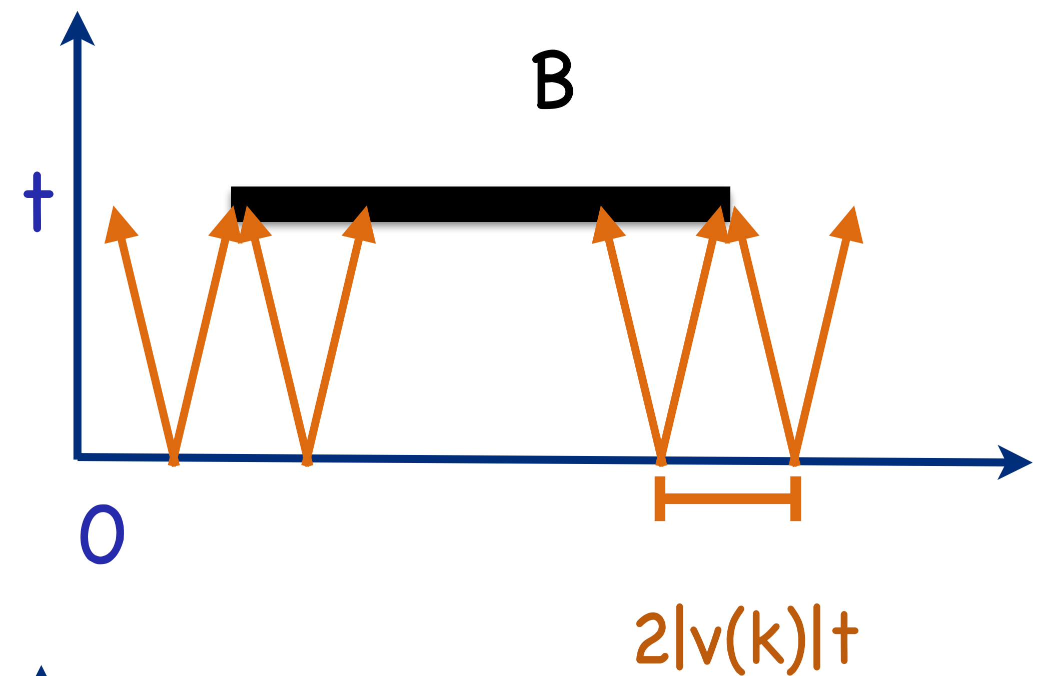


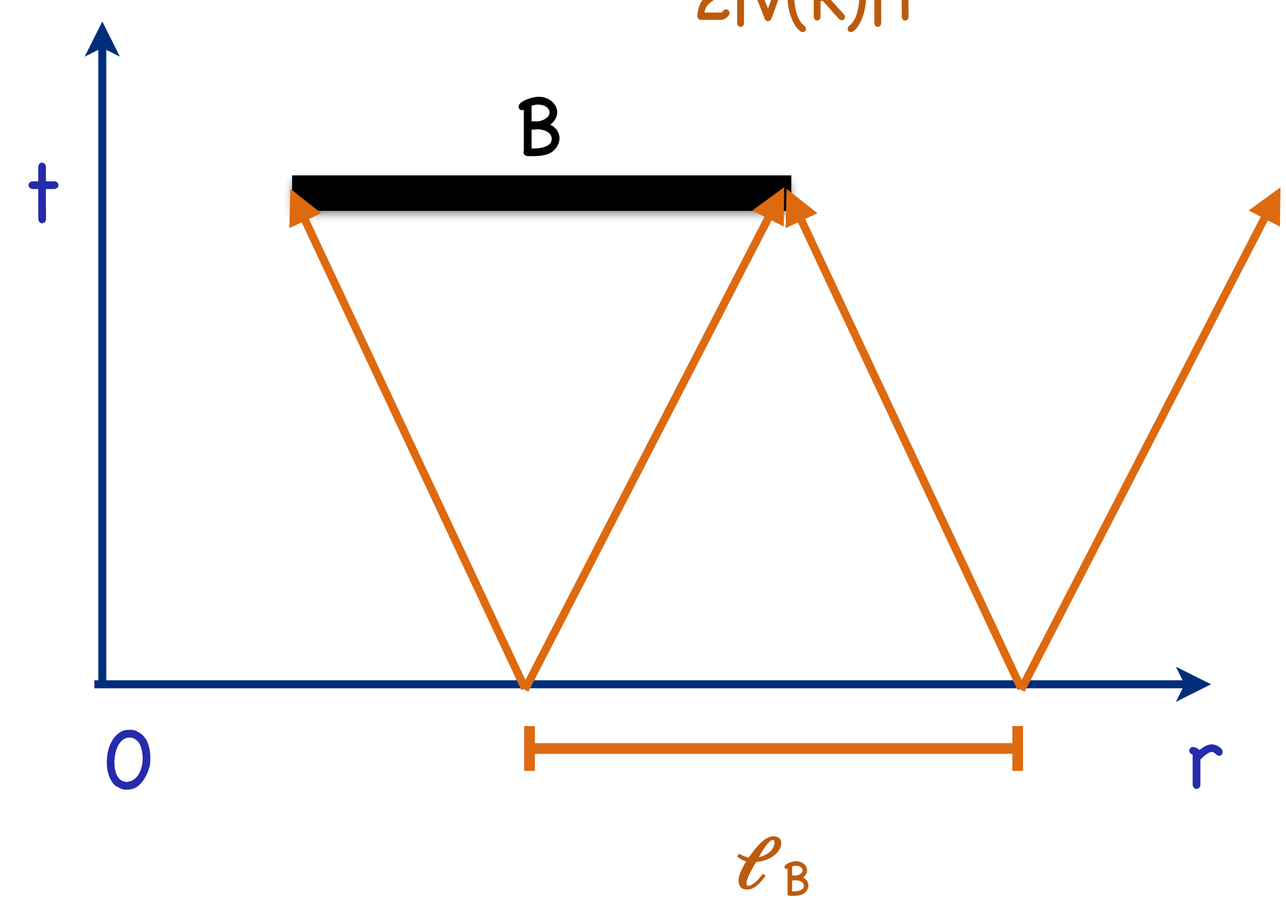
⟹ entanglement gets initially generated at the boundaries of B

Entanglement entropy = measure of the number of correlated quasi-particle pairs, so that at time t one QP is inside B, and the other outside

$$S_B(t) \simeq \int \frac{dk}{2\pi}\, f(k)\, \min\left(\ell_B, 2\,|\,v(k)\,|\,t\right)$$

f(k) quantifies entanglement carried by each pair





Free theories:

$$f(k) = -n(k)\ln[n(k)] - [1 - n(k)]\ln[1 - n(k)]$$

$$n(k) = \langle \Psi(0) | \hat{n}(k) | \Psi(0) \rangle$$

Thermodynamic entropy density

$$s = \int \frac{dk}{2\pi} f(k)$$

EE reduces to thermodynamic entropy in the steady state

$$\lim_{t \to \infty} S_B(t) = s|B|$$