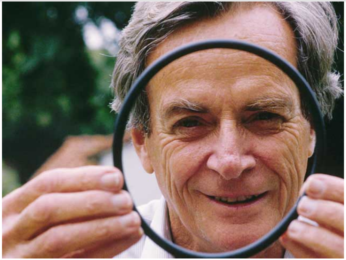
Collegium Urbis Nov Eborac

ಸೈದ್ಧಾಂತಿಕ ವಿಜ್ಞಾನಗಳ ಅಂತರರಾಷ್ಟ್ರೀಯ ಕೇಂದ್ರ

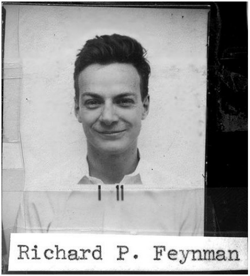
100 Years of Feynman

and 30 without him: some reminiscences

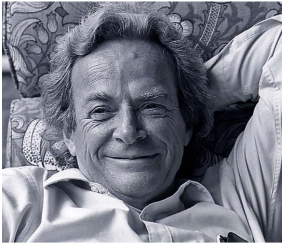


or, Feynman's last adventure in integrability

Feynman needs no introduction



May 11, 1918 - February 15, 1988



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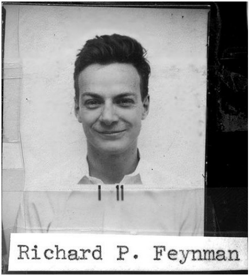

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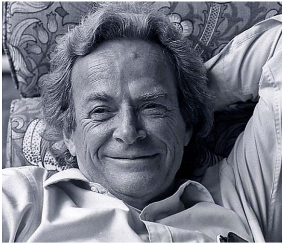

- Dominated physics with his creativity, insights and style
- Created the image of the playful, irreverent physicist relying heavily on his smarts and intuition
- Projected a sense of excitement about science and life in general. His *joie de vivre* was infectious
- A great teacher and communicator, he charmed audiences with his quick-witted, pseudo-naïve, no-nonsense chatter
- Achieved cult status well beyond the physics community (sometimes to the exasperation of his peers!)

The making of a Legend

The making of a Legend

The Great Counterpoint
A perennial Toccata and Fugue

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Serious and exacting

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Street-smart wisecracker

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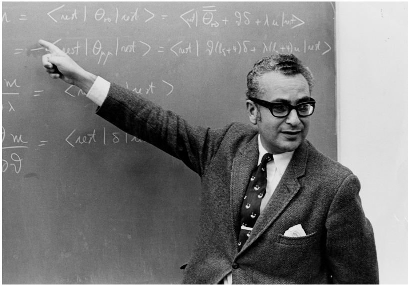
Rock Star

Admired and feared

Admired and adored

Admired and feared

Admired and adored



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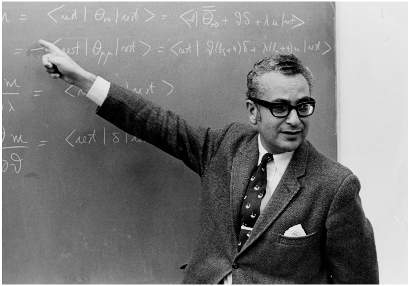

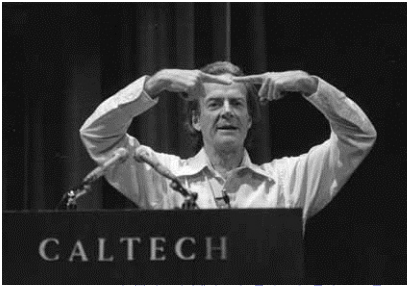

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Bethe Ansatz and Integrability

1/22/87
Lunch Talk

Bethe Ansatz. Thacker Rev. Mod Phys 53 2; p253 (1981)

~~From~~ From time to time many different two ~~one~~ dimensional (x,t) field theories have been proposed as models to learn from. Every once in a while they surprisingly are solved.

Non linear Schrod = Yang gas $(\bar{\psi}\psi)^2 + c(\psi^*\psi)^2$ with colors;
Thirring $|\partial_\mu\psi|^2 - g(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) - m_0^2\bar{\psi}\psi$ Sine Gordon $(\partial\phi)^2 - \cos\phi$
Gross Neveu $(\partial_\mu\psi_a)^2 - g[(\bar{\psi}^a\psi_a)^2 - (\bar{\psi}^a\gamma_5\psi_a)^2]$ } Running Coupling Const.
Σ: $(\partial_\mu\vec{\varphi})^2$ but $\vec{\varphi}\cdot\vec{\varphi} = 1$

~~All sol~~ & Two dimensional stat mech Onsager Baxter.

All solved by same method – guess as to form of Wave Fuct. Bethe Ansatz
Bethe ~~1930~~ 1931 Spin waves.

Mystery When will it work? → Classical Solitons. | Zamoldikov & Z. ... Faddeev Σ model N=4 | OTHER METHODS ...

Why study? (1) QCD & formulation of Field theory running coupling const.
(2) ~~Useful~~ Tool useful in other examples Kondo
(3) Know how to solve every problem which has been solved.
(4) Fun.

What is Bethe Ansatz?

Spin waves $H = -\frac{1}{2}\sum_{adj} \sigma_x\sigma_x' + \sigma_y\sigma_y' + \Delta\sigma_z\sigma_z'$ $N = \Sigma\sigma_z$ is const of motion

$$h = -\Sigma\left(\sigma_+\sigma_-' + \sigma_-\sigma_+' + \frac{\Delta}{2}(\sigma_z\sigma_z' - 1)\right)$$

does nothing two spins same, else

$$h\,\alpha\beta = (-1)\beta\alpha + \Delta(\alpha\beta)$$

STATES. All α. $E=0$

One β. If at x amp C_x $\quad \epsilon C_x = -C_{x+1} - C_{x-1} + 2\Delta C_x \quad C_x = e^{ikx}$

$\epsilon(k) = 2\Delta - 2\cos k$.

Two β.
positions x, y $\quad x < y$

1/22/87
Lunch Talk

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- A few stuck with him...
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- Becomes interested in Calogero model through a review of Sutherland
- Figures a "field theory" proof of commutativity of integrals

$$\langle \phi(x_i)\phi(x_j)\rangle = V(x_i - x_j) \ , \quad V(x) = \frac{g}{x^2}$$

$$I_n = \left\langle \underline{\prod^n (p+\phi)} \right\rangle := \sum_{j_a \,\mathrm{distinct}} \left\langle \prod_{a=1}^{n} (p_{j_a} + \phi(x_{j_a})) \right\rangle$$

(1)

Dear Mr Sutherland,

You asked me to send you what I knew of the conservation laws for a system of N particles all mutually interacting on a line by a potential $V(ij) = V(x_i - x_j)$ where x_i are the positions of particles i. If p_i is the momentum of the ith then the total momentum $I_1 = \sum_i p_i$ (1) is conserved. So is the total energy, and that means $\frac{1}{2}I_1^2 - \text{Energy} = I_2$ is conserved.

$$I_2 = \underline{p_i p_j} - \underline{V(ij)} \quad (2)$$

where by underlining a term I mean the sum of all distinctly different terms of the form underlined, each taken only once where the indices take all values from 1 to N, except that all indices must be different. Thus if $N=3$, $\underline{p_i V(jk)}$ would mean (for $N=3$) $p_1V(23) + p_2V(13) + p_3V(12)$ since $V(ij) = V(ji)$ they are not distinct; while $\underline{p_i p_j p_k}$ is simply the one term $p_1 p_2 p_3$]

Next trying to find an I_3 by trying to start with the term $\underline{p_i p_j p_k}$ that is a constant of the motion (commutes with I_2) we find, as you discovered,

$$I_3 = \underline{p_i p_j p_k} - \underline{p_i V(jk)} \quad (3)$$

provided V satisfies a special condition (which we call C_2):

$$C_2: \quad \underline{V'(ij)V(jk)} = 0 \quad (4) \qquad \wp(x)$$

which you showed implies V is Weierstrasses elliptic function $\{V'(x) = dV(x)/dx$

A special simple case is $V = 1/x^2$ and we can consider the general solution as a kind of generalization into complex x_i, doubly periodic.

The condition $C_1: \sum_j V'(ij) = 0$ is obvious for $V'(12) = -V'(21)$.)

Then it is easy to verify that

$$I_4 = \underline{p_i p_j p_k p_l} - \underline{p_i p_j V(kl)} + \underline{V(ij)V(kl)} \quad (5)$$

is also conserved, as well as an entire string of I_k up to I_N.

In this letter I give my proof that I_2 commutes with them all, and that they all commute with each other. [Actually in the general case so far I only checked the classical case with Poisson Brackets]. But this is only true if the potential satisfies beside C_2 also a sequence of other conditions

$$C_3: \quad \underline{V'(ij)V(jk)V(kl)} = 0 \quad (6)$$

$$\vdots$$

$$C_m: \quad \underline{V'(ij)V(jk)\dots V(rs)V(st)} = 0 \quad (7)$$ (with a V' and $m-1$ V's).

[That C_3 is necessary you can see by trying to commute I_3, I_4 directly by hand]

You can imagine my surprise [that is the demonstration of our lack of deep understanding of these things, – always a surprise!] when I checked C_3 (for $N=4$ is enough) by writing out the 24 terms for $V = 1/x^2$ and doing the algebra and watching it all cancel out. I have since found a recursive proof for them all for $V = 1/x^2$ which, I think, implies them also for $\wp(x)$.

Obviously each C_m needs only be checked as an identity for $N = m$ particles.

My entry in the subject: extended kinematical quantities

$$I_{n,m} = \left\langle \underline{\prod_n x \prod_m (p + \phi)} \right\rangle$$

$$\frac{d}{dt} I_{n,m} = (m+1)\, I_{n-1,m+1}$$

Shows that scattering is as for classical free particles

Letter 2.

~~P.S.~~ while the other ~~this paper we have~~ letter was being written, with a colleague of mine alexis Polys – we ~~we~~ have found more for the case $V(x) = \alpha / x^2$, ~~working in the class~~

Define $F(k, m) = \langle \prod^{k} \dot{x} \times \prod^{m} (p+q) \rangle$; ie k factors of $\dot{x}_i$ and m of $p+q$ and the mean taken on q.

Then the commutator with $H = \frac{1}{2} p^2 + V(x_{ij})$ ~~as well~~ written $\frac{d}{dt}$ we find

$$\frac{d}{dt} F(k, m) = (m+1) F(k-1, m+1)$$

whence $\frac{d^k}{dt^k} F(k, 0) = k!\, F(0, k) = k!\, I_k$

a constant of the motion.

Hence, in the classical case $F(k,0)$ is a kth order polynomial in time t.

But $F(k,0)$ is the sum of the values of $x_i(t)$ taken k at a time, ~~the~~ starting

Thus ~~Hence~~ if $P(z)$ ~~is~~ a polynomial of Nth order (for N particles)

as z^N with coefficients of the z^{N-k} being kth order polynomials in t (ie, $(-1)^k F(k,0)$) then the N solutions of $P(x(t)) = 0$ for $x(t)$ give the motions $x_i(t)$ of the N particles.

I have checked it for $N = 3$ (and, of course 2) and it works!

In all the ~~polynom~~ polynomials in t there are $N(N+3)/2$ constants, whereas there are only $2N$ degrees of freedom so there are $N(N-1)/2$ conditions among the ~~poly~~ coefficients. We have not found a *simple* way to find these constants, but we know (write P_t for $\partial P(z,t)/\partial t$, $P_z = \partial P/\partial z$ etc.)

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Define $F(k,m) = \langle \prod^k x \prod^m (p+\phi) \rangle$; ie k factors of x_i, m of $p+\phi$ and the mean taken on ϕ.

Then the commutator with $H = \frac{1}{2}p^2 + \underline{V(ij)}$ written $\frac{d}{dt}$ we find

$$\frac{d}{dt} F(k,m) = (m+1) F(k-1, m+1)$$

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Hence, in the classical case $F(k,0)$ is a kth order polynomial in t. ~~to power k~~ But $F(k,0)$ is the sum of the values of $x_i(t)$ taken k at a time. Thus if $P(z)$ is a polynomial of Nth order (for N particles) starting as Z^N with coefficients of the Z^{N-k} being kth order polynomials in t (ie, $(-1)^k F(k,0)$) then the N solutions of $P(x(t)) = 0$ for all t give the $x_i(t)$ of the N particles.

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CALIFORNIA INSTITUTE OF TECHNOLOGY

CHARLES C. LAURITSEN LABORATORY OF HIGH ENERGY PHYSICS

PASADENA, CALIFORNIA 91125

May 28, 1987

Dr. Bill Sutherland
Department of Physics
University of Utah
Salt Lake City, UT 84112

Dear Dr. Sutherland:

While the other letter was being written, with a colleague of mine, Alexios Polychronakos, we have found more for the case $V(x) = \alpha/x^2$. Define $F(k,m) = \left\langle \prod^k x \prod^m (p+\phi) \right\rangle$; i.e., k factor of x_i and m of $p+\phi$ and the mean taken on ϕ.

Then the commutator with $H = \frac{1}{2}p^2 + \underline{V(ij)}$ written $\frac{d}{dt}$ we find

$$\frac{d}{dt}F(k,m) = (m+1)F(k-1,\, m+1).$$

$$\frac{d^k}{dt^k}F(k,0) = k!F(0,k) = k!I_p$$

a constant of the motion.

Hence, in the classical case $F(k,0)$ is a k^{th} order polynomial in time t. But $F(k,0)$ is the sum of the values of $x_i(t)$ taken k at a time. Thus if $P(z)$ is a polynomial of N^{th} order (for N particles) starting as Z^N with coefficients of the Z^{N-k} being k^{th} order polynomials in t (i.e.,$(-1)^k F(k,0)$) then the N solutions of $P(x(t)) = 0$ for all t give the motions $x_i(t)$ of the N particles.

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$$P_t P_z^2 + 2P_z P_t P_{zt} - P_t^2 P_{zz} = \frac{d}{4}[P_z^2 P_{zzzz} - 2P_z P_{zz} P_{zzz} + P_{zz}^3].$$

For $N = 3$ the motions $x_1(t)$, $x_2(t)$, $x_3(t)$ in the center of mass $(x_1 + x_2 +$

Alex – I'm cleaning up some files and thought these might be useful to you someday. Hope you're OK. Cheers, Helen

~~if A,~~ if the constants satisfy

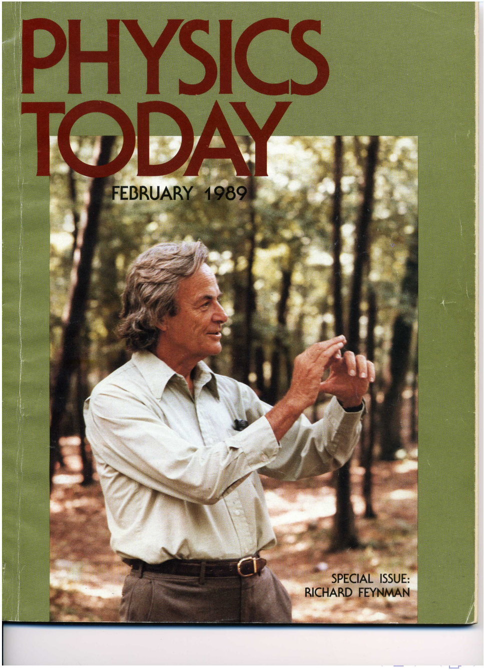
~~1800~~ $$\alpha = \frac{9}{16} a^2 V^2 \left(\frac{4}{9} - 3A^2 - \frac{1}{3} \beta^2 / a^2 \right).$$

Are these the characteristic equation for a simple matrix linear in time? What is the quantum solution? What if V is 0? Don't tell us - we are having fun - that is why we haven't looked at your papers yet. Just tell us if ~~some some of~~ any of this is unknown.

Yours -

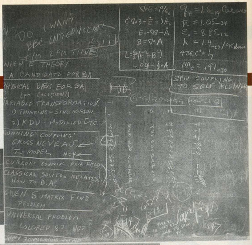
Letter 3

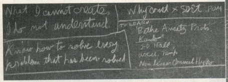
~~We work out stuff faster than we can get the letter.~~

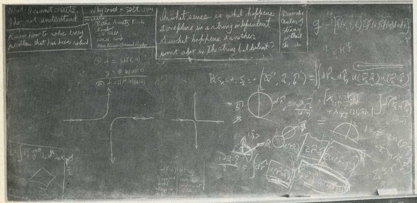

PHYSICS
TODAY
FEBRUARY 1989
SPECIAL ISSUE:
RICHARD FEYNMAN

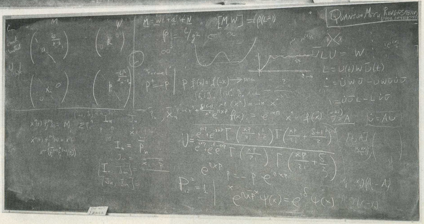
FEYNMAN'S OFFICE: THE LAST BLACKBOARDS

ALL PHOTOS ROBERT PAZ/ARCHIVES, CALIFORNIA INSTITUTE OF TECHNOLOGY



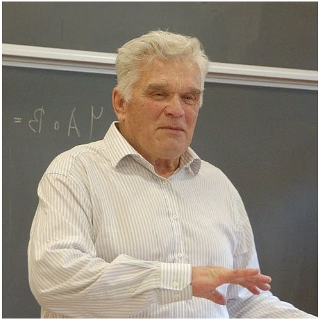






Postscript: Faddeev and the quest for the lost notes

Ludvig Faddeev



23 March 1934 - 26 February 2017

Letter courtesy of Hiroshi Ooguri
and Caltech Archives

Russian Academy of Sciences
"Uspekhi Fizicheskikh Nauk"
("Physics-Uspekhi") journal
119991, Moscow, Leninskii prospekt., 53. Phone (7-499) 132-62-65, Fax (7-499) 132-63-48.
E-mail: keldysh@ufn.ru, maria@ufn.ru

January 30, 2013

To whom it may concern
California Institute of Technology
On-line Archive of California
(OAC)

Dear Colleagues,

The Editorial Board of "Uspekhi Fizicheskikh Nauk" ("Physics-Uspekhi" in English version) journal (one of the leading Russian journal in physics, former Editor-in-Chief Nobel Prize Winner Vitaly Ginzburg) needs your kind permission for our scientific and managing editor ***Dr. Maria Aksenteva*** to get the opportunity to see some documents from your archive.

One of the world-recognized mathematician Prof. Ludvig Faddeed is now writing an interesting review for our journal. Several years ago, just after the death of Prof. Richard Feynmann, Prof. Faddeev has visited Caltech and saw some hand-written notes by Prof. R. Fenmann in which he emphasized the importance of Bethe Ansatz for some fundamental physical problems.

According to the opinion of Prof. Faddeev it would be very desirable to support his reminiscence with the copy of this Feynmann's notes. Via Internet Prof. Faddeev has found in OAC two files, which may be are just what he wants to find:
http://www.oac.cdlib.org/findaid/ark:/13030/kt5n39p6k0/.

1. "Lunch Talk on Bethe Ansatz", Jan22, 1987 in Section 5, box 52 folder 11., 8 pages.
2. "Working notes and calculations Box 62 folder 1 black binder. 1987-88".

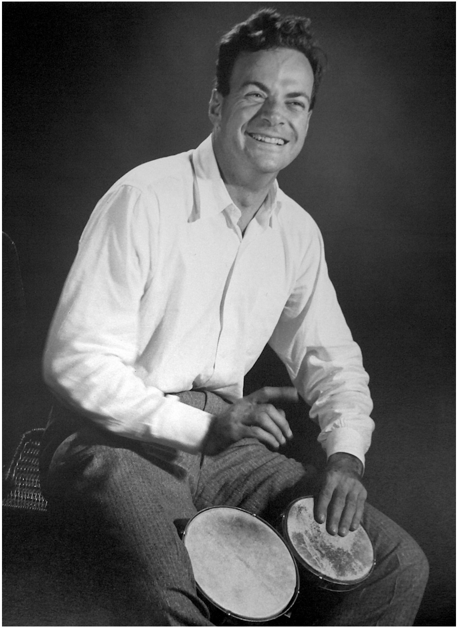
But it is impossible to investigate these folders via Internet.

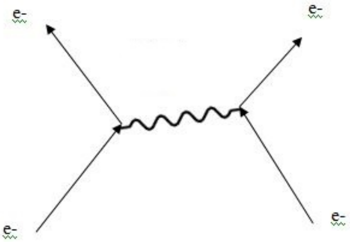
Due to the fact, that Dr. Maria Aksenteva is going to visit California in February we hope, that it would be possible for her to look at these folders.

Thank you in advance for your kind help and support. We would greatly appreciate your kind positive reply.

With kind regards,
on behalf of UFN Editorial Board
Academician Rudenko O.V.
Associate Editor of the "Uspekhi Fizicheskikh Nauk"
("Physics-Uspekhi") journal (www.ufn.ru)

Feynman's legacy





Thank You