Information Paradox and Holography

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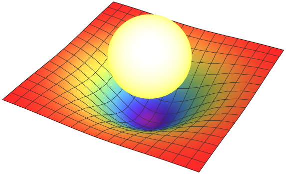
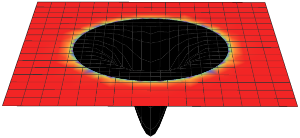
5 January 2018

Background: Black-Holes

Soon, after the discovery of general relativity, it was understood that a massive-enough star would collapse to form a black hole.



$\Rightarrow$



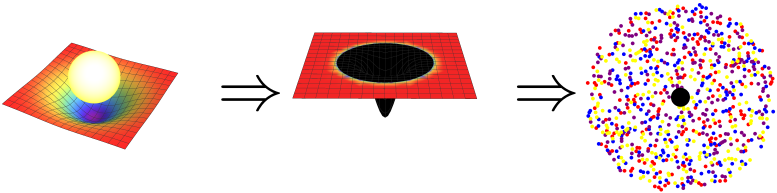
Classically, black-holes only grow! The area of black-holes never decreases

Hawking Radiation

- In 1975, Hawking pointed out that quantum effects cause black holes to radiate with a temperature $T = \frac{\hbar c^3}{8\pi k_B G M}$.

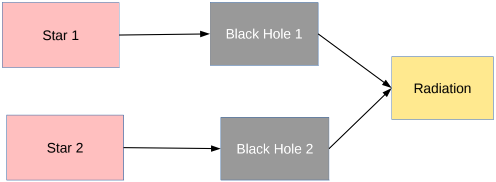
  [Hawking, 1975]

- Hawking radiation should eventually causes a black hole to evaporate.



The Information Paradox

- Hawking radiation depends on few macroscopic parameters; not on what constituted the black hole in the first place.
- Seemingly, for different inputs, we get the same output.

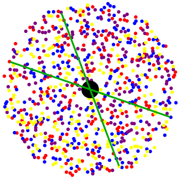


- Conflict with the unitarity of quantum mechanics: time evolution should be reversible.

The Information Paradox

- The information paradox is a central issue in putting together quantum mechanics and gravity.

- Common criticism was that Hawking's original paradox was not precise: maybe small corrections to Hawking radiation preserve information?

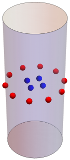


Progress on the Information Paradox

- Past few years have seen progress: information paradox can be framed in precise settings.
- Most interesting versions of this paradox appear in holography.
- These puzzles are teaching us about novel aspects of quantum gravity.

Holography

gravitational interactions in anti-de Sitter space $\Leftrightarrow$ unitary non-gravitational interactions on the boundary of the space



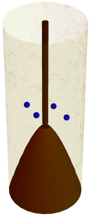
[Maldacena, 1997]

Can holography correctly describe bulk physics — including the formation and evaporation of black-holes — in detail?

Bulk Requirements

In the presence of a bulk-black-hole, gravity predicts correlation functions of local bulk fields

$$\langle \Psi | \phi^{\text{bulk}}(x_1) \ldots \phi^{\text{bulk}}(x_n) | \Psi \rangle = G(x_1 \ldots x_n)$$



Can we find boundary degrees of freedom, with the same correlators?

(overdetermined system — see Seiberg's talk.)

Summary of Recent Information Paradox Arguments

Information paradox becomes

$\nexists \phi(x)$, s.t. in a generic state $|\Psi\rangle$,

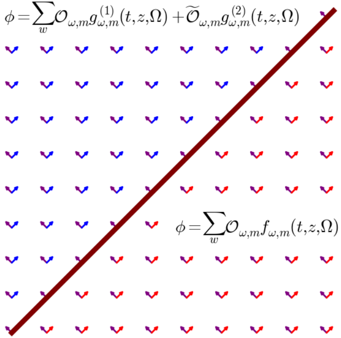
$$\langle \Psi | \phi(x_1) \ldots \phi(x_n) | \Psi \rangle = G(x_1 \ldots x_n)$$

[Mathur, AMPSS, Papadodimas, S.R., 2009–15]

"There are no operators in the boundary theory with the correlation functions predicted by bulk gravity."

Sketch of Refined Information Paradox

- Assume that an operator $\phi(x)$ exists, where x can be inside or outside the horizon.
- Then, by an appropriate integral transform, we can extract $\widetilde{\mathcal{O}}_{\omega,m}$.



Sketch of the Refined Information Paradox

- Bulk gravity tells us that

$$[H, \widetilde{\mathcal{O}}_\omega] = \omega \widetilde{\mathcal{O}}_\omega$$
$$\langle \Psi | \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger | \Psi \rangle = e^{\beta\omega} \langle \Psi | \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega | \Psi \rangle$$

- If the black hole state is approximately thermal

$$\begin{aligned} \langle \Psi | \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger | \Psi \rangle &\approx \frac{1}{Z(\beta)} \mathrm{Tr}(e^{-\beta H} \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger) \\ &= \frac{1}{Z(\beta)} e^{-\beta\omega} \mathrm{Tr}(e^{-\beta H} \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega) \\ &\approx e^{-\beta\omega} \langle \Psi | \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega | \Psi \rangle ? \end{aligned}$$

What gives?

These contradictions require several assumptions.

1. Theory is unitary.
2. Classical general relativity is valid at the horizon scale.
3. Microcanonical and canonical ensemble are equivalent.
4. Operators inside and outside the horizon commute exactly at spacelike separation.
5. Local observations inside the horizon are described by the same operator in all microstates.

Without the highlighted assumptions, possible to explicitly construct operators with the desired correlators.

[Papadodimas, S.R, 2013–15]

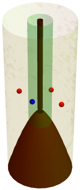
Physical Lessons from the Information Paradox

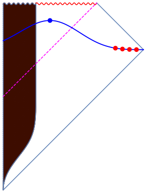
The information paradox appears to teach us that quantum gravity is unusual because

- At times, nonlocal effects may be important even at macroscopic distance scales.
- Local operators may require a state-dependent description.

Limits of Locality

Proposal is that non-local effects take the following form





$$\phi(x_{in}) \cong P(\phi(x_{\text{out},1}), \phi(x_{\text{out},2}), \ldots)$$

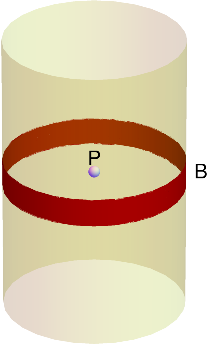
Proposal is a realization of an idea called black-hole complementarity: degrees of freedom inside the black hole are scrambled versions of degrees of freedom outside.

['t Hooft, Susskind, Thorlacius, Verlinde^2, 1984–1993]

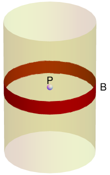
We now have explicit realizations of complementarity in some controlled systems. We also have indirect evidence of loss of locality in other settings.

Empty AdS Complementarity

Nonlocality can be examined explicitly in empty AdS. Operator at center of AdS can be written as complicated polynomial of operators that are uniformly spatially separated!



Empty AdS Complementarity



$$\phi(P) = \sum_{n,m} c_{nm} |n\rangle \langle m|$$

$$X_n |0\rangle = |n\rangle$$

$$\mathcal{P} = \sum_{n=0}^{n_c} \frac{(-\alpha_c H)^n}{n!}.$$

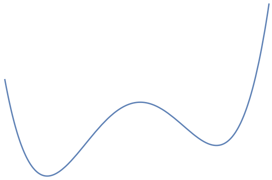
$$\boxed{\phi(P) = \sum_{n,m \ll N} c_{nm} X_n[\phi(B)] \mathcal{P}[\phi(B)] X_m^{\dagger}[\phi(B)]}$$

[Bryan, Banerjee, Papadodimas, S.R, 2016]

A Path Integral Argument

- Nonlocality in gravity is more generic. Locality is only defined about a saddle point of the QG path-integral.

$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}$$



- Breakdown of perturbation theory $\Rightarrow$ change in saddle-point $\Rightarrow$ different semi-classical metric with different notion of locality.

Breakdown of String Perturbation Theory

Motivates the study of limits in which gravitational perturbation theory breaks down.

String perturbation theory breaks down for a large number of external particles, even if the energy per particle stays small.

Supports the idea that locality breaks down if we probe spacetime at “too many” points — even if these points are widely separated.

Unitarity Bounds vs Factorial Growth

- Within perturbation theory, tree amplitudes must satisfy

$$\int d\Pi_{\frac{n}{2}} \left| M^{\text{tr}}(\frac{n}{2} \rightarrow \frac{n}{2}) \right|^2 \leq 2 \left| M^{\text{tr}}(\frac{n}{2} \rightarrow \frac{n}{2}) \right| .$$

- Tree-level string amplitudes grow as

$$\boxed{M^{\text{tr}} \propto g_s^{n-2} n!} \leftarrow \text{key point!}$$

- Factorial growth violates unitarity bounds at

$$n \propto \left(\frac{M_{\text{pl}}}{E} \right)^{d-2} .$$

[Sudip Ghosh, S.R., 2017]

String Amplitudes using Punctured Spheres

- Write tree-amplitudes as integrals over the moduli space of a n-punctured sphere with uniform negative curvature,

$$M^{\text{tr}} = g_s^{n-2} \int_{\mathcal{M}_n} d\mu_{\text{WP}} (\det P_1^{\dagger} P_1)^{\frac{1}{2}} (\det \Delta)^{\frac{-d}{2}} \langle V_1 \ldots V_n \rangle.$$

(See talks by Sen and Mahan.)

- At large n (following Gross, Periwal) argue that the dominant contribution comes from $V_{0,n}$ where

$$V_{g,n} = \int_{\mathcal{M}_{g,n}} d\mu_{\text{WP}}$$

Volumes of Weil-Petersson moduli spaces studied extensively in the Maths literature.

Volume of Moduli Space

- For any fixed n and large g

$$V_{g,n} \longrightarrow (4\pi^2)^{2g+n-3}(2g+n-3)!\frac{1}{\sqrt{g\pi}}\left(1+\mathrm{O}\left(\frac{1}{g}\right)\right).$$

[Zograf, Mirzakhani, 2008–2013]

- At large $g+n$

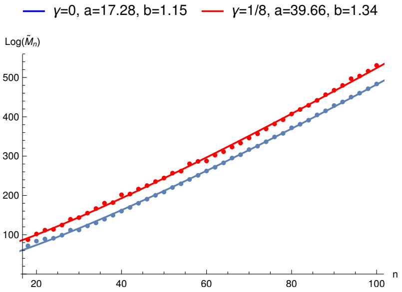
$$\frac{\log(V_{g,n})}{(n+2g)\log(n+2g)} \to 1.$$

With $g = 0$

$$\boxed{V_{0,n} \propto n!}$$

Numerical results: Factorial Growth

Can verify $n!$ growth through numerics

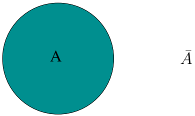


$$\log(\tilde{M}_n) = \log(M_n^{\text{tr}}) - (n-2)\log(4\pi g_s) + n\log(d-2)$$
$$= a + bn + \log((n-3)!).$$

Open Questions: Entanglement in Gravity

This nonlocality leads to several interesting questions. Consider the local von Neumann entropy (See Balents' talk.)

$$S_A = -\mathrm{Tr}(\rho_A \log(\rho_A))$$



But, if nonlocal effects are important, then degrees of freedom in A and $\overline{A}$ are not independent

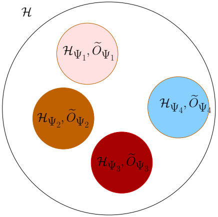
$$H \neq H_A \otimes H_{\overline{A}}$$

Requires new formulation of quantum information measures.

[Sudip Ghosh, SR, 2017]

State-Dependent Observables in Gravity

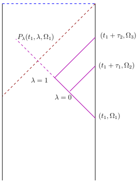
Idea is that observables in the BH interior are not defined globally but locally, in a neighbourhood of the Hilbert space.



This is called state-dependence.

Origins of State-Dependence

Local observables must be specified relationally in gravity because the metric can fluctuate. For example: jump from the boundary, wait for a certain proper time, measure the field.



This is a semi-classical definition: we want a quantum operator that has the right semi-classical behaviour.

Overcompleteness of Semi-Classical Geometries

- Semi-classical geometries correspond to coherent states — overcomplete basis!
- Demanding correct semi-classical behaviour on all coherent states ⇔ overspecifying a linear operator!

  [Papadodimas, S.R., Jafferis, Berenstein, Miller, 2015–17]
- State-dependent operators required to reproduce expected semi-classical behaviour.

Open Questions

State-dependence is very unusual but poorly understood. Several interesting questions remain. For example

1. What is the measurement theory of state-dependent observables?
2. Is state-dependence consistent with the Born rule?

[Marolf, Polchinski, 2015]

[S.R., 2016]

Summary

- The information paradox is a central problem in quantum gravity.
- New ideas allow us to make the paradox precise.
- Refined versions of the information paradox reveal novel physical effects

  1. The loss of locality in gravity.
  2. State-dependence of observables.

Empirical Tests?

These are very delicate effects. Beyond the reach of current experiments! But, in 1916, Einstein wrote, about gravitational waves,

$$A = \frac{\varkappa}{24\pi}\sum_{\alpha\beta}\left(\frac{\partial^3 J_{\alpha\beta}}{\partial t^3}\right)^2. \qquad (21)$$

Würde man die Zeit in Sekunden, die Energie in Erg messen, so würde zu diesem Ausdruck der Zahlenfaktor $\frac{1}{c^4}$ hinzutreten. Berücksichtigt man außerdem, daß $\varkappa = 1.87 \cdot 10^{-27}$, so sieht man, daß A in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß.

"so in all conceivable cases, A must have a practically vanishing value."

So even very delicate effects can be interesting and experimentally verified (given enough time!)

Thank you

Appendix

Definition of Mirror Operators

- Given an equilibrium state $|\Psi\rangle$, define anti-linear conjugate map

$$SA_\alpha|\Psi\rangle = A_\alpha^\dagger|\Psi\rangle$$

- This map exists because $|\Psi\rangle$ is separating for the truncated algebra.

- This is the same map that appears in the Tomita-Takesaki theory of modular isomorphisms of von Neumann algebras.

- Define mirror operators on the little Hilbert space, $\mathcal{H}_\Psi$

$$\widetilde{\mathcal{O}}_\omega = Se^{\frac{-\beta H}{2}} O_\omega e^{\frac{\beta H}{2}} S$$

Avoiding Paradoxes with State-Dependence

- Observer falling in state $|\Psi\rangle$ measures

$$\langle\Psi|A_{\alpha}[H, \widetilde{\mathcal{O}}_{\omega}^{\{\Psi\}}]|\Psi\rangle = \omega\langle\Psi|A_{\alpha}\widetilde{\mathcal{O}}_{\omega}^{\{\Psi\}}|\Psi\rangle$$

- Within correlators

$$\langle\Psi|\widetilde{\mathcal{O}}_{\omega}^{\{\Psi\}}(\widetilde{\mathcal{O}}_{\omega}^{\{\Psi\}})^{\dagger}|\Psi\rangle = e^{\beta\omega}\langle\Psi|(\widetilde{\mathcal{O}}_{\omega}^{\{\Psi\}})^{\dagger}\widetilde{\mathcal{O}}_{\omega}^{\{\Psi\}}|\Psi\rangle$$

- But given a basis of equilibrium states we cannot replace

$$\sum_i\langle\Psi_i|\widetilde{\mathcal{O}}_{\omega}^{\{\Psi_i\}}(\widetilde{\mathcal{O}}_{\omega}^{\{\Psi_i\}})^{\dagger}|\Psi_i\rangle = \mathrm{Tr}\left(e^{-\beta H}\widetilde{\mathcal{O}}_{\omega}\widetilde{\mathcal{O}}_{\omega}^{\dagger}\right),$$

since there is no globally defined mirror-mode.

Fat-tail obstruction to state-independence

- Function on phase space $f(z)$ lifted to operator via

$$\hat{f} = \int f(z)|z\rangle\langle z|dz$$

- This relies on

$$|\langle z|z'\rangle|^2 = e^{-|z-z'|^2}$$

- Here

$$|\langle\Psi|\Psi_T\rangle| = e^{\frac{-CT^2}{2\beta^2}}, \quad T \ll 1$$
$$|\langle\Psi|\Psi_T\rangle| = \mathrm{O}\left(e^{\frac{-S}{2}}\right), \quad T \gg 1$$