

# The Information Paradox and the Infalling Observer

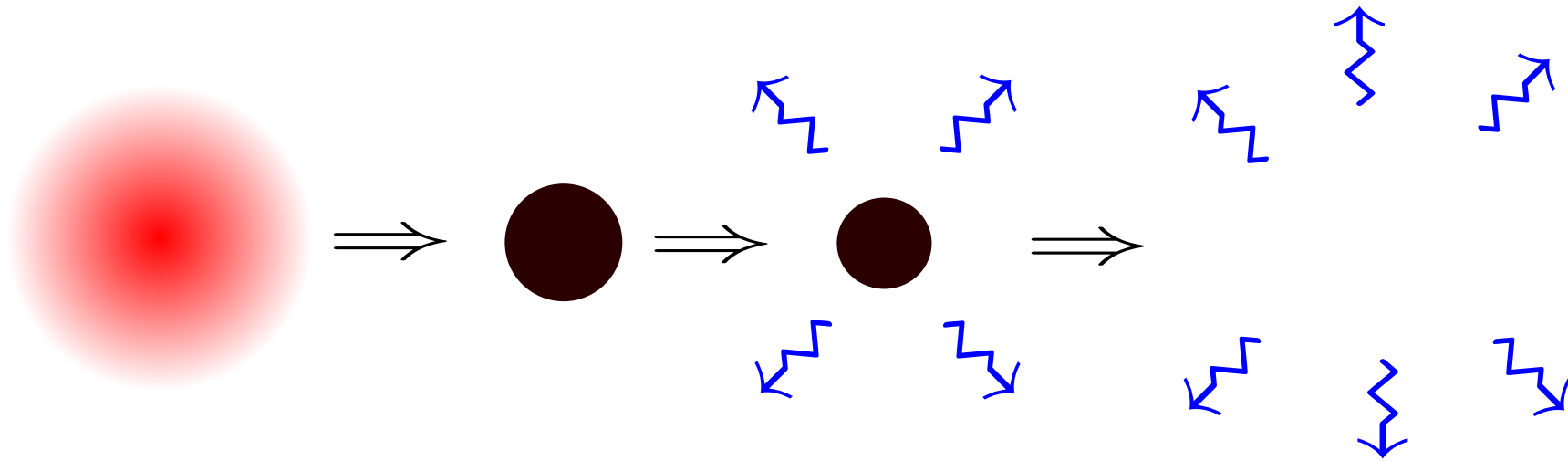
Kyriakos Papadodimas  
University of Groningen

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TIFR, Mumbai

based on arXiv:1211.6767, K.P and Suvrat Raju

# Hawking Radiation vs Unitarity

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gas cloud  
in pure state

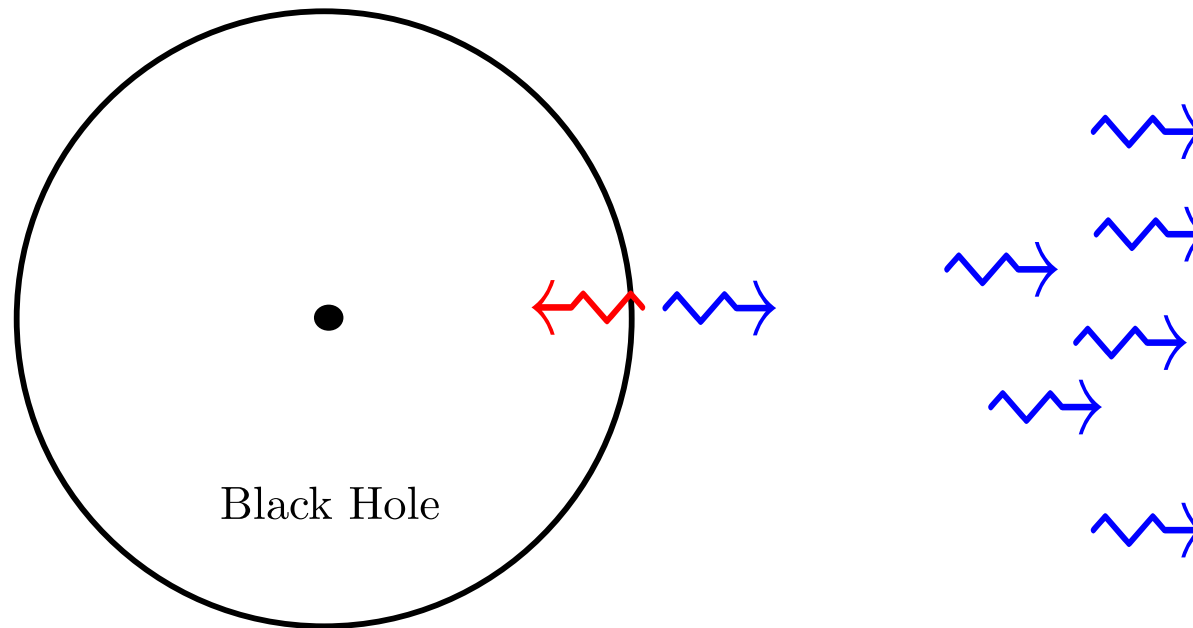
Hawking radiation

$|\Psi_0\rangle \Rightarrow \dots \Rightarrow \rho_{\text{thermal}}$

**INCONSISTENT WITH UNITARY EVOLUTION**

# Information Paradox

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- For unitarity: final state must carry information of initial state
- (In some sense) Hawking quanta are created near the horizon
- **If horizon is featureless and we have locality, how is information transferred to outgoing radiation?**

# Information Paradox

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We have tension between

- Unitarity
- Locality
- Equivalence Principle (smooth horizon)

**CAN SMALL CORRECTIONS RESOLVE THE PARADOX?**

# Information Paradox

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Will try to argue that the answer is: **YES**

**An “acceptably small” amount of non-locality is sufficient to restore unitarity and at the same time preserve the smoothness of the horizon**

# Modification of black hole geometry?

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- (Many) Proposals to modify interior of black hole. Relevant for our discussion:

## **A. FUZZBALL PROPOSAL** (Mathur, Avery, Chowdhury,...)

- quantum effects inside black hole spread all the way to horizon
- interior black hole geometry  $\neq$  Schwarzschild solution
- black hole microstates have no horizon
- infalling observer (partly) feels deviations from semi-classical General Relativity

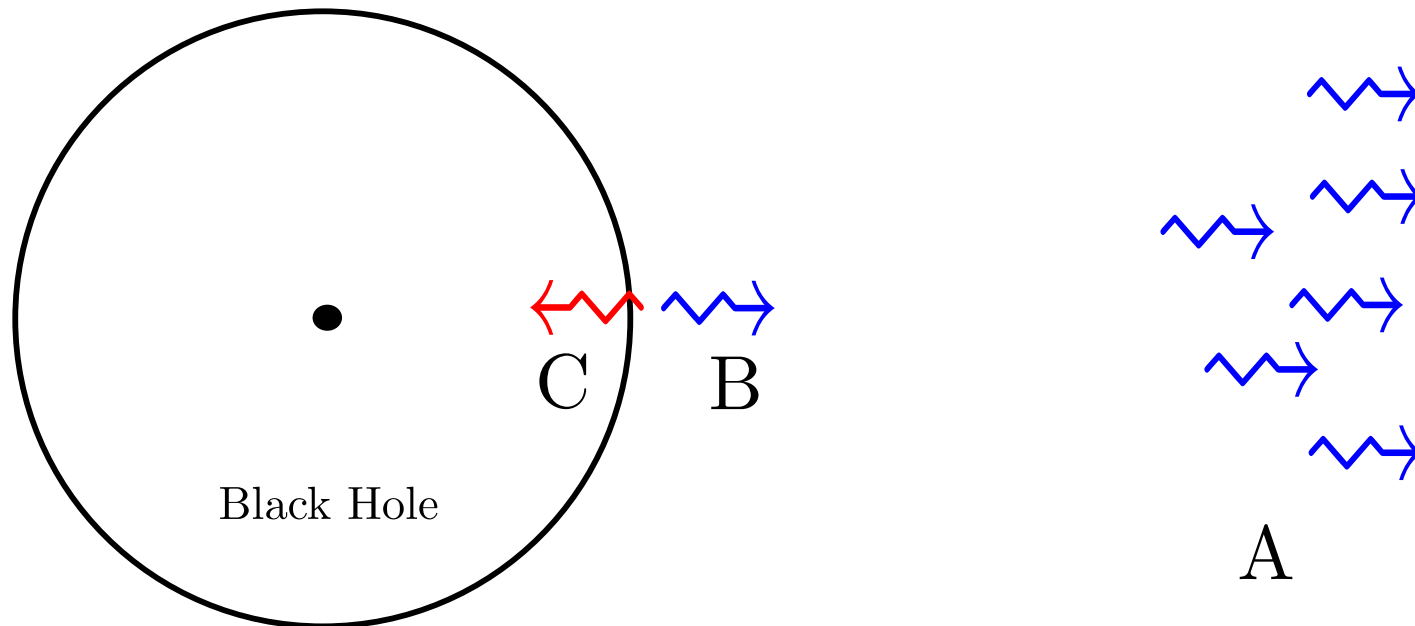
## **B. FIREWALLS** (Almheiri, Marolf, Polchinski, Sully)

- horizon is not smooth
- infalling observer burns when crossing the horizon (firewall)

# Sharpened version of the information paradox (Mathur, AMPS)

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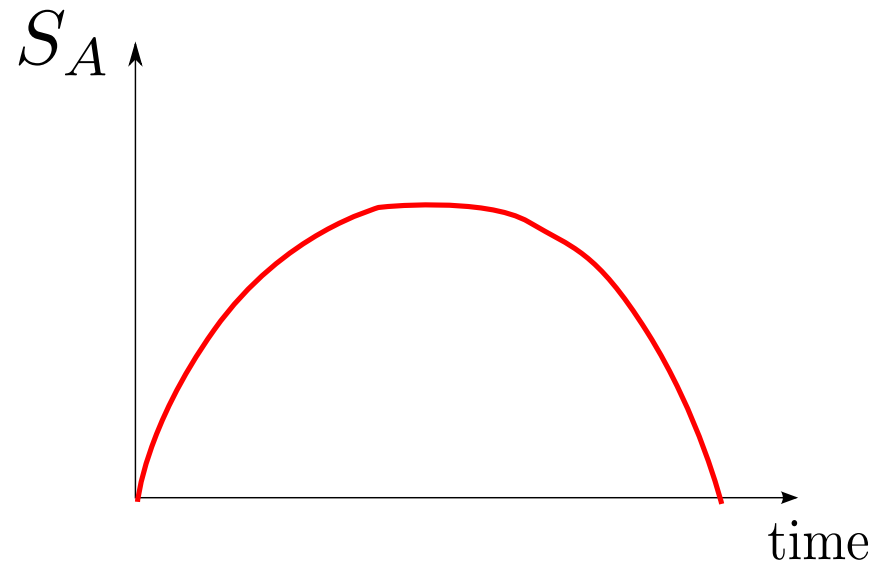
Consider the process of Hawking radiation



- A: old radiation, far from black hole
- B: newly created Hawking particle, outgoing
- C: ingoing partner of B

# Sharpened version of the information paradox (Mathur, AMPS)

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Consider the entropy of radiation  $A$  as a function of time

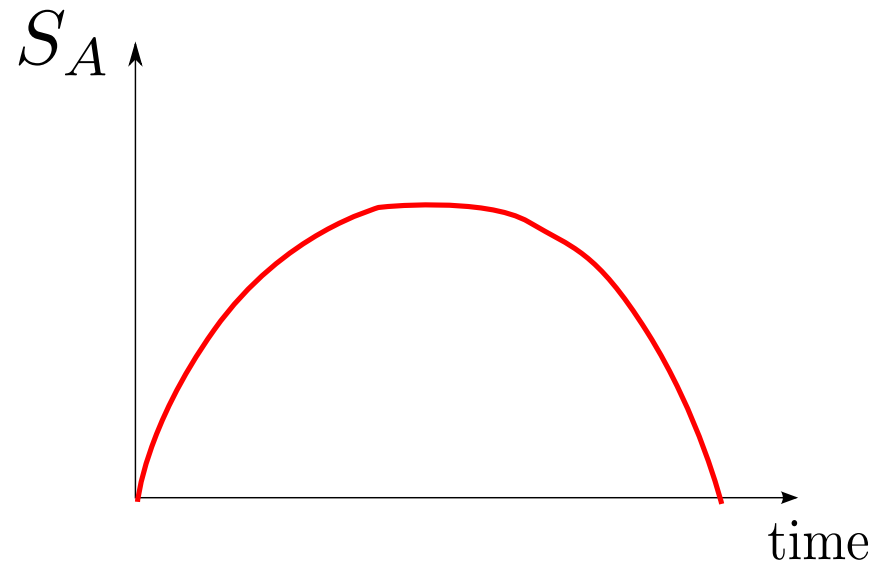
$$S_A = -\text{Tr}(\rho_A \log \rho_A)$$

If initial state is pure then  $S_A$  must go to zero after complete evaporation of the black hole



# Sharpened version of the information paradox (Mathur, AMPS)

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In the beginning adding a  $B$  to  $A$  increases the entropy i.e. we expect

$$S_{AB} > S_A$$

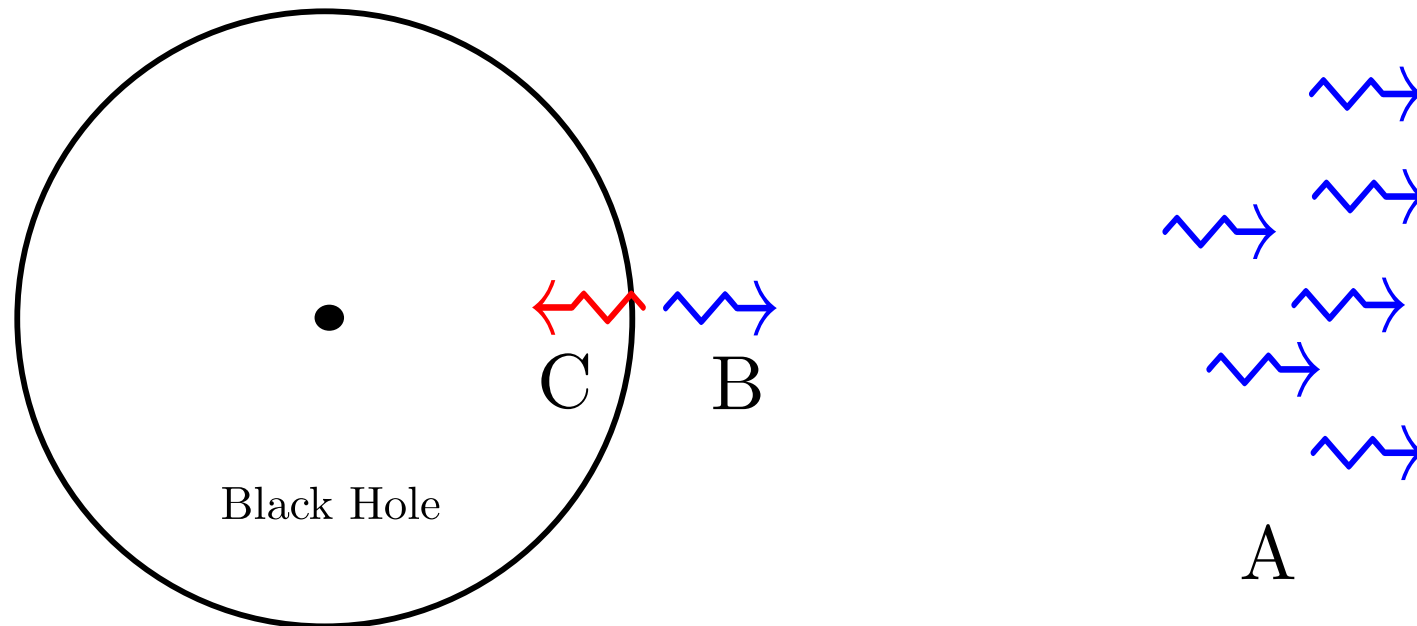
but eventually this must turn around and for an **old black hole** we expect

$$S_{AB} < S_A$$

# Sharpened version of the information paradox (Mathur, AMPS)

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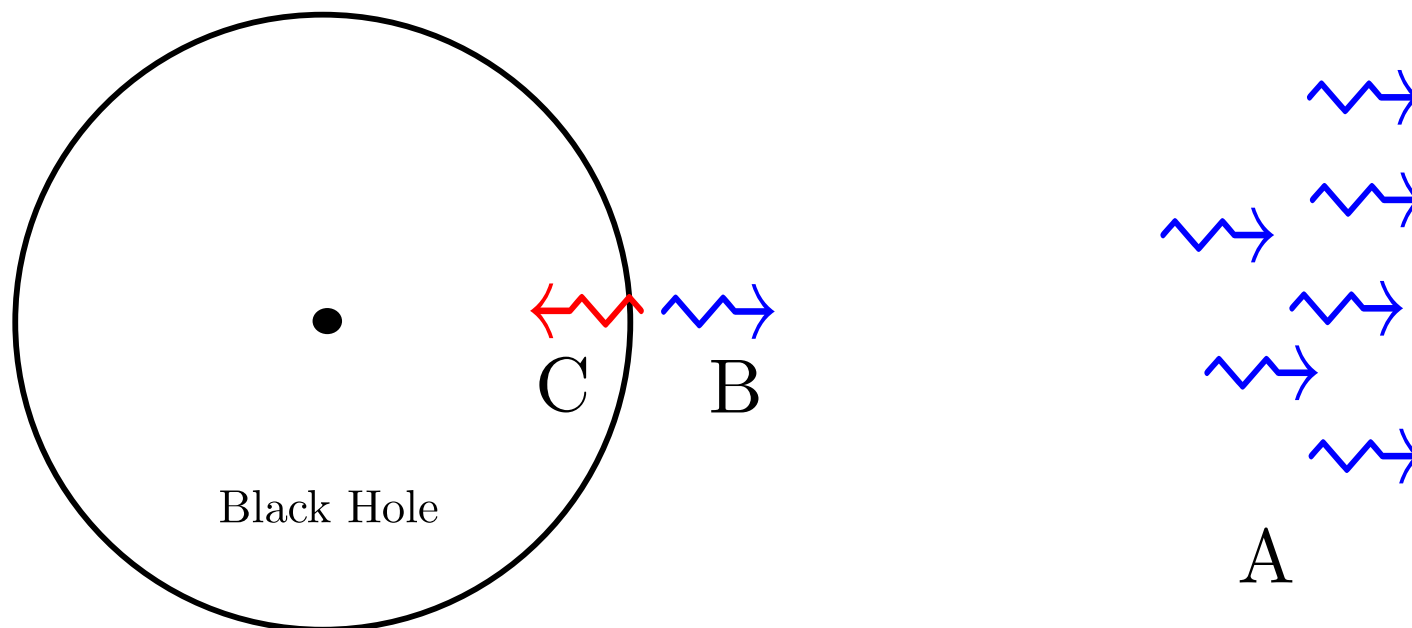
Consider the process of Hawking radiation



- For **free infall**:  $B$  must be fully entangled to  $C$
- For **information recovery**:  $B$  must be very entangled to  $A$
- Are these two statements compatible?

# Sharpened version of the information paradox (Mathur, AMPS)

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**Strong subadditivity theorem:** for 3 independent systems A,B,C we have

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have  $S_{BC} \approx 0$  and  $S_C \approx \log 2$  which would imply

$$S_{AB} > S_A$$

# Modification of black hole geometry?

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- Both the **Fuzzball** and the **Firewall** proposals predict that an infalling observer will notice something dramatic when crossing the horizon
- In contradiction with prediction of General Relativity
- Based on an **indirect** argument i.e. on the information paradox. **Do not directly address the experience of infalling observer.**

## Free infall or not?

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- So, does an infalling observer notice something or not?
- To settle this question: should use a non-perturbative formulation of Quantum Gravity
- **AdS/CFT:** Non-perturbative description of Quantum Gravity in AdS via holographically dual gauge theory

# Black Holes in AdS/CFT

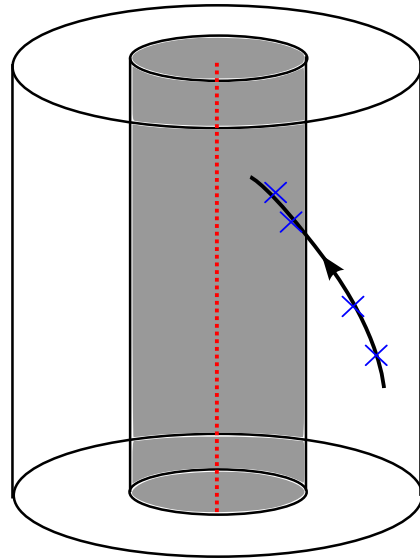
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Main goals:

- Is the region behind the horizon encoded in the boundary CFT?
- Understand what happens to an observer falling into a black hole
- Address the information paradox

# An infalling observer in AdS

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- Consider a big black hole in AdS and an observer freely falling towards it
- The observer performs local experiments (blue crosses in diagram)
- We will **reconstruct** these experiments from the boundary gauge theory
- We will show that the results of these experiments are the same as those of semi-classical GR and hence **AdS/CFT seems to be in conflict with (some versions of) the Fuzzball and Firewall proposals\***

# Reconstructing local observables in empty AdS

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In AdS/CFT we know that

“S-matrix elements” in AdS  $\Leftrightarrow$  Correlation functions in CFT

Local bulk correlators in AdS  $\Leftrightarrow$  ?



Our first goal:

**Construct local bulk observables from CFT**

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Hamilton, Kabat, Lifschytz, Lowe,...)

# Reconstructing local observables in empty AdS

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- Large  $N$  CFTs contain in their spectrum **generalized free fields** i.e. (composite) local operators  $\mathcal{O}(x)$  whose correlators factorize

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_{2n}) \rangle = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle \dots \langle \mathcal{O}(x_{2n-1}) \mathcal{O}(x_{2n}) \rangle + \dots$$

- Factorization  $\approx$  “superposition principle”. However, the operators  $\mathcal{O}$  **do not satisfy any linear equation of motion in the CFT**.
- Hence, they are not **free fields**, but rather **generalized free fields**
- Excitations created by  $\mathcal{O}$  behave like **ordinary free particles** in a higher dimensional AdS spacetime

# Reconstructing local observables in empty AdS

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- First we define the Fourier modes of  $\mathcal{O}_{\omega, \vec{k}}$  by

$$\mathcal{O}(t, \vec{x}) = \int dt d\vec{x} \left( \mathcal{O}_{\omega, \vec{k}} e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.} \right)$$

- Conformal invariance fixes the 2-point function to be

$$\langle \mathcal{O}(t, \vec{x}) \mathcal{O}(0, \vec{0}) \rangle = \left( \frac{-1}{t^2 - \vec{x}^2 - i\epsilon} \right)^\Delta$$

- From this we find

$$\mathcal{O}_{\omega, \vec{k}} |0\rangle = 0, \quad \omega > 0$$

and

$$[\mathcal{O}_{\omega, \vec{k}}, \mathcal{O}_{\omega', \vec{k}'}^\dagger] = \mathcal{N} \theta(\omega^2 - \vec{k}^2) (\omega^2 - \vec{k}^2)^{\Delta-d/2} \delta(\omega - \omega') \delta(\vec{k} - \vec{k}')$$

# Reconstructing local observables in empty AdS

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- From this commutation relation we see that the modes  $\mathcal{O}_{\omega, \vec{k}}$  create a **freely generated Fock space** of excitations.
- For an ordinary free field we have dispersion relation  $\omega^2 = \vec{k}^2 + m^2$ .
- For the generalized free fields, excitations labeled by the **independent** parameters  $\omega$  and  $\vec{k}$ .
- $\Rightarrow$  excitations behave like higher dimensional excitations
- Behave like ordinary free particles in AdS

# Reconstructing local observables in empty AdS

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- Consider AdS in Poincare patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- and a scalar field satisfying  $\square\phi = m^2\phi$ .
- We take  $m^2$  to be related to the conformal dimension  $\Delta$  of  $\mathcal{O}$  by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + d^2/4}$$

- For each value of  $\omega, \vec{k}$  we find a solution of the Klein-Gordon equation of the form

$$f_{\omega, \vec{k}}(t, \vec{x}, z) = e^{-i\omega t + i\vec{k}\vec{x}} z^{d/2} J_{\Delta-d/2}(\sqrt{\omega^2 - \vec{k}^2} z)$$

# Reconstructing local observables in empty AdS

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- We construct non-local CFT operators as

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left( \mathcal{O}_{\omega, \vec{k}} f_{\omega, \vec{k}}(t, \vec{x}, z) + \text{h.c.} \right)$$

Notice that while these are labeled by the coordinate  $z$ , they are really operators in the CFT. They are smeared, nonlocal operators.

- Using the previous results we can show that they satisfy

$$\square_{\text{AdS}} \phi_{\text{CFT}} = m^2 \phi_{\text{CFT}}$$

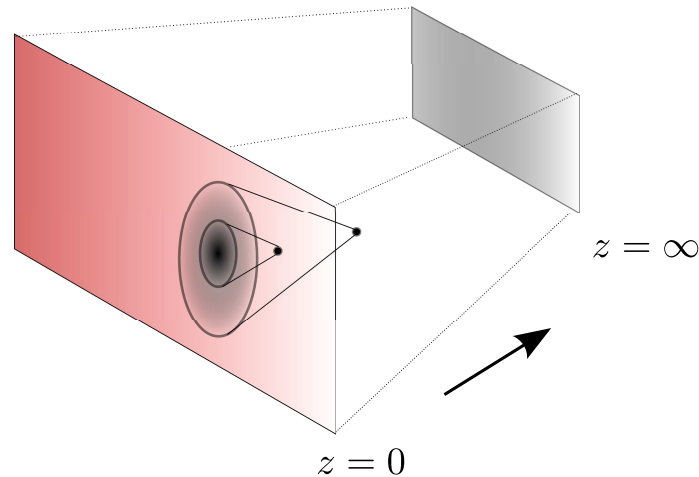
and

$$[\phi_{\text{CFT}}(t, \vec{x}, z), \phi_{\text{CFT}}(t', \vec{x}', z')] = 0$$

for points  $(t, \vec{x}, z)$  and  $(t', \vec{x}', z')$  spacelike **with respect to the AdS metric**.

# Reconstructing local observables in empty AdS

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- From the point of view of the CFT, coordinate  $z$  is an "auxiliary" parameter, which controls the smearing of the operators
- We can explicitly see how AdS space emerges from the lower dimensional CFT, as the combination of the coordinates  $t, \vec{x}$  together with the extra parameter  $z$
- **In particular, we can "understand" the  $d + 1$  causal structure from the  $d$ -dimensional CFT**

# Reconstructing local observables in empty AdS

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- We can also interchange the order of the Fourier transforms to write

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int dt' d\vec{x}' K(t, \vec{x}, z ; t', \vec{x}') \mathcal{O}(t', \vec{x}')$$

where  $K$  is some Kernel — sometimes called the *transfer function*.

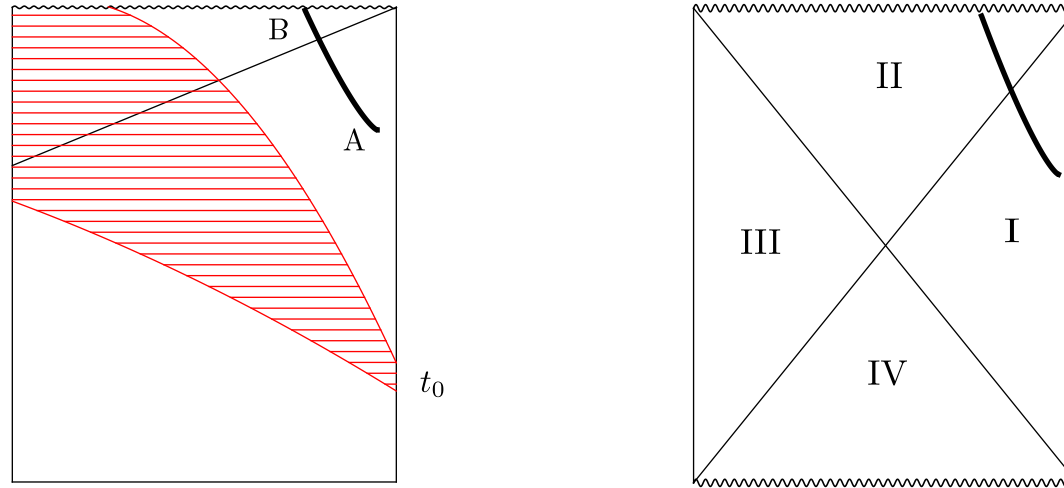
Notice: it is not the same thing as the bulk-to-boundary propagator.

- To summarize, using the operators  $\phi_{\text{CFT}}(t, \vec{x}, z)$  we can study local bulk questions in AdS from the point of view of the dual gauge theory.
- We want to repeat the same construction in the presence of a black hole in AdS, and then push the operators  $\phi_{\text{CFT}}(t, \vec{x}, z)$  behind the horizon and see what we get



# Black Holes in AdS

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In AdS/CFT, Black Hole = Quark-Gluon Plasma (QGP) in gauge theory

BH formed by collapse  $\approx$  Eternal Black Hole in AdS

Typical (QGP) pure state  $|\Psi\rangle$   $\approx$  Thermal ensemble in gauge theory

# CFT Correlators at finite temperature

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We use the notation

$$\langle A_1 \dots A_n \rangle_\beta = \frac{1}{Z(\beta)} \text{Tr} \left( e^{-\beta H} A_1 \dots A_n \right)$$

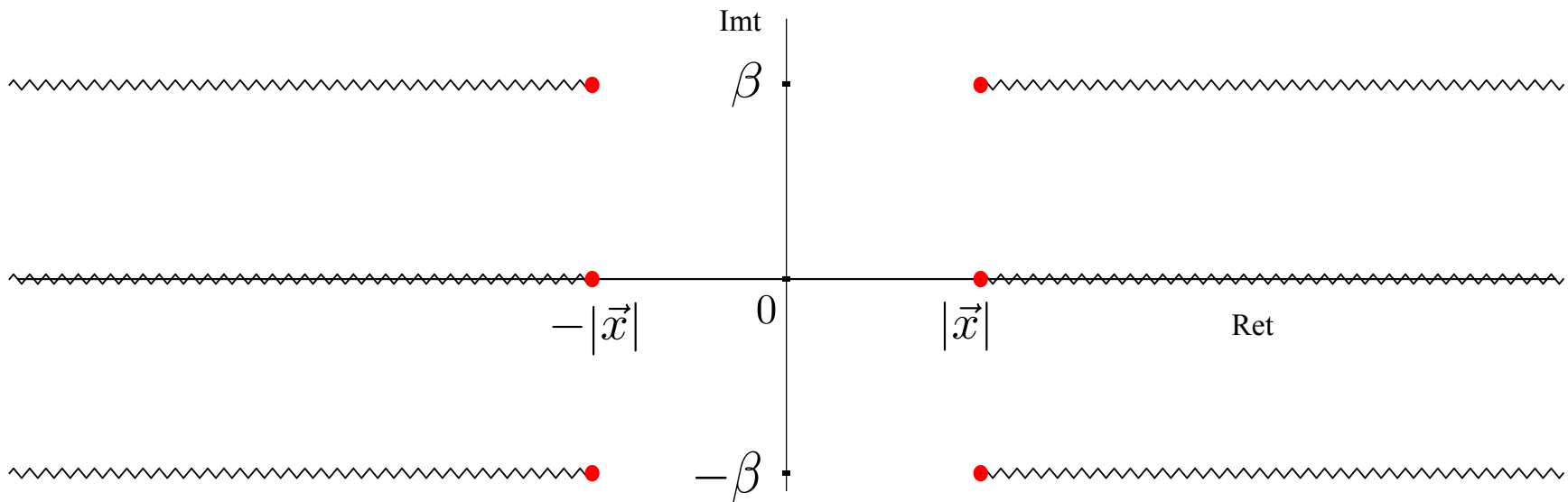
- At large  $N$  thermal correlators factorize

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_{2n}) \rangle_\beta = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \dots \langle \mathcal{O}(x_{2n-1}) \mathcal{O}(x_{2n}) \rangle_\beta + \dots$$

- Of course  $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \neq \langle 0 | \mathcal{O}(x_1) \mathcal{O}(x_2) | 0 \rangle$
- Factorization can fail if we scale the parameters of the correlator with  $N$  (for example: number of insertions, distances  $x_i$ , dimension of operators etc.)

# CFT Correlators at finite temperature

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- Consider the 2-point function  $G_\beta(t, \vec{x}) = \langle \mathcal{O}(t, \vec{x}) \mathcal{O}(0, \vec{0}) \rangle_\beta$
- Satisfies the **KMS condition**

$$G_\beta(t - i\beta, \vec{x}) = G_\beta(-t, -\vec{x})$$

- In Fourier space

$$G_\beta(-\omega, \vec{k}) = e^{-\beta\omega} G_\beta(\omega, \vec{k})$$

# CFT Correlators at finite temperature

---

- If we again define the Fourier modes  $\mathcal{O}_{\omega, \vec{k}}$  by

$$\mathcal{O}(t, \vec{x}) = \int dt d^{d-1}x \left( \mathcal{O}_{\omega, \vec{k}} e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.} \right)$$

- we find that they satisfy an oscillator algebra

$$[\mathcal{O}_{\omega, \vec{k}}, \mathcal{O}_{\omega', \vec{k}'}^\dagger] = \left( G_\beta(\omega, \vec{k}) - G_\beta(-\omega, \vec{k}) \right) \delta(\omega - \omega') \delta(\vec{k} - \vec{k}')$$

- but now the (canonically normalized) oscillators are thermally populated

$$\langle \hat{\mathcal{O}}_{\omega, \vec{k}}^\dagger \hat{\mathcal{O}}_{\omega, \vec{k}} \rangle_\beta = \frac{1}{e^{\beta\omega} - 1}$$

(this is the CFT analogue of the “thermal atmosphere” of the black hole)

# Reconstructing the region outside the black hole

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- Consider a black hole in AdS given by the metric

$$ds^2 = \frac{-h(z)dt^2 + dx^2 + h^{-1}(z)dz^2}{z^2}, \quad h(z) = 1 - \frac{z^d}{z_0^d}$$

- Look for solutions of the Klein-Gordon equation of the form

$$f_{\omega, \vec{k}}(t, \vec{x}, z) = e^{-i\omega t + i\vec{k}\vec{x}} \psi_{\omega, \vec{k}}(z)$$

- For every  $(\omega, \vec{k})$  there is a unique solution, normalizable at the boundary  $z = 0$ .
- These are the usual "Schwarzschild modes" that we get when we quantize a scalar field near a black hole. We identify

$$f_{\omega, \vec{k}}(t, \vec{x}, z) \quad \Leftrightarrow \quad \mathcal{O}_{\omega, \vec{k}}$$

# Reconstructing the region outside the black hole

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- As before, we can write nonlocal CFT operators

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left( \mathcal{O}_{\omega, \vec{k}} f_{\omega, \vec{k}}(t, \vec{x}, z) + \text{h.c.} \right)$$

- which behave like local fields around a black hole

$$(\square - m^2)\phi_{\text{CFT}} = 0$$

$$[\phi_{\text{CFT}}(t, \vec{x}, z), \phi_{\text{CFT}}(t', \vec{x}', z')] = 0 \quad , \quad \text{for spacelike points}$$

- and more generally

$$\langle \phi_{\text{CFT}}(P_1) \dots \phi_{\text{CFT}}(P_n) \rangle_{\beta} = \langle \phi_{\text{gravity}}(P_1) \dots \phi_{\text{gravity}}(P_n) \rangle_{\text{Hartle Hawking}}$$

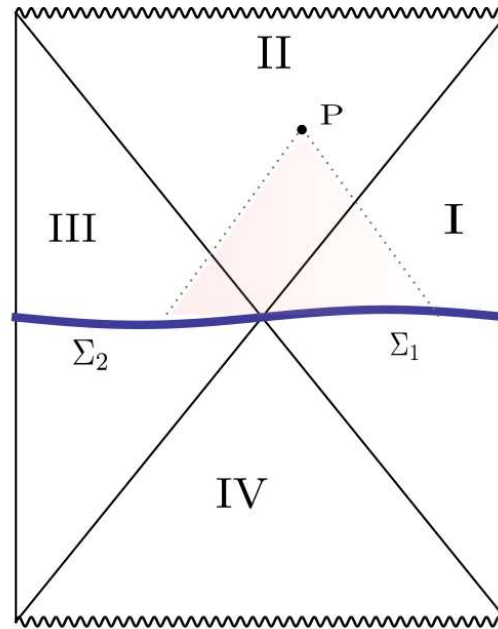
# Reconstructing the region outside the black hole

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- We have understood how to reconstruct the region outside the black hole from the point of view of the gauge theory
- We can write local observables in gravity as non-local operators in the gauge theory

# Falling behind the horizon

- Penrose diagram of (eternal) AdS black hole



- Cauchy slice for points in II is  $\Sigma_1 \oplus \Sigma_2$
- To reconstruct local operator at  $P$  we need **both** modes on  $\Sigma_1$  and  $\Sigma_2$

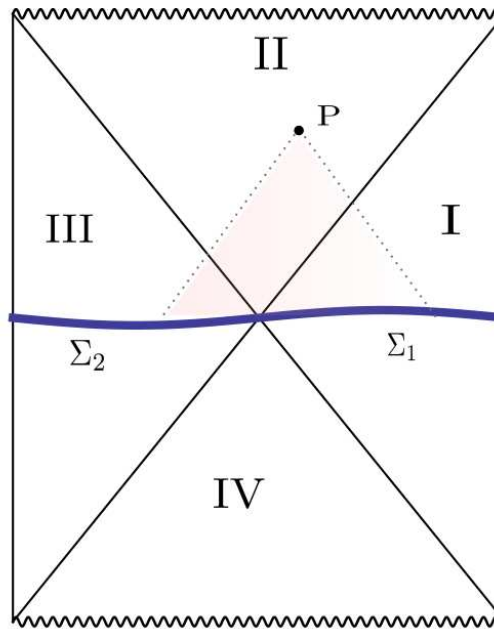
$$\text{Modes on } \Sigma_1 \quad \Leftrightarrow \quad \mathcal{O}_{\omega, \vec{k}}$$

$$\text{Modes on } \Sigma_2 \quad \Leftrightarrow \quad ?$$



# Falling behind the horizon

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- Maldacena: eternal black hole = 2 copies of CFT in entangled state
- In this formalism, modes on  $\Sigma_2$  are the operators  $\tilde{\mathcal{O}}_{\omega, \vec{k}}$  in the second copy of the CFT
- Do we really need the two entangled copies?
- If we work **with a single CFT**, what is the meaning of the operators  $\tilde{\mathcal{O}}_{\omega, \vec{k}}$ ?

# Coarse-graining and doubling of operators

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- Consider complicated (ergodic) system in pure state  $|\Psi\rangle$
- Intuitive expectation  $\Rightarrow$  system "thermalizes"
- For some observables  $\{A_i\}$ - called **coarse-grained observables**, their correlators on  $|\Psi\rangle$  come close to thermal correlators

$$\langle \Psi | A_1 \dots A_n | \Psi \rangle \approx \text{Tr} \left( e^{-\beta H} A_1 \dots A_n \right)$$

- This is not true for all observables, there are also **fine grained observables** which do not thermalize
- Hilbert space has the form

$$\mathcal{H} = \mathcal{H}_{\text{coarse}} \otimes \mathcal{H}_{\text{fine}}$$

- $\mathcal{H}_{\text{fine}}$  plays the role of a **heat bath** for  $\mathcal{H}_{\text{coarse}}$

# Coarse graining and doubling of operators

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- Every state  $|\Psi\rangle$  can be written as

$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$

where  $|\Psi_i^c\rangle, |\Psi_j^f\rangle$  are orthonormal basis of  $\mathcal{H}_{\text{coarse}}$  and  $\mathcal{H}_{\text{fine}}$  respectively

- If  $\mathcal{H}_{\text{coarse}}$  thermalizes, it means that the reduced density matrix

$$\rho_{\text{coarse}} = Z_c^{-1} e^{-\beta H_{\text{coarse}}}$$

- which means we can redefine our orthonormal basis such that

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

# Coarse graining and doubling of operators

---

- The state  $|\Psi\rangle$  can be written as

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

- Consider a **coarse-grained** operator acting on  $\mathcal{H}_{\text{coarse}}$  as

$$A = \sum_{ij} a_{ij} |\hat{\Psi}_i^c\rangle \otimes \langle \hat{\Psi}_j^c|$$

- Then we **define** a new operator

$$\tilde{A} = \sum_{ij} a_{ij}^* |\hat{\Psi}_i^f\rangle \otimes |\hat{\Psi}_j^f\rangle$$

acting on the fine-grained Hilbert space.

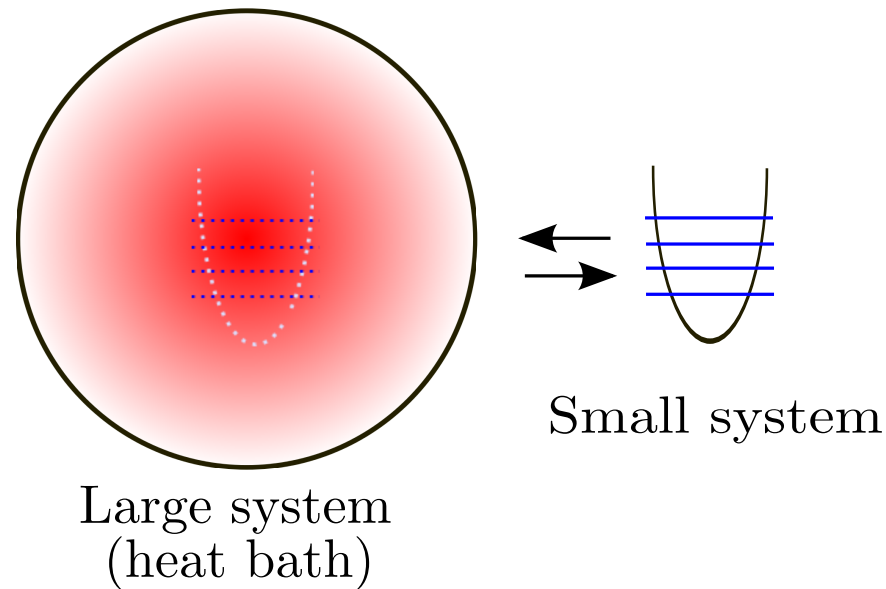
## Coarse graining and doubling of operators

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- We started with a set of coarse-grained operators  $A_i$  which thermalize.
  
- The operators  $\tilde{A}_i$  constructed as above, have the properties
  1. Their operator algebra  $\tilde{A}_i$  is isomorphic to that of  $A_i$
  2. Operators  $A_i$  commute with operators  $\tilde{A}_i$

# Coarse graining and doubling of operators

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## SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH

- For us the Quark-Gluon-Plasma is the heat bath
- The glueball operators  $\mathcal{O}_i$  are the coarse-grained observables
- They are mirrored in the QGP, which leads to new operators  $\tilde{\mathcal{O}}_i$
- This mirroring involves the fine-degrees of freedom

## Coarse graining and doubling of operators

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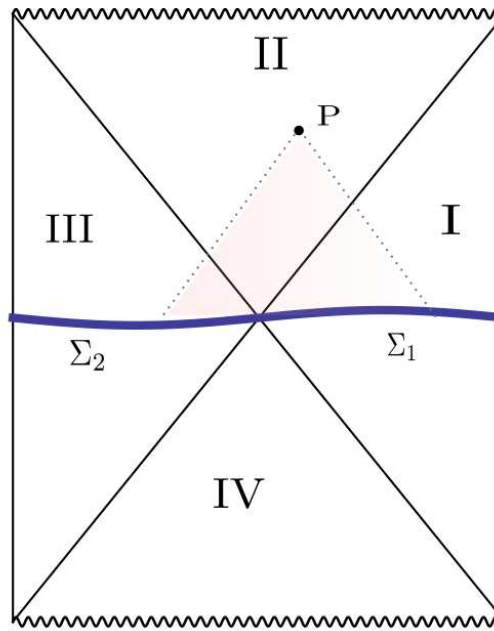
- At large  $N$ , correlation functions of the mirrored operators  $\tilde{\mathcal{O}}$  on a pure state, agree with those of analytically continued operators

$$\mathcal{O}(t + i\beta/2)$$

- However the  $\tilde{\mathcal{O}}$ , **as operators acting on pure states**, were defined via the coarse/fine-grained decomposition

# Falling behind the horizon

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Modes on  $\Sigma_1$   $\Leftrightarrow$   $\mathcal{O}_{\omega, \vec{k}}$

Modes on  $\Sigma_2$   $\Leftrightarrow$   $\tilde{\mathcal{O}}_{\omega, \vec{k}}$

where  $\tilde{\mathcal{O}}_{\omega, \vec{k}}$  are the Fourier transforms of the mirrored operators  $\tilde{\mathcal{O}}$



## Local operators behind the horizon

---

Using both  $\mathcal{O}_{\omega, \vec{k}}$  and  $\tilde{\mathcal{O}}_{\omega, \vec{k}}$  we can write local observables behind the horizon of the black hole.

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left[ \mathcal{O}_{\omega, \vec{k}} g_{\omega, \vec{k}}^{(1)}(t, \vec{x}, z) + \tilde{\mathcal{O}}_{\omega, \vec{k}} g_{\omega, \vec{k}}^{(2)}(t, \vec{x}, z) + \text{h.c.} \right]$$

here  $g^{(1),(2)}$  are solutions of the Klein-Gordon equation in region  $II$

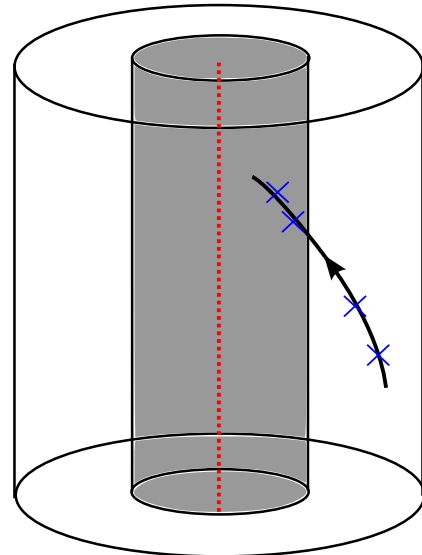
In the large  $N$  limit, correlators of  $\phi_{\text{CFT}}(t, \vec{x}, z)$  on a typical pure state  $|\Psi\rangle$  (corresponding to a black hole microstate) agree with those computed in semiclassical gravity.

**We have reconstructed both the exterior and the interior of the black hole from the dual gauge theory**

## Fate of the infalling observer

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Using the operators  $\phi_{\text{CFT}}$  we can reconstruct the experiments of the infalling observer



**MAIN CONCLUSION:** For a big black hole, an infalling semi-classical observer does not notice anything special when crossing the horizon. In particular, he does not see a firewall or a fuzzball.

## Various subtleties

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- Sensitivity to pure state  $|\Psi\rangle$ ?
- Including  $1/N$  corrections?
- Spread of transfer function as we approach the horizon?
- Sensitivity to late times - Poincare recurrences?

If large  $N$  expansion at finite temperature holds, we can(?) show that they are under control.

# The information paradox

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- We argued that infalling observer can use effective field theory (EFT) to describe physics near the horizon and does not notice anything special.
- Mathur and AMPS argue that free infall cannot be compatible with unitary evaporation (information paradox)
- Sharpened version of information paradox (strong subadditivity argument)

**What does our construction teach us about the information paradox?**

# Naive version of the information paradox

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- Hawking's computation  $\Rightarrow$  mixed (thermal) state  $\rho_{\text{Hawking}}$
- Starting from pure state  $|\Psi\rangle$  we end up with mixed state  $\rho_{\text{Hawking}}$
- Inconsistent with unitary evolution in Quantum Mechanics

## Naive version of the information paradox

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However, consider what happens when a normal object burns (say a piece of coal)

- Outgoing photons seem to be thermal to a very good approximation.
- How is unitarity preserved? Where is the information of the original piece of coal stored in the outgoing radiation?
- ANSWER: It is encoded in very small correlations (entanglement) between the outgoing photons.
- While final state **looks like** a thermal density matrix  $\rho_{\text{thermal}}$  in reality it is a pure state.

**SMALL CORRECTIONS TO LEADING THERMAL APPROXIMATION CAN RESTORE UNITARITY**

## Naive version of the information paradox

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- Imagine that outgoing photons can be in 2 states. For  $N$  photons we have

$$2^N \text{ states}$$

- Density matrix of outgoing radiation is of size  $2^N \times 2^N$ .

- Consider

$$\rho_{\text{exact}} = \rho_{\text{thermal}} + 2^{-N} \rho_{\text{correction}}$$

where  $\rho_{\text{correction}} \sim \mathcal{O}(1)$ .

- Can easily check that  $\rho_{\text{exact}}$  of this form, can correspond to a **pure state**

**EXPONENTIALLY SMALL CORRECTIONS CAN RESTORE  
UNITARITY**

## Resolution of (naive version of) the information paradox

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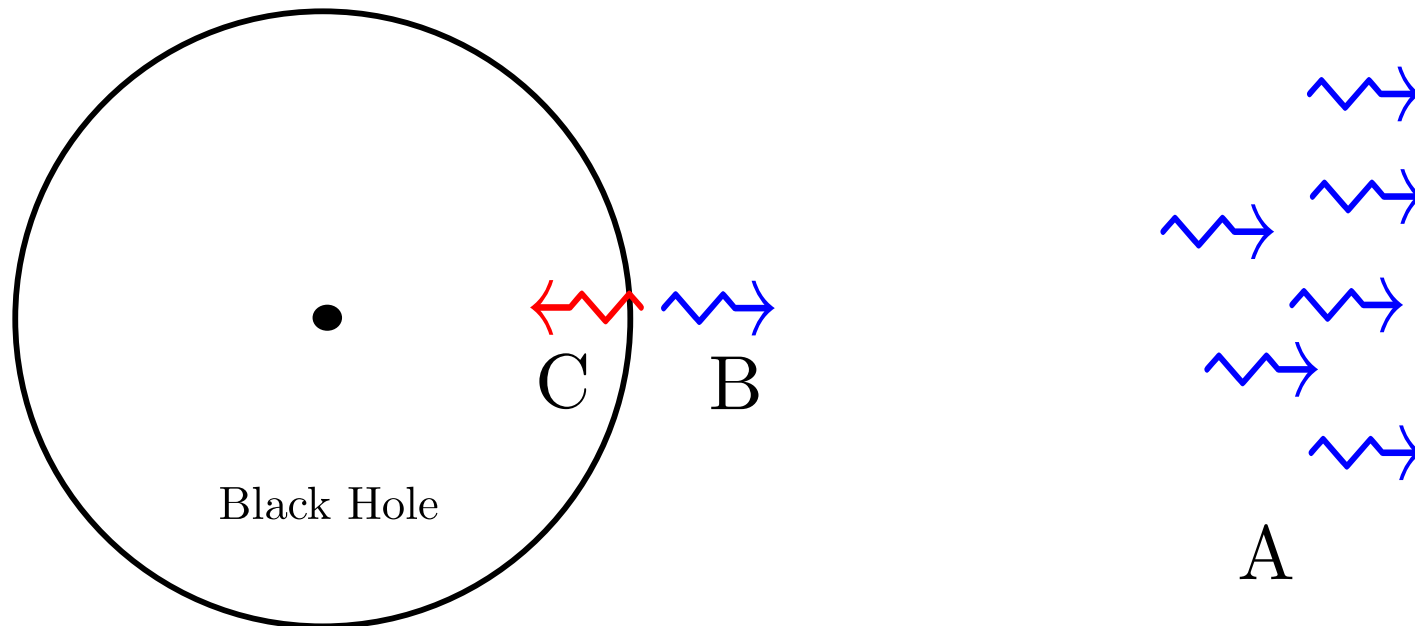
- Hawking's computation is only **the leading order** result
- We certainly expect corrections to the leading order computation, from quantum gravity effects (saddle points etc.)
- Even extremely small corrections are able to restore unitarity due to the large number of particles



# Sharpened version of the information paradox (Mathur, AMPS)

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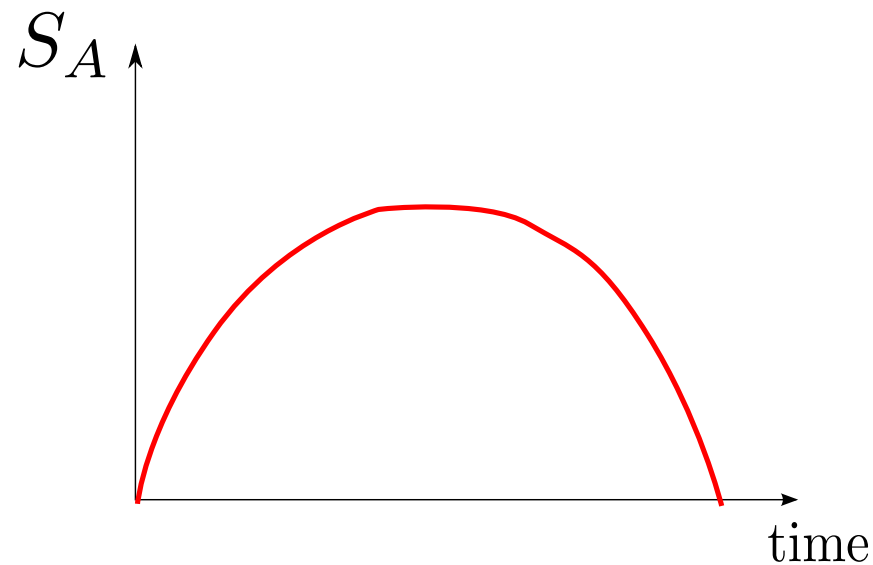
Consider the process of Hawking radiation



- A: old radiation, far from black hole
- B: newly created Hawking particle, outgoing
- C: ingoing partner of B

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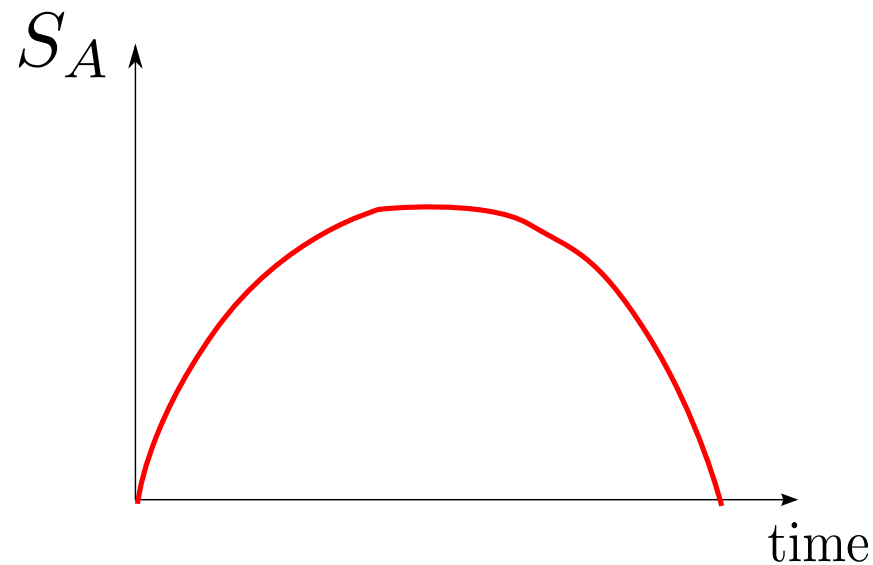
Consider the entropy of radiation  $A$  as a function of time

$$S_A = -\text{Tr}(\rho_A \log \rho_A)$$

If initial state is pure then  $S_A$  must go to zero after complete evaporation of the black hole

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In the beginning adding a  $B$  to  $A$  increases the entropy i.e. we expect

$$S_{AB} > S_A$$

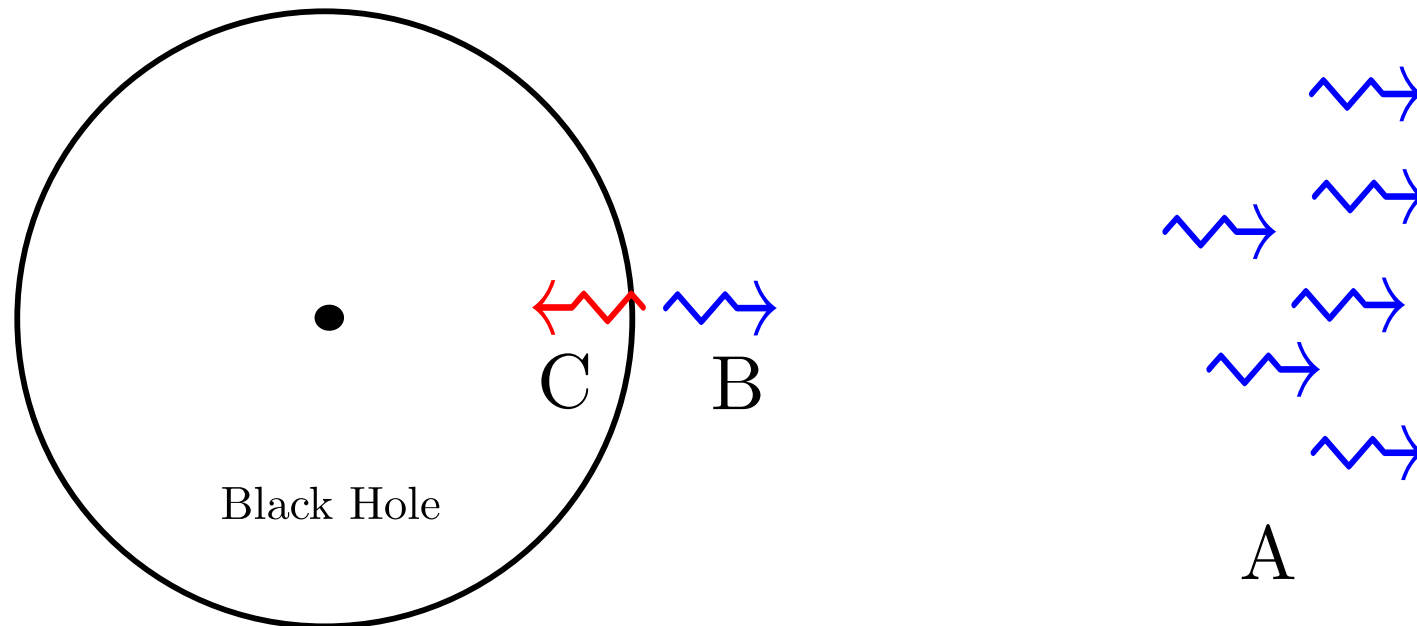
but eventually this must turn around and for an **old black hole** we expect

$$S_{AB} < S_A$$

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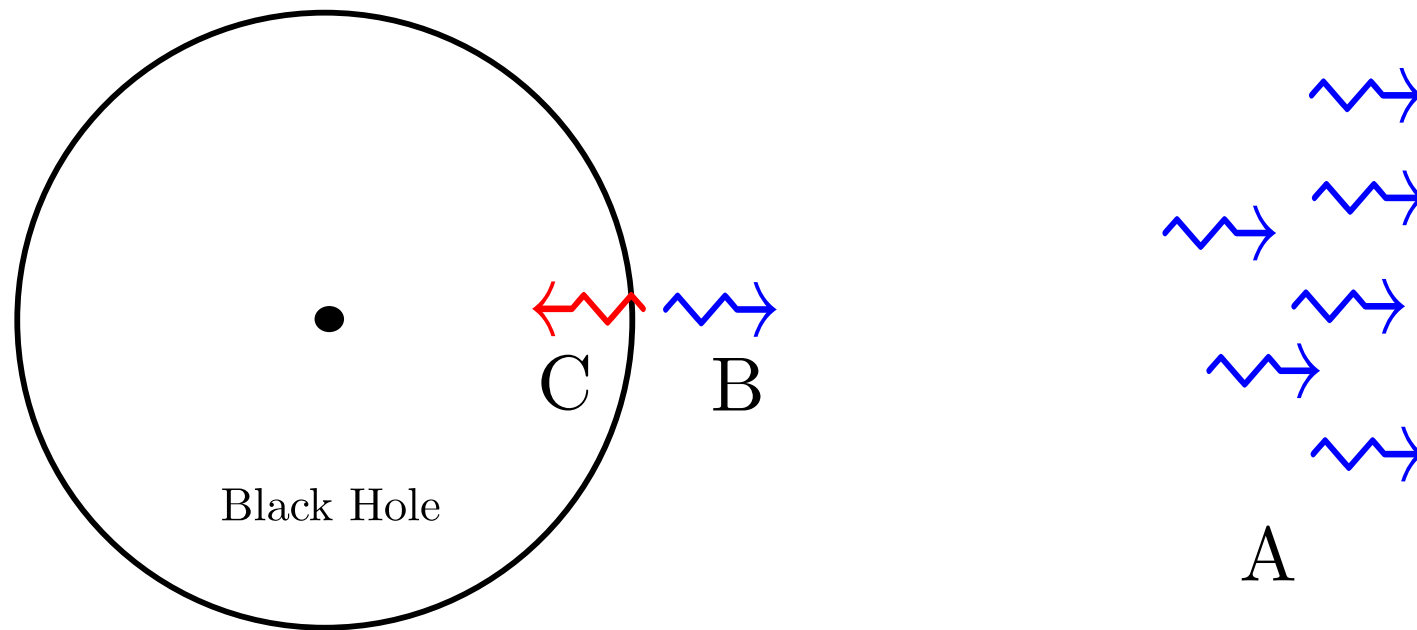
Consider the process of Hawking radiation



- For **free infall**:  $B$  must be fully entangled to  $C$
- For **information recovery**:  $B$  must be very entangled to  $A$
- Are these two statements compatible?

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**Strong subadditivity theorem:** for 3 independent systems A,B,C we have

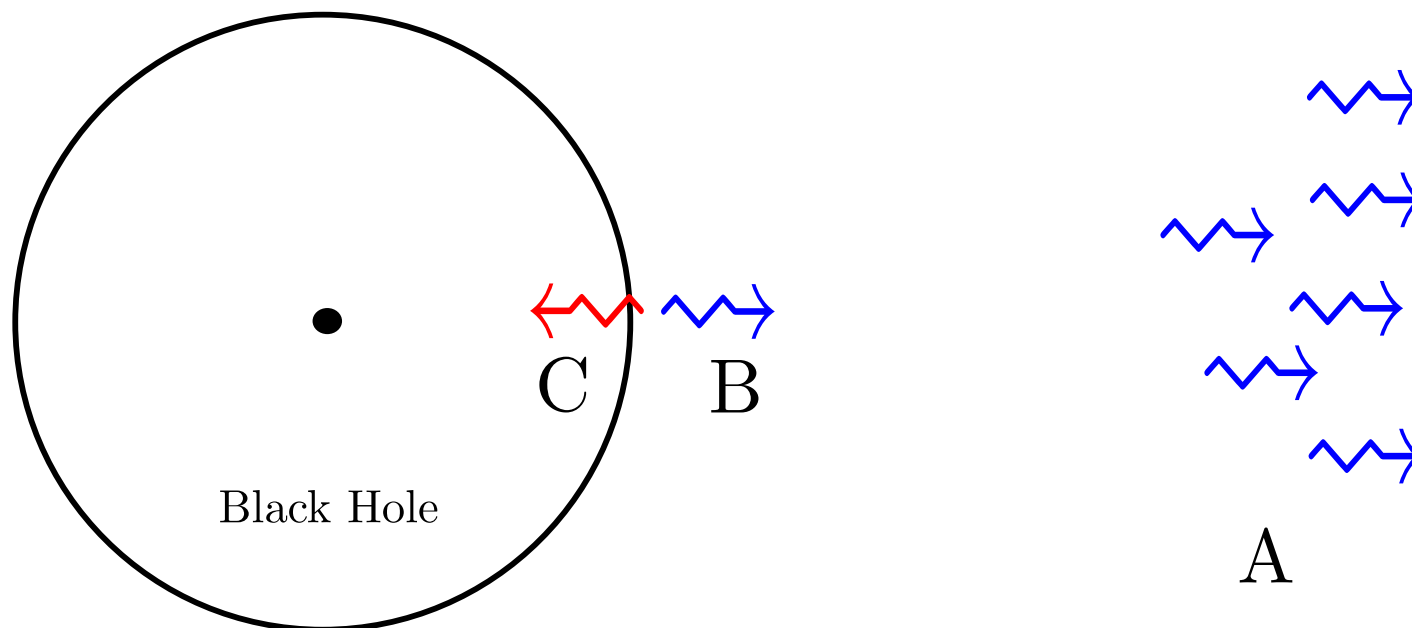
$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have  $S_{BC} \approx 0$  and  $S_C \approx \log 2$  which would imply

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# Sharpened version of the information paradox (Mathur, AMPS)

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- **Refined version of information paradox:** If we have smooth horizon ( $S_{BC} \approx 0$ ) then entropy of radiation keeps increasing.
- Critically based on theorem of strong subadditivity.
- What is the resolution suggested by our construction?

# Resolution of the (refined) information paradox

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**Systems A,B,C are not really independent**



**Strong subadditivity theorem cannot be applied to A,B,C**



**There is no paradox**



## Resolution of the (refined) information paradox

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- In our language the  $C$  is the tilde operator  $\tilde{\mathcal{O}}_{\omega, \vec{k}}$
- These operator came to existence because of splitting the Hilbert space into coarse- and fine-grained components.
- The outgoing radiation  $A$  is the coarse-grained part described by the operators  $\mathcal{O}_{\omega, \vec{k}}$ .
- **However**, the splitting into fine-and coarse-graining makes sense only if

$$\dim(\mathcal{H}_{\text{coarse}}) < \dim(\mathcal{H}_{\text{fine}})$$

- After Page time (half-evaporation), size of  $A$  is comparable to  $\mathcal{H}_{\text{fine}}$
- Hence, it is not possible to include in our coarse-grained Hilbert space **AT THE SAME TIME** all the  $A$  **AND** reconstruct  $C$  as part of the fine grained Hilbert space

## Resolution of the (refined) information paradox

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Other ways to say the same thing

- System C is not independent, but rather a **highly scrambled** version of (part of) A
- Locality for **simple measurements** is preserved. For  $P_1$  inside horizon and  $P_2$  outside

$$[\phi(P_1), \phi(P_2)] \approx 0$$

up to very small corrections.

- However if  $\mathcal{W}$  is complicated operator acting outside the horizon which measures all of the As then

$$[\phi(P_1), \mathcal{W}] \neq 0$$

- We can not reconstruct local  $n$ -point functions when  $n$  scales with  $N$  (or the entropy of the black hole)
- Consistent with the fact that boundary large  $N$  expansion does not hold when  $n \sim N^\#$ .

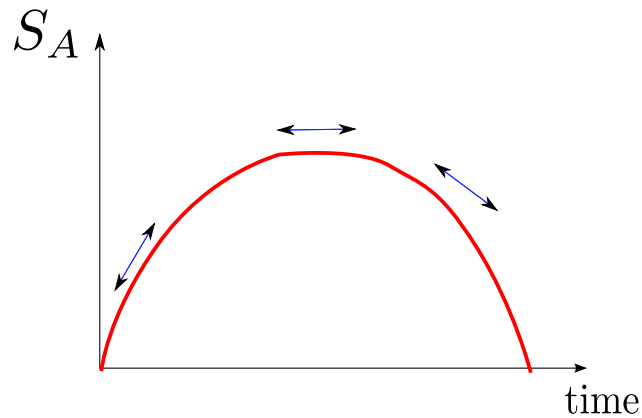
## Toy model: spin chain

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- Previous statements can be explicitly checked in toy model:  $N$  spins in random pure state
- Take  $p \ll N$  of them as the coarse-grained Hilbert space
- For any operator  $\mathcal{A}$  acting on the  $p$  spins, we can **explicitly** construct  $\tilde{\mathcal{A}}$  acting on the  $N - p$  spins
- When  $p \ll N$  we have  $[\mathcal{A}, \tilde{\mathcal{A}}] \approx 0$ .
- Can see explicitly that this breaks down when  $p \sim \frac{N}{2}$

# Final conclusion about black hole evaporation

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- Effective field theory (EFT) on evaporating black hole background works fine, as long as we do not try to compute **too complicated** correlation functions (for instance  $S$ -point functions, where  $S$  is the black hole entropy)
- By appropriately splitting the full Hilbert space into fine-and coarse-grained components, we can describe various parts of the evaporation process using EFT, but not all of them simultaneously.
- In spirit this is similar to the idea of complementarity.

# On complementarity

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How we understand complementarity

- There is a large Hilbert space describing **both** the inside and the outside of the black hole
  - However, it is not a **direct product** of inside  $\times$  outside degrees of freedom
  - We can construct operators acting on this Hilbert space and describing observables outside the black holes
  - We can construct operators acting on the same Hilbert space and describing observables inside the black holes
  - The two sets of operators do not commute
  - For few, light observables, they may approximately commute.
  - But not for too accurate measurements, or measurements involving too many insertions
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# Conclusions

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- **Local bulk physics from CFT:** found local observables both outside and inside the black hole
- **Infalling observer:** does not notice anything special
- **Interior of black hole:** coarse-grained observables effectively doubled in fine grained degrees of freedom. Black hole interior is a combination of both.
- **Information paradox:** small corrections can restore unitarity.
- **Strong subadditivity argument (Mathur, AMPS):** does not apply because A,B,C are not independent systems. C is a highly scrambled rewriting of A
- **Non-locality:** The amount of non-locality necessary is small.

## Future directions

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- Understand  $1/N$  corrections in more details
- Concrete computation in  $\mathcal{N} = 4$  SYM at small  $\lambda$ ?
- Dynamical toy model for thermalization/complementarity
- Use local operators to study black hole singularity from gauge theory

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**THANK YOU**