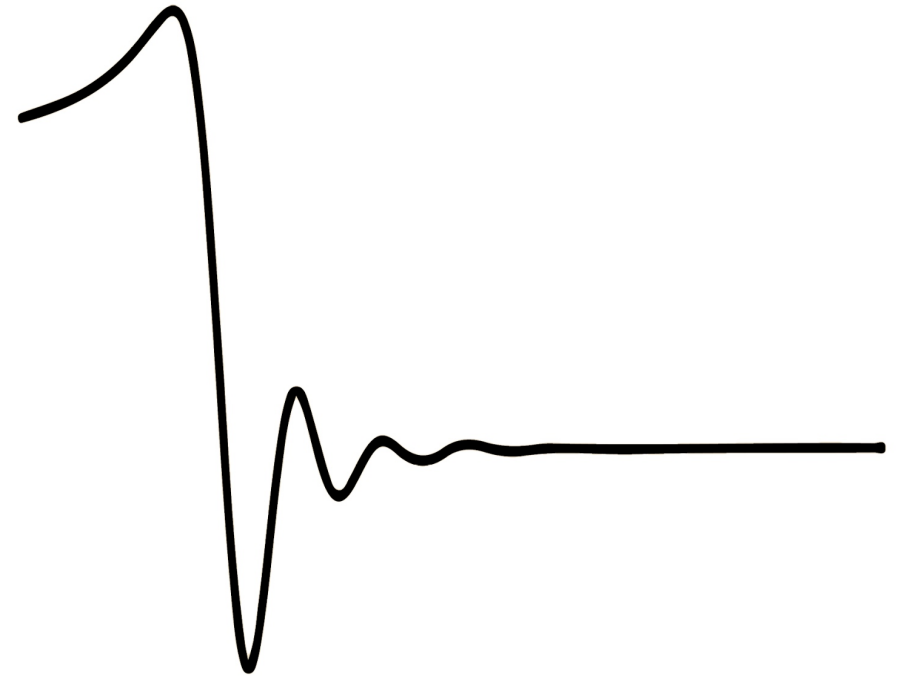


*Exotic **compact** objects and their gravitational-wave signatures*



Andrea Maselli



References

- S. Chandrasekhar,
The Mathematical theory of black holes
(Oxford University Press)
- M. Maggiore,
Gravitational Waves
(Cambridge University Press)
- B. S. Sathyaprakash, B. Schutz,
*Physics, Astrophysics and Cosmology with
Gravitational waves*
Living Reviews in. Relativity 12.2 (2009)
- E. Berti, V. Cardoso & A. Starinets,
Quasinormal modes of black holes and black branes
Classical and Quantum Gravity 26: 163001 (2009)
- E. Berti,
*A Black-Hole Primer: Particles, Waves, Critical
Phenomena and Superradiant Instability*
arXiv: 1410.4481 [gr-qc]
- T. Regge and J. A. Wheeler
Stability of a Schwarzschild Singularity
Phys. Rev. 108, 4 (1957)
- Frank J., Zerilli
*Gravitational Field of a Particle Falling in Schwarzschild
Geometry Analyzed in Tensor Harmonics*
Phys. Rev. D 2, 10 (1970)
- K. D. Kokkotas
Axial Modes of Relativistic Stars
MNRAS 268, 4 (1994)
- V. Ferrari and K. D. Kokkotas
*Scattering of particles by neutron stars: Time
evolutions for axial perturbations*
Phys. Rev. D 62, 10 8 (2000)
- V. Cardoso and P. Pani, Nature Astronomy 1: 586 (2017)
arXiv: 1707.03021 [gr-qc]
- Z. Mark, A. Zimmerman et al,
A recipe for echoes from exotic compact objects
Phys. Rev. D 96, 084002 (2017)

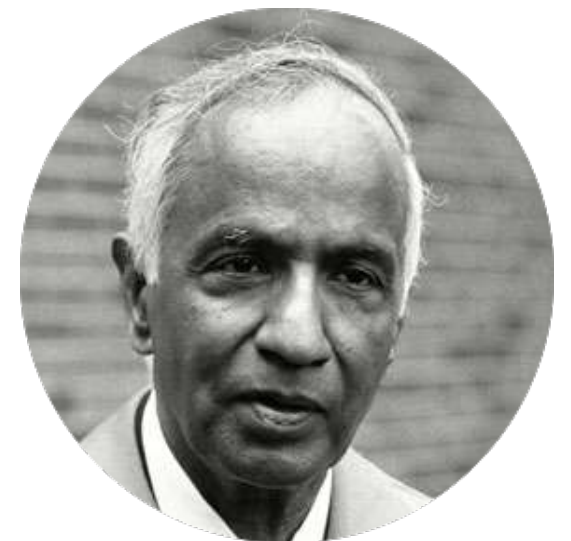
BH sociology



*“As you see, the **war** treated me kindly enough, in spite of the heavy gunfire, to allow me to get away from it all and take this walk in the land of your **ideas**: ...”*

Karl Schwarzschild to Albert Einstein
Letter dated 22 December 1915

*“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realisation that an **exact** solution of Einstein’s equations of general relativity provides the absolutely exact representation of untold numbers of black holes that populate the universe.”*



S. Chandrasekhar
The Nora and Edward Ryerson lecture, Chicago April 22 1975

History of (black) holes

1783

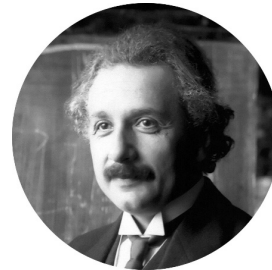


Mitchell

1916



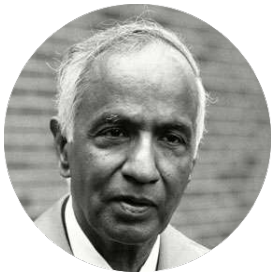
Schwarzschild



Einstein



Eddington



Chandrasekhar

1963



Kerr



Wheeler



Oppenheimer



Penrose



Carter



Hawking

2017

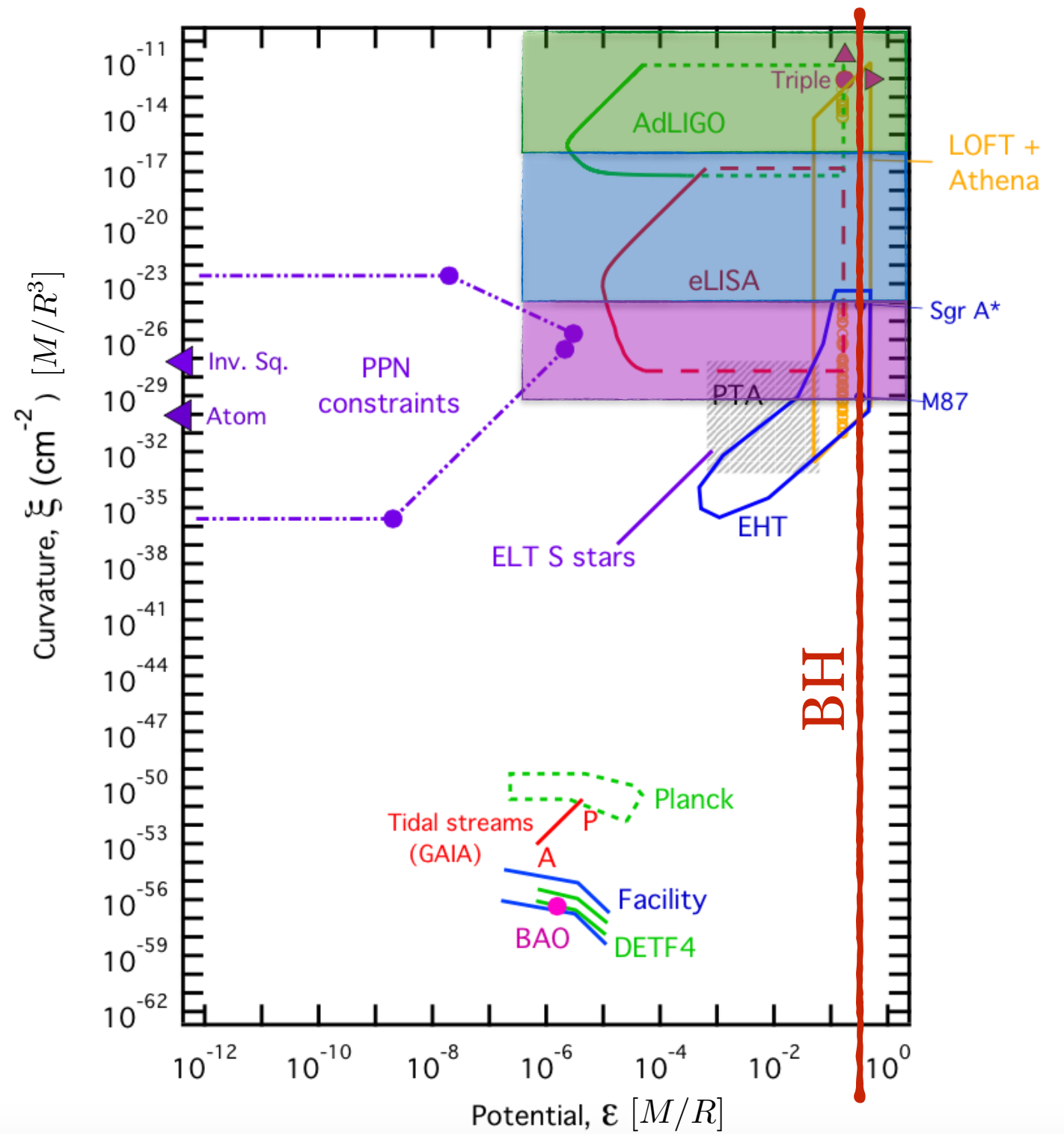


Thorne

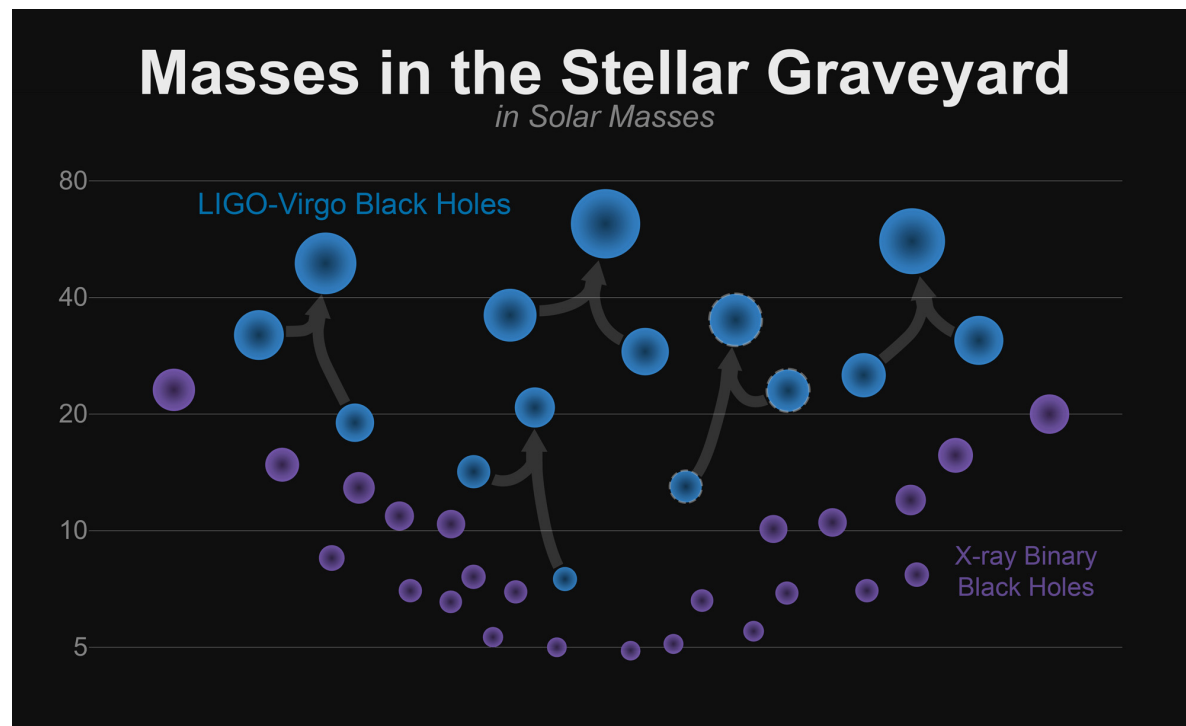
Many more...

BHs vs experiments

stellar
intermediate
supermassive



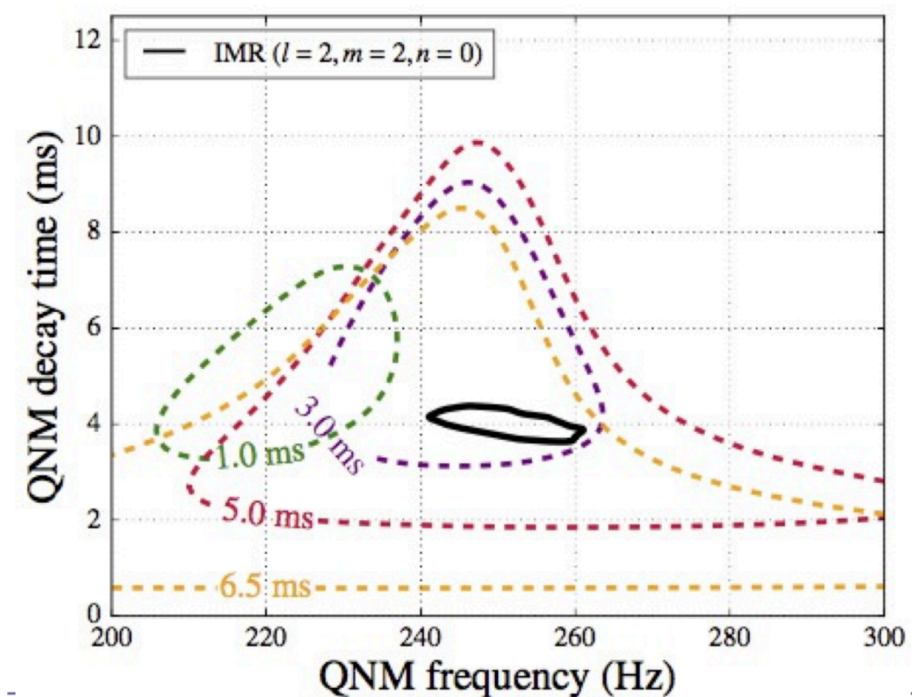
BHs in the sky



- ✓ Black holes form in nature as binary systems
- ✓ BBH coalesce within the Hubble time

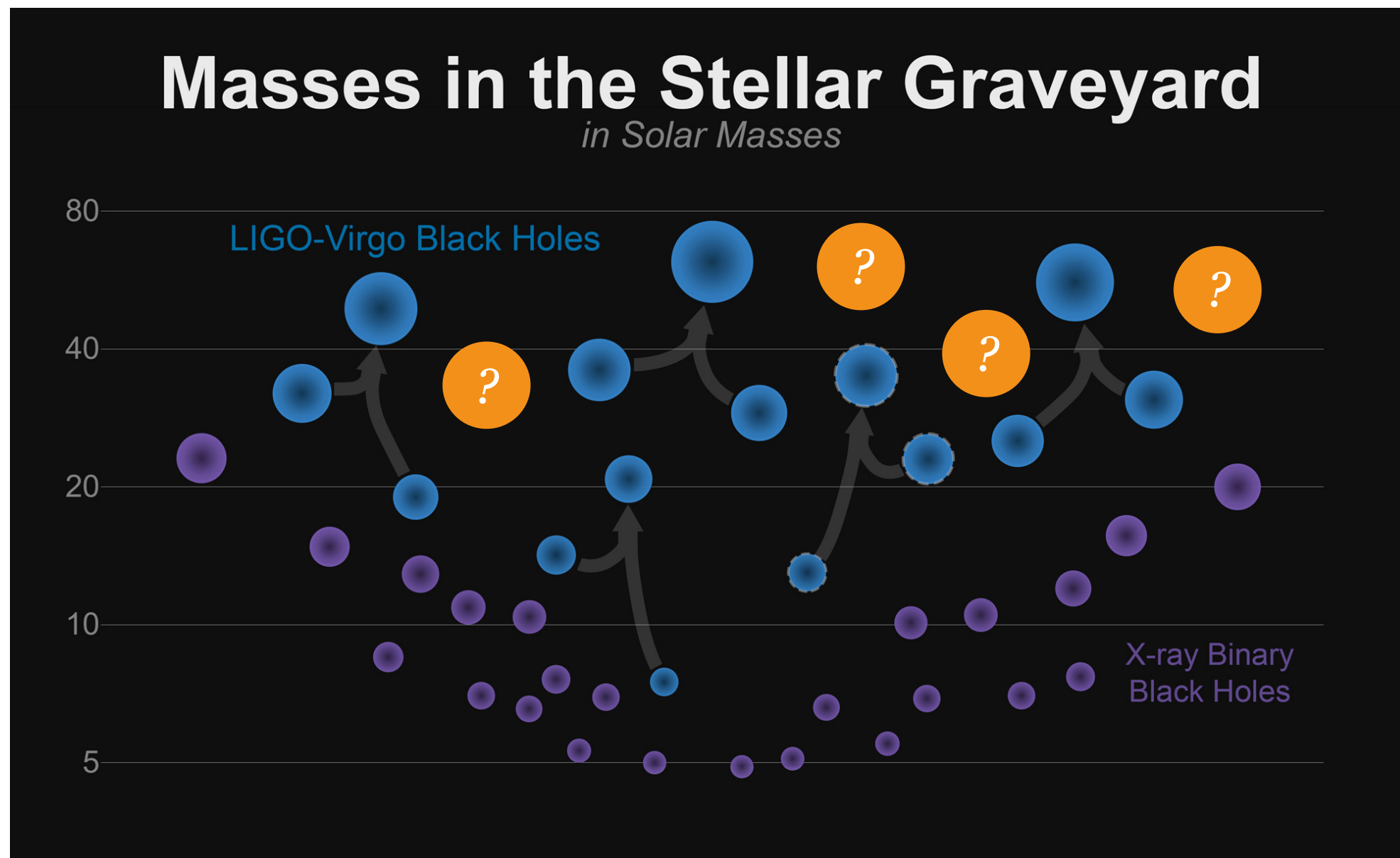
First attempt to detect BH
Quasi-Normal-Modes

[Abbott et al. LVC 2016-2018]



Everything clear ?

Are they alone in the Universe?



Why do we look for alternatives?

- ✓ BH exterior is pathology-free, but interior is not. Singularities and Cauchy horizons, signalling breakdown of predictability
- ✓ Quantum effects are not fully understood, namely information loss, which could lead to new end-states
- ✓ An horizon does exist in any spacetime harbouring a singularity (*cosmic censorship*)
 - ✓ “*Remarkable claims require remarkable evidences*”
- ✓ Observational limits in tests strong field regime around BHs
- ✓ If we don't devise alternative, we don't find them

Everything clear ?

BHs as endpoints of stellar evolution ?

- ✓ Unexpected processes may avoid their formation
- ✓ Other **E**xotic **C**ompact **O**bjects may be the output of stellar collapse, which form without an event horizon



- ✓ Can we distinguish ECOs with no horizon, compact enough to mimic a BH?

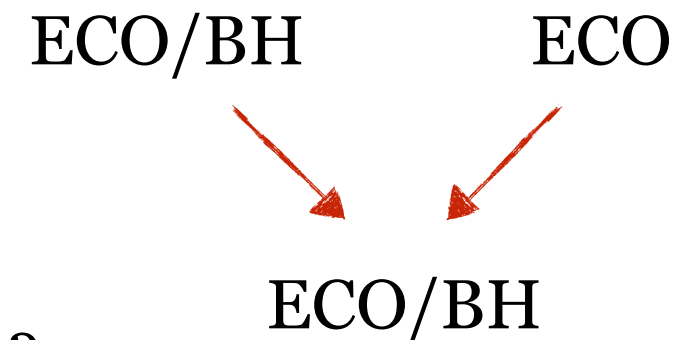


- ✓ GWs from coalescing binaries may provide new answers

ECOs: formation and evolution

If they form in Nature

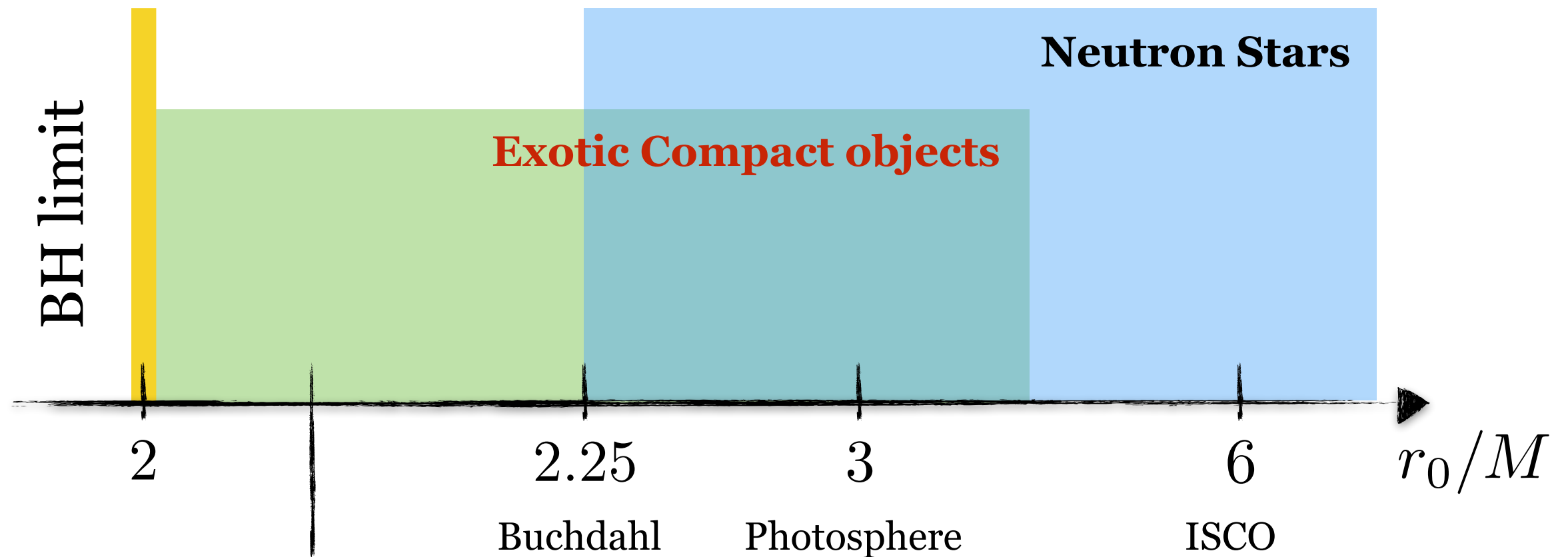
- ✓ Stability requirements
- ✓ Single sources v.s. binary systems
 - ✓ Which are the main evolutionary channels?
 - ✓ Detection rates, masses, distances: which detector?
- ✓ Realistic astrophysical environments: accretion disk?
 - ✓ Favour electromagnetic observables



ECOs vs BHs

A first order classification

[Cardoso, Pani, '17]



$$R_{\text{ECO}} = 2M(1 + \epsilon)$$

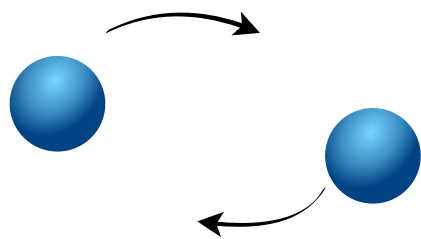
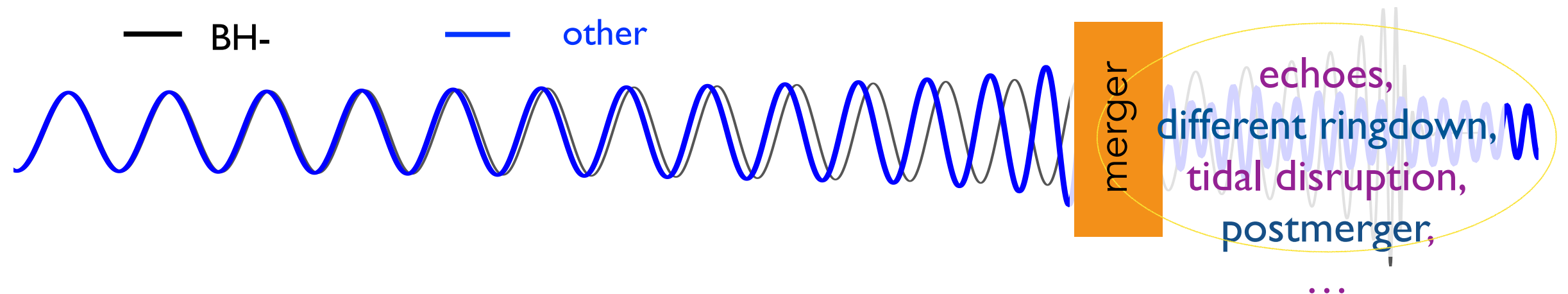
$\epsilon = 10^{-30}$

$$M_{\text{BH}}/R_{\text{BH}} = 0.5 \quad \text{v.s.} \quad M_{\text{ECO}}/R_{\text{ECO}} = 0.49(99\dots)$$

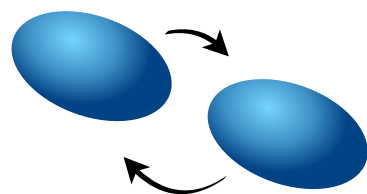
[The following text is heavily obscured by heavy black redaction bars.]



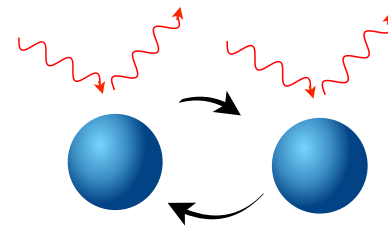
Where do we look for?



~point masses:
same signal
for all objects



tidal effects
+
spins
deformations



absence of horizon
absorption
effects

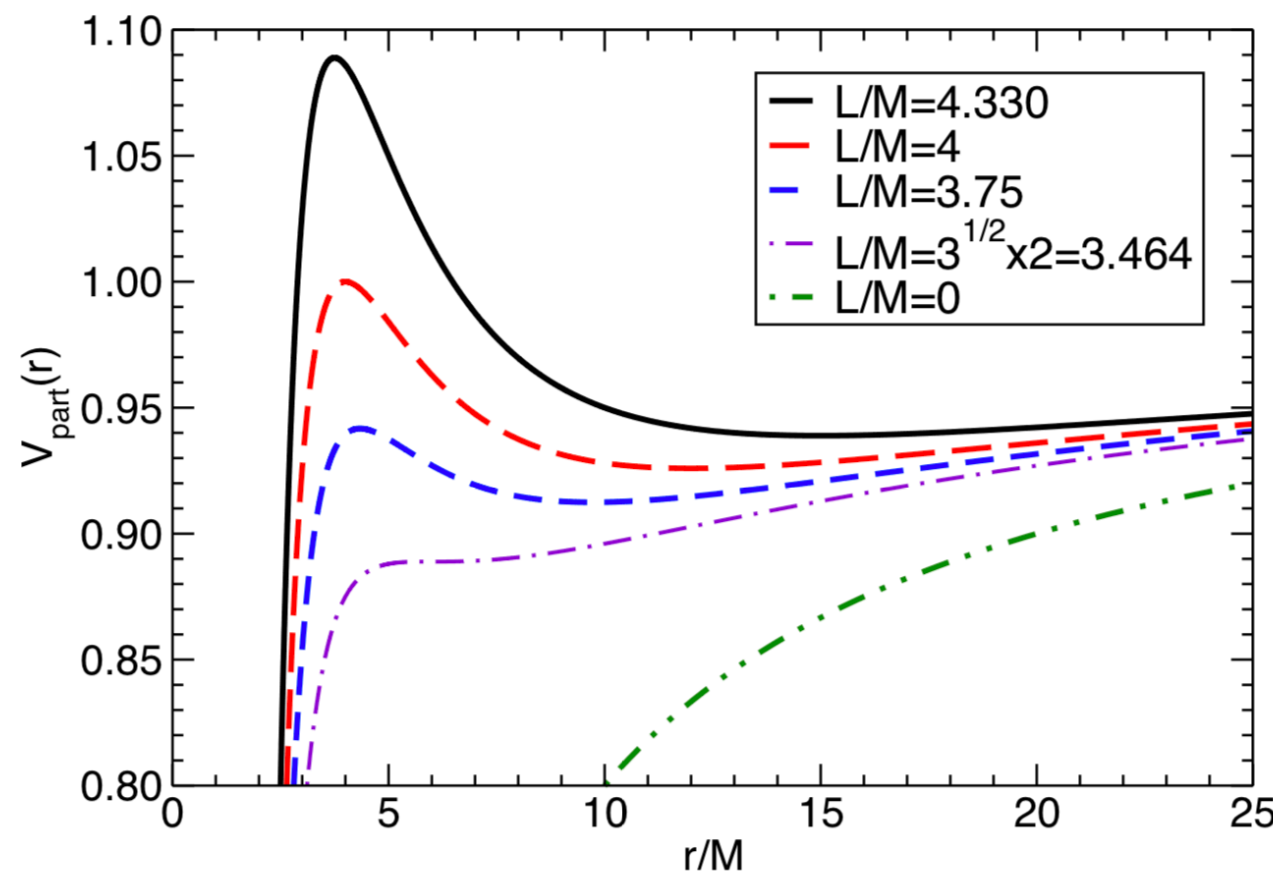


modified
QNMs and
echoes

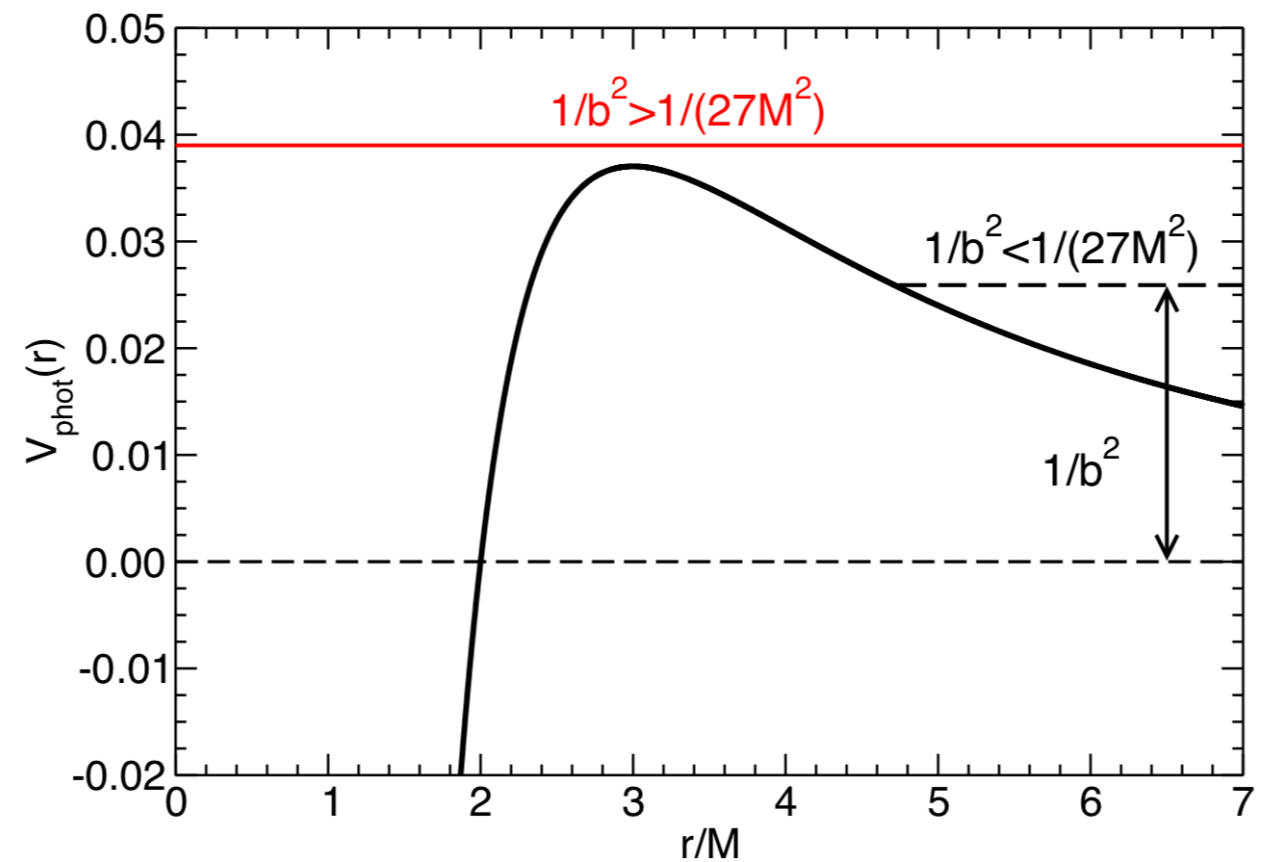
*Geodesics of
spherically
symmetric spacetimes*

Effective potentials

massive

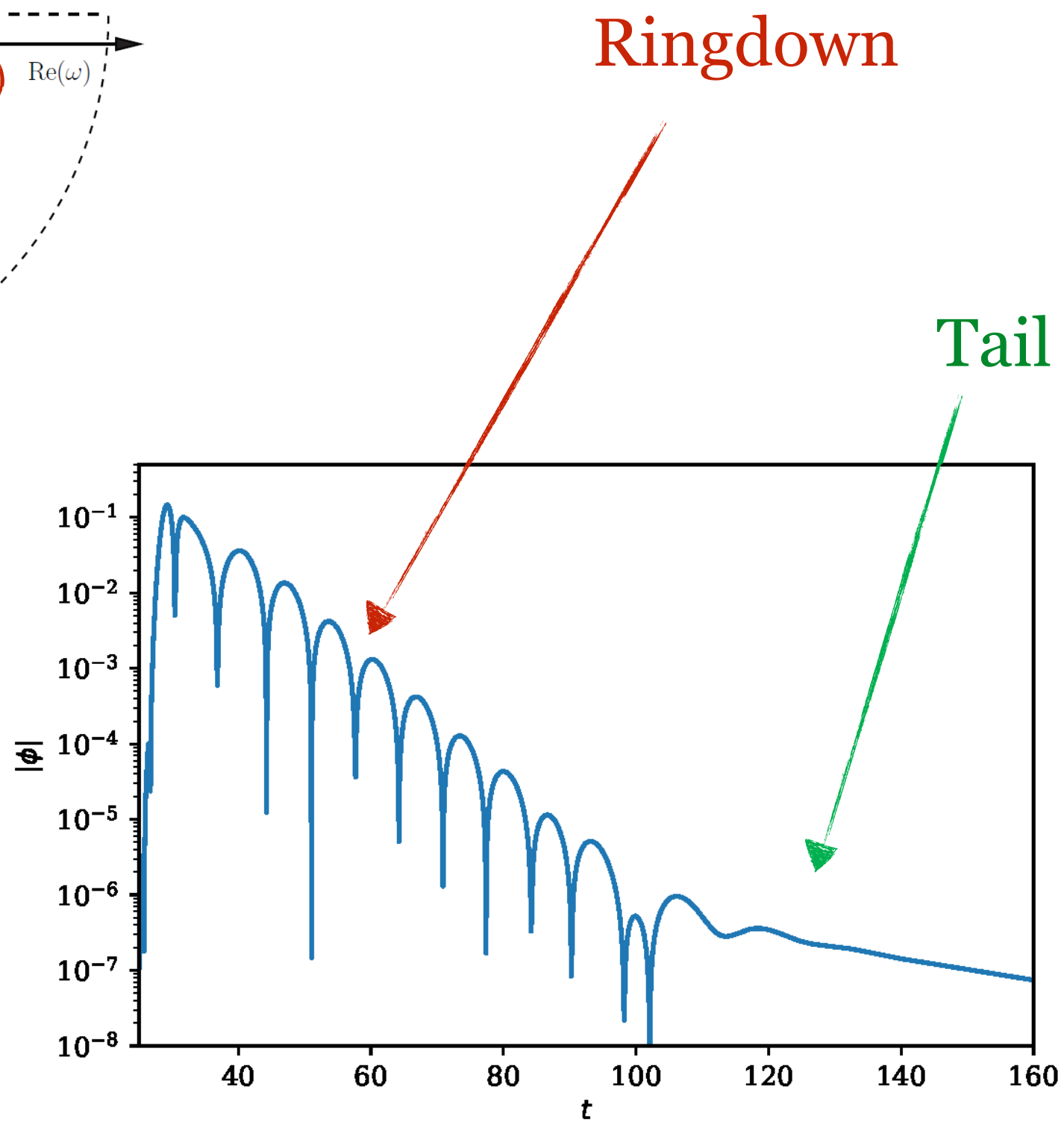
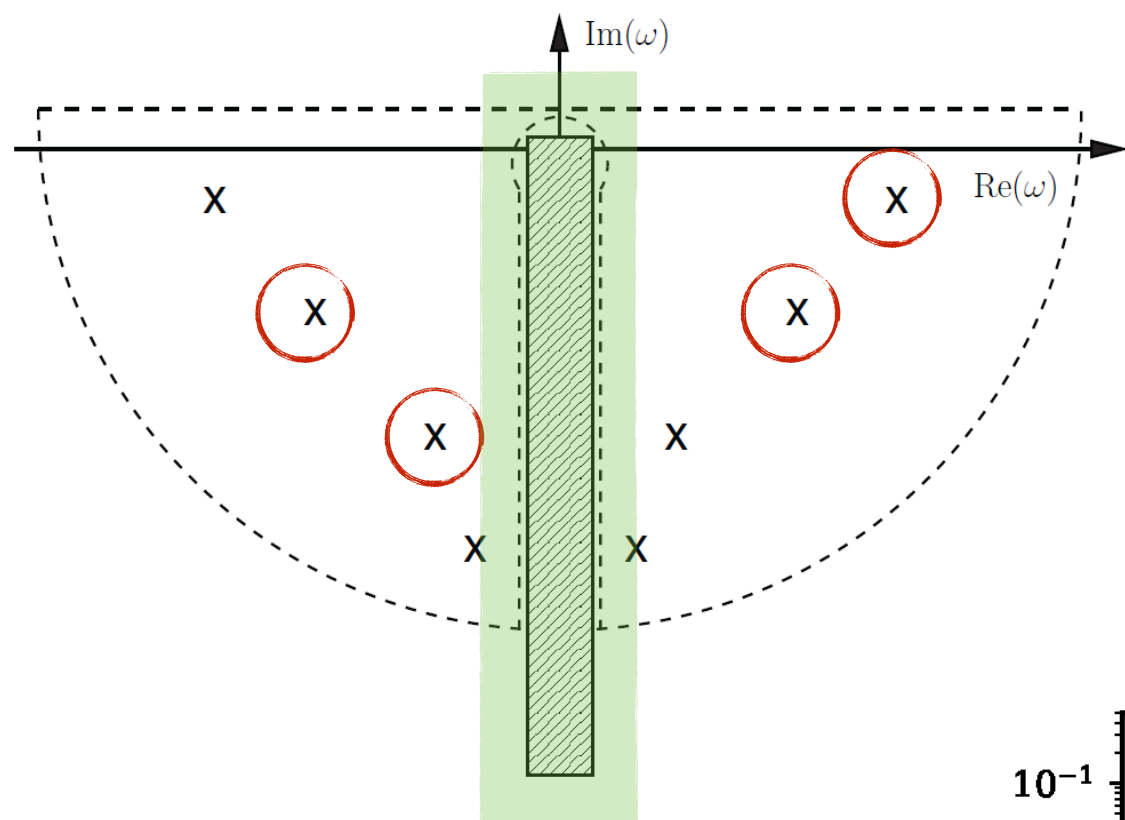


massless

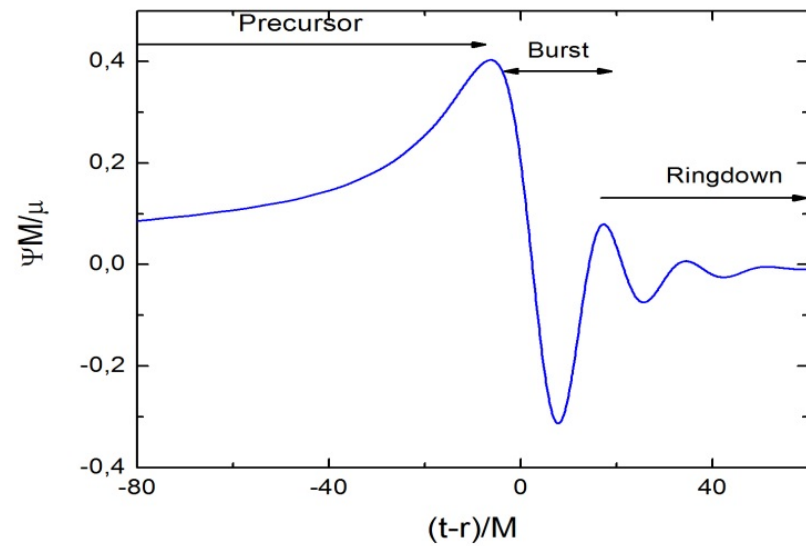


*Black Hole
Quasi Normal
Modes*

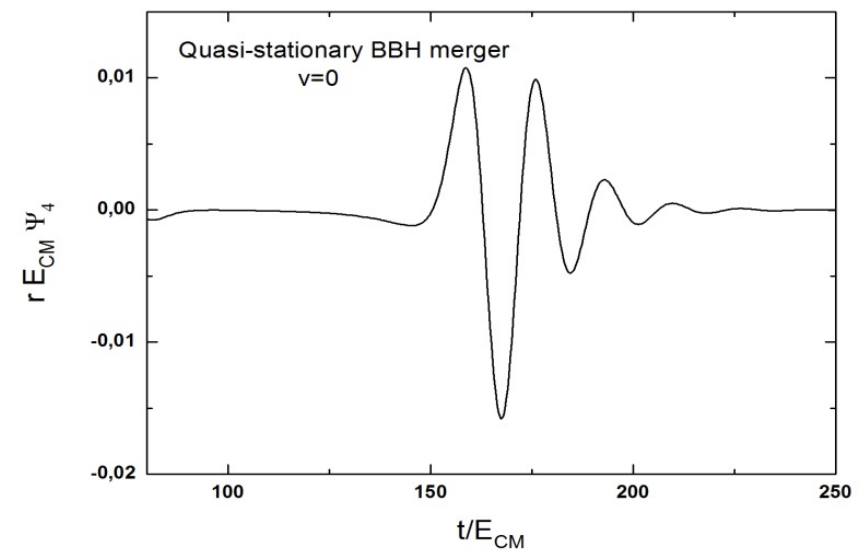
BH - QNMs



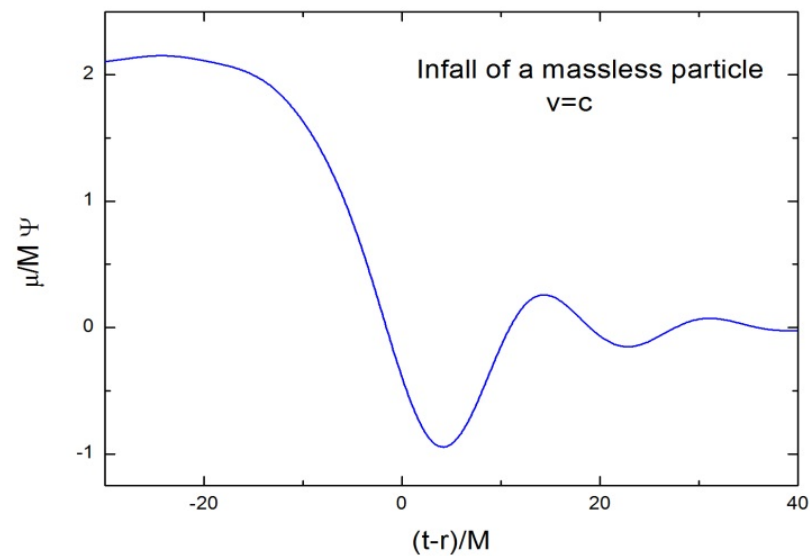
Some examples



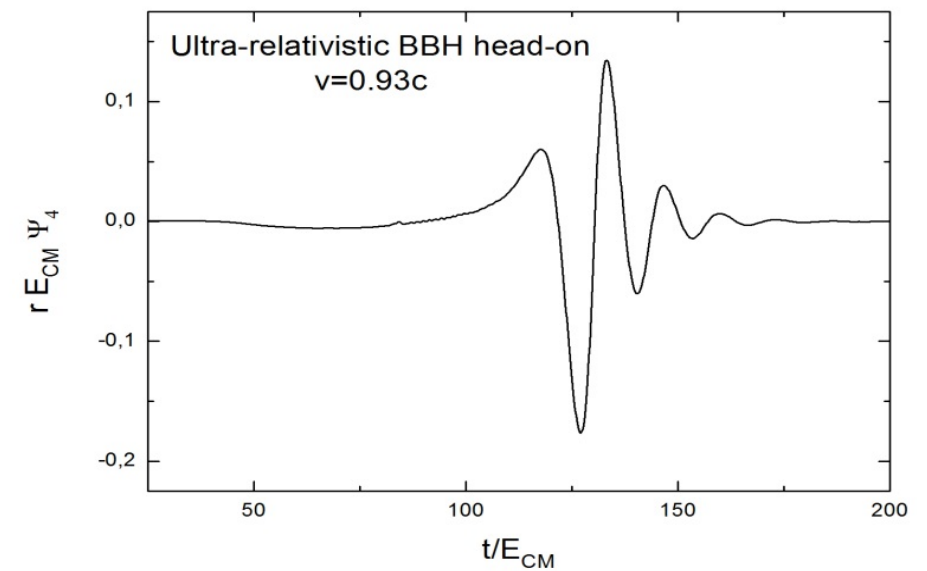
[Davis et al, 1971]



[Anninos et al, 1993]



[Cardoso, Lemos, 2002]



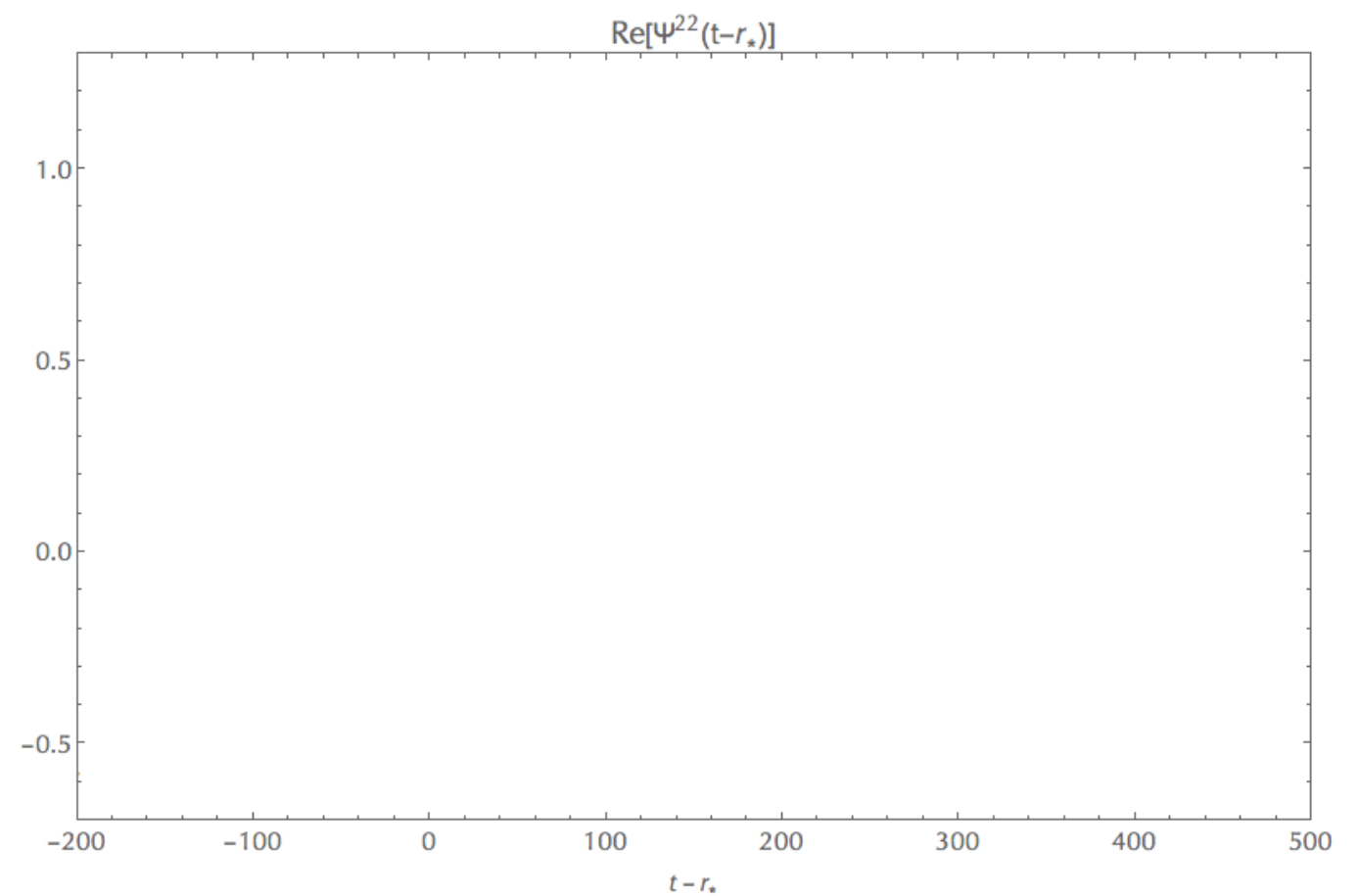
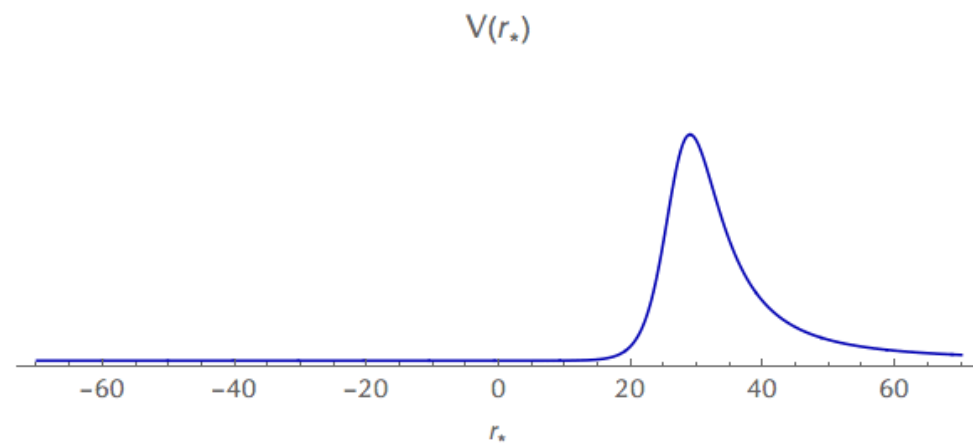
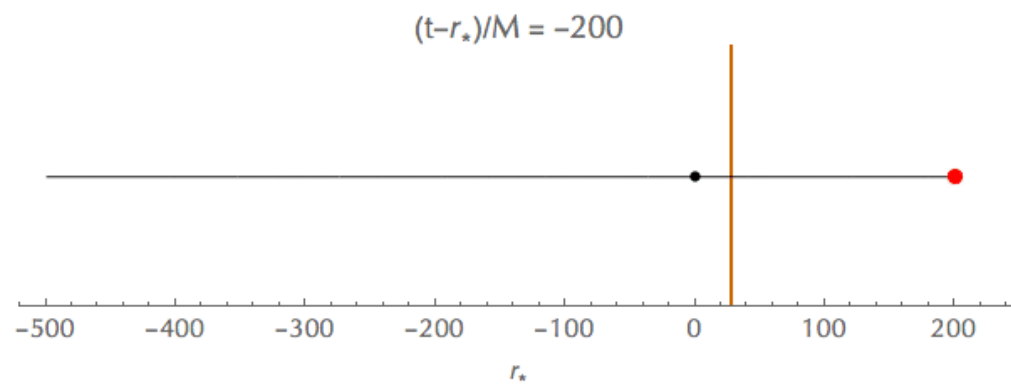
[Sperhake et al, 2008]

$$e^{-0.0898 t/M_{BH}} \sin(0.374 t/M_{BH})$$

Light-ring-connection

Excitation of ringdown happening close to crossing of light ring

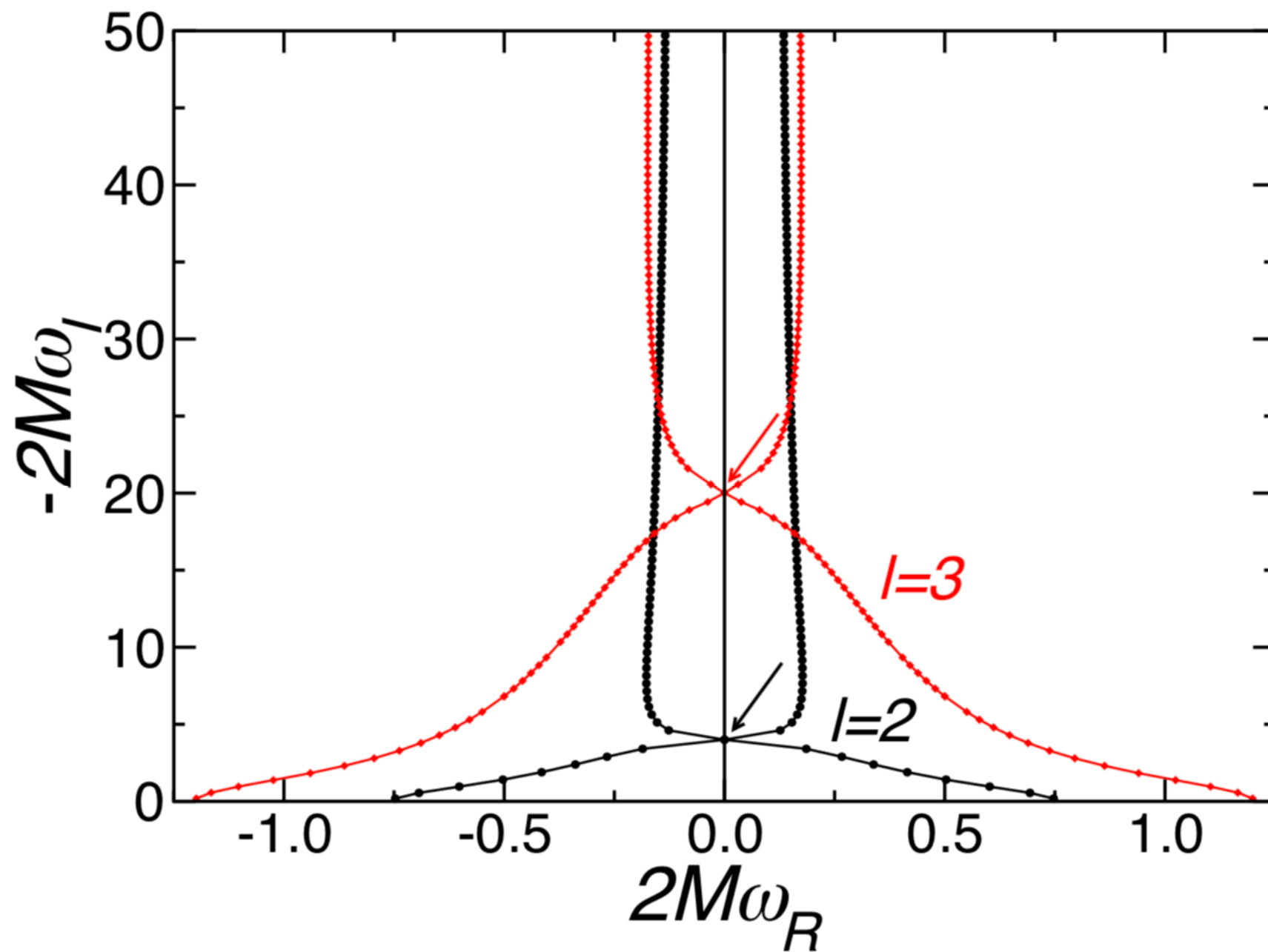
$$\varepsilon = 1.5, \mathcal{J} = 0$$



Radiated energy

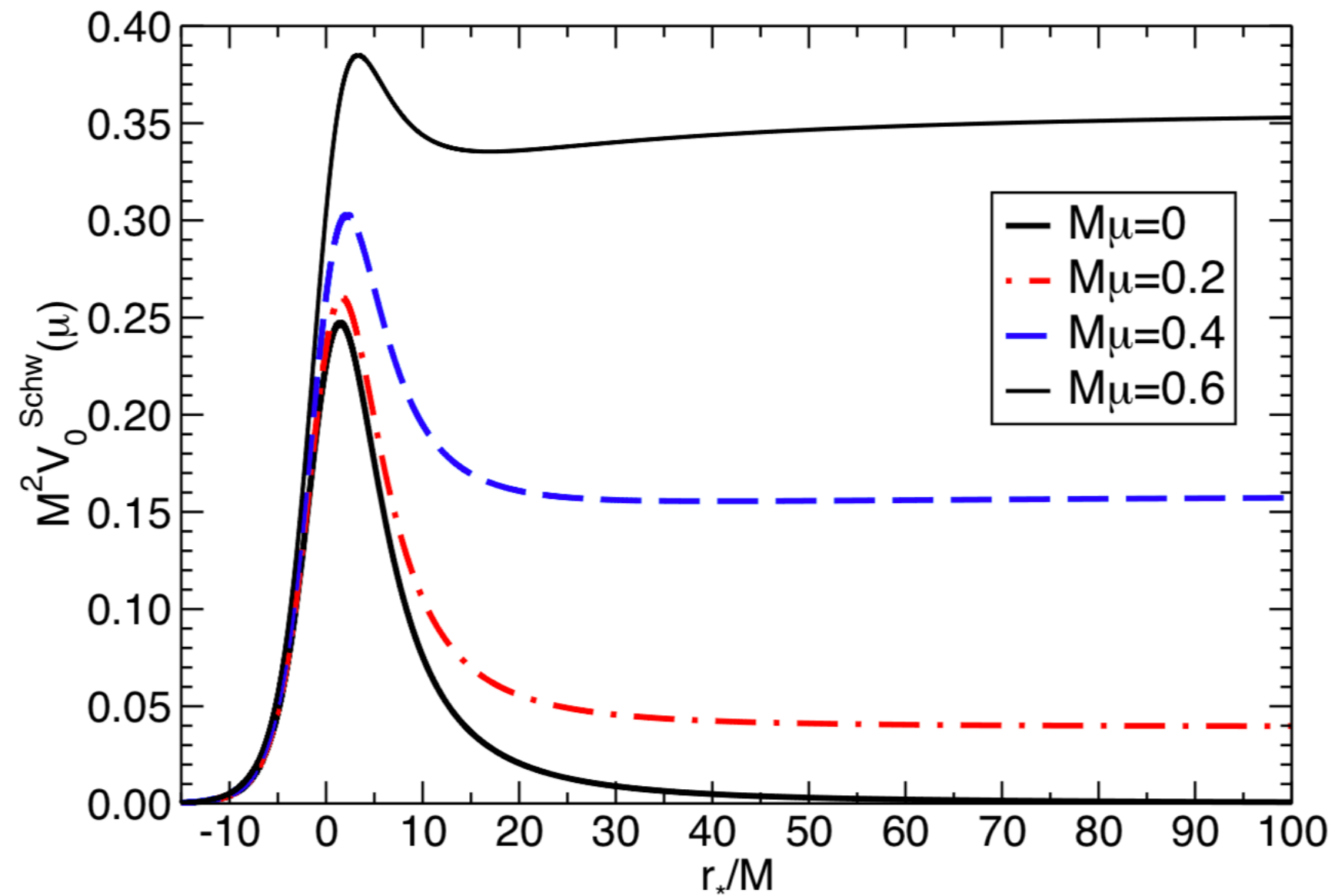
		Radiated energy	Numerical Relativity
Head-on	Point particle	$E_{\text{rad}} = 0.010 \frac{\mu^2}{M}$	$E_{\text{rad}} = 0.010 \frac{\mu^2}{M}$
	Equal mass	$E_{\text{rad}} = 0.00065M$	$E_{\text{rad}} = 0.00057M$
Q-Circular	Point particle	$E_{\text{rad}} = 0.057\mu$	Not done
	Equal mass	$E_{\text{rad}} = 0.014M$	$E_{\text{rad}} = 0.048M$

Schwarzschild frequencies



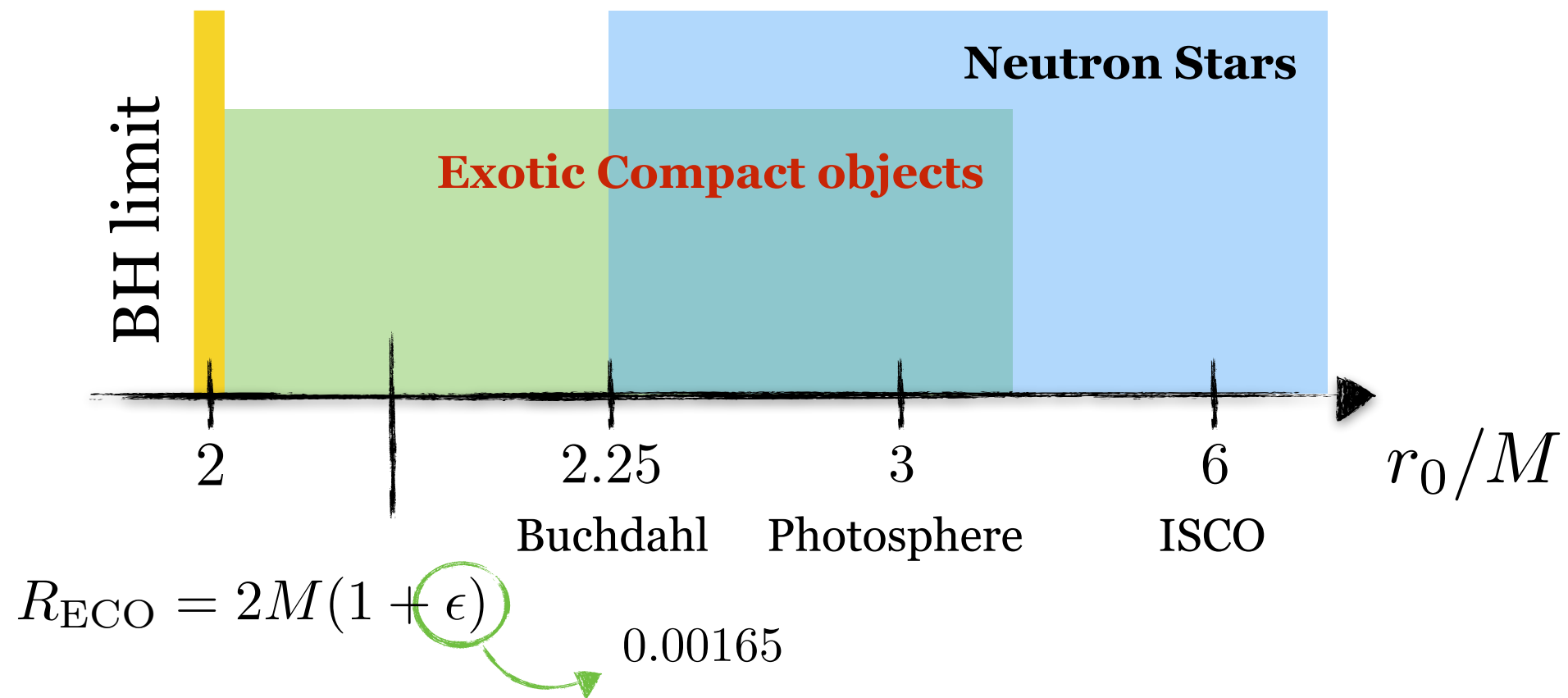
Scalar Waves

Scalar wave - Potential



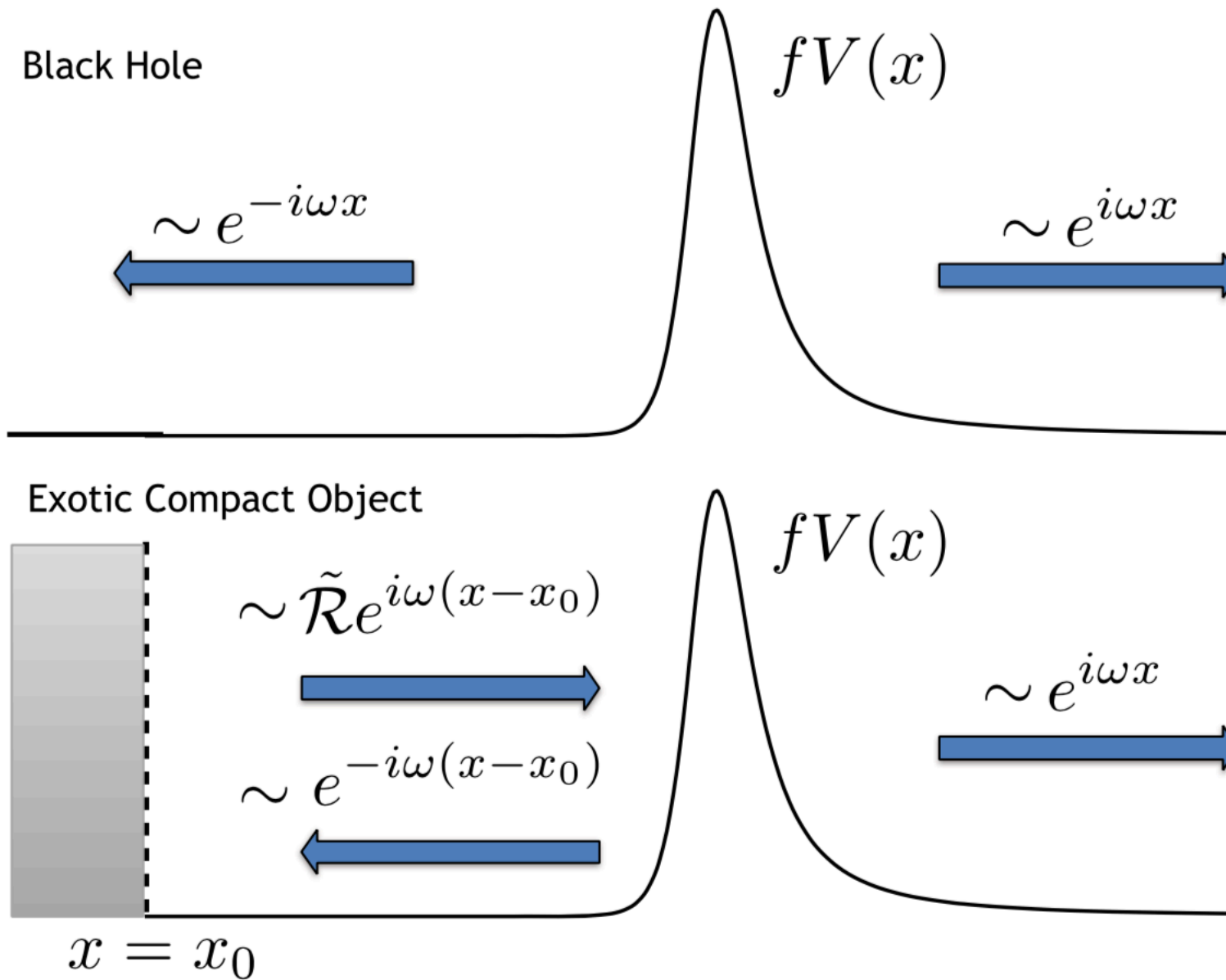
ECOs
and echoes

ECO vs BH

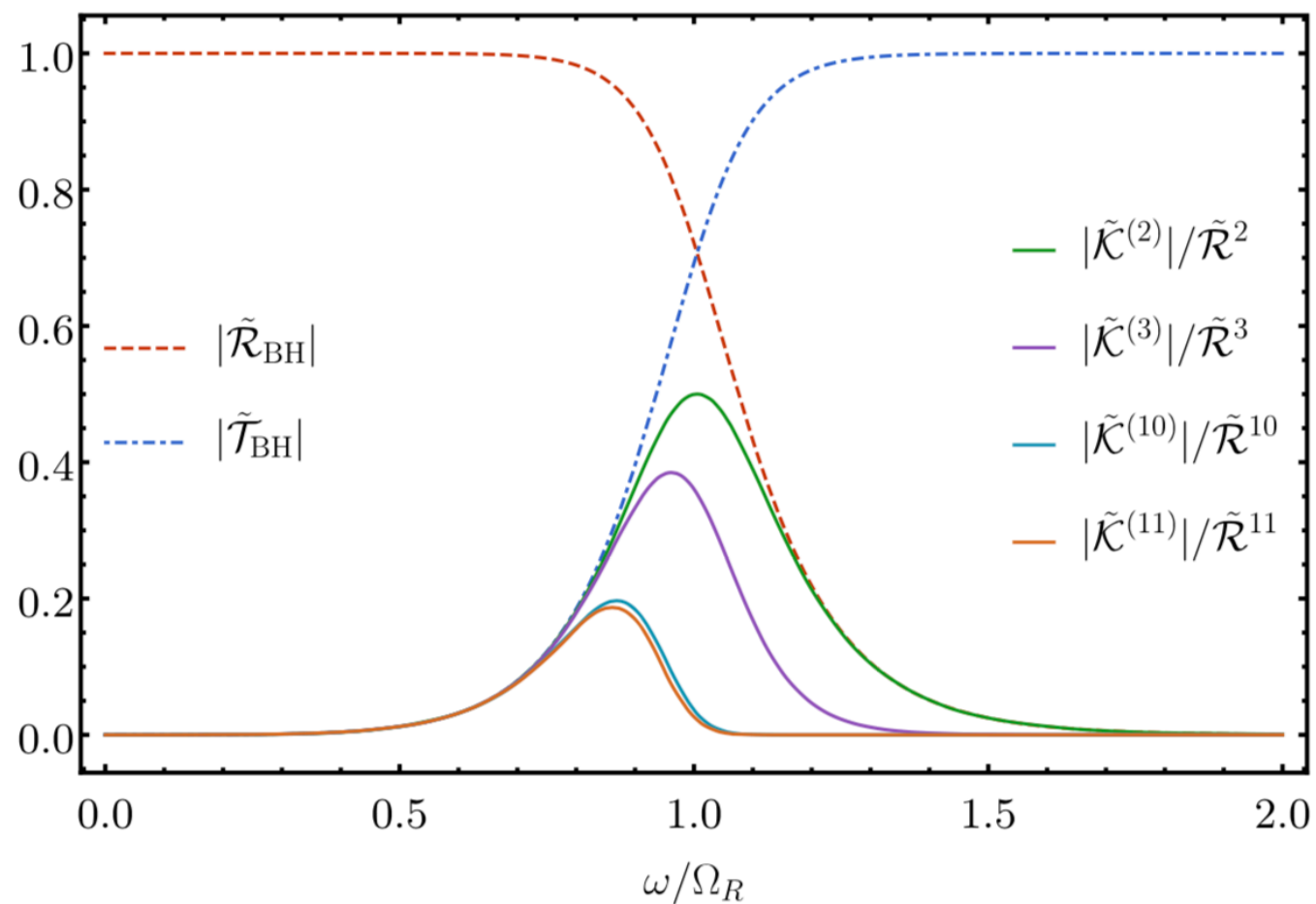


- ✓ QNMs spectrum different from the BH case
- ✓ New structure may be visible in the waveform: the **echoes**

ECO: the boundary conditions



ECO: Reflection and Transmission



low frequencies

$$(M\omega)^2 \ll V$$

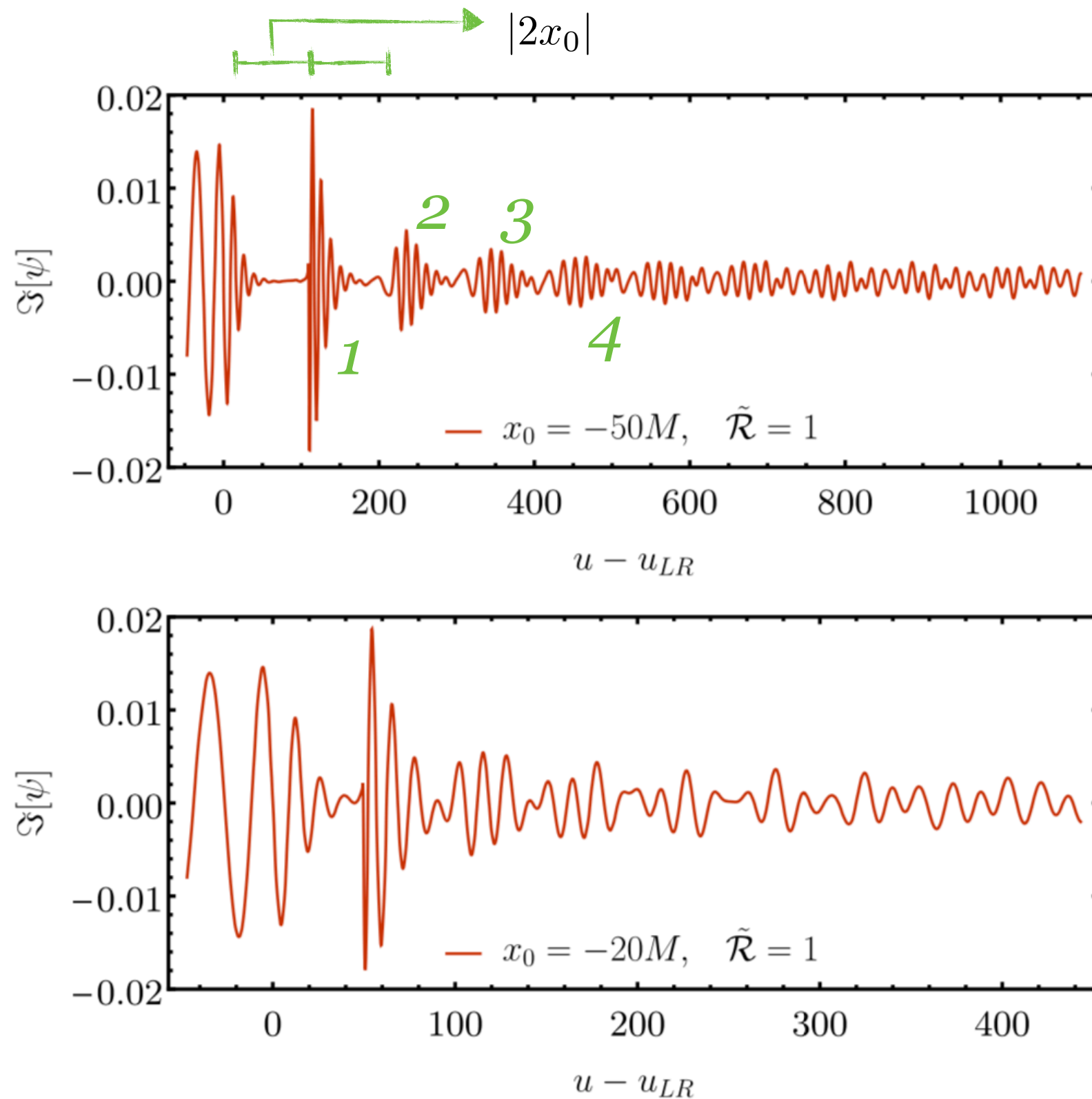
$$|\mathcal{T}_{\text{BH}}| \rightarrow 0 \quad |\mathcal{R}_{\text{BH}}| \rightarrow 1$$

high frequencies

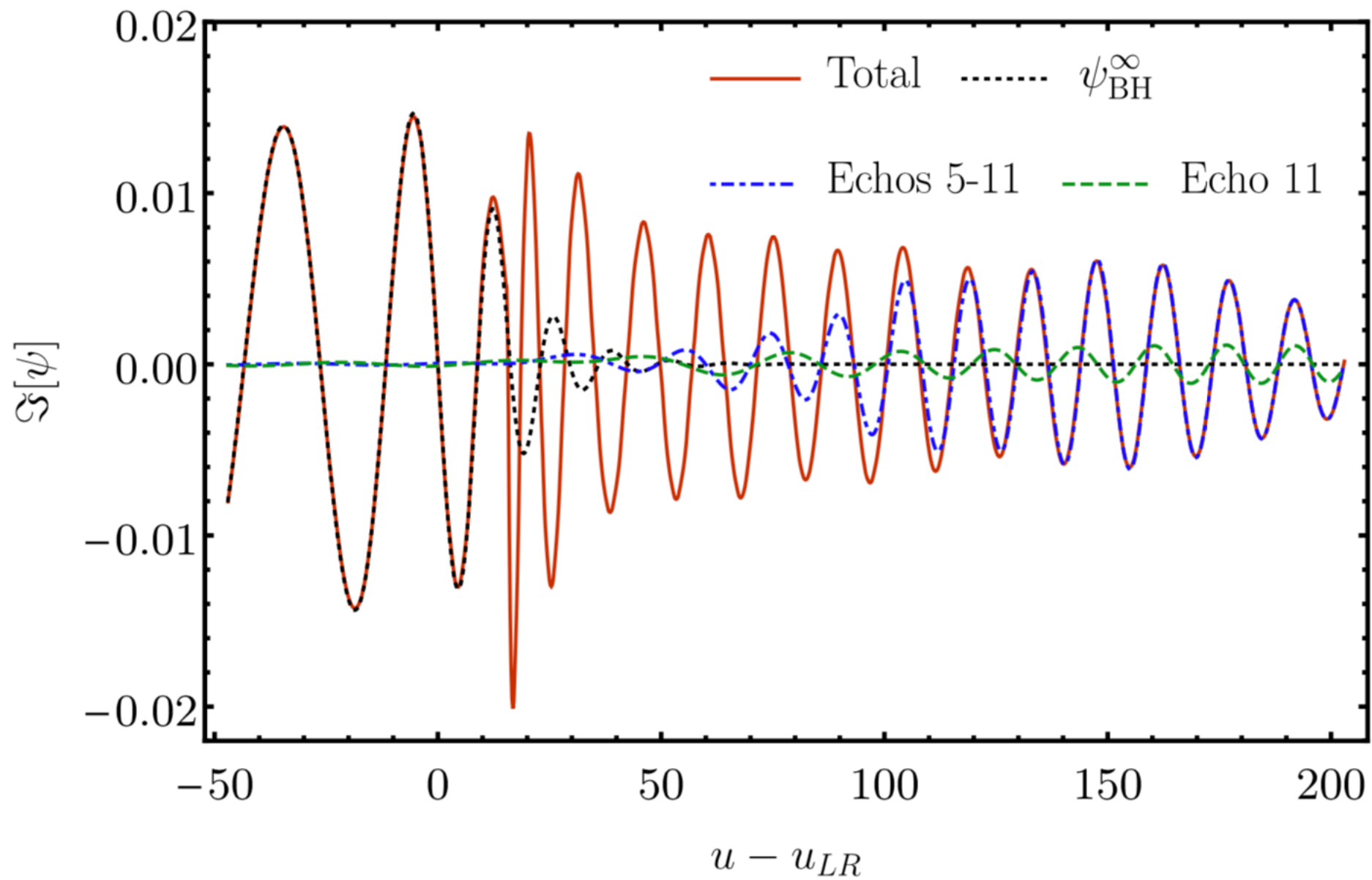
$$(M\omega)^2 \gg V$$

$$|\mathcal{R}_{\text{BH}}| \rightarrow 0 \quad |\mathcal{T}_{\text{BH}}| \rightarrow 1$$

Echoes from ECO

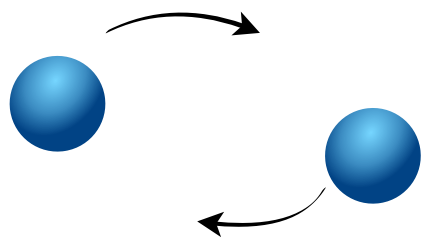
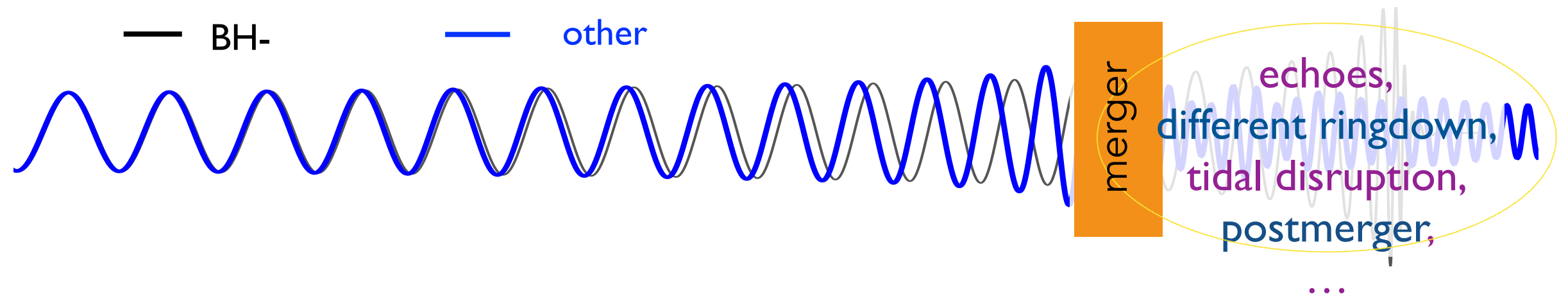


Echoes from ECO

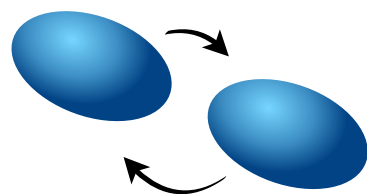


ECOs
phenomenology

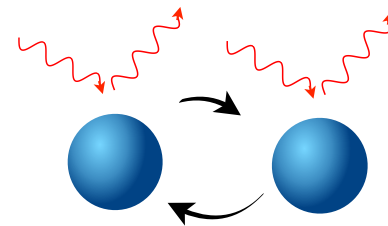
Where do we look for?



~point masses:
same signal
for all objects



tidal effects
+
spins
deformations



absence of horizon
absorption
effects



modified
QNMs and
echoes

Tidal Love Numbers

Tidal interactions leave the footprint of the ECO's structure within GW signals [learning from NS]

*Hinderer, '08; Binnington & Poisson '09
Damour & Nagar '10*

- ✓ Deformation properties encoded within the **Love numbers**

The diagram illustrates the relationship between the ECO's quadrupole, the tidal equation, and the external tidal field. It features a central equation with two arrows pointing outwards to its components.

$$Q_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$

ECO's quadrupole external tidal field

- ✓ λ depends on the **internal** composition of the ECO
- ✓ λ enters within the gravitational waveform

Tidal effects: the recipe

Polar-type perturbation of background metric

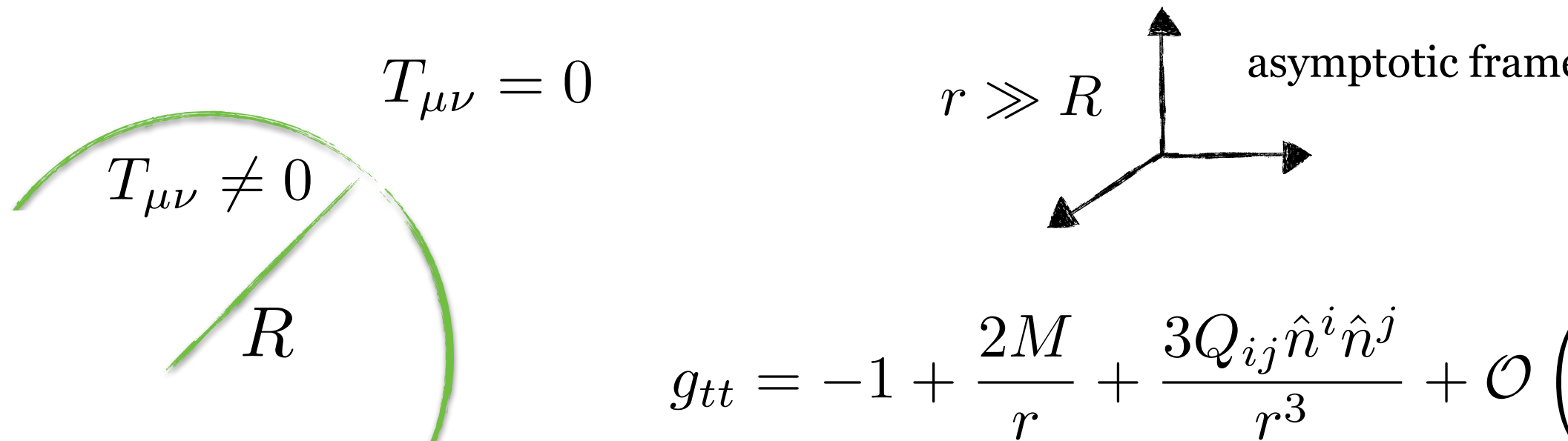
[Regge & Wheeler '57]

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

$$\begin{pmatrix} -e^{\nu(r)} & 0 & 0 & 0 \\ 0 & e^{\lambda(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} + \begin{pmatrix} -e^{\nu(r)} H_0(r) & 0 & 0 & 0 \\ 0 & e^{\lambda(r)} H_2(r) & 0 & 0 \\ 0 & 0 & r^2 K(r) & 0 \\ 0 & 0 & 0 & r^2 K(r) \sin^2 \theta \end{pmatrix} Y_{lm}(\theta, \phi)$$

- ✓ Cook everything within Einstein's equations $G_{\mu\nu} = kT_{\mu\nu}$
- ✓ Solve at linear order in the perturbations
- ✓ Numerically integrate from center to outside with appropriate boundary conditions

Tidal effects: the recipe



The diagram illustrates the matching between an interior region and an exterior asymptotic frame. On the left, a green arc represents a curved surface with radius R . Inside this region, the stress-energy tensor is non-zero, $T_{\mu\nu} \neq 0$. Outside the surface, the stress-energy tensor is zero, $T_{\mu\nu} = 0$. To the right, a 3D coordinate system with three axes is shown, labeled "asymptotic frame". The radial coordinate r is much greater than the radius R , $r \gg R$. Below these diagrams, the metric component g_{tt} is given by the following expansion:

$$g_{tt} = -1 + \frac{2M}{r} + \frac{3Q_{ij}\hat{n}^i\hat{n}^j}{r^3} + \mathcal{O}\left(\frac{1}{r^3}\right) - \mathcal{E}_{ij}x^ix^j + \mathcal{O}(r^3)$$

✓ Matching interior & exterior solution

$$Q_{ij} = \frac{2}{3} \kappa_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$

✓ Deformation properties encoded within the **Love numbers**

Tidal Love Numbers

k_2 (or λ) depends on the compactness $\mathcal{C} = M/R$, only

Neutron stars

$$\mathcal{C} \in [0.1 \div 0.2]$$

$$k_2 \neq 0$$

GW170104



constrain the NS
equation of state

Black holes

$$\mathcal{C} = 1/2$$

$$k_2 = 0$$

ECOs

$$\mathcal{C} < 0.5 \quad k_2 \neq 0$$

$$\mathcal{C} \rightarrow 0.5 \quad k_2 \rightarrow 0$$

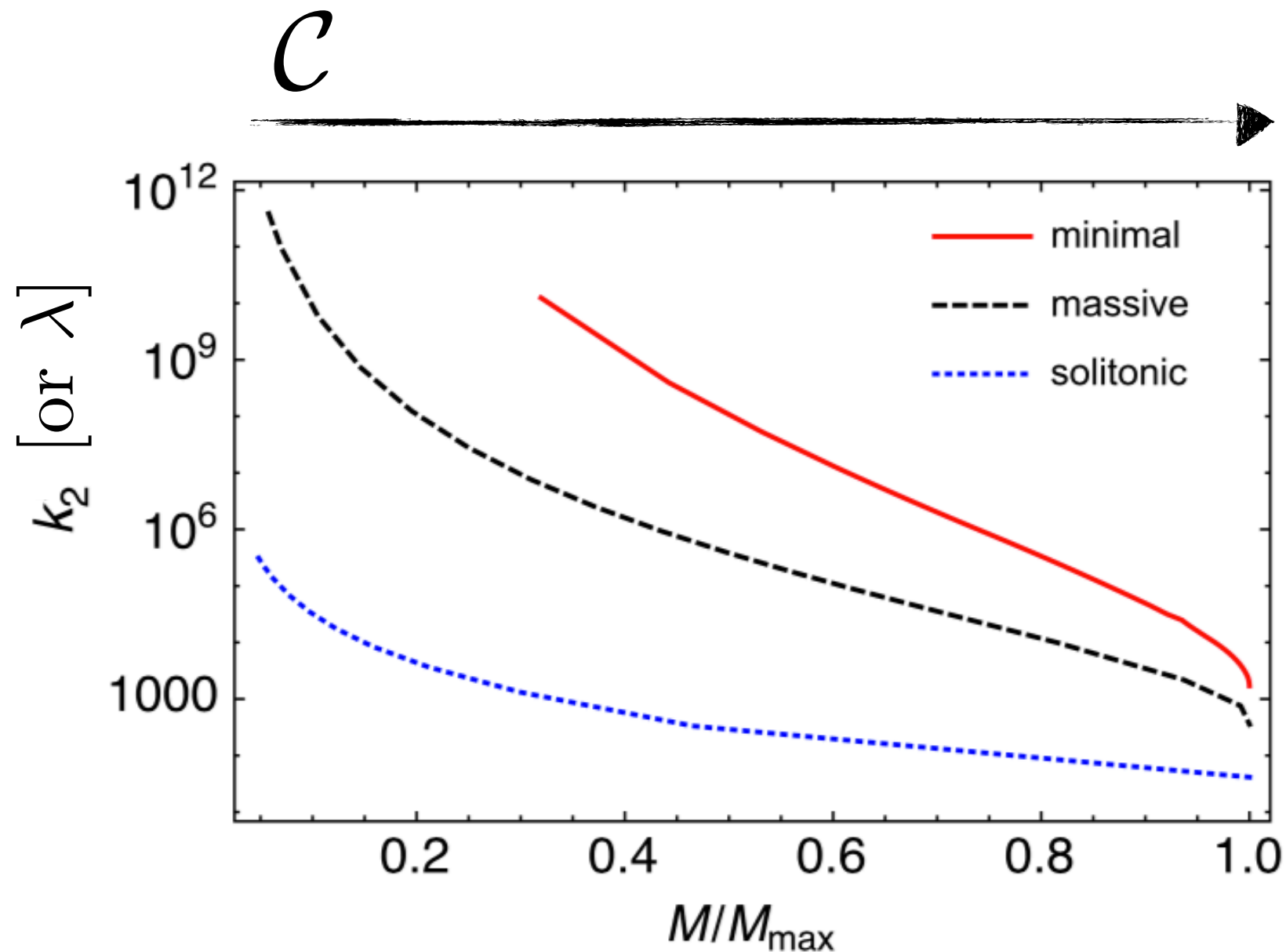


signature to distinguish
ECO and BH's inspiral

[Cardoso et al., '17, Maselli et al., '17
Sennett et al., 17]

The boson star case

Monotonic function of the compactness $\mathcal{C} = M/R$



✓ λ decreases as the compactness grows and goes to zero

[see LIGO 1805.11581 for the NS game]

ECO Love numbers

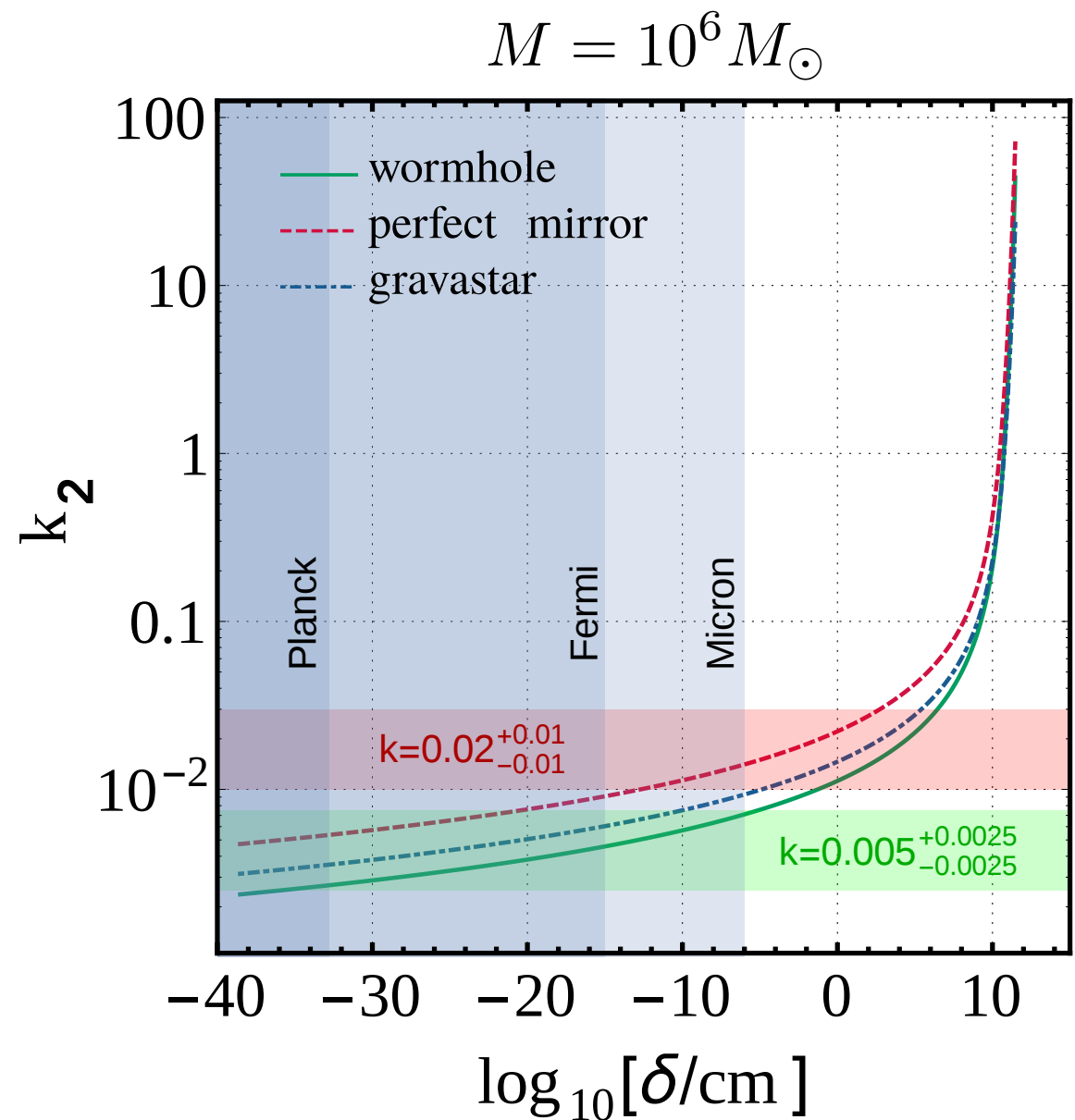
- ✓ k_2 vanishes logarithmically in the BH limit $k_2 \sim 1/|\log \delta|$
- ✓ k_2 translate in distance ECO surface from its Schwarzschild radius

$$\delta \equiv r_0 - 2M \sim 2M e^{-1/k_2}$$

$$k_2 \simeq 0.005$$



$$\delta \simeq 10^{-33} \text{cm} \sim \ell_P$$



Where do we look in?

Tidal effects add linearly to the GW phase $h(f) = \mathcal{A}e^{i[\psi_{\text{PP}}(f) + \psi_{\text{T}}(f)]}$

$$\psi_{\text{PP}} \propto \left[1 - \left(\frac{743}{336} + \frac{11}{4}\nu \right) \frac{(m\pi f)^{2/3}}{c^2} + \frac{\dots}{c^3} + \dots + \frac{\dots}{c^7} \right]$$


Newt


1 PN


1.5 PN

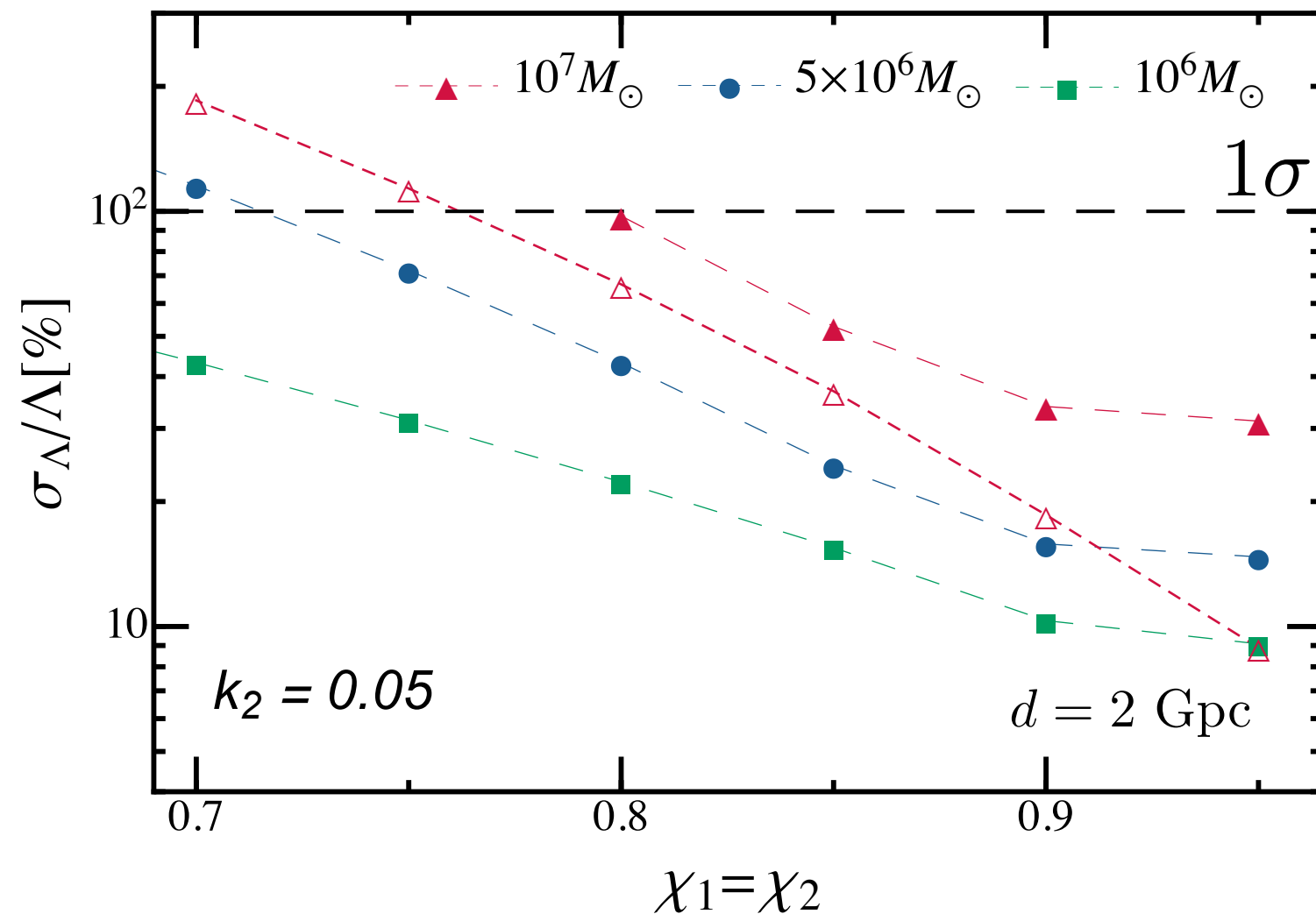
$$\psi_{\text{T}} \propto \frac{1}{26} \left[\left(1 + \frac{12m_2}{m_1} \right) \lambda_1 + \left(1 + 12\frac{m_1}{m_2} \right) \lambda_2 \right] \frac{(m\pi f)^{10/3}}{c^{10}} + \frac{\dots}{c^{11}}$$


 Λ

- ✓ The *average* tidal deformability enters the signal at the leading order

Detectability with LISA

- ✓ Spins lead to more than one order of magnitude improvement



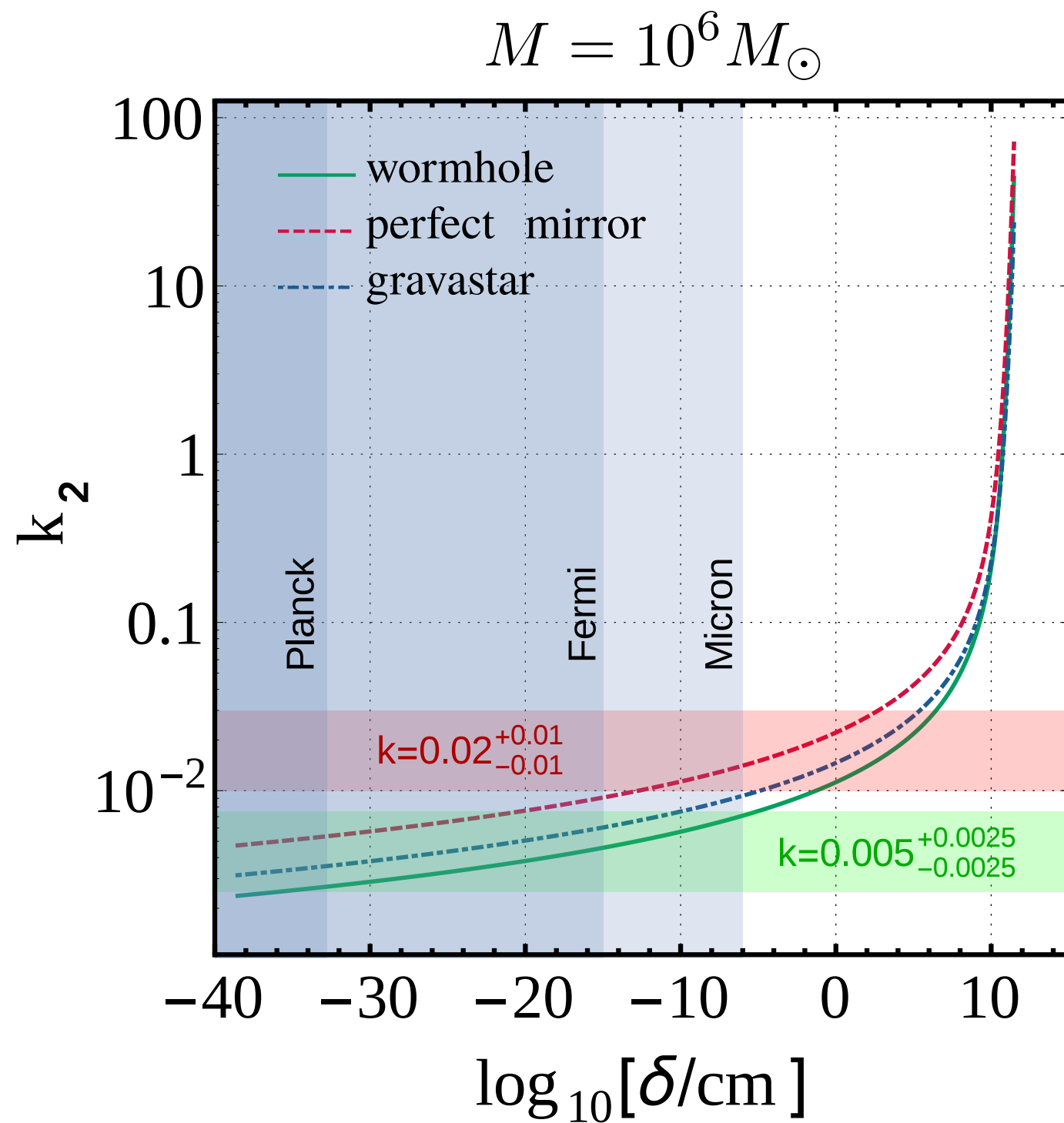
● $\sigma_\Lambda/\Lambda \sim 1/k_2$

$k_2 = 0.005 \text{ @ } d = 2 \text{ Gpc}$

● $\sigma_\Lambda/\Lambda \sim d$

$k_2 = 0.05 \text{ @ } d = 20 \text{ Gpc}$

ECO Love numbers



Modified Quadrupole

Test based on the no-hair theorem

[Krishnendu et al., '17]

✓ Agnostic deformation of the quadrupole

$$Q = -m^3 \chi^2 \kappa$$

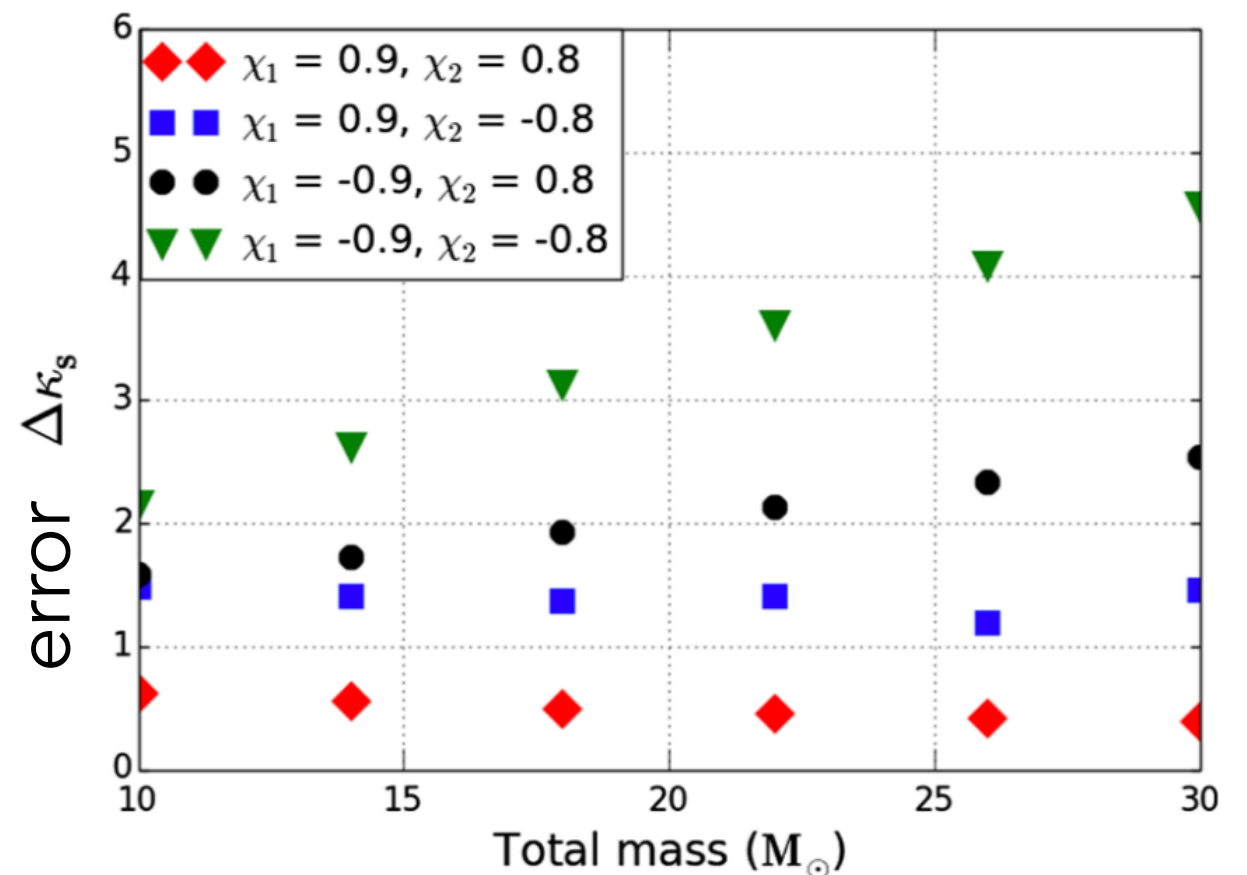
10-150 boson stars

1 BH

✓ Q enters the GW signal at 2 PN through

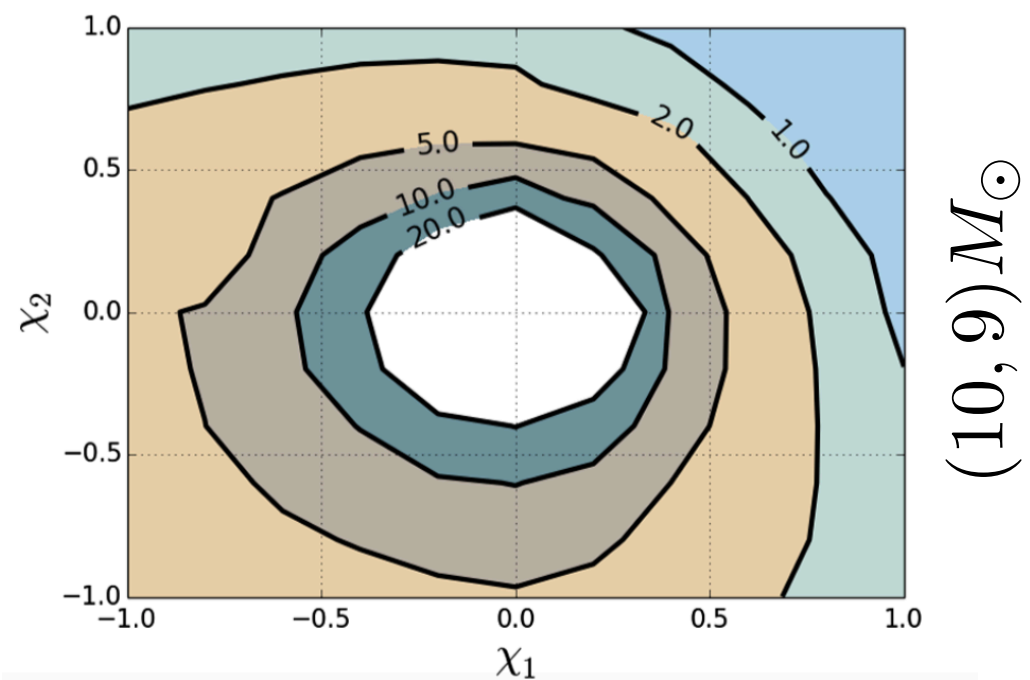
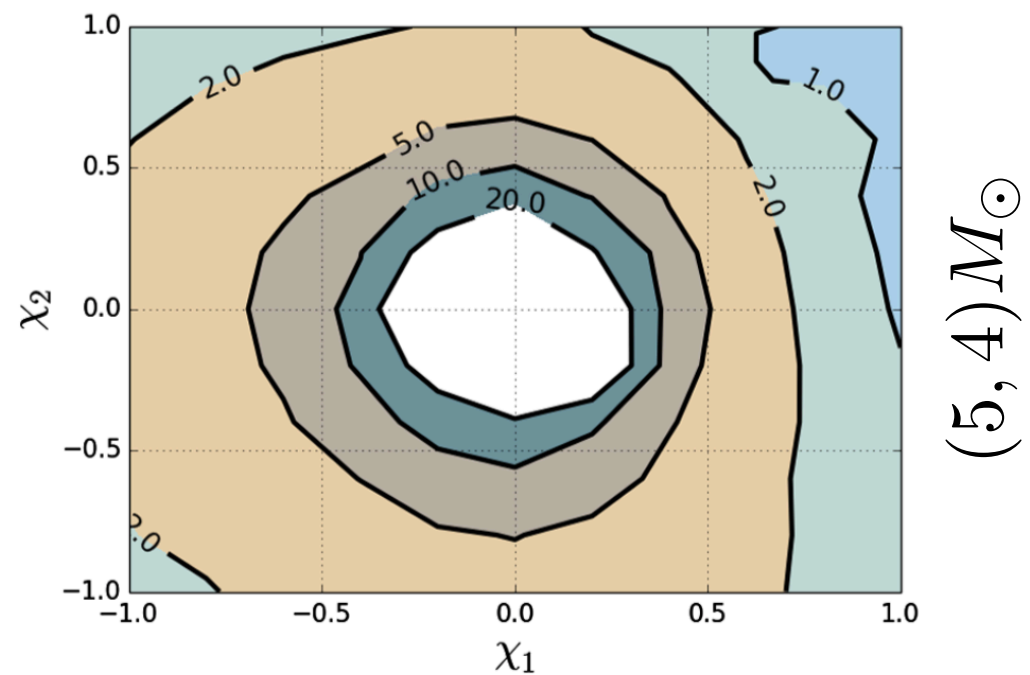
$$\kappa_s = (\kappa_1 + \kappa_2)/2$$

$$\kappa_a = (\kappa_1 - \kappa_2)/2$$

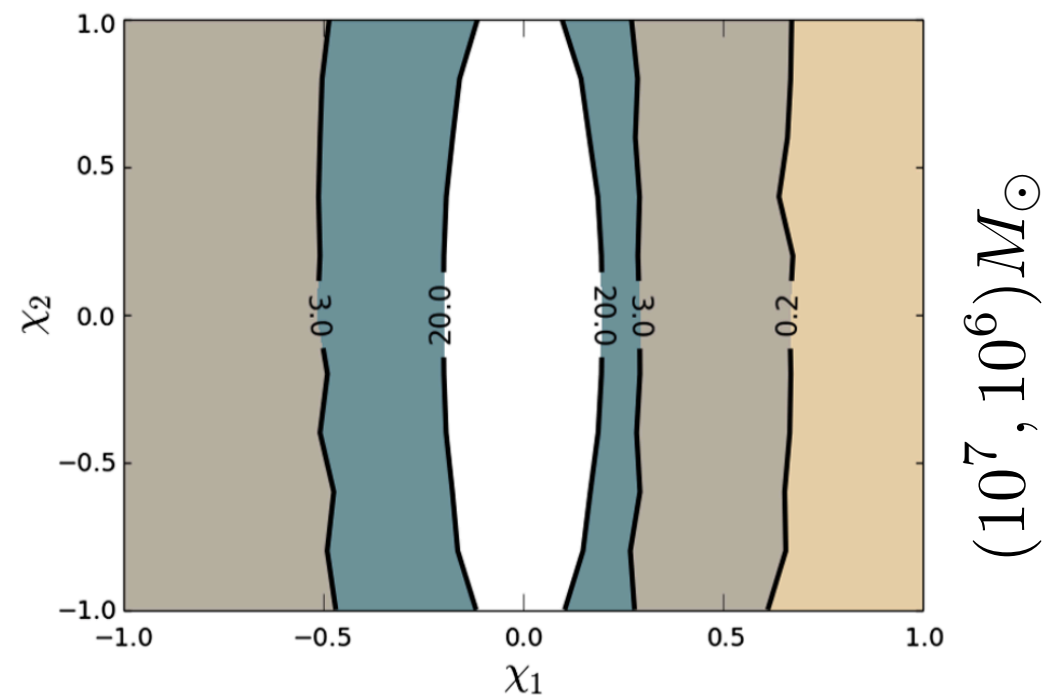
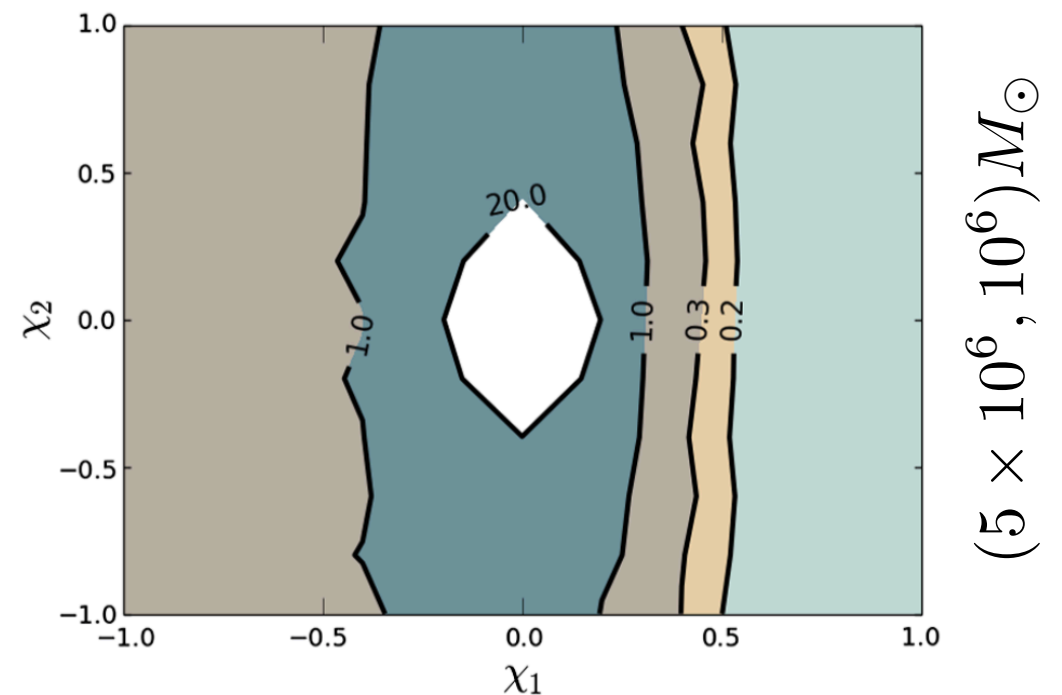


Quadrupole errors

LIGO - SNR = 10



LISA - d = 3Gpc



Tidal Heating

BH as classical viscous fluid

- ✓ Small contribution to the orbital evolution given by dissipation of energy and angular momentum at the horizon

$$\psi_{\text{TH}}^{\text{BH}} =$$

2.5PN  $G(m_i) \frac{(m\pi f)^{8/3}}{c^8}$  4PN

- ✓ For known matter GW absorption is negligible

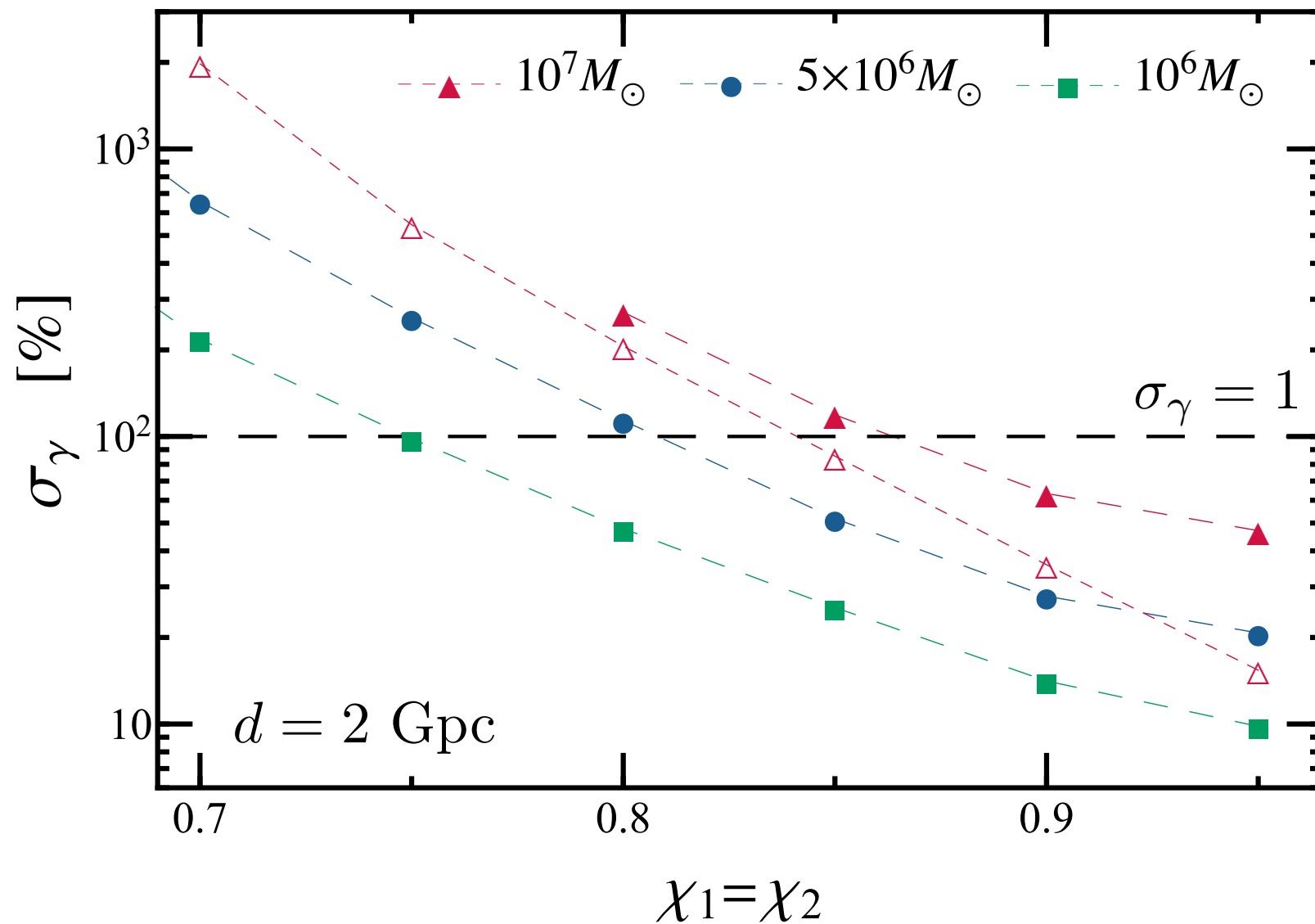
$$h(f) = \mathcal{A} e^{i[\psi_{\text{PP}}(f) + \psi_{\text{TH}}(f)]}$$

$$\psi_{\text{TH}} = \gamma \times \psi_{\text{TH}}^{\text{BH}}$$

$$\gamma = 0 \quad \longleftrightarrow \quad \text{ECO}$$

$$\gamma = 1 \quad \longleftrightarrow \quad \text{BH}$$

Detectability: tidal heating



- ✓ Spin enhancement is more dramatic
- ✓ Chances to put interesting constraints up to 20 Gpc