Quantum dynamics with stochastic resets

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Outline

1. *Motivation: a reference to classical counterpart*
2. *Formalism*
3. *Application to quantum quench*
4. *Steady states: non-integrable models*
5. *Application to cold atom systems*
6. *Integrable models*
7. *Periodic drives and correlation functions*
8. *Conclusion*

Motivation

Several recent studies of reset dynamics in classical systems.

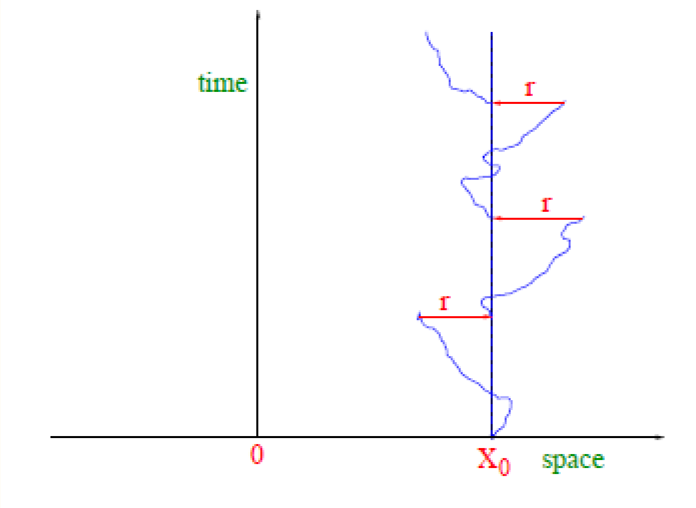
Consider a general classical system evolving under its own natural dynamics.

Now consider interruption of such dynamics at random times during which the system is reset to initial condition.

Question: What happens to the system at long times under such dynamics?

Answer: This leads to realization of non-trivial stationary states.

A simple specific example (Evans and Majumdar, PRL 2011)



A single particle, diffusing in one dimension with a diffusion constant D , is being reset to its original position x_0 at random times τ

The equation for the probability of finding the particle at (x,t) given the initial condition (x_0,0) is determined by the equation

$$\frac{\partial p(x,t|x_0)}{\partial t} = D\frac{\partial^2 p(x,t|x_0)}{\partial x^2} - rp(x,t|x_0) + r\delta(x-x_0)$$

This leads to a non-Gaussian stationary distribution for a finite rate of reset r at long times

$$p_{\rm st}(x|x_0) = \frac{\alpha_0}{2}\exp(-\alpha_0|x-x_0|)$$

$$\alpha_0 = \sqrt{r/D}$$

Application to target search

Such reset averaged dynamics leads to interesting results for first passage problems

In these problems one typically finds the mean first passage time for a particle to hit a target during its evolution.

For the single diffusive particle, for diffusive dynamics with no reset and origin set as the target position, the mean first passage time is infinite.

Reset averaged dynamics gives a finite first passage time and thus provides scope of more efficient search algorithm.

$$T(x_0) = \frac{1}{r}\left(\exp(\alpha_0 x_0) - 1\right)$$

T diverges for r=0 as $1/r^{1/2}$ which is the reset-free result. It also diverges when r diverges since the particle has less time to evolve and reach origin between resets.

This leads to an optimal reset rate which leads to fastest first passage time →

$$\frac{z^*}{2} = 1 - e^{-z^*}$$

$$z^* = 1.59362.....$$
$$r^* = (z^*)^2 D/x_0^2$$

$z=\alpha_0 x_0$

Several other applications:

a) *Efficient search algorithms for Multiparticle systems*

b) *Novel stationary states*

c) *Novel stochastic thermodynamics and fluctuation theorems*

Question: What happens for the case of quantum systems whose unitary time evolution is interrupted by random stochastic resets which takes the system back to its initial state?

Formalism

What do we mean by reset averaged quantum dynamics

There are two equivalent ways of thinking about averaging over resets

Ensemble average

1. Make N copies of a quantum system.
2. Let them evolve starting from the same initial condition.
3. For each copy, an observer makes a measurement after a random time τ
4. The time τ is a random number drawn from a probability distribution $p(\tau)$
5. Average over these measurements.
6. For large N the system reaches a steady state which we shall study.

Time average

1. Consider a single copy of the quantum system and let it evolve .
2. After unitary evolution for a random time τ, drawn from $p(\tau)$, the observer makes the first measurement.
3. Immediately after this measurement, one resets the system to the initial state.
4. The process is repeated and the system evolves in this manner for a total time t and one averages all measured values.
5. For large t, the system reaches a steady state which we shall study.

Since each unitary time evolution is independent, time average == ensemble average

Generic quantum system under reset

Consider a quantum time dependent Hamiltonian H(t) which controls time evolution of a quantum system in the absence of resets

$$|\psi(t)\rangle_{r=0} = U(0,t)\,|\psi(0)\rangle$$

$$U(t_1,t_2) = T_t \exp[-i\int_{t_1}^{t_2} H(t')dt'],$$

In the presence of resets characterized by a rate r, the dynamics is modified as

$$|\psi(t+dt)\rangle = \begin{cases} |\psi(0)\rangle, & \text{with prob. } r\,dt \\ [1 - iH(t)\,dt]\,|\psi(t)\rangle & \text{with prob. } 1 - r\,dt \end{cases}$$

The system either undergoes a reset with probability r dt or undergo Hamiltonian evolution with probability 1-rdt.

Our goal is to compute the density matric of the system at time t after averaging over all resets

$$\rho(t) = E\left[\hat{\rho}(t)\right]$$

→ ***E denotes over average over all reset histories***

To do this we first need to find the probability distribution for the resets.

To this end, we note that a reset is a Poisson process characterized by a rate r.

Thus the probability that there is will be no reset up to a time t is exp(-r t).

Thus the probability that there will be at least one reset within a time t is (1 – exp(-r t))

This allows us to define a probability distribution function (pdf)

$$\int_0^t p(\tau|t)d\tau \ = \ 1 - e^{-rt} \quad \Rightarrow \quad p(\tau|t)\, d\tau = r\, e^{-r\tau}\, d\tau$$

Thus the probability distribution for the reset time τ is given by

$$p(\tau|t) = r\, e^{-r\,\tau} + e^{-r\,t}\, \delta(\tau - t) \quad 0 \leq \tau \leq t\,.$$

Note that the last term which gives the probability of no resets till t, vanishes exponentially with increasing time t.

Now consider the evolution of the system after a time τ where the reset occurs

$$\hat{\rho}(\tau|t) \;=\; U^{\dagger}(0,\tau)\,\rho_0\,U(0,\tau)$$

Thus the reset averaged density matrix for evolution till a time t can be written as

$$\rho(t) \;=\; \int_0^t re^{-r\tau}\,\rho(\tau|t)\,d\tau \\ +e^{-rt}\,U^{\dagger}(0,t)\,\rho_0\,U(0,t)$$

For large measurement time t one finds a stationary state density matrix

$$\rho_{\mathrm{stat}} \;=\; \int_0^{\infty} re^{-r\tau}\,U^{\dagger}(0,\tau)\rho_0 U(0,\tau)d\tau$$

Question: Does such reset average gives rise to interesting steady states?

Reset dynamics following a quantum quench

Digression: Steady state following a quantum quench: no reset

Consider the time evolution of a quantum system following a quantum quench.

The initial state of the system can be expressed in the basis of eigenstates of the final Hamiltonian as

$$|\psi(0)\rangle = \sum_\alpha c_\alpha|\alpha\rangle, \quad H|\alpha\rangle = \epsilon_\alpha|\alpha\rangle$$

The matrix elements of the density matrix in this basis at any time t can be written as

$$\rho_{\alpha\beta}(t) = c_\alpha^* c_\beta e^{-i\omega_{\beta\alpha}t} \qquad \omega_{\beta\alpha} = (\epsilon_\beta - \epsilon_\alpha).$$

The time average of the density matrix in the steady state (provided it is reached) is

$$\begin{aligned}\overline{\rho}_{\alpha\beta}(t) &= \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\rho_{\alpha\beta}(t)\\ &= |c_\alpha|^2\delta_{\alpha\beta}\end{aligned}$$

Thus the system is described a diagonal density matrix. Thus in steady state for any operator O, we can write

$$\langle O\rangle = \sum O_{\alpha\beta}\rho_{\beta\alpha} = \sum |c_\alpha|^2 O_{\alpha\alpha} = O_D,$$

This signifies complete loss of phase information of the initial state.

Steady state following a quantum quench with reset

Consider now the same quantum system which undergoes unitary dynamics interrupted by resets.

The initial wavefunction can once again be written as $|\psi(0)\rangle = \sum_{\alpha} c_\alpha |\alpha\rangle,$

One now track the density matrix evolution. The density matrix evolves unitarily for random times τ *followed by measurements and immediate resets .*

Between any two resets, the density matrix is given by (time measured from last reset)

$$\rho_{\alpha\beta}(t) = c_\alpha^* c_\beta e^{-i\omega_{\beta\alpha} t}$$

Thus the steady state density matrix, averaged over resets, is given by

$$\begin{aligned}
\overline{\rho}_{\alpha\beta} &= \int_0^\infty d\tau r e^{-(r+i\omega_{\beta\alpha})\tau} c_\alpha^* c_\beta \\
&= (\rho_0)_{\alpha\beta} \frac{r}{r + i\omega_{\beta\alpha}} \quad \text{for } \beta \neq \alpha \\
&= \rho_D = (\rho_0)_{\alpha\alpha} \quad \text{for } \alpha = \beta
\end{aligned}$$

$$
\begin{aligned}
\overline{\rho}_{\alpha\beta} &= \int_0^\infty d\tau r e^{-(r+i\omega_{\beta\alpha})\tau} c_\alpha^* c_\beta \\
&= (\rho_0)_{\alpha\beta} \frac{r}{r+i\omega_{\beta\alpha}} \quad \text{for } \beta \neq \alpha \\
&= \rho_D = (\rho_0)_{\alpha\alpha} \quad \text{for } \alpha = \beta
\end{aligned}
$$

The steady state density matrix has finite off-diagonal elements. It is not diagonal in the energy basis of the final Hamiltonian.

It is characterized by the reset rate r and retains phase infomation of the initial state.

For r -> 0, one gets back the diagonal density matrix corresponding to pure unitary evolution

For large r, the density matrix approaches the initial density matrix which Is a manifestation of the quantum Zeno effect.

The expectation value of a generic operator recieives contribution from off-diagonal elements . The system does not thermalize under such dynamics.

$$
\begin{aligned}
\langle O \rangle &= O_D + \sum_{\alpha\beta} |c_\beta||c_\alpha| O_{\alpha\beta} \frac{r}{r^2+\omega_{\alpha\beta}^2} \\
&\quad \times [r\cos(\theta_{\alpha\beta}) + \omega_{\alpha\beta}\sin(\theta_{\alpha\beta})]
\end{aligned}
$$

Digression: Imperfect reset

$$\begin{aligned}\overline{\rho}_{\alpha\beta} &= \int_0^\infty d\tau r e^{-(r+i\omega_{\beta\alpha})\tau} c_\alpha^* c_\beta \\ &= (\rho_0)_{\alpha\beta} \frac{r}{r+i\omega_{\beta\alpha}} \quad \text{for } \beta \neq \alpha \\ &= \rho_D = (\rho_0)_{\alpha\alpha} \quad \text{for } \alpha = \beta\end{aligned}$$

For imperfect resets ca is no longer determined by a single initial set.

If the reset is completely random c_α will take completely random values

The average of these coefficients with some distributions will lead to vanishing of the off-diagonal elements.

However, in a generic case, one expects the reset not be completely random.

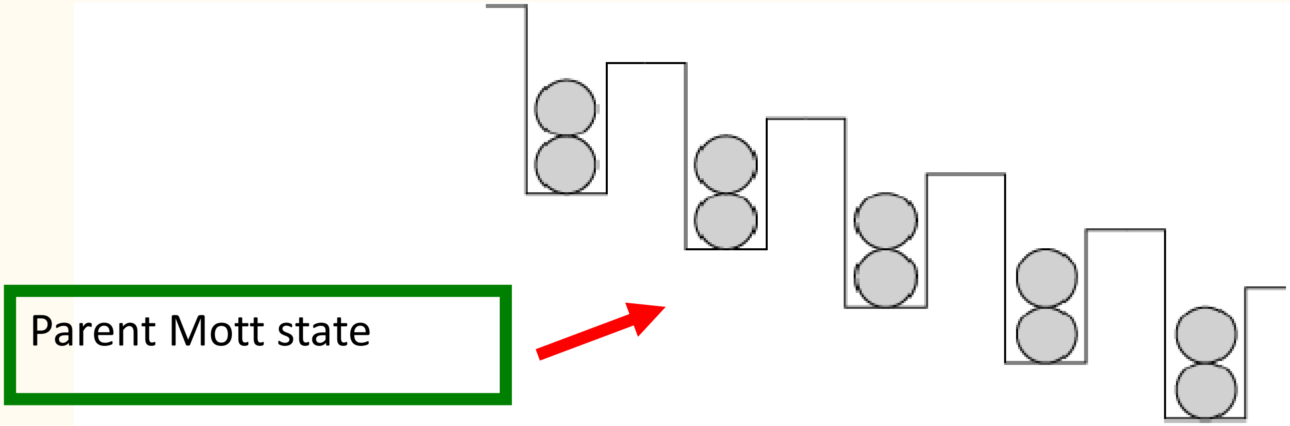
In this case c_α will have a peaked distribution about a mean position.

The average over c_α s in such cases will lead to finite value of the off-diagonal elements of the density matrix.

The effect is robust against small imperfection in the reset protocol.

A concrete model: Tilted Bose-Hubbard model in an optical lattice

Construction of an effective model: 1D



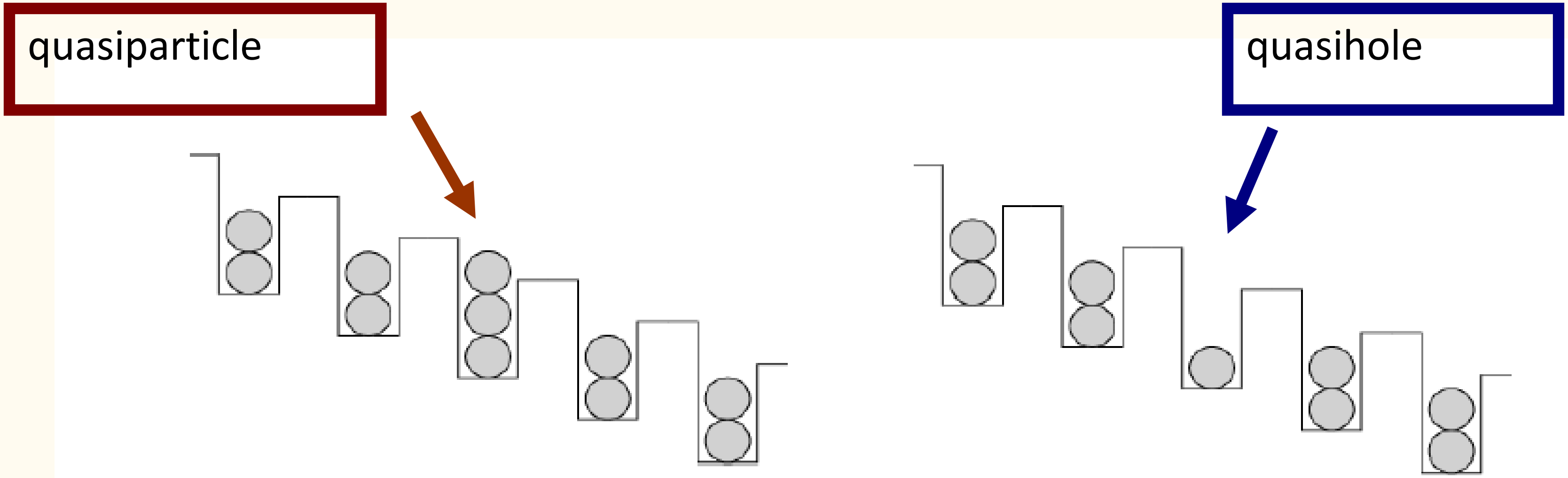
$$H = -w\sum_{\langle ij \rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

$$n_i = b_i^\dagger b_i$$

$$\left|U - E\right|, w \ll E, U$$

Describe spectrum in subspace of states resonantly coupled to the Mott insulator

Charged excitations



Effective Hamiltonian for a quasiparticle in one dimension (similar for a quasihole):

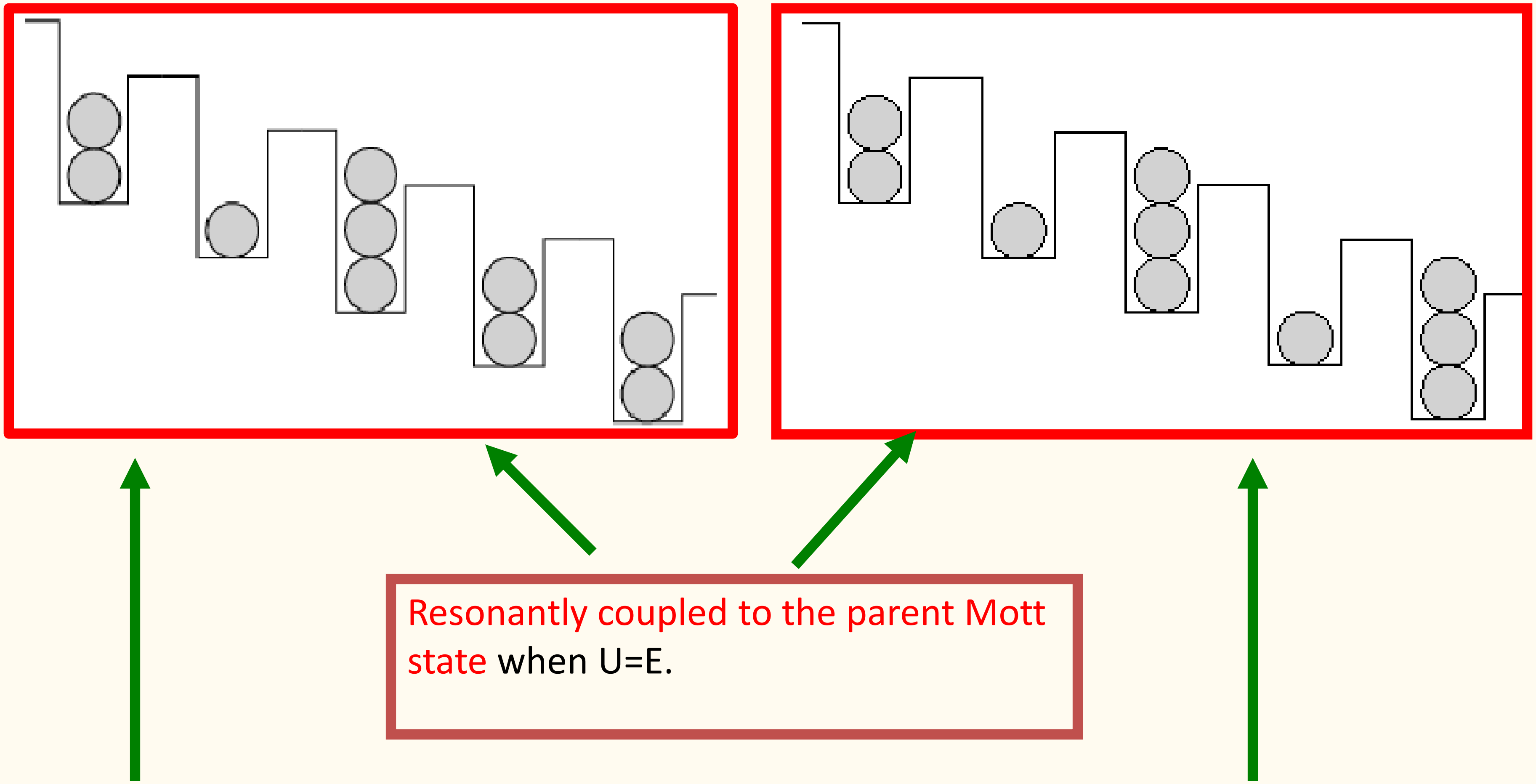
$$H_{\rm eff} = -\sum_j \left[3w\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + Ejb_j^\dagger b_j \right]$$

Exact eigenvalues $\varepsilon_m = Em \quad ; \quad m = -\infty \cdots \infty$

Exact eigenvectors $\psi_m(j) = J_{j-m}(6w/E)$

All charged excitations are strongly localized in the plane perpendicular electric field.
Wavefunction is periodic in time, with period h/E (Bloch oscillations)
Quasiparticles and quasiholes are not accelerated out to infinity

Neutral dipoles



Resonantly coupled to the parent Mott state when U=E.

Neutral dipole state with energy U-E.

Two dipoles which are not nearest neighbors with energy 2(U-E).

Effective dipole Hamiltonian: 1D

$d_\ell^\dagger \Rightarrow$ Creates dipole on link ℓ

$$H_d = -\sqrt{6}w \sum_\ell \left(d_\ell^\dagger + d_\ell \right) + (U - E) \sum_\ell d_\ell^\dagger d_\ell$$

Constraints: $\quad d_\ell^\dagger d_\ell \leq 1 \quad ; \quad d_{\ell+1}^\dagger d_{\ell+1} d_\ell^\dagger d_\ell = 0$

Determine phase diagram of H_d as a function of *(U-E)/w*

Note: there is no explicit dipole hopping term.

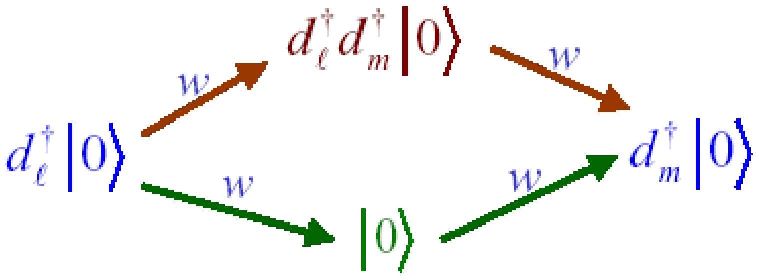
However, dipole hopping is generated by the interplay of terms in H_d and the constraints.

Weak Electric Field

Ground state is dipole vacuum (Mott insulator) $|0\rangle$

First excited levels: single dipole states $d_\ell^\dagger |0\rangle$

<u>Effective hopping between dipole states</u>



If both processes are permitted, they exactly cancel each other.

The top processes is blocked when ℓ, m are nearest neighbors

$\Rightarrow$ A nearest-neighbor dipole hopping term $\sim \dfrac{w^2}{U-E}$ is generated

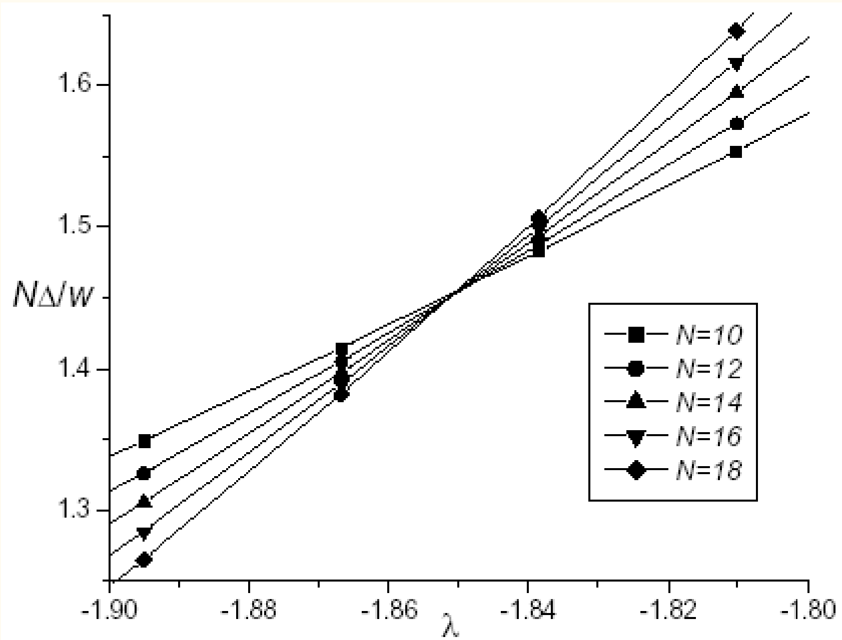
Strong Electric field

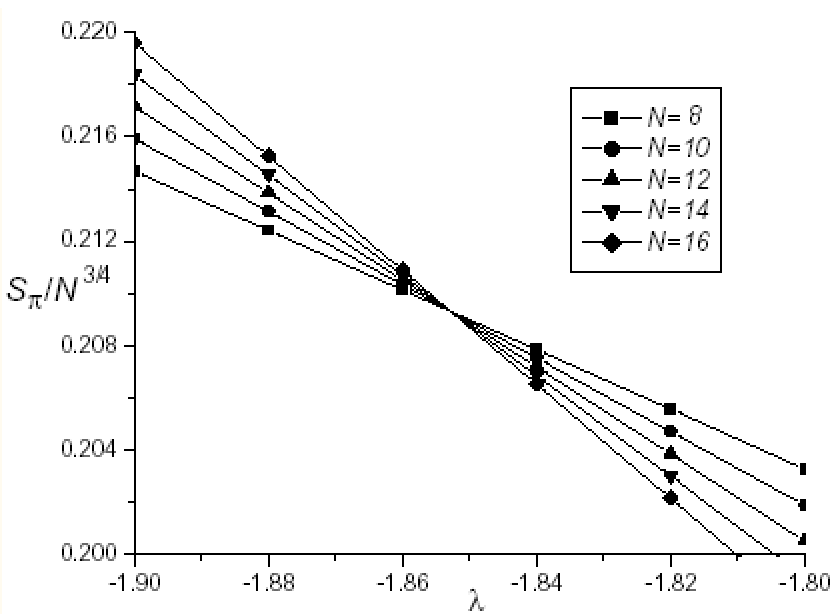
- The ground state is a state of maximum dipoles.
- Because of the constraint of not having two dipoles on consecutive sites, we have two degenerate ground states

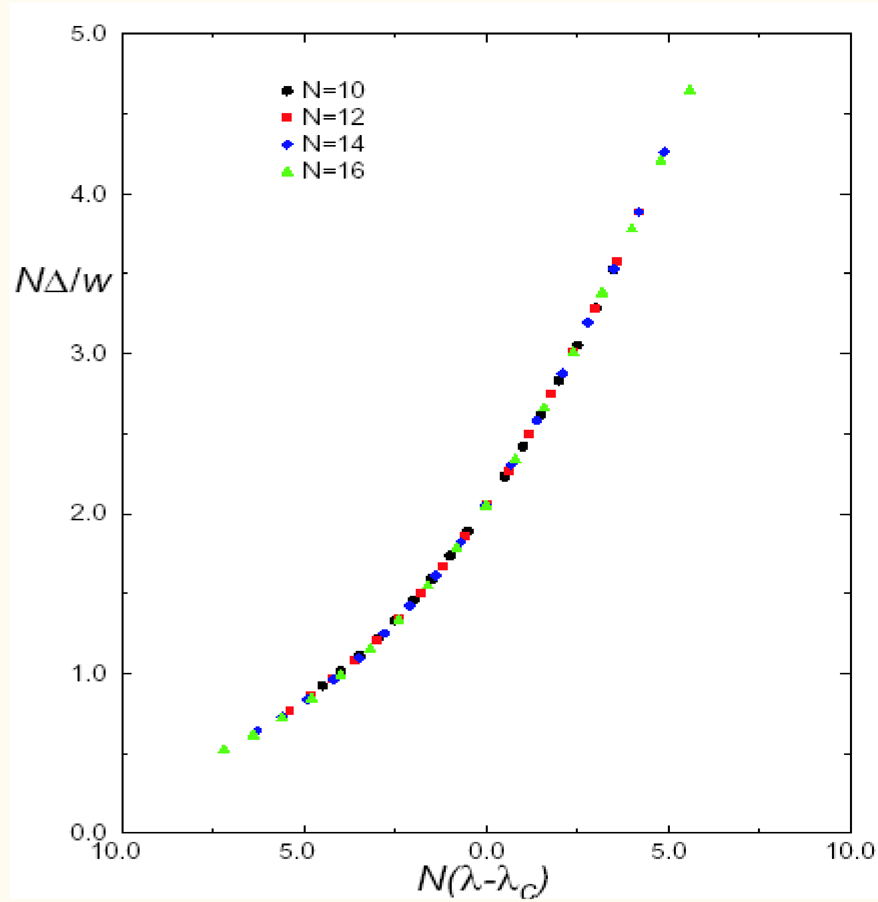
$$\cdots d_1^\dagger d_3^\dagger d_5^\dagger d_7^\dagger d_9^\dagger d_{11}^\dagger \cdots |0\rangle \quad or \quad \cdots d_2^\dagger d_4^\dagger d_6^\dagger d_8^\dagger d_{10}^\dagger d_{12}^\dagger \cdots |0\rangle$$

- The ground state breaks Z2 symmetry.
- The first excited state consists of band of domain walls between the two filled dipole states.
- Similar to the behavior of Ising model in a transverse field.

Intermediate electric field: QPT







Quantum phase transition at E-U=1.853w. Ising universality.

Measurements appropriate for the Mott state

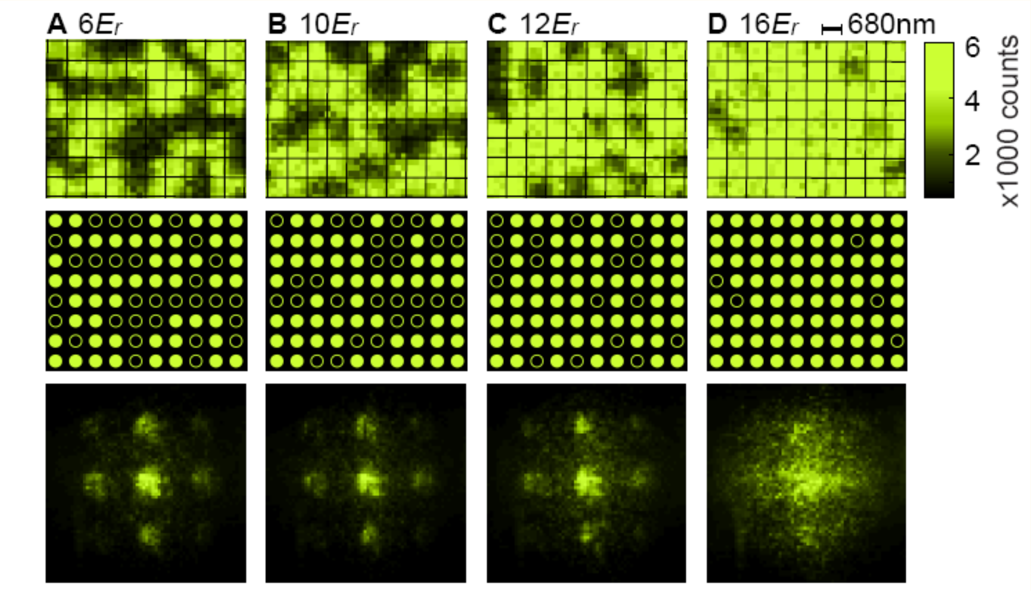


FIG. 1. Single-site imaging of atom number fluctuations across the superfluid-Mott insulator transition. (**A** – **D**) Images within each column are taken at the same final 2D lattice depth of (**A**) $6E_r$, (**B**) $10E_r$, (**C**) $12E_r$ and (**D**) $16E_r$. Top row: in-situ fluorescence images from a region of 10×8 lattice sites within the $n = 1$ Mott shell that forms in a deep lattice. In the superfluid regime (**A**, **B**), sites can be occupied with odd or even atom numbers, which appear as full or empty sites respectively in the images. In the Mott insulator, occupancies other than 1 are highly suppressed (**D**). Middle row: results of the atom detection algorithm [16] for images in the top row. A full (empty) circle indicates the presence (absence) of an atom on a site. Bottom row: time of flight fluorescence images after 8ms expansion of the cloud in the 2D plane as a result of non-adiabatically turning off the lattice and the transverse confinement (averaged over 5 shots and binned over 5×5 lattice sites).

1. *Prepare the state with 2D ^{87}Rb BEC of 10^5 atoms ($F=m_f=1$): use magnetic trap potential.*

2. *Project a square optical lattice on it with λ=680nm and ramp the lattice depth.*

3. *After the state is prepared, increase the lattice depth to freeze the atoms.*

4. *Apply light so that light-assisted collision eject pair of atoms from each site.*

5. *Image the remaining atoms to detect parity of occupation.*

6. *For SF, $p_{odd} = 0.5(1-e^{-\langle n\rangle}) < 0.5$ while for Mott state p_{odd}=0,1.*

Realizing the tilted Bose-Hubbard model

Experimental generation of a linearly varying Zeman field along x to generate the tilt.

One can describe the system using dipoles or spins via the following transformation

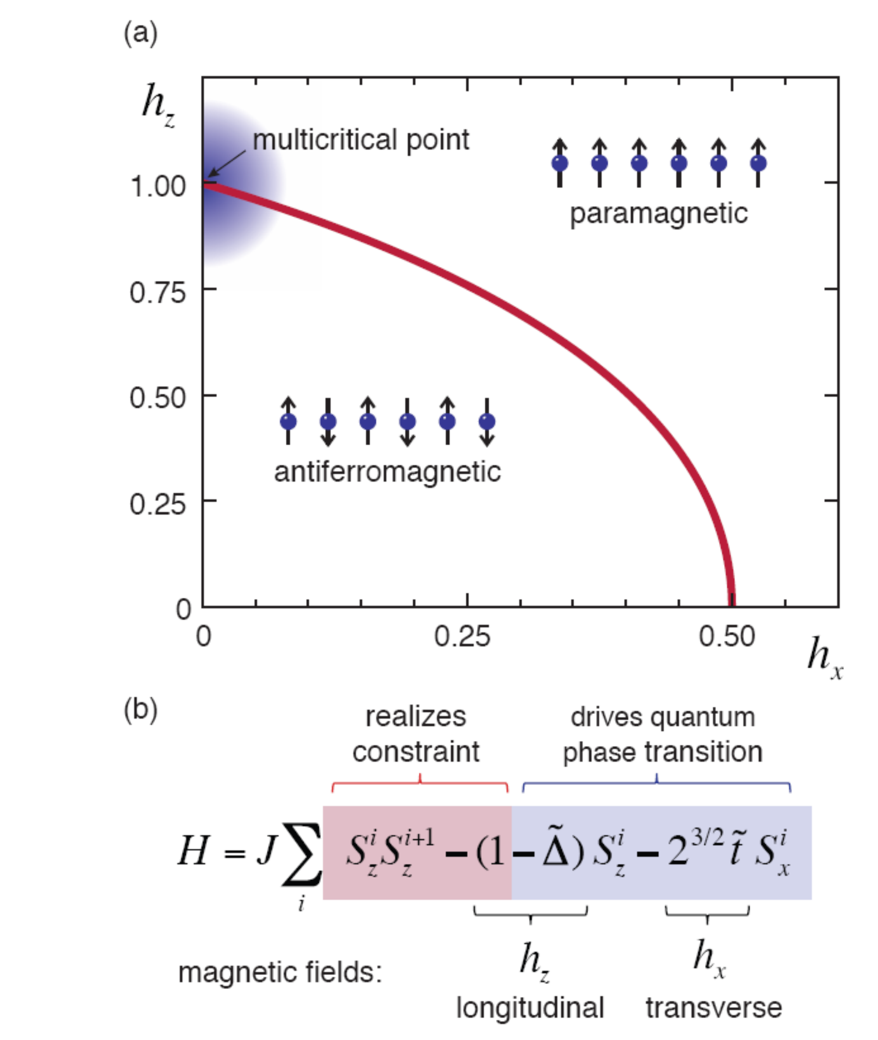
$$S_z^j = \frac{1}{2} - d_j^\dagger d_j,\, S_x^j = \frac{1}{2}\left(d_j^\dagger + d_j\right),\ \text{and}\ S_y^j = \frac{i}{2}\left(d_j^\dagger - d_j\right)$$

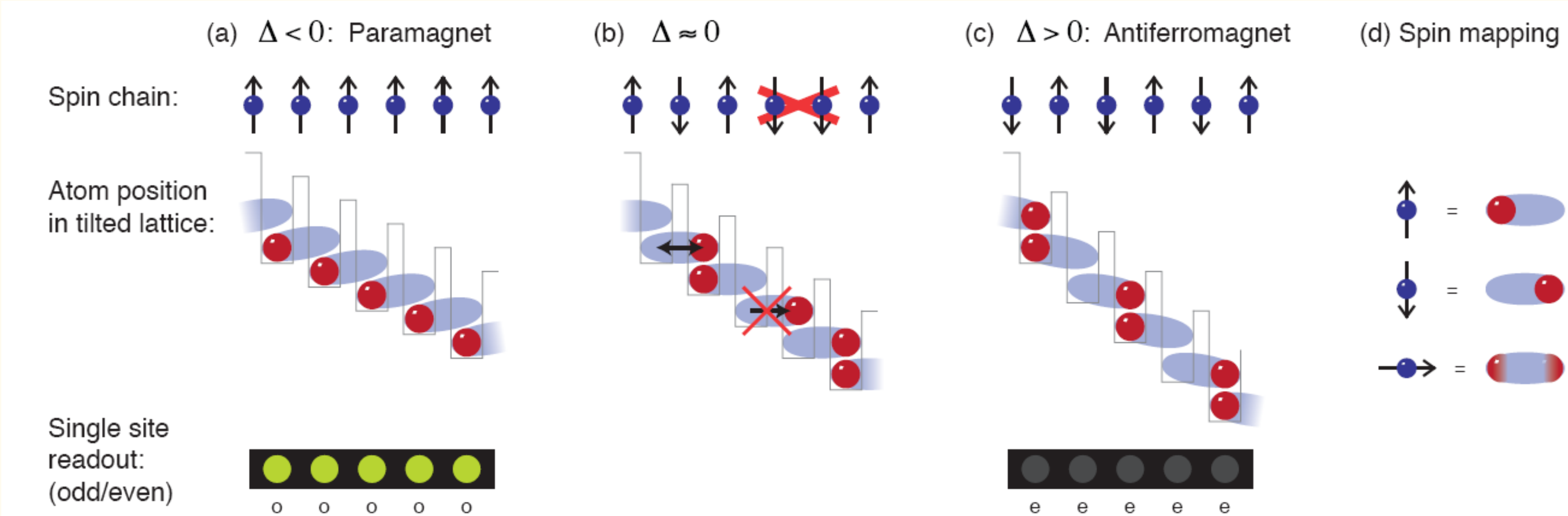
The dipole Hamiltonian can then be mapped to the spin Hamiltonian where the constraint is realized by the J term.

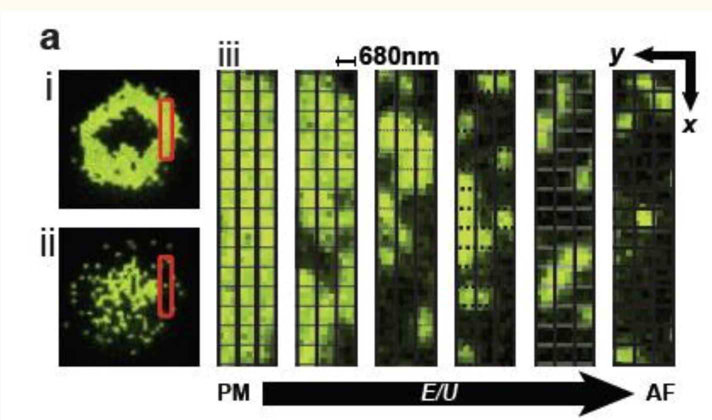
$$H = -\sqrt{M(M+1)}t\sum_j\left(d_j^\dagger + d_j\right) + (U-E)\sum_j d_j^\dagger d_j$$

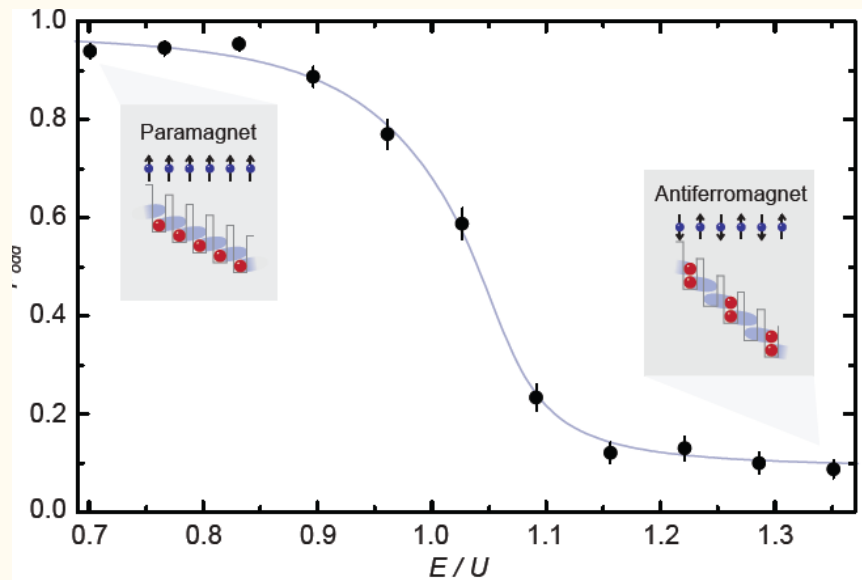
⇓

$$\begin{aligned} H &= J\sum_j S_z^j S_z^{j+1} - 2\sqrt{M(M+1)}t\sum_j S_x^j \\ &\quad - (J-\Delta)\sum_j S_z^j \\ &= J\sum_j\left(S_z^j S_z^{j+1} - h_x S_x^j - h_z S_z^j\right) \end{aligned}$$




(a) Δ < 0: Paramagnet
(b) Δ ≈ 0
(c) Δ > 0: Antiferromagnet
(d) Spin mapping
Spin chain:
Atom position
in tilted lattice:
Single site
readout:
(odd/even)
o o o o o
e e e e e


a
i
ii
iii
680nm
y
x
PM
E/U
AF


1.0
0.8
0.6
0.4
0.2
0.0
0.7
0.8
0.9
1.0
1.1
1.2
1.3
E / U
Paramagnet
Antiferromagnet

Post Quench dynamics of dipoles with reset

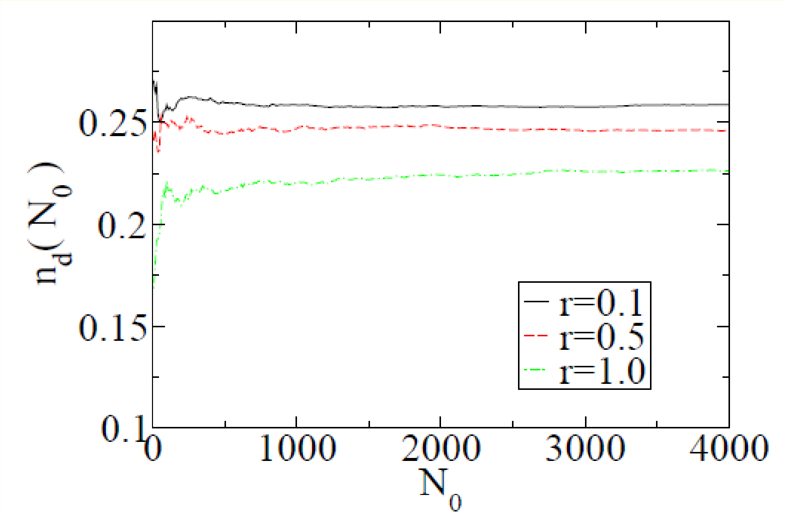
Start from the dipole vacuum state and quench to U=E at t=0.

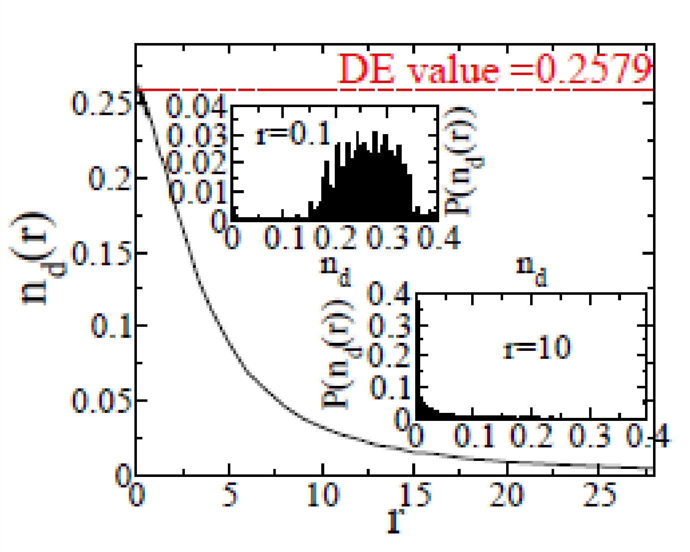
Compute the dipole excitation density

$$n_d(t) = \frac{1}{N}\sum_{\alpha\beta} c_\alpha c_\beta e^{-i\omega_{\beta\alpha}t}\langle\alpha|\sum_\ell n_\ell|\beta\rangle, \qquad H[\mathcal{E}_f]|\alpha\rangle = \epsilon_\alpha|\alpha\rangle$$

At t=0, n_d=0 since the initial ground state has zero dipole density.

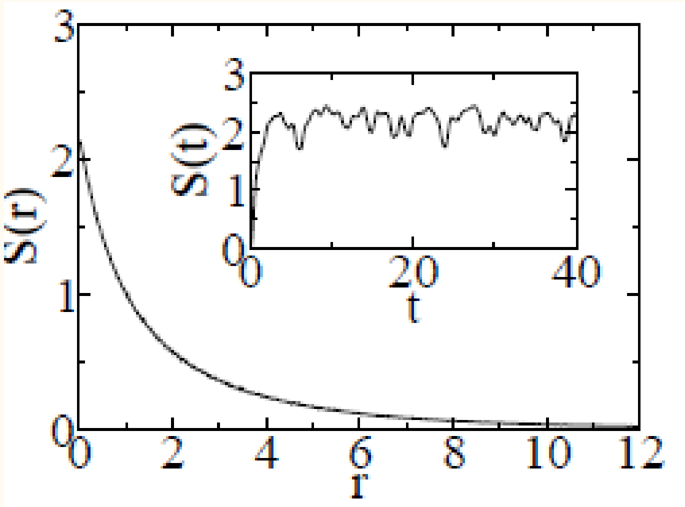
For dynamics without reset, one can compute the steady state dipole density After the quench to be $n_d(t\to\infty) = n_d^{\rm de} = 0.2575$





One reaches a steady state at large N_0

Crossover between DE and Zeno behaviors



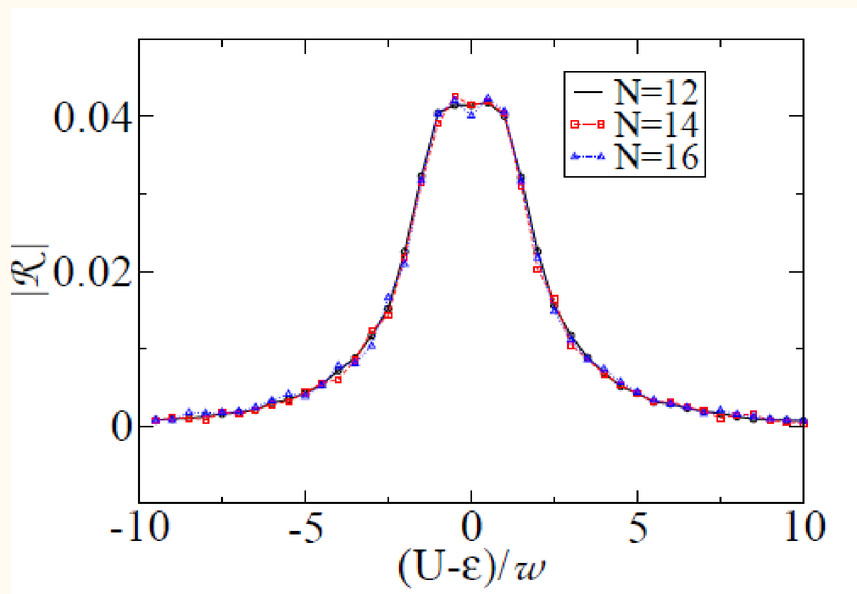
The half-chain steady state entanglement entropy S Shows a crossover between DE (S=2.18…) and Zeno (S-0) behavior.

The initial slope of n_d depends on the final value of U-E.

$$\mathcal{R} = \frac{dn_d(r)}{dr} = \sum_{\alpha>\beta} c_\alpha c_\beta \langle\alpha|n_d|\beta\rangle \frac{2\omega_{\beta\alpha}^2 r}{(r^2+\omega_{\beta\alpha}^2)^2}.$$

For r=0 (DE limit) and very large r (Zeno limit) the slope vanishes.

For a given finite r (say r=1) It peaks around the critical point where the number of states with which the initial state has finite overlap is maximal.



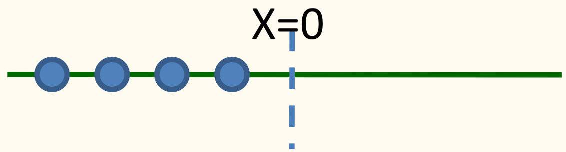
Reset dynamics with integrable models

Non-interacting Fermions on a chain

Consider the problem of a bunch of spinless fermions on a chain whose Hamiltonian is

$$H = -(1/2)\sum_m (c_m^\dagger c_{m+1} + \text{h.c.})$$

The initial state is chosen to be such that the fermions occupy left half of the chain.



How does the fermion density evolve as a function of time?

Exact solution of the equation of motion for the operators

$$c_m(t) \;=\; e^{iHt} c_m(0) e^{-iHt} = \sum_n i^{n-m} J_{n-m}(t) c_n(0),$$

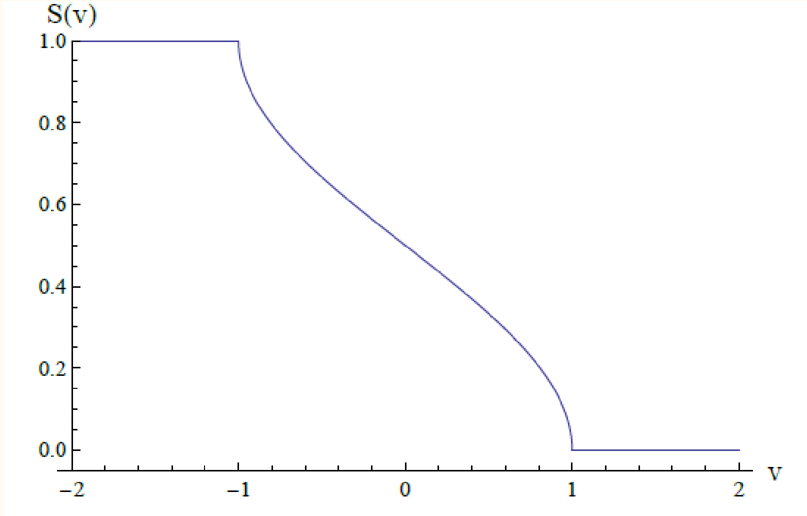
The density at any time t is given by

$$n_m(t) = \sum_{k=m}^{\infty} J_k^2(t)\,. \quad n_m(t) = 1 - n_{1-m}(t) \quad \text{for } m > 0\,.$$

At late times the density profile is given by a scaling function

$$n_m(t) \to S\left(\frac{m}{t}\right)$$

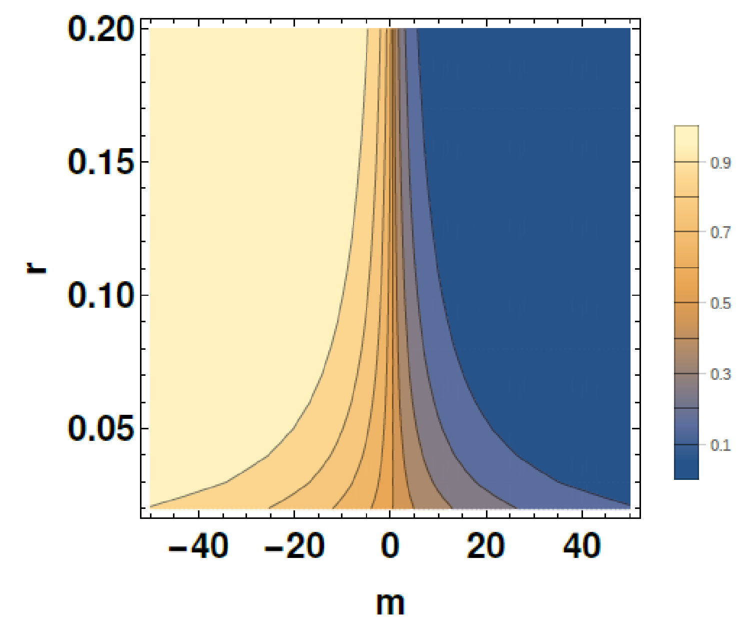
$$\begin{aligned} S(v) &= \frac{1}{\pi}\cos^{-1}(v) \quad \text{for } 0 < v < 1 \\ &= 0 \quad\quad\quad\quad\quad\;\; \text{for } v \geq 1 \end{aligned}$$



What is the stationary state of fermions under reset

The stationary state density profile of fermions in the presence of reset

$$n_m(r) \;=\; \langle m|\rho_{\rm stat}|m\rangle = r\int_0^\infty d\tau\, e^{-r\tau}\, n_m(\tau)$$



A crossover between the unitary and Zeno behavior.

The density profile for the fermions for small r and large m with fixed x=mr is given by a scaling function F

$$n_m(r) \to F(r\,m)\,.\qquad F(x) = \frac{1}{\pi}\int_x^\infty K_0(|y|)\,dy\,.$$

$$F(-x) = 1 - F(x)\,.$$

F(0)=1/2 which predicts the eventual uniform distribution for r=0

For large x, the tail of the density distribution matches with numerics

$$F(x) \to \frac{e^{-x}}{\sqrt{2\pi x}}$$

A class of integrable models

Free fermionic models in d dimensions with matrix structure of the Hamiltonian

$$H = \sum_{\vec{k}} \psi^{\dagger}_{\vec{k}} \left[(g(t) - b_{\vec{k}})\tau_3 + \Delta_{\vec{k}}\tau_1 \right] \psi_{\vec{k}},$$

Two component fermion creation operator
$\psi_{\vec{k}} = (c_{\vec{k}}, c^{\dagger}_{-\vec{k}})^T$;

Tuning parameter: chosen to be a periodic function of time according to a chosen protocol

Pauli matrices in Particle-hole space

H represents, for different realizations of g(t), Δ_k and b_k, Ising model in d=1, Kitaev model in d=2 , and Dirac fermions describing quasiparticles of Graphene and topological insulators (also in d=2).

Study of reset dynamics of H when the periodic drive, characterized by number of periods n and frequency ω, is interrupted after a random integer number of periods.

Floquet Hamiltonian

For stroboscopic measurements at the end of n drive periods, the system Is described by the Floquet Hamiltonian

$$U_{\vec{k}} = e^{-iH_{\vec{k}F}T}$$

For the present class of integrable models U_k is 2 by 2 matrix. Thus one may write

$$H_{\vec{k}F} = \vec{\sigma}\cdot\vec{\epsilon}_{\vec{k}}. \text{ where } \vec{\epsilon}_{\vec{k}} = (\epsilon_{1k}, \epsilon_{2k}, \epsilon_{3k}).$$

$$U_{\vec{k}} = e^{-i(\vec{\sigma}\cdot\vec{n}_{\vec{k}})\phi_{\vec{k}}}, \quad n_{\vec{k}} = \frac{\vec{\epsilon}_{\vec{k}}}{|\vec{\epsilon}_{\vec{k}}|} \quad ,\phi_{\vec{k}} = T|\vec{\epsilon}_{\vec{k}}|$$

One can express the Floquet Hamiltonian in terms of the parameters of U and hence in terms of the initial and final wavefunctions for each k

$$\begin{aligned}
\epsilon_{\vec{k}1} &= -|\vec{\epsilon}_{\vec{k}}|\sin(\theta_{\vec{k}})\sin(\gamma_{\vec{k}})\mathrm{Sgn}[\sin(\phi_{\vec{k}})]/D_{\vec{k}} \\
\epsilon_{\vec{k}2} &= -|\vec{\epsilon}_{\vec{k}}|\sin(\theta_{\vec{k}})\cos(\gamma_{\vec{k}})\mathrm{Sgn}[\sin(\phi_{\vec{k}})]/D_{\vec{k}} \\
\epsilon_{\vec{k}3} &= -|\vec{\epsilon}_{\vec{k}}|\cos(\theta_{\vec{k}})\sin(\alpha_{\vec{k}})\mathrm{Sgn}[\sin(\phi_{\vec{k}})]/D_{\vec{k}} \\
D_{\vec{k}} &= \sqrt{1-\cos^2(\theta_{\vec{k}})\cos^2(\alpha_{\vec{k}})} \\
|\vec{\epsilon}_{\vec{k}}| &= \arccos[\cos(\theta_{\vec{k}})\cos(\alpha_{\vec{k}})]/T
\end{aligned}$$

→ *Exact expression for the Floquet Hamiltonian*

Interpretation of the transition: Evolution matrix

The unitary evolution in the presence of a periodic drive after n drive cycles leads to

$$\psi^i = \prod_{\vec{k}} \psi^i_{\vec{k}} = \prod_{\vec{k}} (u^i_{\vec{k}}, v^i_{\vec{k}})^T \longrightarrow \psi^f = \prod_{\vec{k}} \psi^f_{\vec{k}} = \prod_{\vec{k}} (u^{nf}_{\vec{k}}, v^{nf}_{\vec{k}})^T.$$

The parametrization of U_k follows from its unitary nature: θ, α, and γ are real quantities

$$\psi^f_{\vec{k}} = U^n_{\vec{k}} \psi^i_{\vec{k}}, \quad \psi'_{\vec{k}} = U_{\vec{k}} \psi^i_{\vec{k}},$$

$$U_{\vec{k}} = \begin{pmatrix} \cos(\theta_{\vec{k}}) e^{i\alpha_{\vec{k}}} & \sin(\theta_{\vec{k}}) e^{i\gamma_{\vec{k}}} \\ -\sin(\theta_{\vec{k}}) e^{-i\gamma_{\vec{k}}} & \cos(\theta_{\vec{k}}) e^{-i\alpha_{\vec{k}}} \end{pmatrix}$$

One can find U_k as a function of initial and final values of the wavefunctions.

$$\sin^2(\theta_{\vec{k}}) = \Big[|u^f_{\vec{k}}|^2 v^{i2}_{\vec{k}} + |v^f_{\vec{k}}|^2 u^{i2}_{\vec{k}} - 2|u^f_{\vec{k}}||v^f_{\vec{k}}| u^i_{\vec{k}} v^i_{\vec{k}} \cos(\mu_{\vec{k}} - \mu'_{\vec{k}}) \Big] \quad (2($$

$$\gamma_{\vec{k}} = \arctan\left(\frac{|u^f_{\vec{k}}| v^i_{\vec{k}} \sin(\mu_{\vec{k}}) + u^i_{\vec{k}} |v^f_{\vec{k}}| \sin(\mu'_{\vec{k}})}{|u^f_{\vec{k}}| v^i_{\vec{k}} \cos(\mu_{\vec{k}}) - u^i_{\vec{k}} |v^f_{\vec{k}}| \cos(\mu'_{\vec{k}})} \right)$$

$$\alpha_{\vec{k}} = \arctan\left(\frac{|u^f_{\vec{k}}| u^i_{\vec{k}} \sin(\mu_{\vec{k}}) - v^i_{\vec{k}} |v^f_{\vec{k}}| \sin(\mu'_{\vec{k}})}{|u^f_{\vec{k}}| u^i_{\vec{k}} \cos(\mu_{\vec{k}}) + |v^f_{\vec{k}}| v^i_{\vec{k}} \cos(\mu'_{\vec{k}})} \right)$$

$$u^f_{\vec{k}} = |u^f_{\vec{k}}| \exp[i\mu_{\vec{k}}] \text{ and } v^f_{\vec{k}} = |v^f_{\vec{k}}| \exp[i\mu'_{\vec{k}}].$$

For an initial state (0,1), this yields the simple result

$$\sin(\theta_{\vec{k}}) = |u_{\vec{k}f}|, \ \alpha_{\vec{k}} = -\mathrm{Arg}(v_{\vec{k}f}) \text{ and } \gamma_{\vec{k}} = \mathrm{Arg}(u_{\vec{k}f}).$$

Relation of Floquet Hamiltonian with elements of correlation matrix

$$C_{ij} = \langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_n = 2 \sum_{\vec{k} \in \mathrm{BZ}/2} |u_{\vec{k}}(t)|^2 \cos(\vec{k} \cdot (\vec{i} - \vec{j}))/L^d \quad (2)$$

$$F_{ij} = \langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_n = 2 \sum_{\vec{k} \in \mathrm{BZ}/2} u^*_{\vec{k}}(t) v_{\vec{k}}(t) \sin(\vec{k} \cdot (\vec{i} - \vec{j}))/L^d$$

The elements of the correlation matrix depend on the final wavefunction

It can be expressed in terms of the initial wavefunction and the elements of the Floquet Hamiltonian after n drive cycles

$$\langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_n = \langle c^\dagger_{\vec{i}} c_{\vec{j}} \rangle_\infty - \frac{1}{(2\pi)^d} \int_{\vec{k} \in BZ/2} d^d k \cos(\vec{k} \cdot (\vec{i} - \vec{j})) \times (1 - \hat{n}^2_{\vec{k}3}) \cos(2n\phi_{\vec{k}}) \quad (7)$$

$$\langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_n = \langle c^\dagger_{\vec{i}} c^\dagger_{\vec{j}} \rangle_\infty + \frac{1}{(2\pi)^d} \int_{\vec{k} \in BZ/2} d^d k \sin(\vec{k} \cdot (\vec{i} - \vec{j})) \times \left[\hat{n}_{\vec{k}3} (\hat{n}_{\vec{k}1} + i\hat{n}_{\vec{k}2}) \cos(2n\phi_{\vec{k}}) + i(\hat{n}_{\vec{k}1} + i\hat{n}_{\vec{k}2}) \sin(2n\phi_{\vec{k}}) \right]$$

All elements of the correlation matrix can be expressed in terms of elements of H_F

We would like to find the reset averaged steady state value of these correlators.

→ Do these values differ from the reset free GGE values?

To this end, we single out the n dependent part of the correlators. These parts vanish in The steady state limit for reset free dynamics.

$$\delta C_{\vec{k}}(n) = f_1(\vec{k}) \cos(2n\phi_{\vec{k}})$$
$$\delta F_{\vec{k}}(n) = (f_2(\vec{k}) \cos(2n\phi_{\vec{k}}) + f_3(\vec{k}) \sin(2n\phi_{\vec{k}}))$$

$$f_1(\vec{k}) = -(1 - n_{\vec{k}3}^2), \quad f_2(\vec{k}) = -i\hat{n}_{\vec{k}3} f_3(\vec{k})$$
$$f_3(\vec{k}) = i(n_{\vec{k}1} + i n_{\vec{k}2}).$$

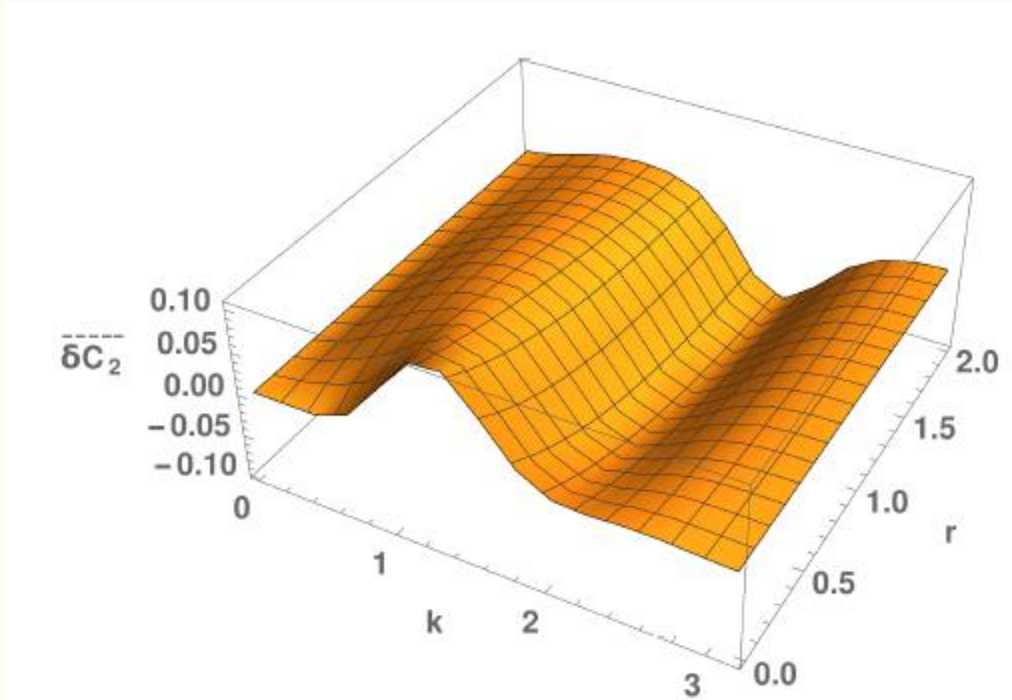
We let the system evolve till some random number of cycles before a reset. This random number is chosen from a distribution $P_r(n)$

$$P_r(n) = r^n \exp[-r]/n!.$$

For unitary dynamics, $\delta C_k(n)$ and $\delta F_k(n)$ vanishes in the steady state. Thus any finite $\delta C_k(n)$ or $\delta F_k(n)$ suggest the presence of a GGE which is different from the r=0 case.

Exact evaluation demonstrating a finite value of reset averaged value of δC_k

$$\overline{\delta C_{2\vec{k}}}(r) = f_1(\vec{k})[e^{-r(1-\cos(2\epsilon_{F\vec{k}}T))} \times \cos[\sin(2\epsilon_{F\vec{k}}T)] - e^{-r}]$$



The reset averaged value of δC_k is finite for all k except for k=0, π

Thus the GGE reached at finite r yields different correlation functions compared to their r=0 counterpart

Conclusion

Quantum dynamics, interrupted by stochastic resets, leads to interesting steady/stationary states.

For non-integrable systems, such states are in general athermal; the off-diagonal elements of the density matrix do not vanish for such states.

They provide a natural interpolation between unitary dynamics and quantum Zeno limit of a generic quantum system.

For integrable models, such as fermion chains in 1D, they lead to novel stationary state density distribution.

For periodically driven Dirac model, it leads to the presence of new GGEs.

Ultracold atoms may serve as a platform for studying such dynamics since some of such systems have been used to study quantum Zeno effect.