A big congratulation to ICTS!

Developments in Superstring Perturbation Theory

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Bangalore, January 2018

Why superstring perturbation theory?

Gravity is undoubtedly one of the forces we see in nature.

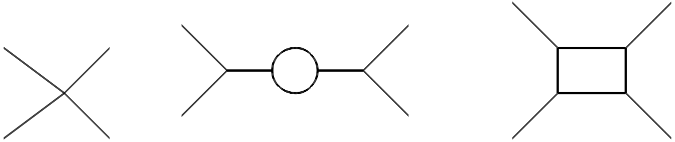
However the standard tools for making a classical theory into quantum theory fails for gravity.

One problem is the lack of a good perturbation theory

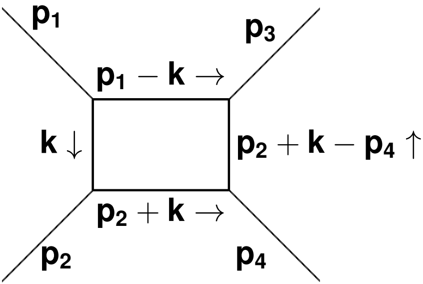
– a Taylor series expansion in powers of a parameter that controls the strength of gravitational force.

Usually relativistic dynamics of point particles is described by quantum field theory.

The scattering amplitudes are obtained as sums of ‘Feynman diagrams’.



Diagrams with larger number of loops correspond to terms that are higher order in the perturbation expansion in powers of the interaction strength



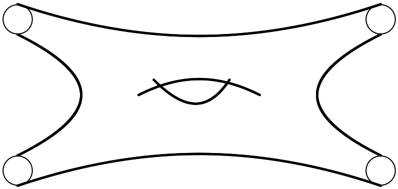
Evaluation of the diagram involves integration over 'loop momenta' k

– often suffer from 'ultraviolet divergences' from the region of large loop momenta, or equivalently coincident vertices.

For gravity these become unmanageable.

String theory replaces the notion of point-like constituents of matter by string-like constituents of matter

Scattering amplitudes are described by different kinds of 'Feynman diagrams'.



The ultraviolet divergences are absent due to the absence of definite interaction vertices.

This intuitive idea can be realized explicitly in a baby version of string theory known as the 'bosonic string theory'.

Scattering amplitudes are expressed as integrals over the

'moduli spaces of punctured Riemann surfaces'

Riemann surface: A two dimensional surface with a specific complex coordinate system / metric (up to overall scale at each point)

Moduli space: the space of all inequivalent complex coordinate system / metric

Punctured Riemann surface: Riemann surfaces with marked points

Moduli space of punctured Riemann surfaces: Moduli space of Riemann surfaces and the coordinates of the punctures

Dimension of the moduli space of Riemann surfaces of genus g (no of handles) and n punctures

$$6g - 6 + 2n$$

– we shall denote this by $M_{g,n}$

In bosonic string theory a g-loop amplitude with n-external states is given by

$$\int_{M_{g,n}} I(\{m\}, \{Q\})$$

The integrand I depends on 6g-6+2n the coordinates $\{m\}$ of $M_{g,n}$ and also on the momenta and other quantum numbers $\{Q\}$ of the n external states.

$I(\{m\}, \{Q\})$ is finite in the interior of the moduli space

$\Rightarrow$ absence of ultra-violet divergences.

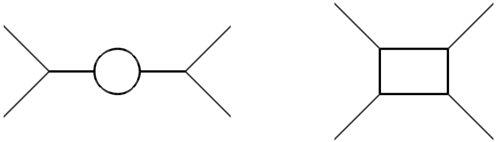
There are infrared divergences from the boundaries of $M_{g,n}$ (to be discussed later)

Additional advantages in string theory

• String theory automatically contains gravity

– no need to add gravity as an additional force.

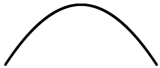
• In quantum field theories, there are many Feynman diagrams that we need to add for a given number of loops



In contrast string theory has only one term (for a given number of loops)

Unfortunately bosonic string theory is not fully consistent due to the existence of ‘tachyon’

The perturbation theory describes perturbation around the maximum of a potential



Some day we may find the minimum of the potential and develop perturbation theory around the minimum, but this is not the case today.

This problem is overcome in superstring theory

– does not have any instability

– shares the good properties with bosonic string theory, e.g. inclusion of gravity and ultra-violet finite perturbation theory.

However there were some technical problems in the perturbation expansion which have been fully resolved only recently.

Spurious poles

g-loop scattering amplitude of m bosonic and 2n fermionic states has the form

$$\int_{M_{g,m+2n}} I(\{m\},\{Q\};y_1,\cdots y_{m+n+2g-2})$$

The integrand I depends on

• 6g-6+2m+4n the coordinates $\{m\}$ of $M_{g,m+2n}$

• the momenta and other quantum numbers $\{Q\}$ of the external states

• the complex coordinates of $(m+n+2g-2)$ additional points $y_1,\cdots y_{m+n+2g-2}$ on the Riemann surface, known as 'locations of picture changing operators'

$$\int_{M_{g,m+2n}} I(\{m\},\{Q\}; y_1, \cdots y_{m+n+2g-2})$$

The result seems to depend on the spurious data $y_1, \cdots y_{m+n+2g-2}$.

Under a change in the variables y_i, the integrand changes by a total derivative in the variables {m}

The change is zero if we can ignore boundary terms.

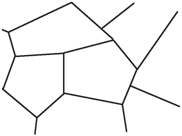
There is however a problem.

The integrand has poles in $M_{g,m+2n}$ whose locations depend on the y_i's, making the integral ill-defined

Solution

A.S.; A.S., Witten

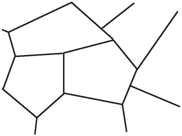
• **Divide the moduli space into a collection of small cells**



• **In each cell choose the y_i's so that I$(\{m\};\{Q\};\{y\})$ does not have any pole in that region.**

• **At the boundary between two cells y_i's jump**

We add correction terms at the boundary to compensate for the jump.



We also need additional correction terms where the codimension one boundaries meet on a codimension two subspaces, and so on

– gives a systematic procedure for computing amplitudes that are independent of the choice of the y_i's in each cell

Infrared divergences

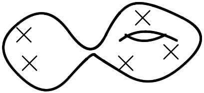
Amplitude

$$\int_{M_{g,m+2n}} I(\{m\},\{Q\};y_1,\cdots y_{m+n+2g-2})$$

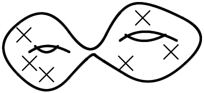
$M_{g,N}$ has boundaries where the Riemann surface degenerates

– the surface develops a narrow neck

Example 1: Two punctures come close



More general degeneration



$$\int_{\mathbf{M_{g,m+2n}}} \mathbf{I}(\{\mathbf{m}\}, \{\mathbf{Q}\}; \mathbf{y_1}, \cdots \mathbf{y_{m+n+2g-2}})$$

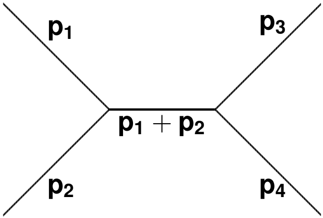
$\mathbf{I}(\{\mathbf{m}\}, \{\mathbf{Q}\}; \mathbf{y_1}, \cdots \mathbf{y_{m+n+2g-2}})$ **diverges when** $\{\mathbf{m}\}$ **approaches one of these boundaries**

$\Rightarrow$ **often** $\int_{\mathbf{M_{g,m+2n}}} \mathbf{I}(\{\mathbf{m}\}, \{\mathbf{Q}\}; \mathbf{y_1}, \cdots \mathbf{y_{m+n+2g-2}})$ **becomes ill-defined or divergent.**

These divergences are analogs of 'infrared divergences' in quantum field theory

– divergences associated with poles in the propagator

Example:

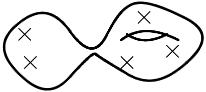


In this Feynman diagram the propagator carrying momentum $p_1 + p_2$ contributes

$$\{(p_1 + p_2)^2 + m^2\}^{-1}, \quad (p_1 + p_2)^2 \equiv -(p_1^0 + p_2^0)^2 + (\vec{p}_1 + \vec{p}_2)^2$$

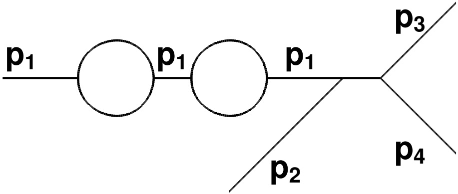
– has pole

In string theory this arises from the region of the moduli space where two punctures approach each other.



However understanding the physical origin of the divergence does not immediately offer a solution.

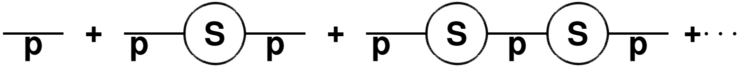
Example: Consider the following diagram in a quantum field theory.



The intermediate propagators carrying momentum p_1 contributes $(p_1^2 + m^2)^{-1}$.

Since the external state carries momentum p_1 we have $p_1^2 + m^2 = 0$, causing this diagram to diverge.

In quantum field theory these divergences are addressed using mass renormalization.



is resummed as

$$\frac{1}{p^2+m^2}+\frac{1}{p^2+m^2}S(p)\frac{1}{p^2+m^2}+\cdots=\frac{1}{p^2+m^2-S(p)}$$

We then look for zeroes of $p^2+m^2-S(p)$ and identify that as 'renormalized' mass^2.

$\Rightarrow$ converts multiple poles into a single pole which can be handled by standard quantum field theory tricks.

In conventional string perturbation theory this option does not exist.

• Since there is only one term in every loop order, we cannot isolate divergent contributions at different loop orders and resum.

• For consistency of the formalism, the value of p^2 for external states is fixed from the beginning, and there is no option for changing it at the loop order.

Both problems have recently been solved using superstring field theory.

Superstring field theory is a quantum field theory with infinite number of fields with the following properties

• Amplitudes are given by sum of Feynman diagrams as in a normal quantum field theory

• Contribution from each Feynman diagram may be expressed as integral over a cell of $M_{g,N}$ with the correct integrand.

• Sum of all Feynman diagrams gives us integration over the union of all the cells

– the whole moduli space.

Since we have a field theory, we can follow the correct rules, e.g. resum self-energy graphs and carry out mass renormalization to avoid the divergences

$\Rightarrow$ **gives a procedure for defining amplitudes in superstring field theory that are free from all divergences.**

Summary and Outlook

We now have a well-defined and unambiguous prescription for computing scattering amplitudes in superstring theory.

Goal: Make this into a practical tool

– develop codes for computing string scattering amplitudes to high order

Main bottleneck: A good parametrization of the moduli space of Riemann surfaces and a good choice of coordinate system on the Riemann surface.

Once we have the practical tools and get a few terms in the perturbation expansion. we can try resummation techniques to get non-perturbative information

– to be reported at

ICTS@20