Using general relativity to study superconductivity

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Claim:

In addition to describing gravitational phenomena (black holes, gravitational waves, etc.) general relativity can also describe other fields of physics including aspects of superconductivity.

Outline

A. Review black hole thermodynamics

B. Gauge/gravity duality (AdS/CFT)

C. Using general relativity to describe superconductivity (including Josephson junctions)

Laws of black hole mechanics

(Carter, Bardeen, Hawking, 1973)

0) For stationary black holes, the surface gravity κ is constant on the horizon

1) Under a small perturbation: $dM = \frac{\kappa}{8\pi G} dA + \Omega dJ$

2) The area of the event horizon always increases

Semiclassical black holes

Hawking coupled quantum matter fields to a classical black hole, and showed that they emit black body radiation with a temperature

$$T = \frac{\hbar \kappa}{2\pi}$$

This implies black holes have an entropy (Bekenstein)

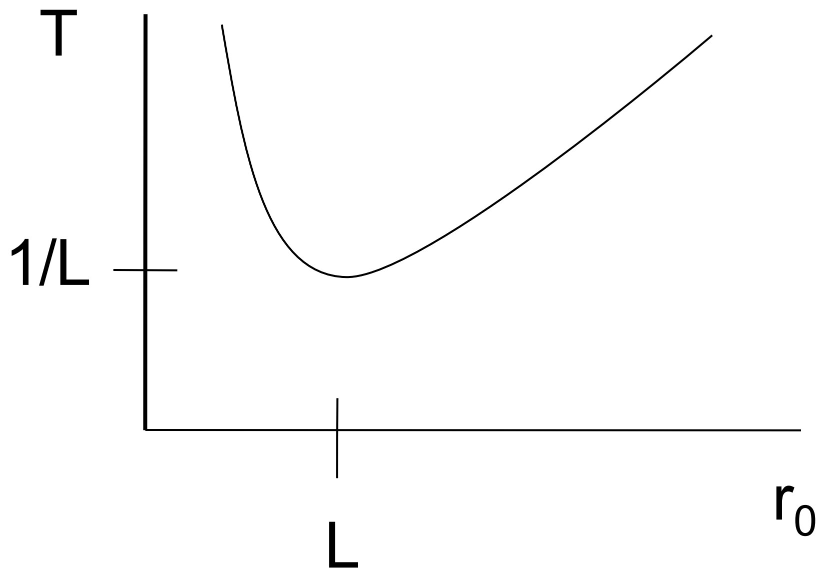
$$S_{BH} = \frac{A}{4\hbar G}$$

Black hole thermodynamics is much better defined with anti-de Sitter (AdS) boundary conditions.

The negative curvature acts like a confining box and there are static black holes in thermal equilibrium with their Hawking radiation.
(Hawking and Page, 1983)

Black holes in AdS exist with various horizon geometry: S^2, R^2, H^2.

Spherical black holes have a minimum temperature in AdS set by the cosmological constant: $\Lambda = -3/L^2$.



There is a phase transition between a gas of particles in AdS at low T and a large BH at high T.

The planar black hole has metric (L = 1)

$$ds^2 = r^2[-f(r)dt^2 + dx_i dx^i] + \frac{dr^2}{r^2 f(r)}$$

where $$f(r) = \left(1 - \frac{r_0^3}{r^3}\right)$$

The Hawking temperature is $T \sim r_0$.

The total energy is $E \sim r_0^3\, V \sim T^3\, V$.

The entropy is $S \sim A \sim r_0^2\, V \sim T^2\, V$.

So the 3+1 BH energy and entropy are exactly like a thermal gas in 2+1 dimensions.

This is the first indication that GR with AdS boundary conditions can be related to nongravitational systems.

Charged (planar) black holes in AdS are similar to Reissner-Nordstrom.

There is an extremal limit with T = 0 and nonzero entropy.

Vector potential: $$A_t = \mu - \frac{\rho}{r}$$

ρ ← charge density

μ ↑ chosen so A_t = 0 at the horizon

Having S ≠ 0 at T = 0 is uncommon. We will see that RN AdS is often unstable at low temperature and the new BH has S = 0 at T = 0.

Gauge/gravity duality

Gauge Theories

These are generalizations of electromagnetism in which the U(1) gauge invariance is replaced by e.g. SU(N).

Our standard model of particle physics is based on a gauge theory.

QCD has SU(3) gauge symmetry. The interactions are weak at high energy but become strong at low energy causing quark confinement.

‘t Hooft argued in the 1970’s that a 1/N expansion of an SU(N) gauge theory would resemble a theory of strings.

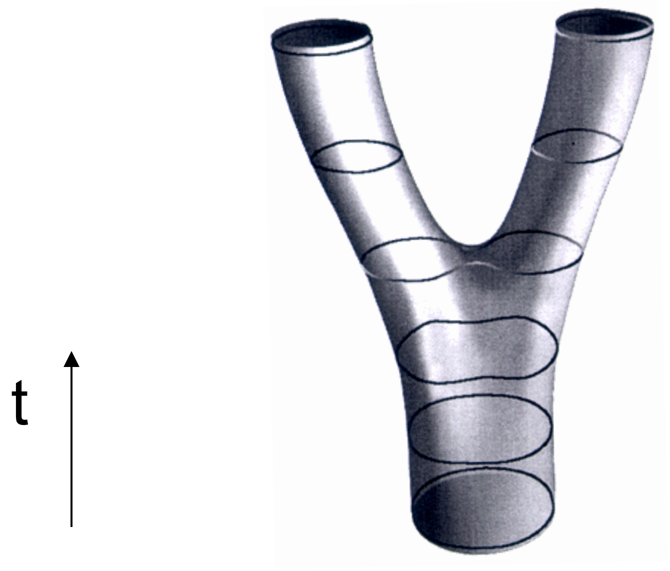
It took more than 20 years for this idea to be made precise.

String Theory

This is a promising candidate for both

1) a complete quantum theory of gravity
2) a unified theory of all forces and particles

It is based on the idea that elementary particles are not pointlike, but excitations of a one dimensional string.



Strings interact with a simple splitting and joining interaction with strength g.

String theory reduces to general relativity (with certain matter) in a classical limit.

Gauge/gravity duality

(Maldacena; Gubser, Klebanov, Polyakov; Witten)

With anti-de Sitter boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living on the boundary at infinity.

When string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

Shows that quantum gravity is holographic

(‘tHooft, Susskind)

AdS can be written (setting L =1)

$$ds^2 = r^2[-dt^2 + dx_i dx^i] + \frac{dr^2}{r^2}$$

The gauge theory lives on the Minkowski spacetime at r = ∞.

Scaling symmetry: $r \rightarrow ar$, $(t, x_i) \rightarrow (t/a, x_i/a)$ so small r corresponds to large distances or low energy in the gauge theory.

Traditional applications of gauge/gravity duality

Gain new insight into strongly coupled gauge theories, e.g., geometric picture of confinement.

Gain new insight into quantum gravity, e.g., quantum properties of black holes

Quantum Black Holes

- What is the origin of black hole entropy?
- Does black hole evaporation lose information? Does it violate quantum mechanics?

Answers from Gauge/Gravity Duality

The gauge theory has enough microstates to reproduce the entropy of black holes.

The formation and evaporation of small black holes can be described by ordinary Hamiltonian evolution in the gauge theory. It does not violate quantum mechanics.

After thirty years, Hawking finally conceded this point in 2004.

In a certain limit, all stringy and quantum effects are suppressed and gravity theory is just general relativity

(with asymptotically anti-de Sitter boundary conditions).

New application of gauge/gravity duality: Condensed matter

In 2007, a few condensed matter effects (e.g. the Hall effect) were reproduced using general relativity.

(Hartnoll, Herzog, Kovtun, Sachdev, Son)

What about superconductivity?

Superconductivity

Superconductivity 101

In conventional superconductors (Al, Nb, Pb, …) pairs of elections with opposite spin can bind to form a charged boson called a Cooper pair.

Below a critical temperature T_c, there is a second order phase transition and these bosons condense.

The DC conductivity becomes infinite.

This is well described by BCS theory.

The new high T_c superconductors were discovered in 1986. These cuprates (e.g. YBaCuO) are layered and superconductivity is along CuO_2 planes.

Highest T_c today (HgBaCuO) is T_c = 134K

New superconductors based on iron and not copper (FeAs) discovered in 2008 have T_c = 56K.

The pairing mechanism is not well understood. Unlike BCS theory, it is not weakly coupled.

Use gauge/gravity duality to try to gain insight into these high T_c superconductors.

Gravity dual of a superconductor

(Hartnoll, Herzog, and G.H., 2008)

Gravity	Superconductor
Black hole	Temperature
Charged scalar field	Condensate

Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.

This is not an easy task.

Gubser (2008) argued that a charged scalar field around a charged black hole would have the desired property. Consider

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\partial\Psi - iqA\Psi|^2 - m^2|\Psi|^2$$

For an electrically charged black hole, the effective mass of Ψ is

$$m^2_{eff} = m^2 + q^2 g^{tt} A_t^2$$

But the last term is negative. This produces scalar hair at low temperature.

Hairy black holes

Look for static, homogeneous solutions:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

$$A = \phi(r)dt, \quad \Psi = \psi(r)$$

Get four coupled nonlinear ODE's which can be solved numerically. At the horizon, $r = r_0$, g and Φ vanish, χ is constant. Asymptotically, metric approaches AdS.

At large radius, the vector potential and charged scalar behave as

$$A_t = \mu - \frac{\rho}{r}, \qquad \psi = \frac{\psi^{(2)}}{r^2}$$

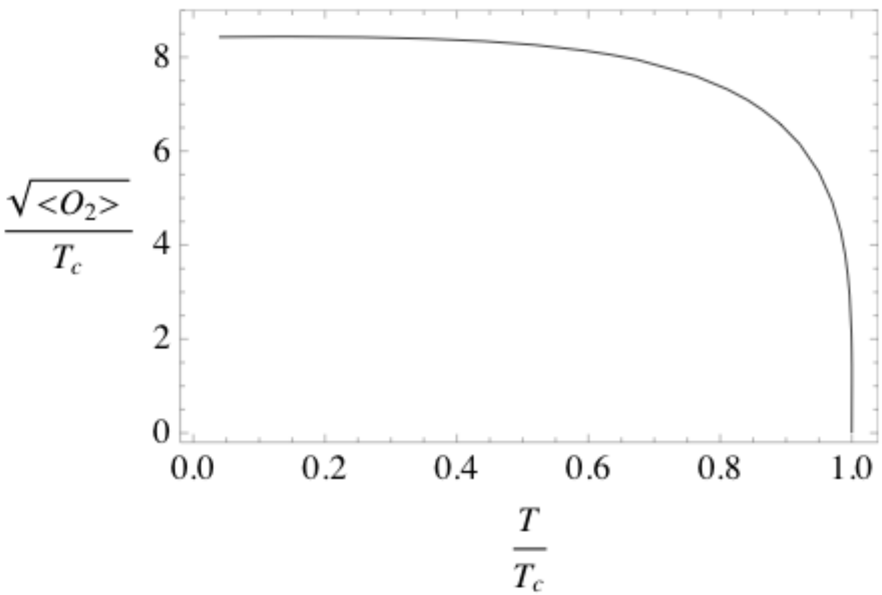
Gauge/gravity duality relates these constants to properties of the dual field theory:

μ = chemical potential, ρ = charge density

There is an operator O_2 dual to ψ, and

$$\langle O_2 \rangle = \psi^{(2)}$$

Condensate (hair) as a function of T



$$T_c \propto \mu$$

As T ⟶ 0, the horizon area vanishes, consistent with a unique ground state.

Conductivity

We want to compute the conductivity as a function of frequency. Start by perturbing the Maxwell field around the black hole. Assume time dependence $e^{-i\omega t}$ and impose ingoing wave boundary conditions at the horizon.

The asymptotic behavior is

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots$$

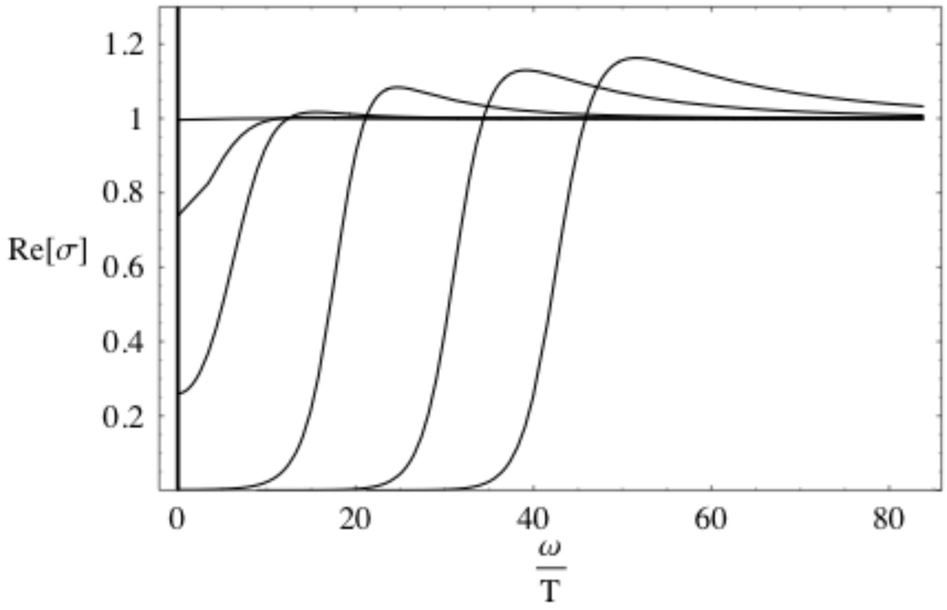
The gauge/gravity duality dictionary says

$$A_x = A_x^{(0)}\,, \qquad \langle J_x \rangle = A_x^{(1)}$$

We obtain the conductivity from Ohm's law

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{\langle J_x \rangle}{\dot{A}_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = -\frac{iA_x^{(1)}}{\omega A_x^{(0)}}$$

Frequency dependent conductivity



Curves represent successively lower temperatures. Get delta-function at $\omega = 0$.

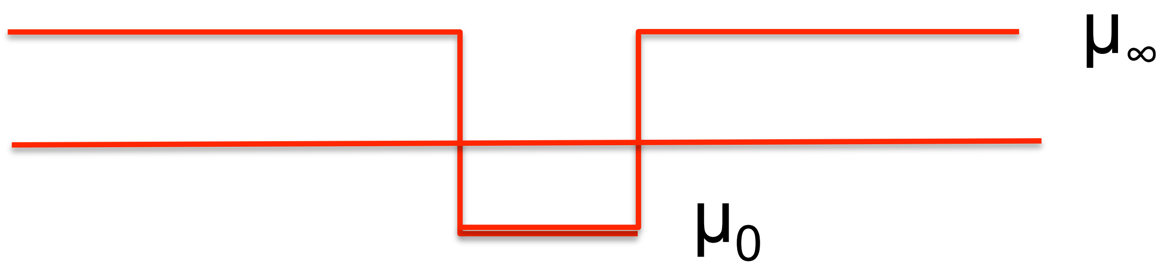
Josephson junctions

A Josephson junction consists of two superconductors separated by a weak link:

Insulator: SIS junctions
Normal conductor: SNS junctions ⬅
Narrow superconducting bridge

Josephson predicted that even without a voltage difference across the junction, $J = J_{max} \sin \gamma$ where γ is the phase difference.

Model this by letting μ be position dependent.
(Santos, Way, G.H., 2011)

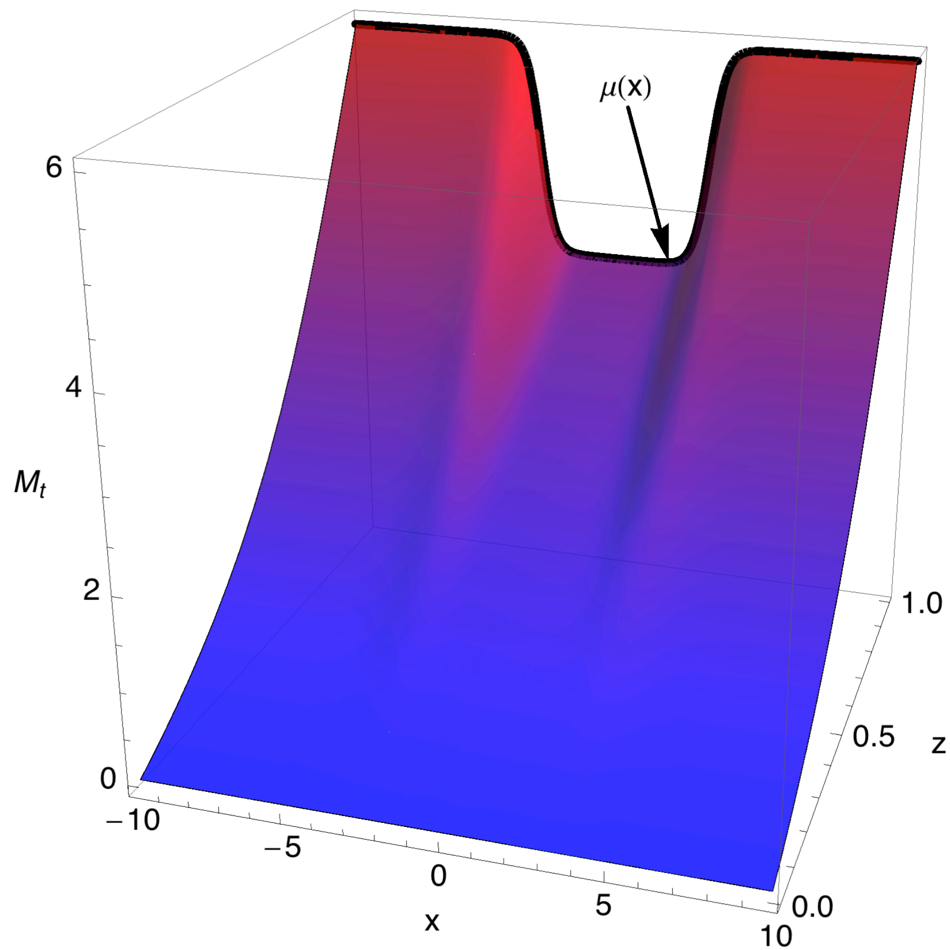


For a range of temperatures, you have two superconductors separated by a normal conductor.

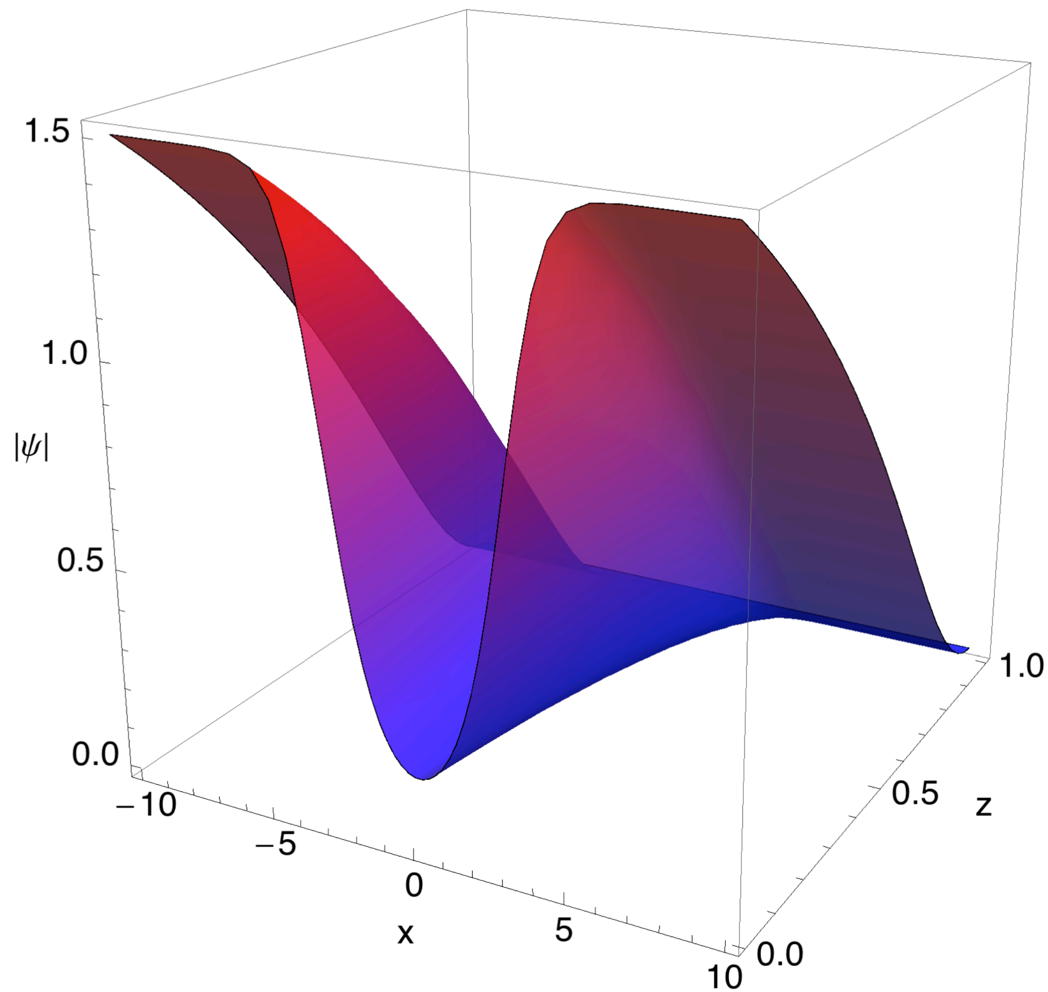
The critical temperature for the junction is the same as the case with constant $\mu = \mu_\infty$

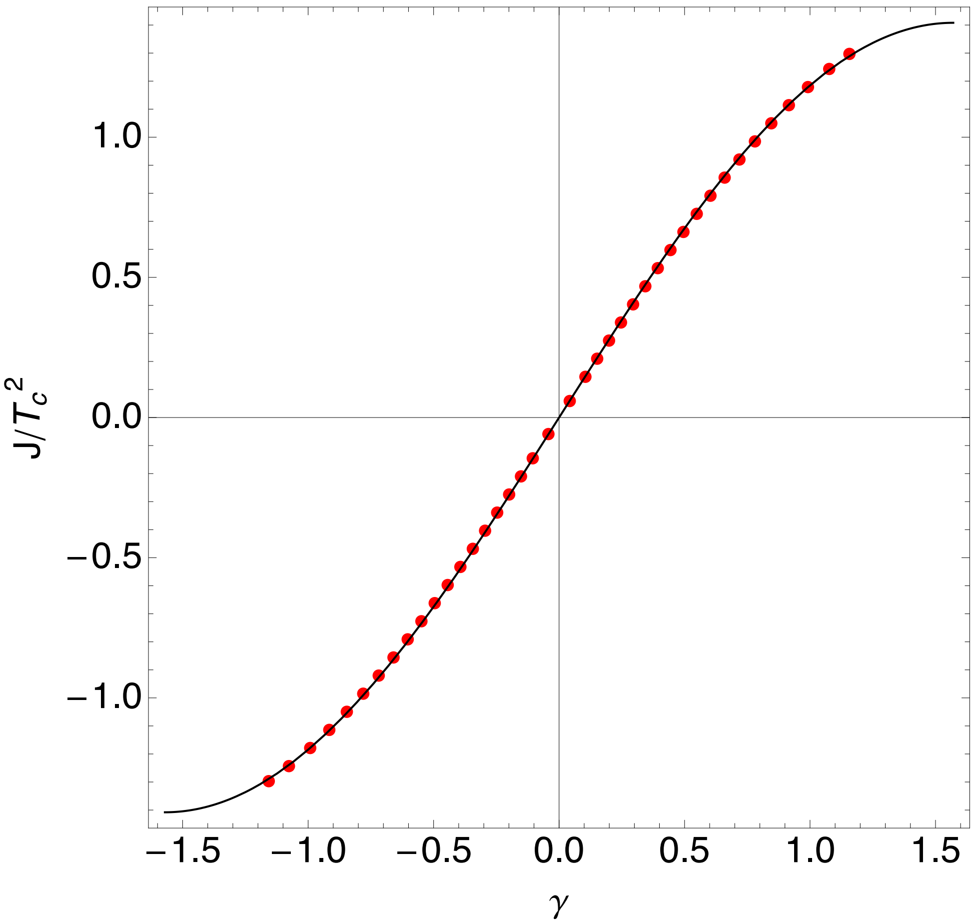
Results

A_t as a function of x and $z = 1 - r_0/r$



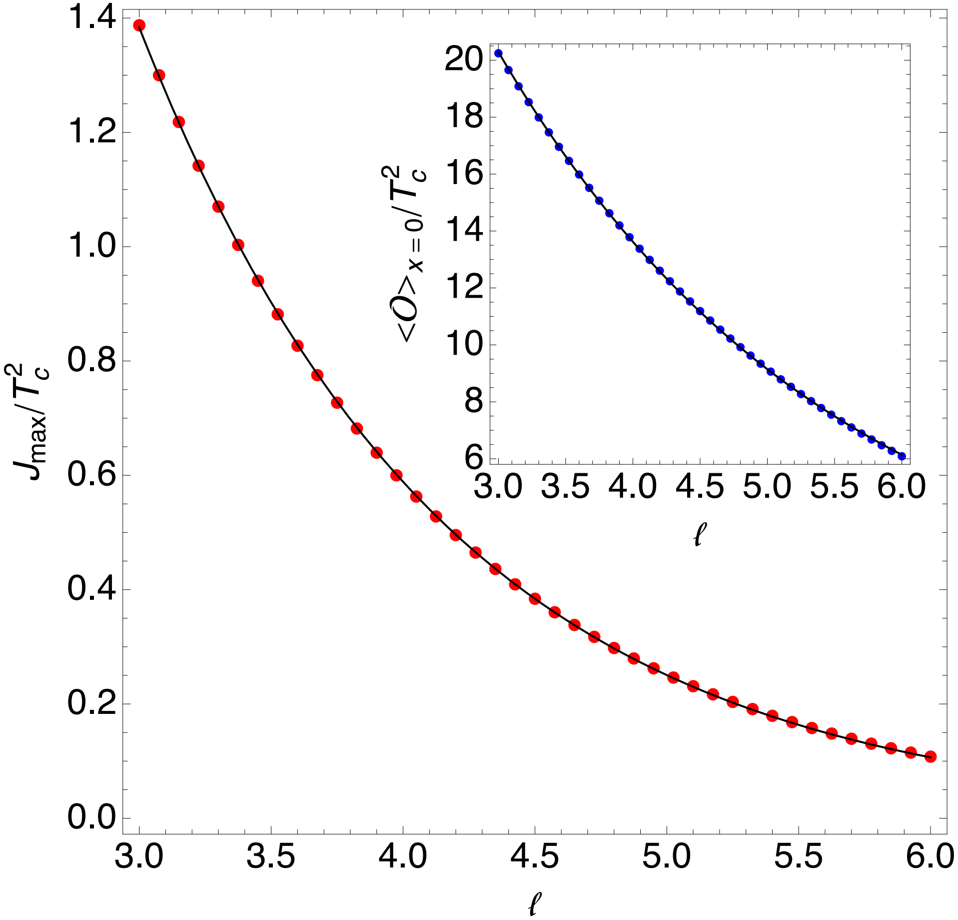
Scalar stays small inside the gap.





Indeed: $J = J_{max} \sin \gamma$

Dependence of J_{max} on width of junction:



Line is fit to

$$J_{\max}/T_c^2 = A_0\, e^{-\frac{\ell}{\xi}}$$

Condensate at center of junction behaves similarly

$$\langle \mathcal{O} \rangle_{x=0}/T_c^2 = A_1\, e^{-\frac{\ell}{2\,\xi}}$$

with $\xi \approx 1.2$

General relativity can also describe:

1) Fermi surfaces
2) Fermi and non-Fermi liquids
3) Quantum critical points

Several condensed matter physicists in the US are now learning general relativity to use this new tool.

Summary

1) Black holes in AdS behave exactly like a thermal system in one lower dimension
2) Gauge/gravity duality predicts general relativity can also describe nongravitational physics.
3) One can indeed recover aspects of superconductivity (including Josephson junctions) from general relativity.

Final Comments

- This duality allows us to compute dynamical transport properties of strongly coupled systems at nonzero T. Condensed matter theorists have very few tools to do this.
- At present this can only be done for theories the condensed matter physicists don’t care about.
- Eventually, one might use gravity to predict new exotic states of matter and then try to look for them in the lab.

- Conversely, (if we are wildly optimistic) one might someday be able to learn about quantum gravity by doing condensed matter experiments!