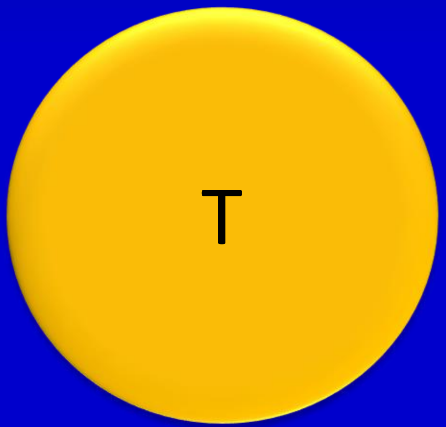
Spinning Up a Neutron Star

G. Srinivasan

Manifestation of Neutron Stars

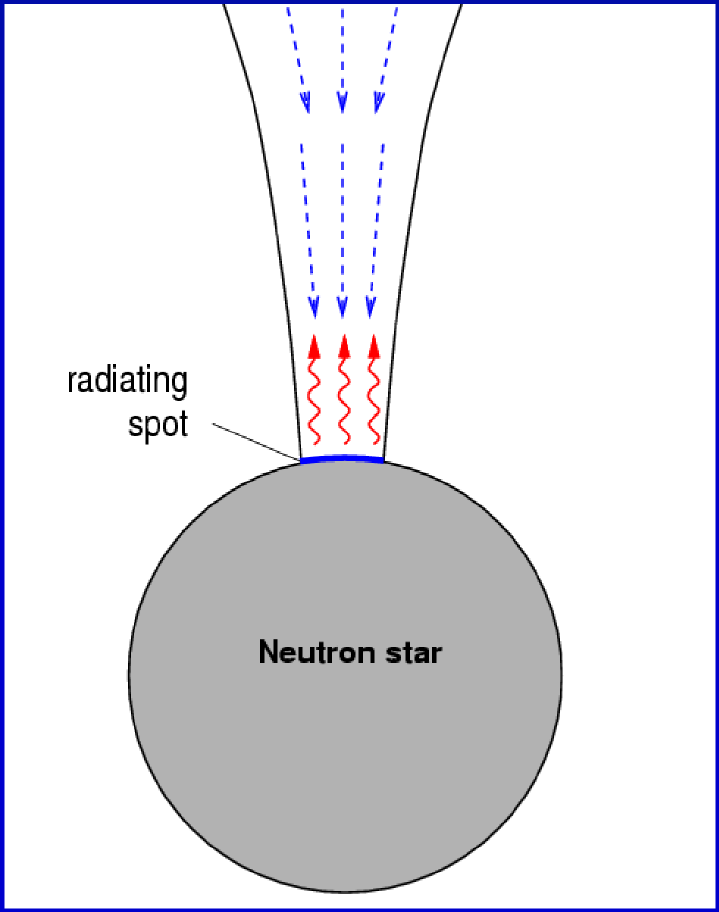


Black Body emission

$$L = 4\pi R^2 \sigma T^4$$

Since the surface temperature is expected to be ~ 10^7 Kelvin the thermal radiation will be in the soft X-ray region.

Manifestation of Neutron Stars



X-ray emission results when matter accretes onto the surface of a neutron star.

The gravitational potential energy is converted to radiation.

The energy released is roughly

$$0.1mc^2$$

$$\textbf{Luminosity} = L = \eta \dot{M} c^2$$

Electric Dipole Radiation

$$\frac{dE}{dt} = \frac{2e^2}{3c^3} a^2 = \frac{2}{3c^3} (\ddot{d})^2$$

Where ‘d’ is the electric dipole moment. For an oscillating charge, the energy loss rate, averaged over a period is given by

$$\frac{dE}{dt} = \frac{1}{3c^3} d^2 \Omega^4$$

$$d = d \cos \Omega t \Rightarrow \left(\ddot{d}\right)^2 = d^2 \Omega^4 \cos^2 \Omega t$$

Magnetic Dipole Radiation

$$L = \frac{dE}{dt} = \frac{2}{3c^3} m_{\mathrm{perp}}^2 \Omega^4 = \frac{1}{3c^3} B^2 R^6 \Omega^4$$

The magnetic moment is related to the magnetic field strength by the relation

$$m = BR^3$$

Luminosity of a Rotating Magnet

$$L = \frac{dE}{dt} = \frac{2}{3c^3} m_{\text{perp}}^2 \Omega^4 = \frac{1}{3c^3} B^2 R^6 \Omega^4$$

$$L \propto B^2$$

$$L \propto \Omega^4$$

A Rotating mass will also radiate Gravitational Waves

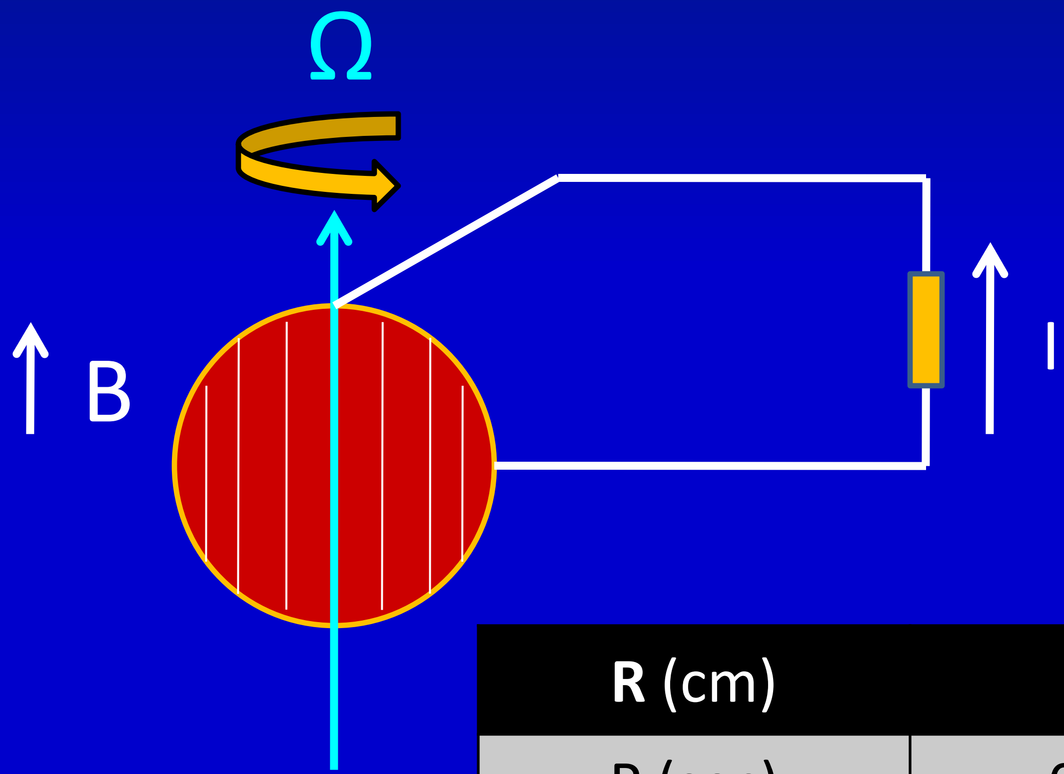
$$-\frac{dE}{dt} = \frac{G}{45c^5}\left(\dddot{Q}\right)^2$$

Here, Q is the Quadrupole Moment. Conservation of mass forbids “Monopole Radiation”. Conservation of momentum of centre of mass forbids “Dipole Radiation”.

Because of c^5 in the denominator and the smallness of G, this radiation is very weak.

Neutron Stars as Pulsars

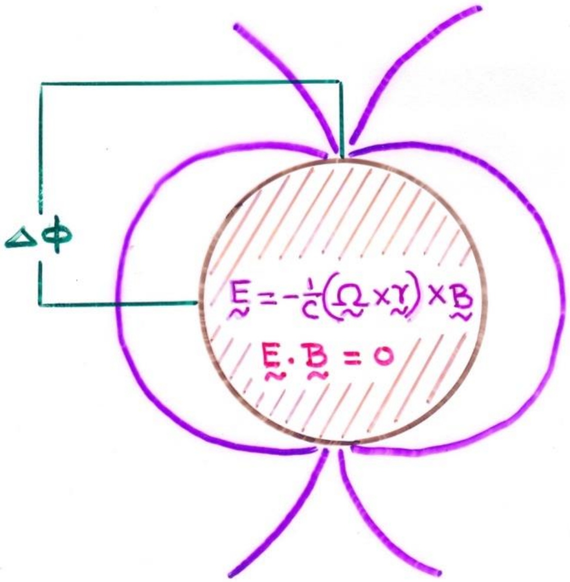
Neutron stars are powerful dynamos!



Faraday's
Unipolar Dynamo

		Neutron Star
R (cm)	**10**	10^6
P (sec)	0.015	**1**
B (Gauss)	**10^4**	**10^{12}**
Δϕ (Volts)	5	**> 10^{16}**

The Unipolar inductor



$\Omega \parallel B$

Potential is 'quadrupolar'

$$\phi = -\left(\frac{B\Omega R^5}{3c\, r^3}\right) P_2(\cos\theta)$$

Surface charge density

$$\sigma = -\frac{B\Omega R}{4\pi c}\cos^2\theta$$

$$E \cdot B = -\left(\frac{\Omega R}{c}\right)\left(\frac{R}{r}\right)^7 B^2 \cos^3\theta$$

Outside the star, there will be an electric field parallel to the magnetic field.

$$\Delta\phi = 3 \times 10^{16}\left(\frac{B_{12}}{P}\right) \text{ Volts}$$

Plasma density:

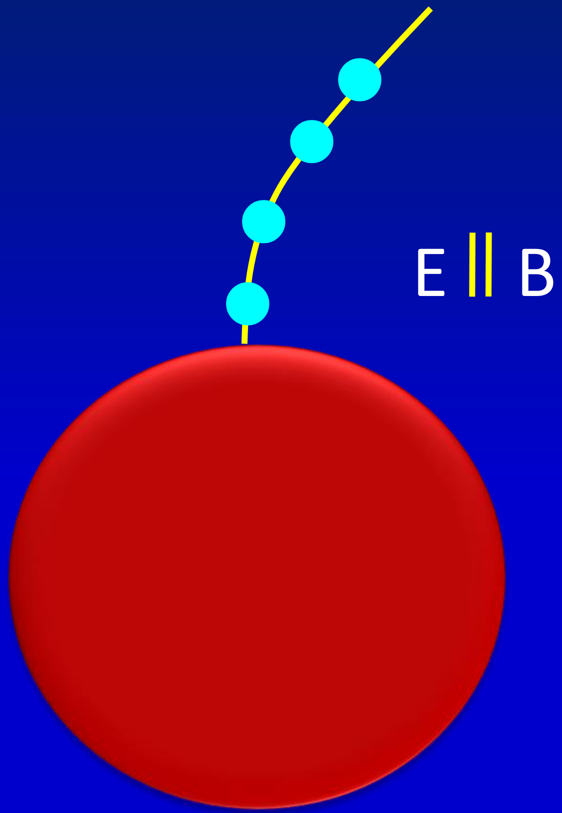
$$n = 7 \times 10^{-2}\left(\frac{B_z}{P}\right) \text{ cm}^{-3}$$

These charges will distribute themselves outside so as to make field lines equipotentials.

Thus $E \cdot B \rightarrow 0$ even outside

Pulsar electrodynamics

- The electrostatic force on a charge at the surface is far greater than all other forces, including nuclear forces.
- Therefore, charges are pulled out of the surface, and accelerated to ultra relativistic speeds by the electric field.
- Since the magnetic field near the surface is very strong, the charges are confined to move along the field lines, rather than cross them.
- **Therefore, the accelerated charges will slide along the field lines like beads on a string.**



Since the magnetic field near the surface is very strong, the charges are confined to move along the field lines, rather than cross them.

Therefore, the charges accelerated by the electric field parallel to the magnetic field will slide along the field lines like beads on a string.

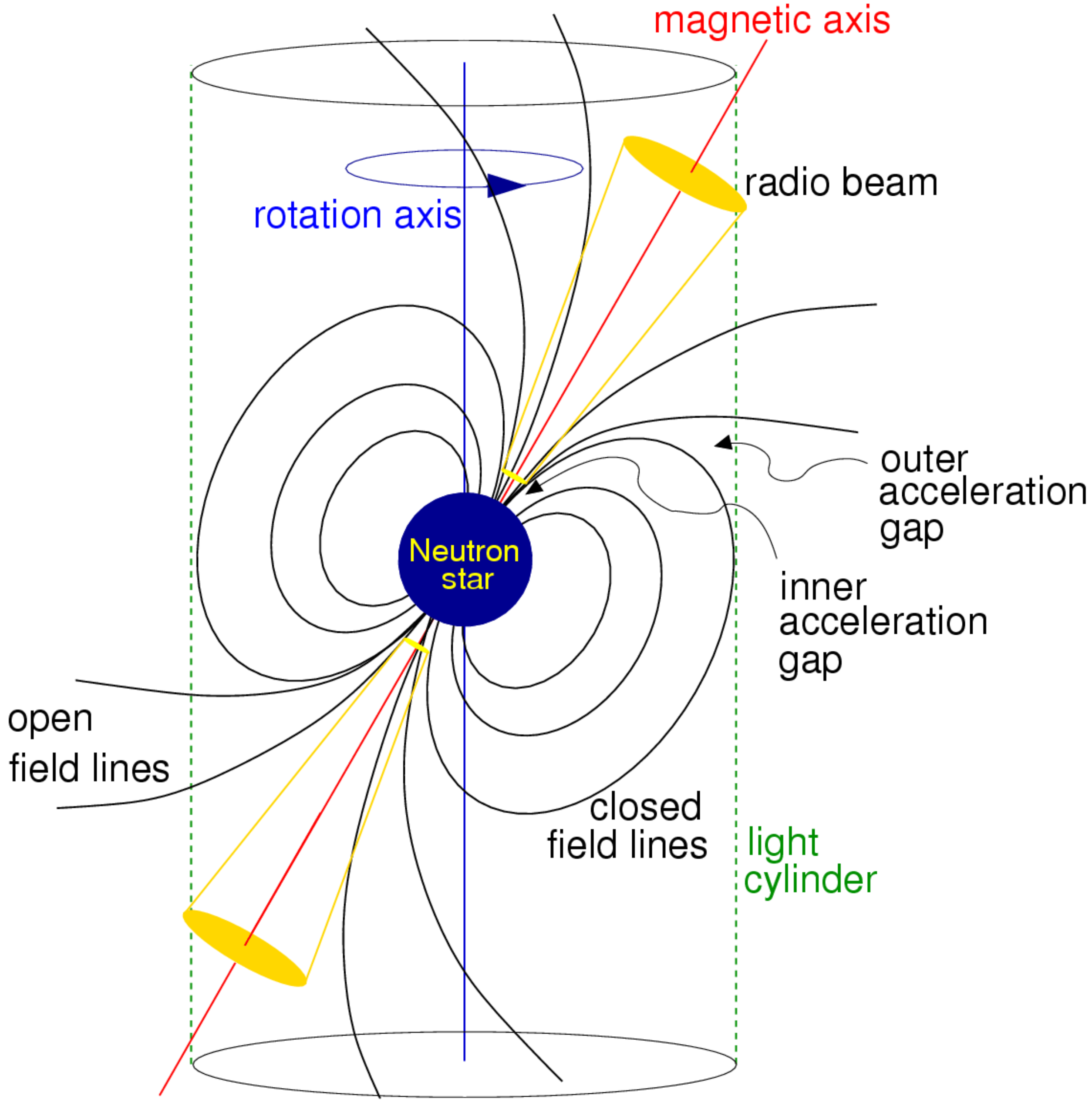
Magnetosphere of the neutron star

- The charges pulled out and accelerated are "trapped" in the magnetic field lines.
- They can move along the field lines, from one pole to the other, and back.
- Since the field lines are anchored to the surface, they will corotate with the star.
- The charges trapped in the field will ALSO corotate with the star.
- This is the magnetosphere of the neutron star.

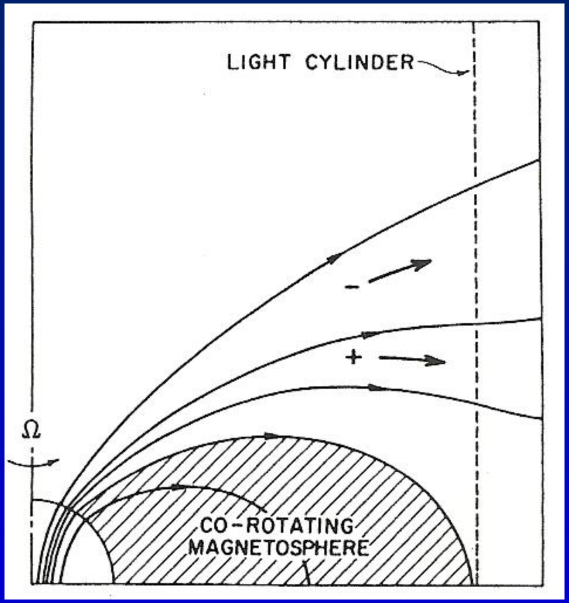
Light Cylinder

- The corotating magnetosphere cannot be larger than a critical size.
- At a critical distance from the star, the corotating charges will be moving at the speed of light.
- Clearly, corotation is not possible beyond this radius.
- This important distance is known as “speed of light radius” or “light cylinder radius”.

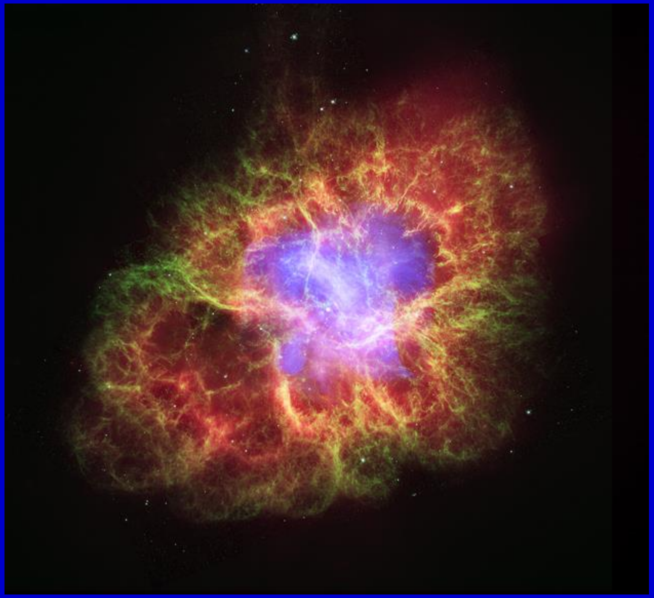
$$v = R\Omega \quad \Rightarrow \quad \boxed{R_{LC} = \frac{c}{\Omega}}$$



magnetic axis
radio beam
rotation axis
outer acceleration gap
inner acceleration gap
Neutron star
open field lines
closed field lines
light cylinder



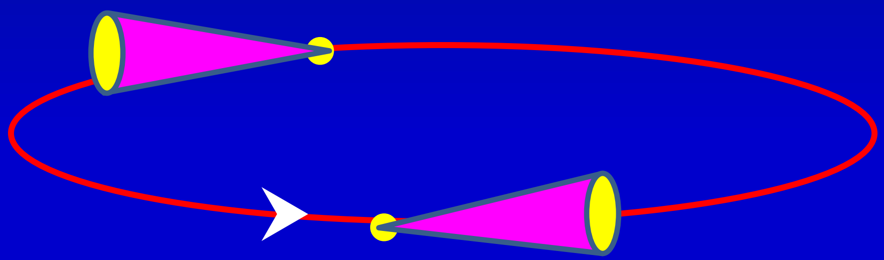
Charges moving along field lines that cross the light cylinder will escape from the star.



These are the charges that fill the CRAB NEBULA.

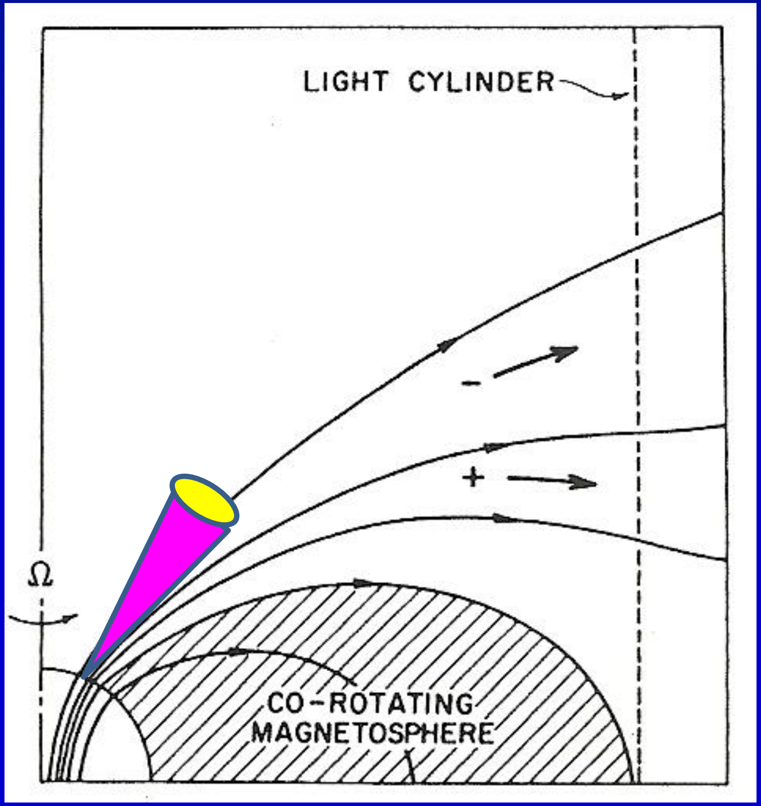
The radiation from the Nebula is the radiation emitted by these particles gyrate in the nebular magnetic field.

Radiation from a relativistic charge

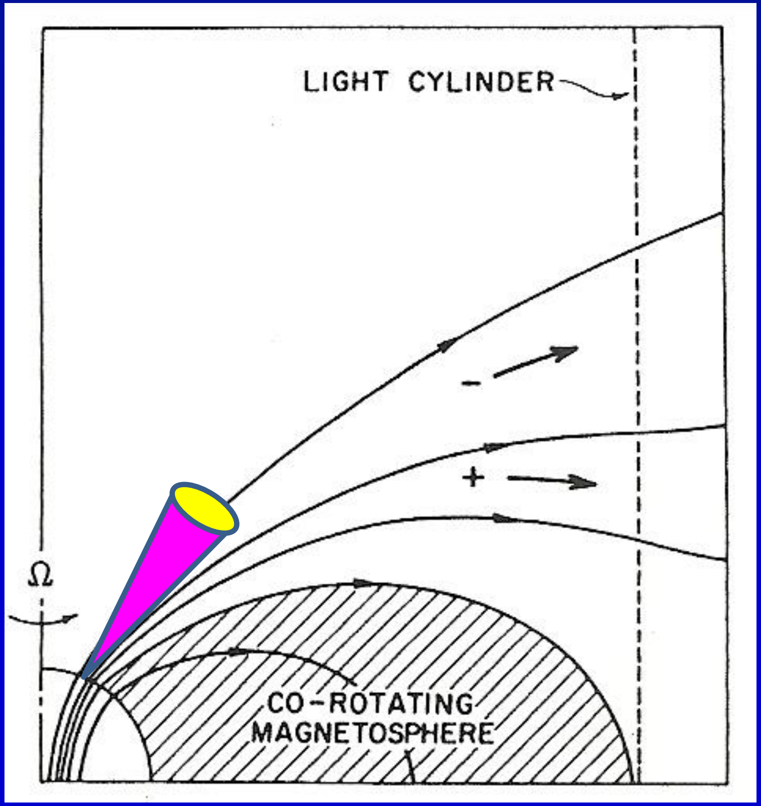


- Radiation from a relativistic charge will be "beamed" in the forward direction.
- The "cone of radiation" will be narrower, faster the particle.
- The radiation will not be monochromatic. It will be linearly polarized.

Radio radiation from pulsars

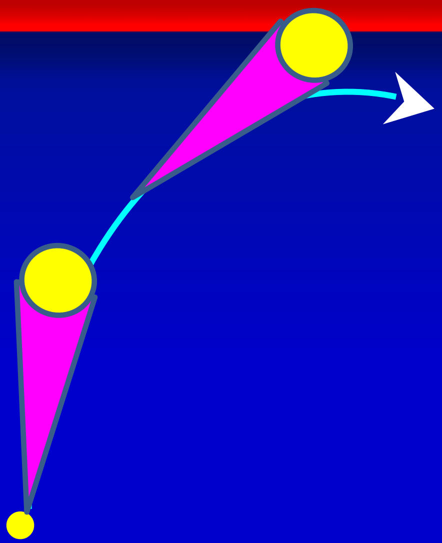


Radio radiation from pulsars



Electrons accelerated near the surface have such enormous energy that they produce **gamma rays** and NOT radio waves!

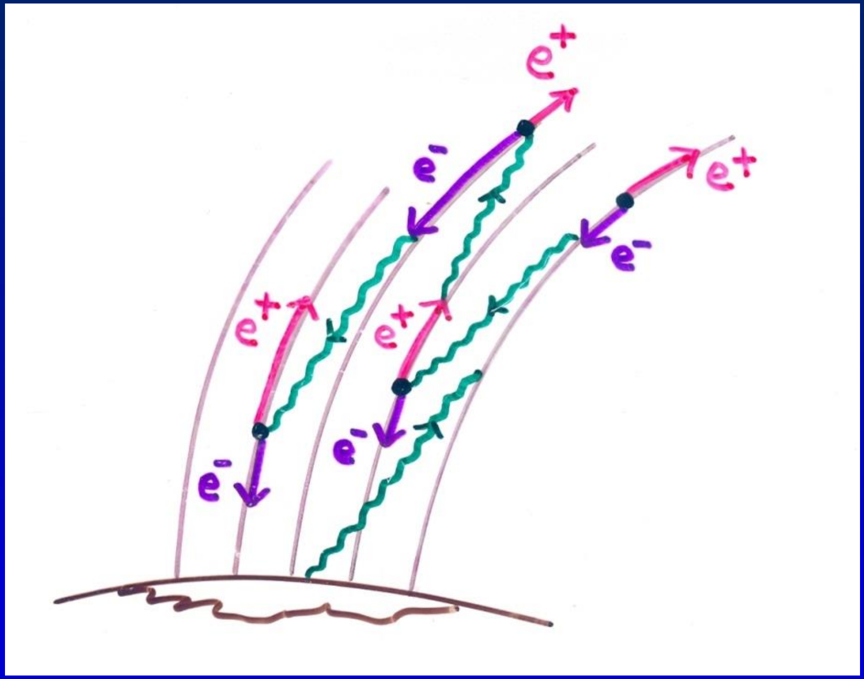
Radiation from a relativistic charge



- Radiation from a relativistic charge will be "beamed" in the forward direction.
- It will be linearly polarized.
- Characteristic frequency will be ~ $\gamma^3 \omega_{gyration}$
- Radiation produced will be ultra high energy gamma rays

Pair creation

A high energy photon can create an electron – positron pair in a strong magnetic field.



Mean free path for pair production

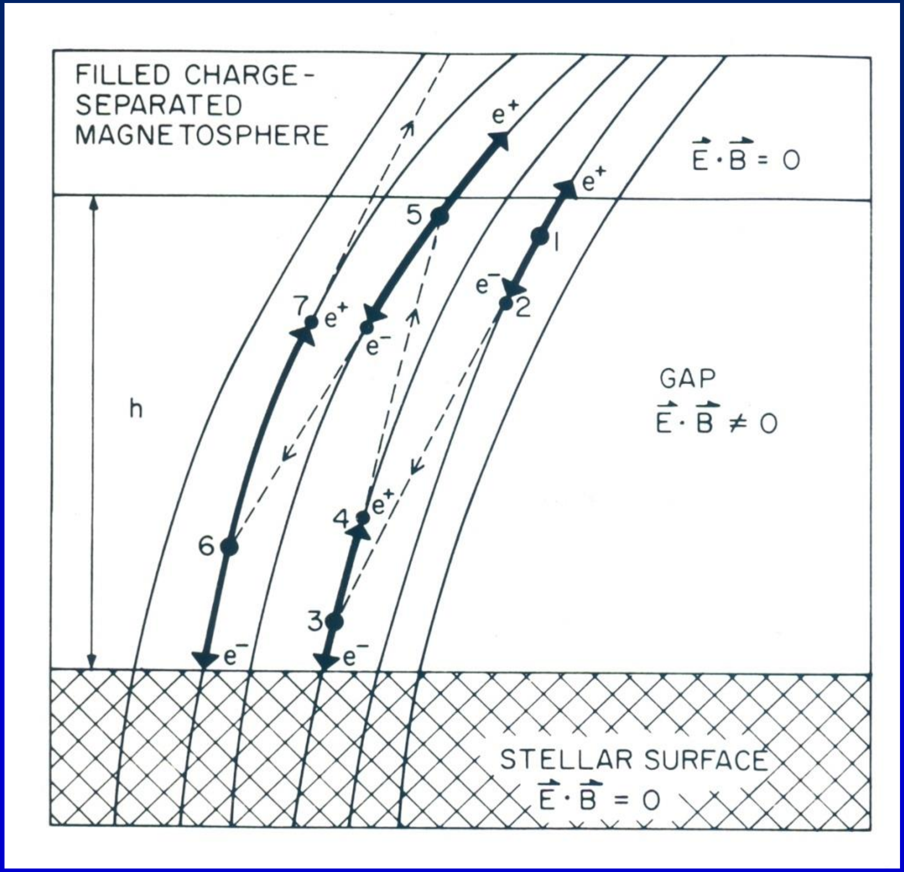
$$\ell = \frac{4.4}{\left(e^2/\hbar c\right)} \cdot \frac{\hbar}{mc} \cdot \frac{B_q}{B_\perp} \cdot e^{\frac{4}{3\chi}}$$

$$\boldsymbol{l = (\ldots.)\frac{1}{B_\perp}\, e^{\frac{(\ldots)}{\omega B_\perp}}}$$

$$\chi \equiv \frac{\hbar\omega}{2mc^2} \cdot \frac{B_\perp}{B_q}$$

$$B_q = \frac{m^2c^3}{e\hbar} = 4.4 \times 10^{13}\,\text{Gauss}$$

Pair production in a magnetic field will lead to a relativistic electron-positron wind. (Sturrock, 1971)



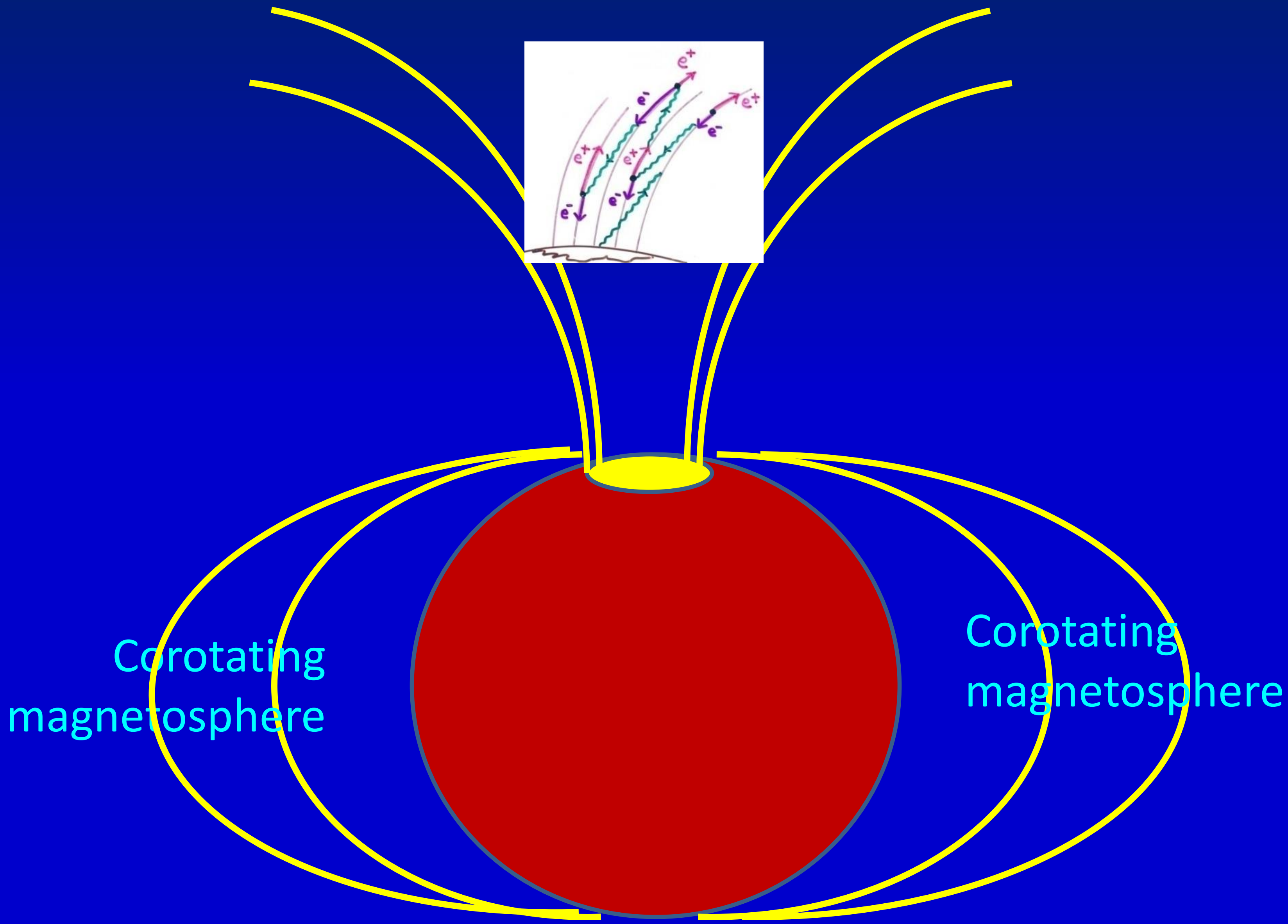
Because the base of the magnetosphere is positively charged, the electrons produced in the cascade will flow to the surface, and the POSITRONS WILL FLOW OUTWARDS.

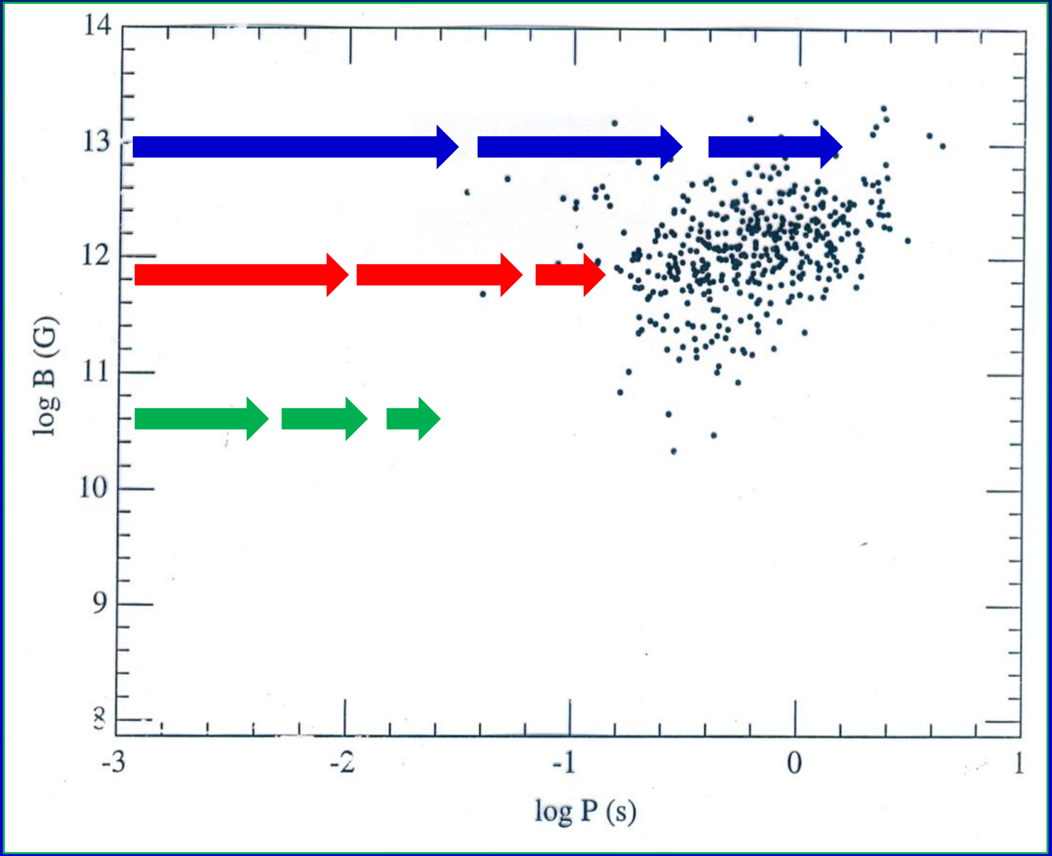
Thus a region will be created above the polar cap where there is VACUUM.

In this region $\vec{E}\bullet\vec{B} \neq 0$

Therefore this region behaves like a PARALLEL PLATE CAPACITOR, with a strong electric field. The potential drop across this VACUUM GAP will be given by

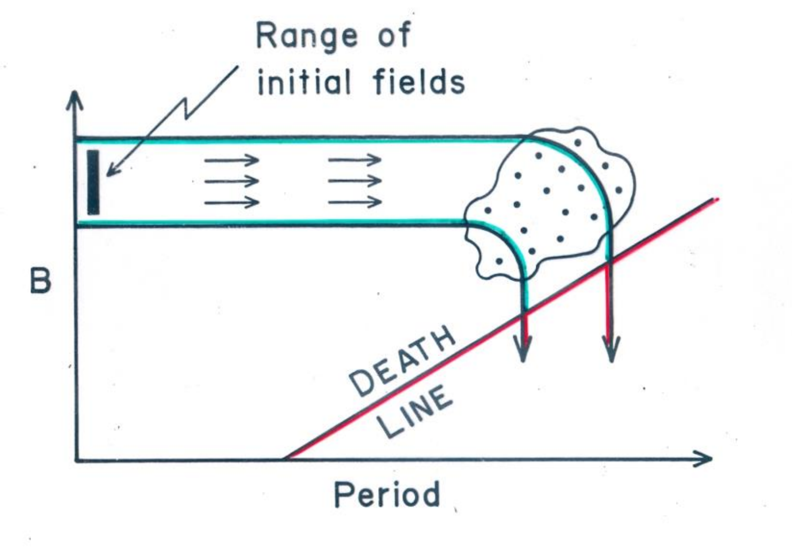
$$\Delta\phi \approx \frac{\Omega B}{c} h^2$$

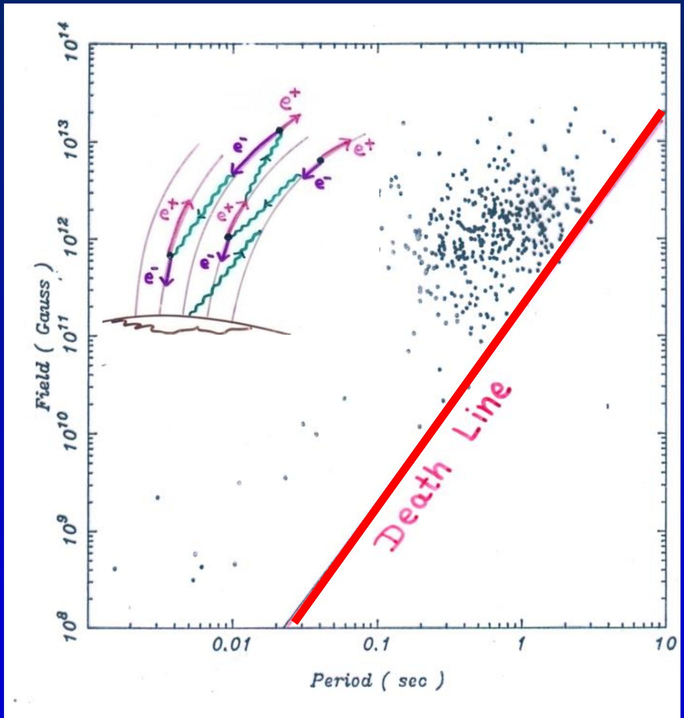

Corotating
magnetosphere
Corotating
magnetosphere



$$-\frac{d}{dt}\left[\frac{1}{2}I\Omega^2\right] = L = \frac{2}{3c^3}\left(B^2R^6\right)\Omega^4$$

$$\dot{P} \propto \frac{B^2}{P}$$





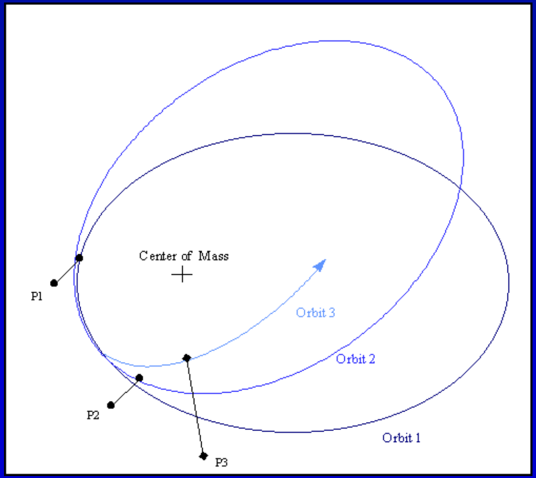
When the voltage generated by the dynamo drops below a **critical value** due to the lengthening of the period, electron-positron pair production will cease, and neutron stars will “die” as pulsars.

The Binary Pulsar

- Discovered by Hulse and Taylor in 1974.
- Orbital Period = 7.75 Hours
- Eccentricity of the orbit = 0.617
- Mass of the pulsar ~ 1.4 solar mass
- Mass of the companion ~ 1.4 solar mass

- Rotation period of pulsar = 59 ms
- Magnetic field = 3×10^{10} Gauss

General Relativity Effects

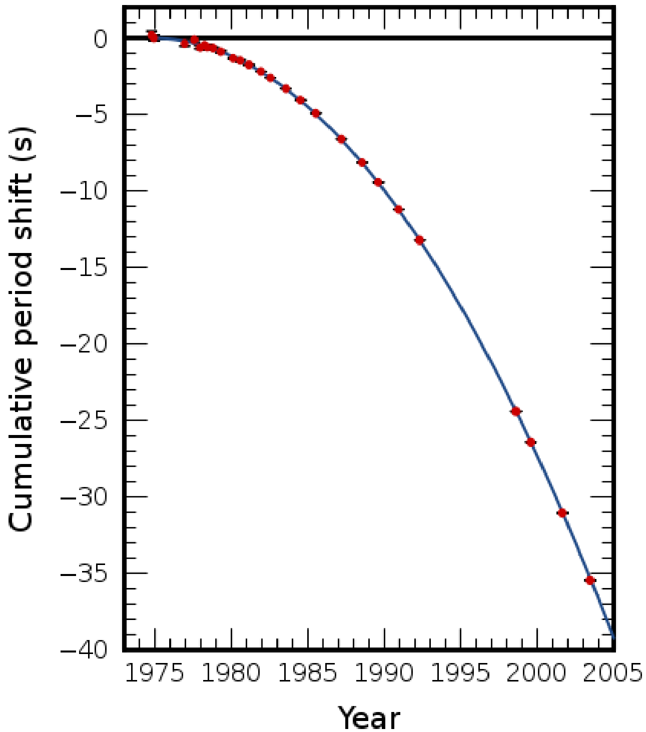


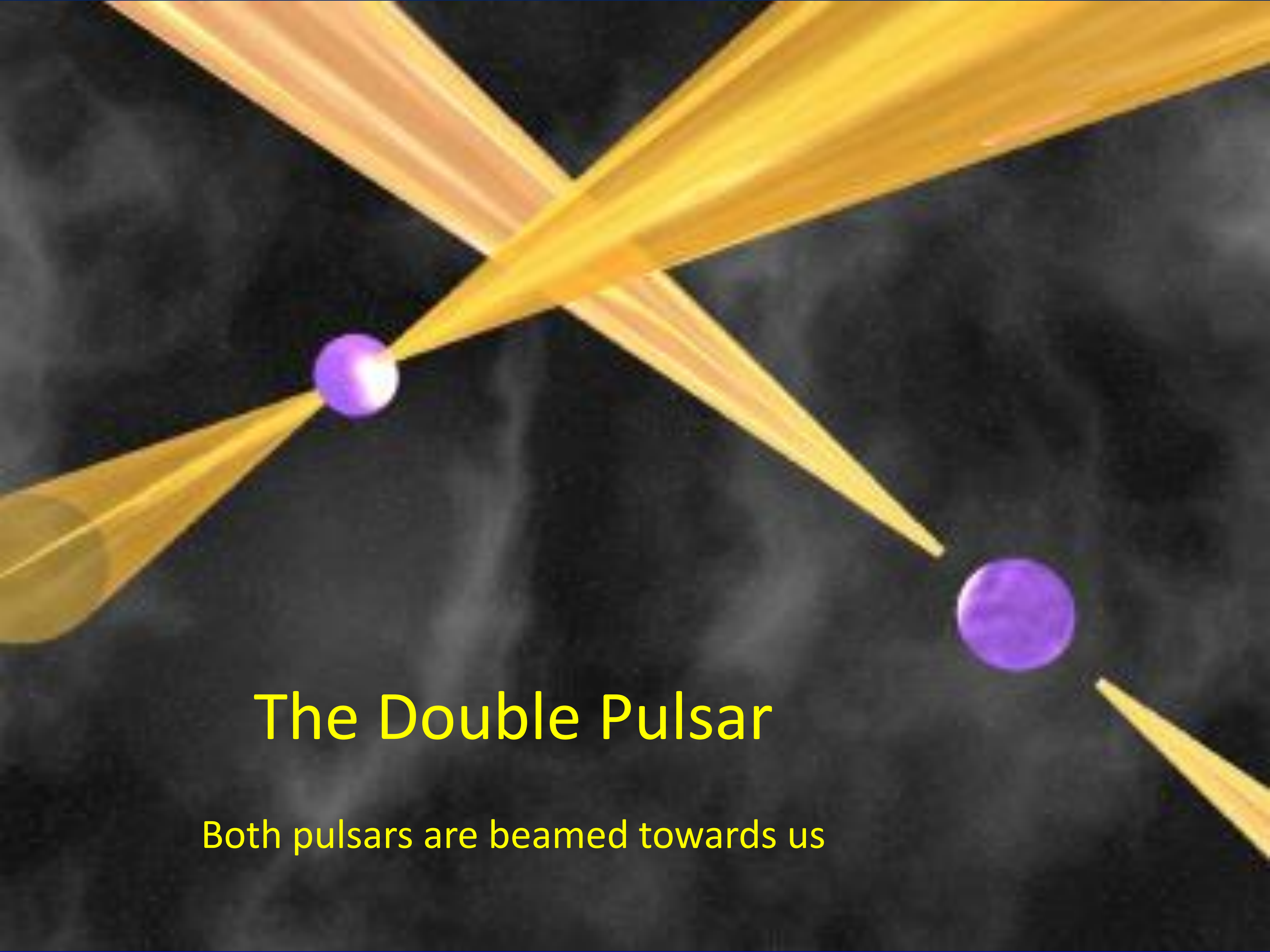
Given two objects, each roughly 1.4 solar mass, separated (at closest approach) by a mere 1 solar radius, this binary system is a wonderful laboratory to test various predictions of General Relativity. And this has been done.

- Precession of the periastron
- Gravitational redshift
- Shapiro delay, etc.
- Not surprisingly, the GR effects are 'jumbo' !
- Periastron precesses by 4.2 deg/year (43" per century for Mercury)!

Shrinking of the orbit due to Gravitational Radiation

This binary pulsar provided the first evidence for gravitational radiation. The decrease in the orbital period due to gravitational radiation agrees spectacularly well with the prediction of GTR.





The Double Pulsar

Both pulsars are beamed towards us

General Relativity Effects

Advance of periastron : 16.9 degrees per year

Gravitational redshift parameter: 0.39(2) ms

Orbital decay due to gravitational radiation: 7 mm / day

Shapiro delay

Geodetic precession

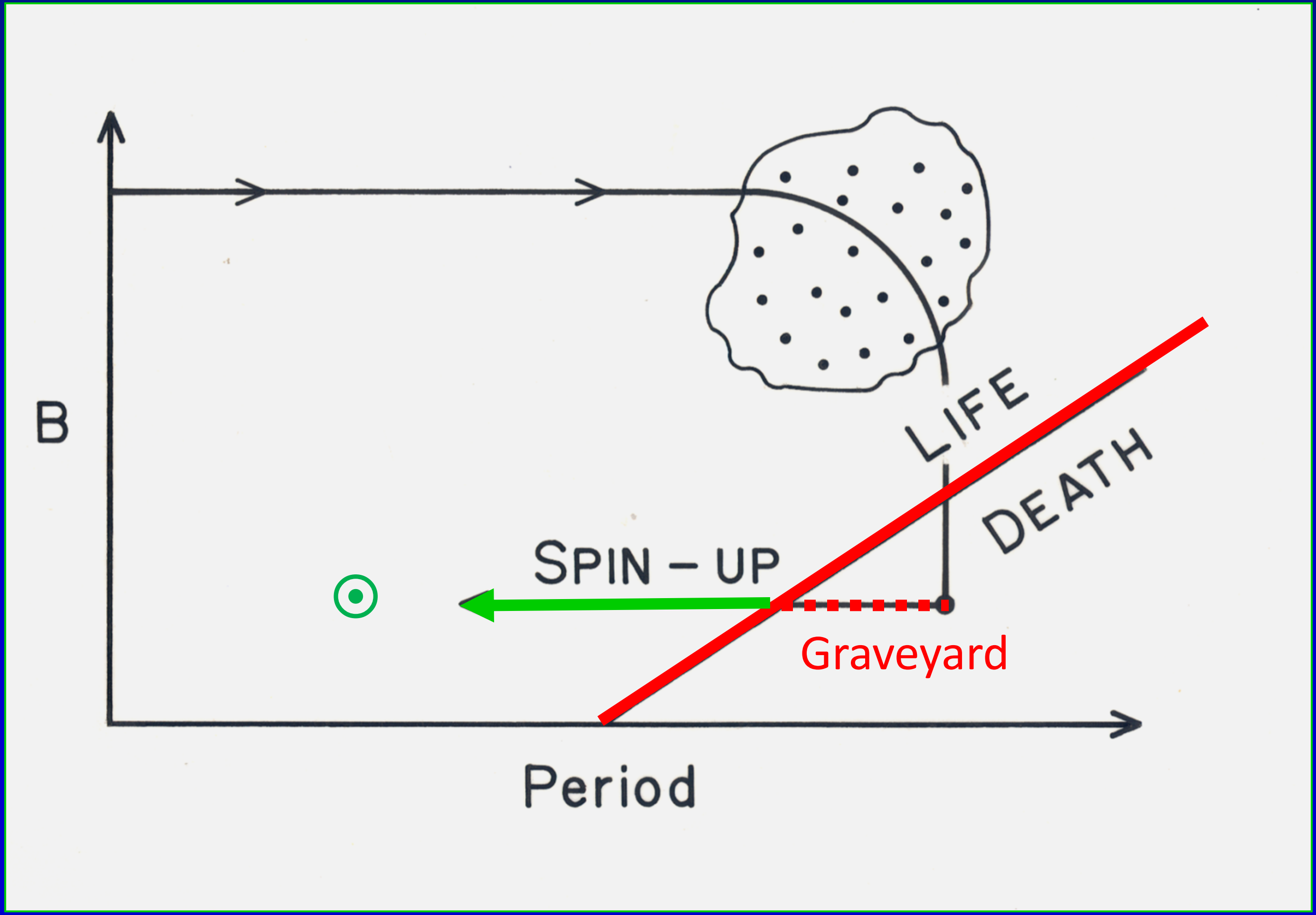
Masses of the pulsars: $M_1 = 1.338 \pm 0.001\ M_\odot$

$M_2 = 1.249 \pm 0.001\ M_\odot$

Curious coincidences!

- Short period of rotation of 59 ms suggests a young pulsar.
- Low magnetic field suggests an OLD pulsar.
- It might just be a coincidence.
- But how come this coincidence occurs for the only pulsar that is in a binary system?!

Reincarnation of pulsars



A dead pulsar, whose field has decayed, is resurrected from its graveyard by being spun up as a short period, low-field pulsar!

To make a rapidly rotating pulsar, with a weak magnetic field, two things must happen:

- The magnetic field must decay, and
- The neutron star has to be spun up.

Massive Binary

$25\ M_\odot$ and $10\ M_\odot$

Birth of the first neutron star

Heavy accretion; Neutron star is spun up.

Second star ends its life. Double neutron star system, or two run away neutron stars

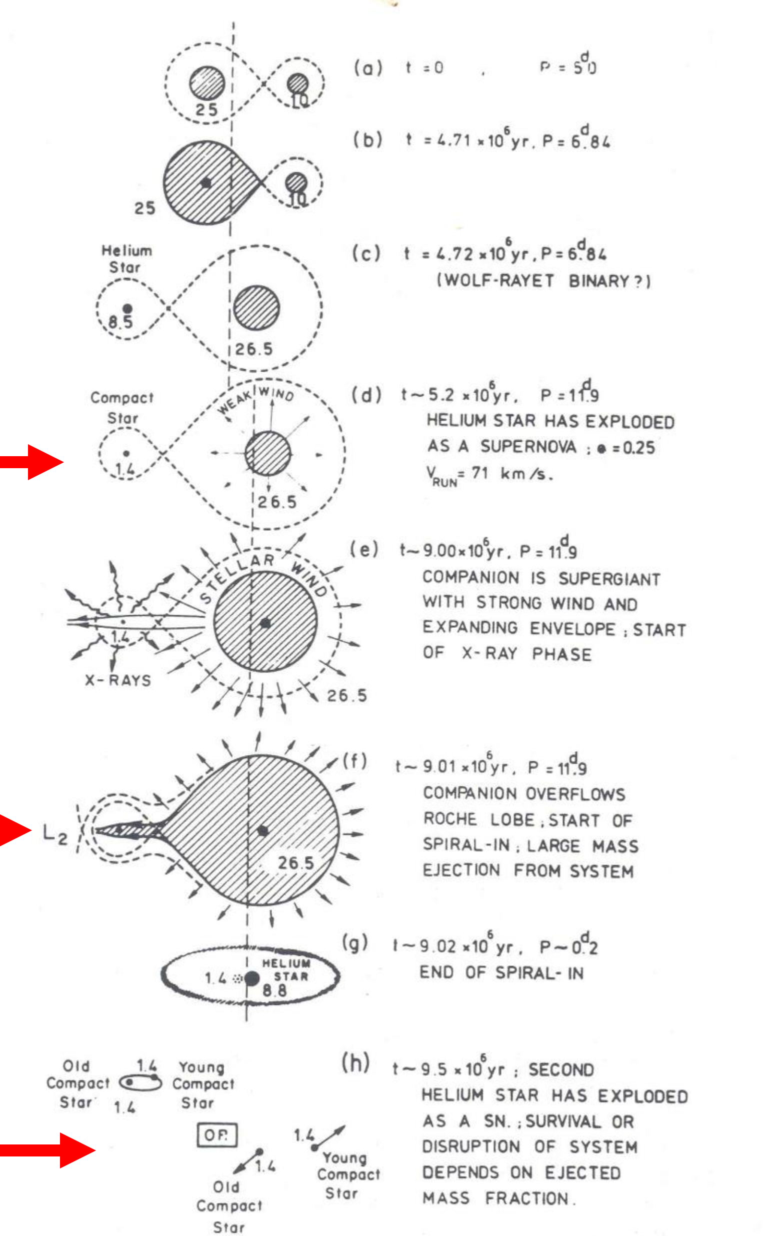
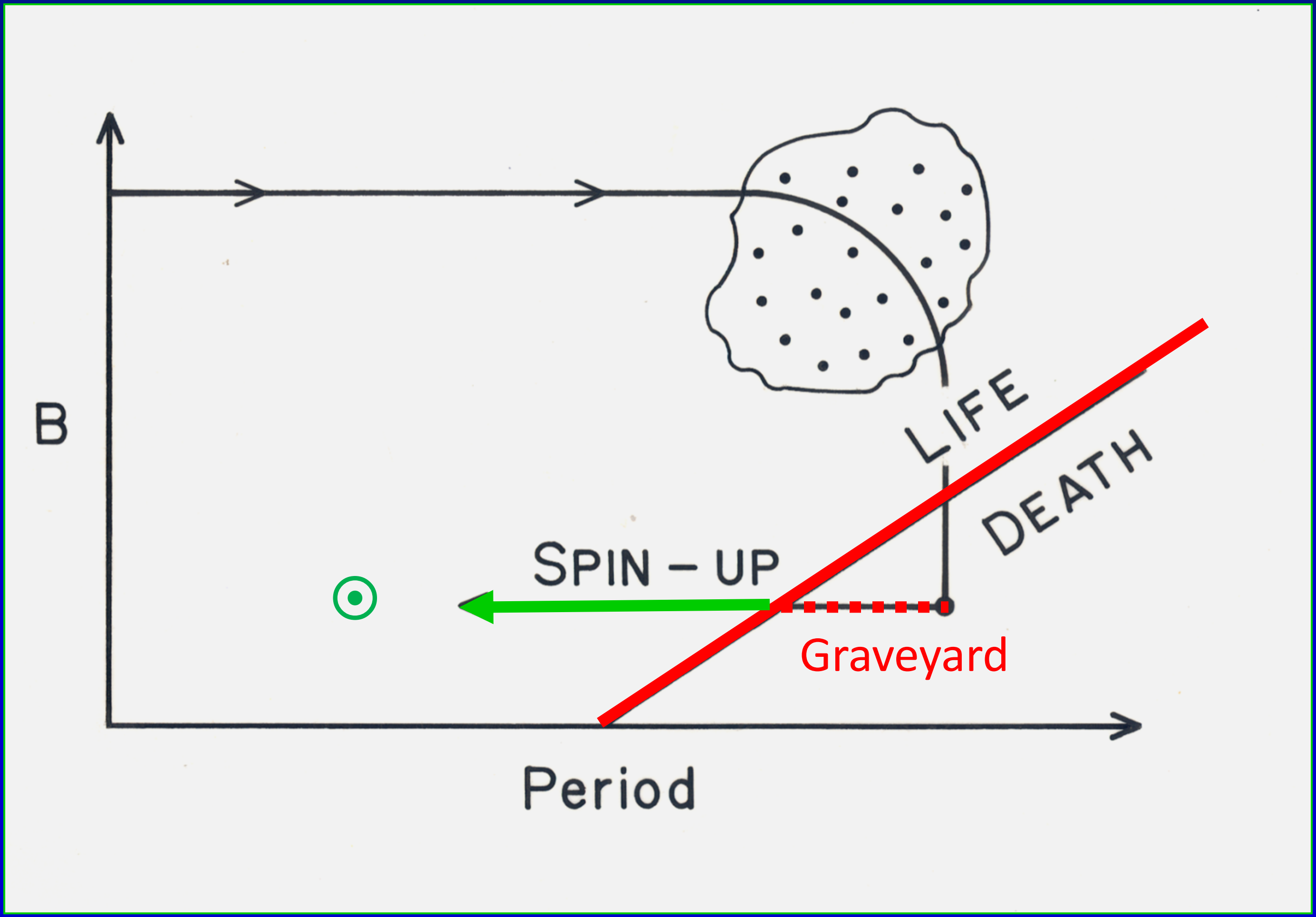


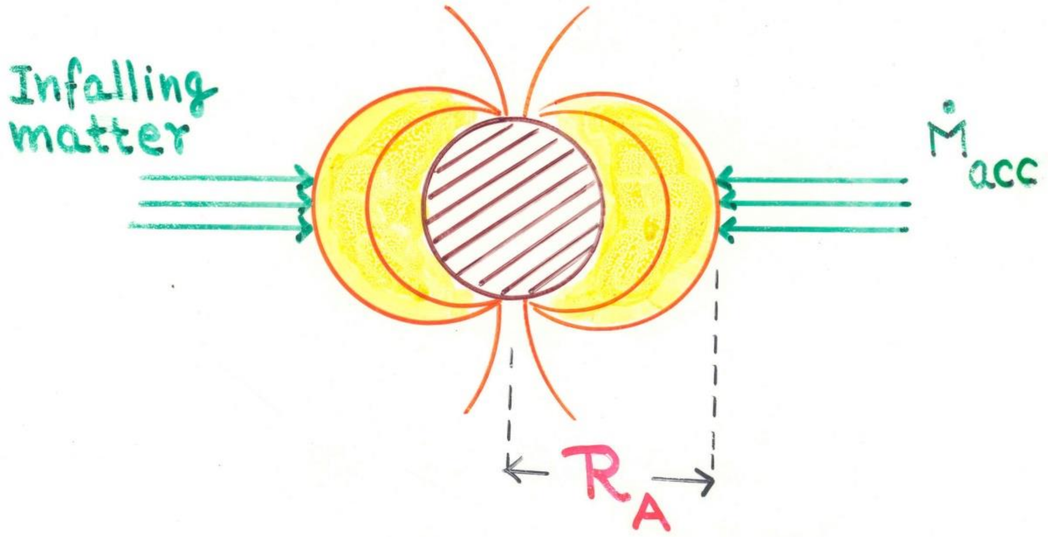
Fig. 45. The evolution of massive binaries (from van den Heuvel 1983).

Spinning up a neutron star



The first-born neutron star is spun up during mass transfer from the giant companion star.

§ Alfven Radius

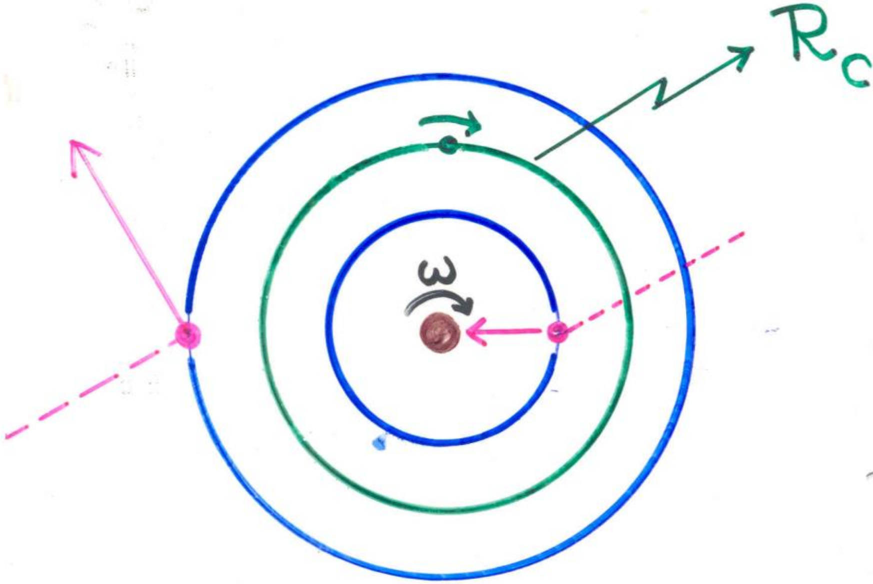


At R_A,

Magnetic Press = Ram Pressure

$$R_A = \left(\frac{B^4 R^{12}}{8GM \dot{M}_a^2} \right)^{1/7}$$

§ CO-ROTATION RADIUS

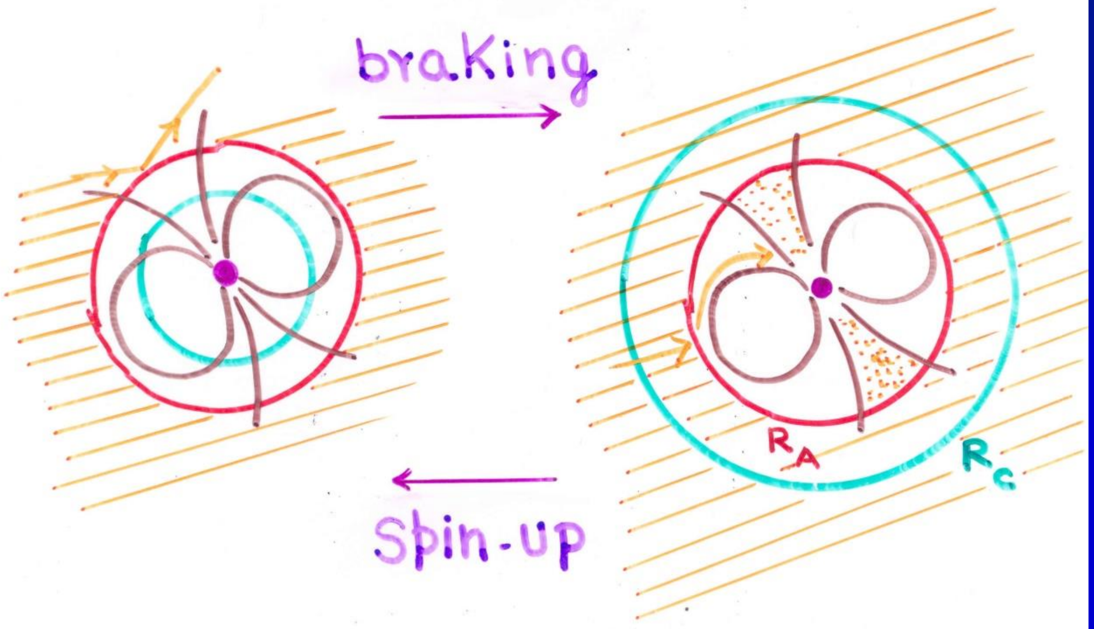


$$mR\omega^2 = \frac{GMm}{R^2}$$

$$R_c = \left(\frac{GM}{\omega^2}\right)^{1/3}$$

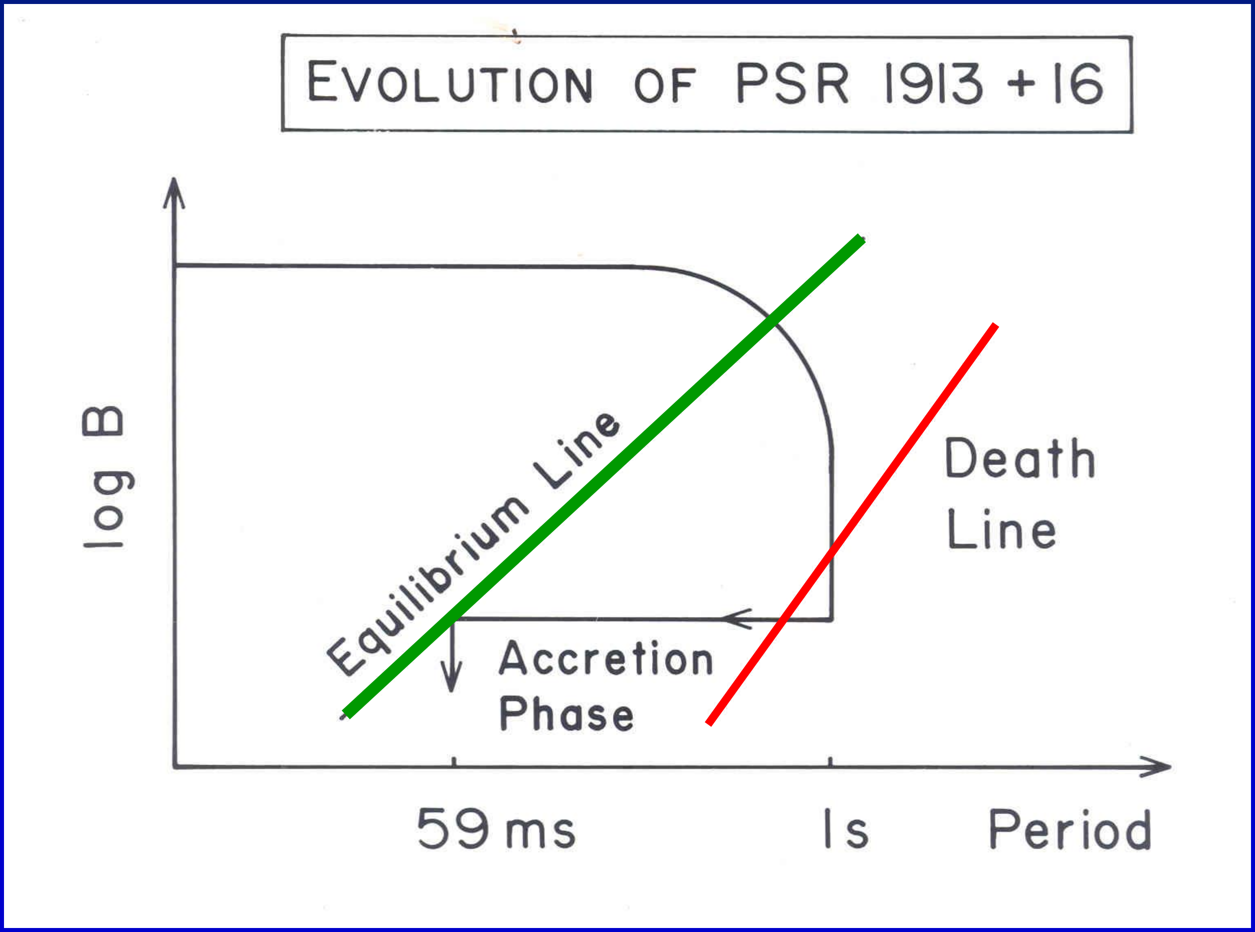
Faster the central star rotates smaller the corotation radius.

Equilibrium Period : Alfven radius = Corotation radius



EQUILIBRIUM PERIOD :

$$P_{eq} = (\dots)\, B^{6/7}\, R^{18/7}\, M^{-5/7}\, \dot{M}_a^{-3/7}$$



PSR 1913+16 must be a recycled pulsar

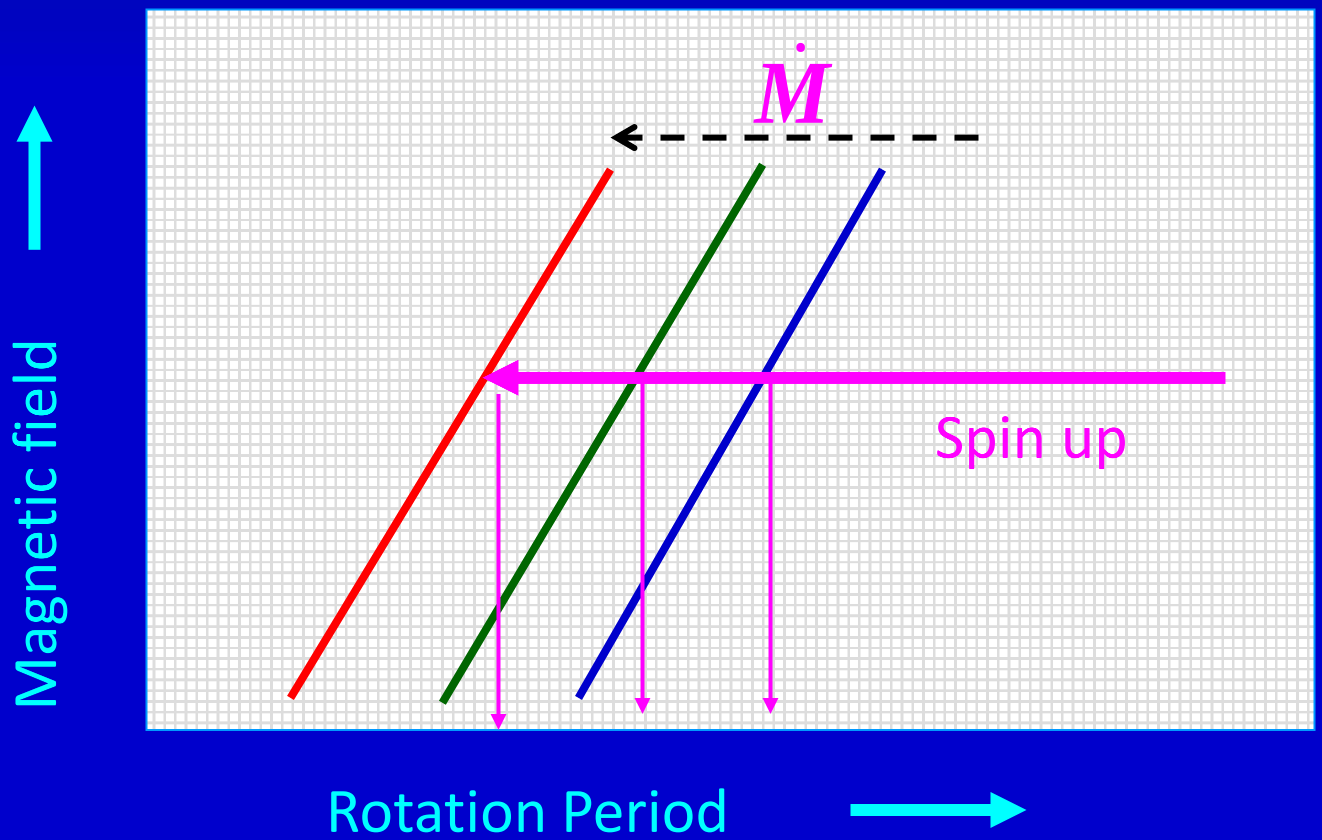
- The first-born neutron star in a close binary will be spun up during the mass transfer phase.

- Question: What will be its period after the spin up phase is over?

The Equilibrium Period

$$P_{eq} = (....)B^{\frac{6}{7}}\dot{M}^{-\frac{3}{7}}$$

This is the Keplerian Period at the inner edge of the accretion disk



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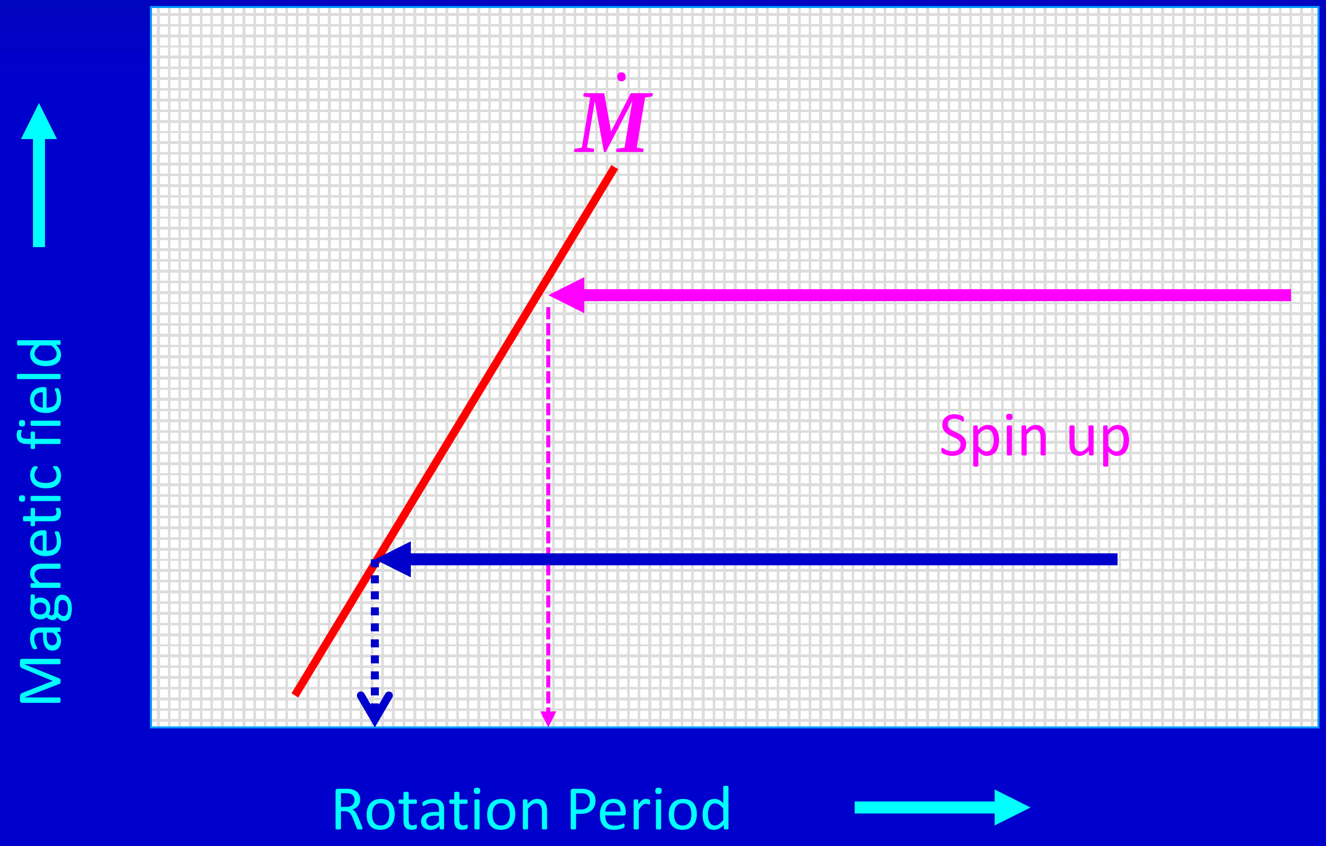
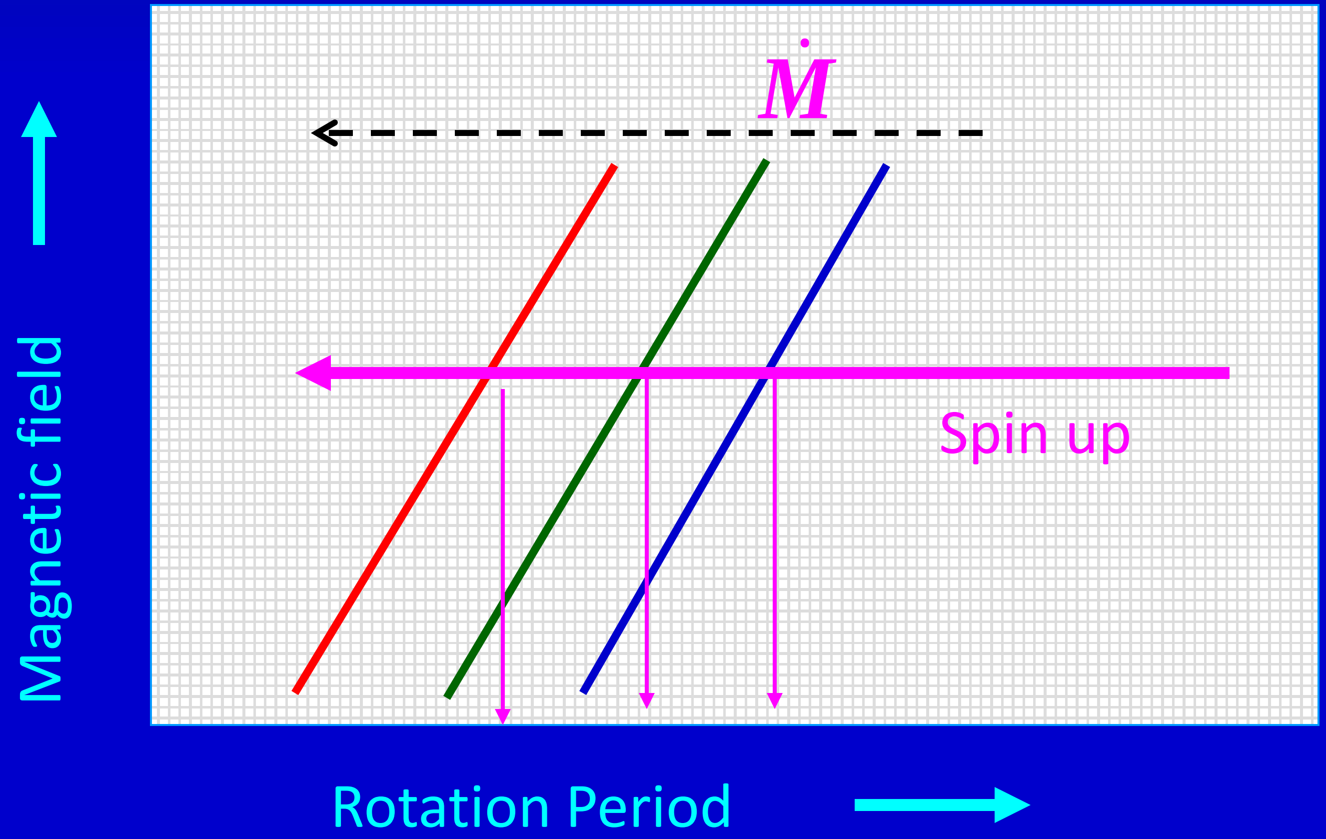
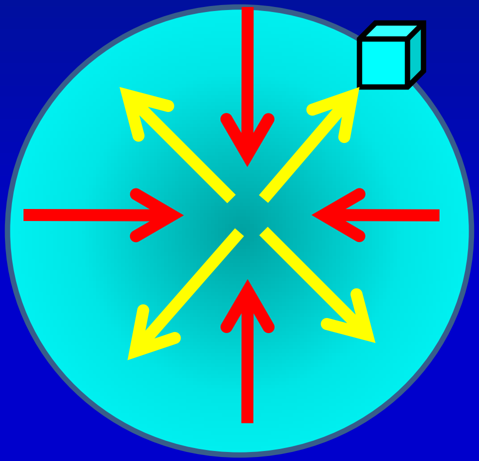


For a given magnetic field, can we spin up a neutron star to arbitrarily small period?

$$P_{eq} = (....) B^{\frac{6}{7}} \dot{M}^{-\frac{3}{7}}$$



Eddington Luminosity Limit



Consider a unit volume, consisting of n electrons and n protons, at a distance r from the centre, in RADIATIVE EQUILIBRIUM – radiative force is precisely equal to the gravitation force acting on it.

Radiative force on the unit volume:

$$F_{rad} = \frac{L}{4\pi r^2 c}\left(n\sigma_{Th}\right)$$

L is the luminosity, and σ_{Th} is the Thompson scattering cross section of the electrons. Radiation force on the proton will be negligible in comparison.

Thompson scattering cross section: $$\sigma_{Th} = \frac{8\pi}{3} r_e^2$$

Classical "radius" of the electron: $$\frac{e^2}{r_e} = m_e c^2$$

Gravitational force on unit volume: $$F_G = \frac{GM(r)(nm_p)}{r^2}$$

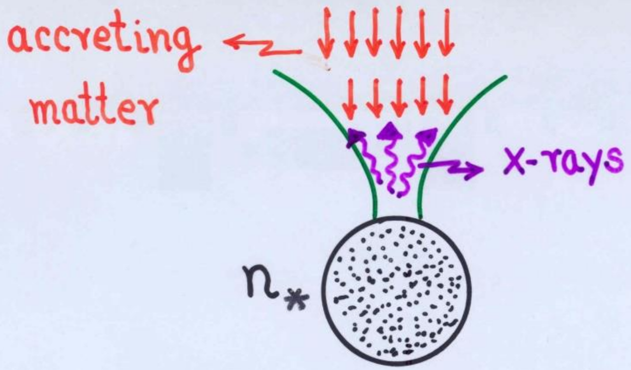
For radiative equilibrium: $$\frac{GM(r)m_p}{r^2} = \frac{L}{4\pi r^2 c}\sigma_{Th}$$

$$L = \left(\frac{4\pi c G m_p}{\sigma_{Th}}\right) M$$

Eddington Luminosity Limit

$$L_{Edd} = \left(\frac{4\pi c G m_p}{\sigma_{Th}} \right) M$$

$$L_{Edd} = 10^{38} erg / s \left(\frac{M}{M_{sun}} \right)$$



- $L_X \propto \eta \dot{M} c^2$

- EDDINGTON LIMITING LUMINOSITY

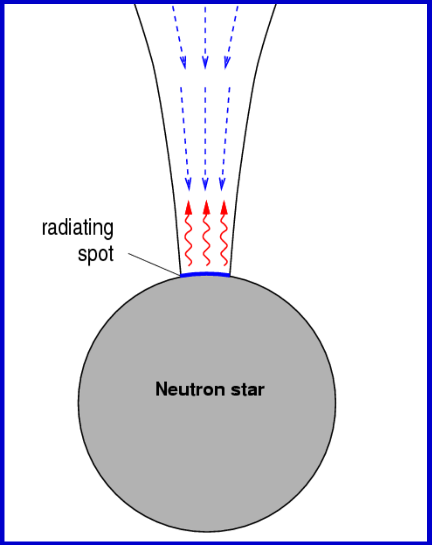
$$L_{Edd} = \frac{4\pi c G m_p}{\sigma_T} M_*$$

- Limiting Accretion Luminosity L_{Edd}

  ⟹ Limiting Accretion Rate $\dot{M}_{Edd}$

- critical $P_{eq}(\dot{M}_{edd}) = (\cdots) B^{6/7}$

Eddington Limit for the accretion rate



$$\text{Luminosity} = L = \eta \dot{M} c^2$$

$$L_{Edd} = \left(\frac{4\pi c G m_p}{\sigma_{Th}} \right) M$$

$$L_{Edd} = \eta \dot{M}_{Edd} c^2$$

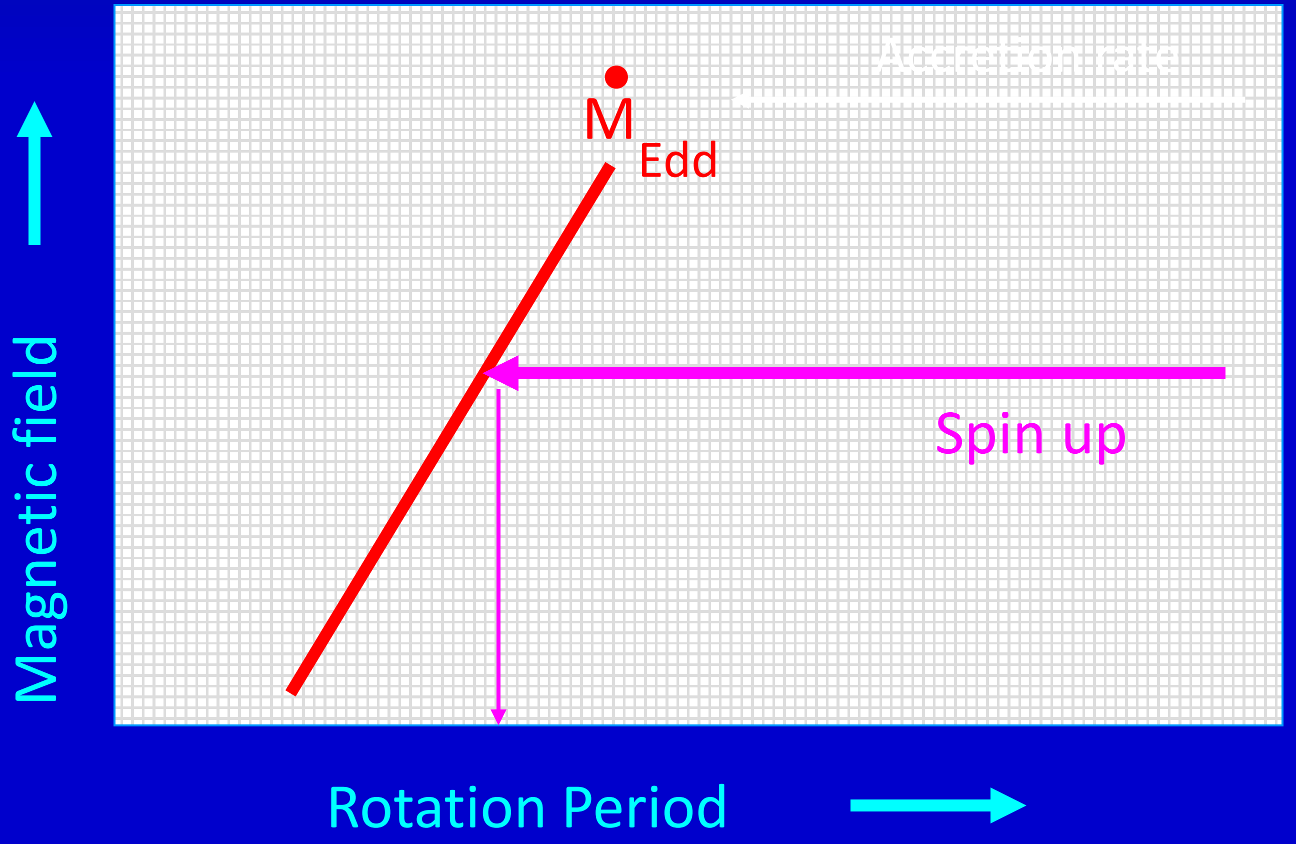
The Minimum Equilibrium Period

Since the Eddington luminosity limit defines an upper limit to the accretion rate, the limiting equilibrium period is determined uniquely by the magnetic field at the time of spin-up.

$$P_{\min} = 1.9ms\left(\frac{B}{10^9 G}\right)^{\frac{6}{7}}$$

The Minimum Equilibrium Period

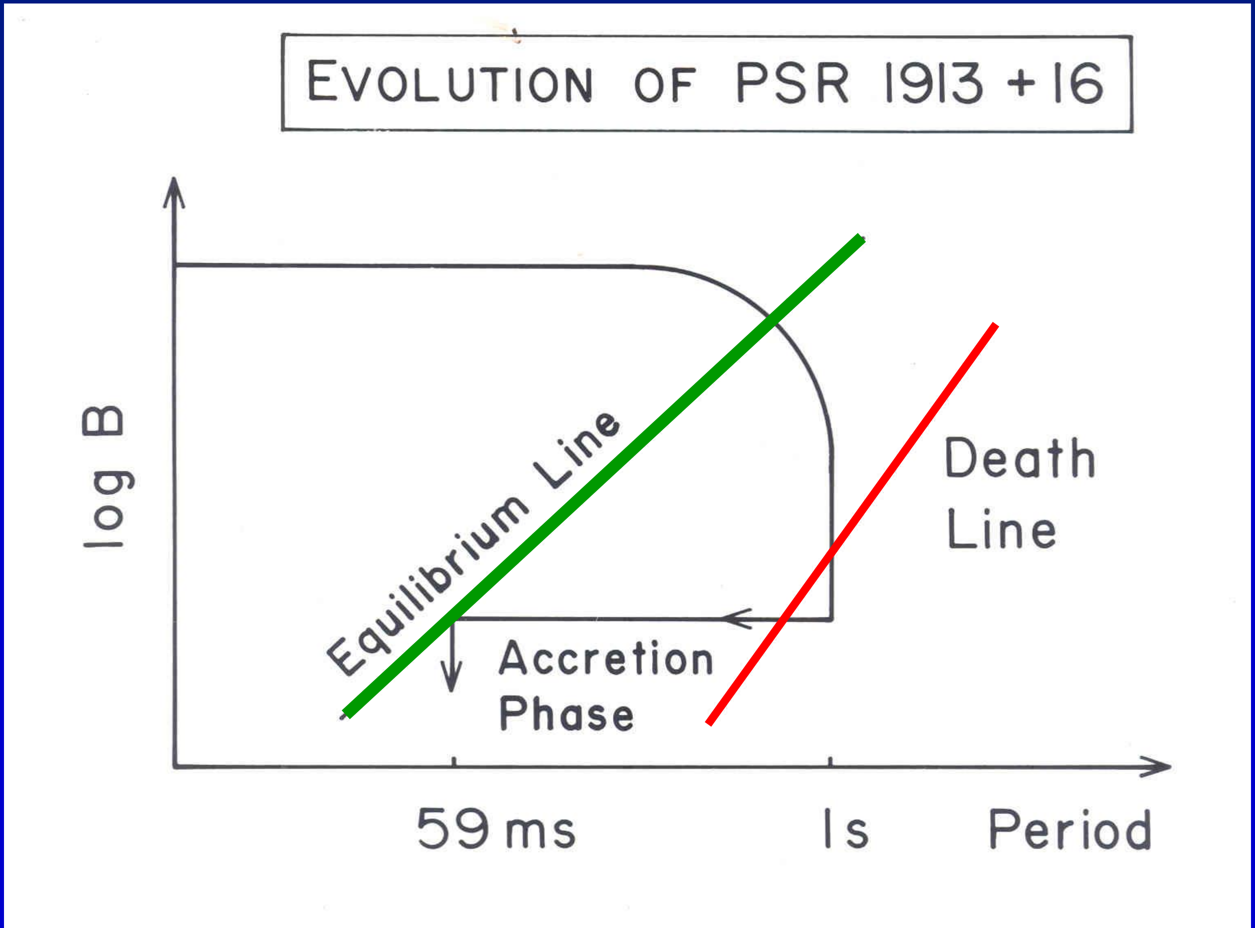
$$P_{\min} = 1.9ms\left(\frac{B}{10^{9}G}\right)^{\frac{6}{7}}$$



The critical equilibrium period

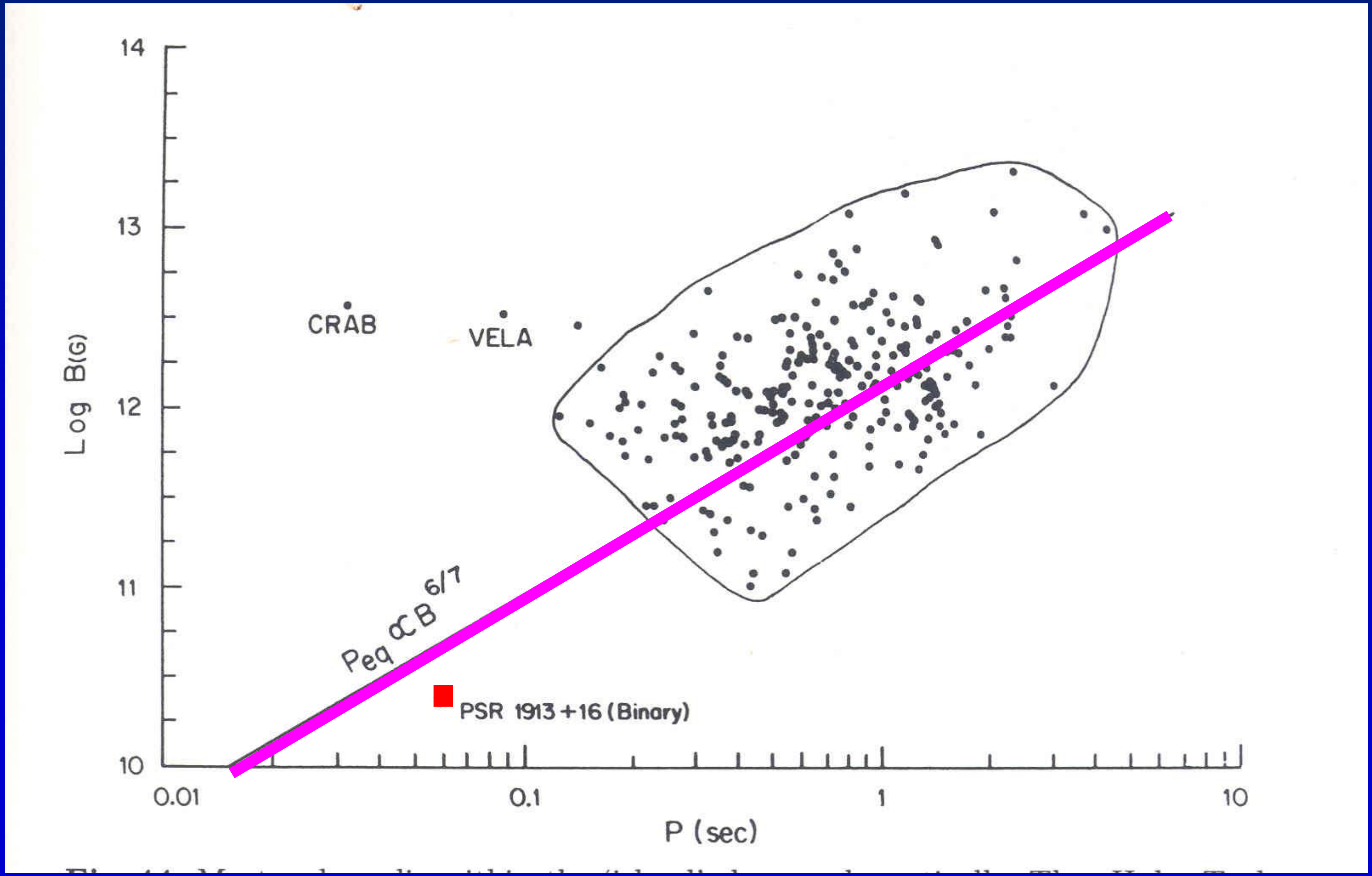
- For a given magnetic field strength, can a neutron star be spun up to arbitrarily short periods, by increasing the accretion rate?

- No! There is an upper limit to the accretion rate, determined by the upper limit to the luminosity generated by the accretion, namely the Eddington Luminosity Limit.



PSR 1913+16 must be a recycled pulsar

The Hulse-Taylor Binary Pulsar



The proximity of the pulsar to the limiting equilibrium period line confirms that this is a *Recycled Pulsar.*

Recycled Pulsars

Two predictions made in 1978:

1. The first born neutron star will be recycled. It will have a low field and short period. Its location in the B – P plot will be close to the “SPIN UP LINE”.

2. The second born neutron star will be a “normal pulsar”, with high field and long period, like the majority of solitary pulsars.

The Double Pulsar

Pulsar A

P = 22.7 ms

B = 6.3 x 10^{9} G

Pulsar B

P = 2.8 s

B = 1.2 x 10^{12} G

Characteristic age = 50 My

Millisecond Pulsars

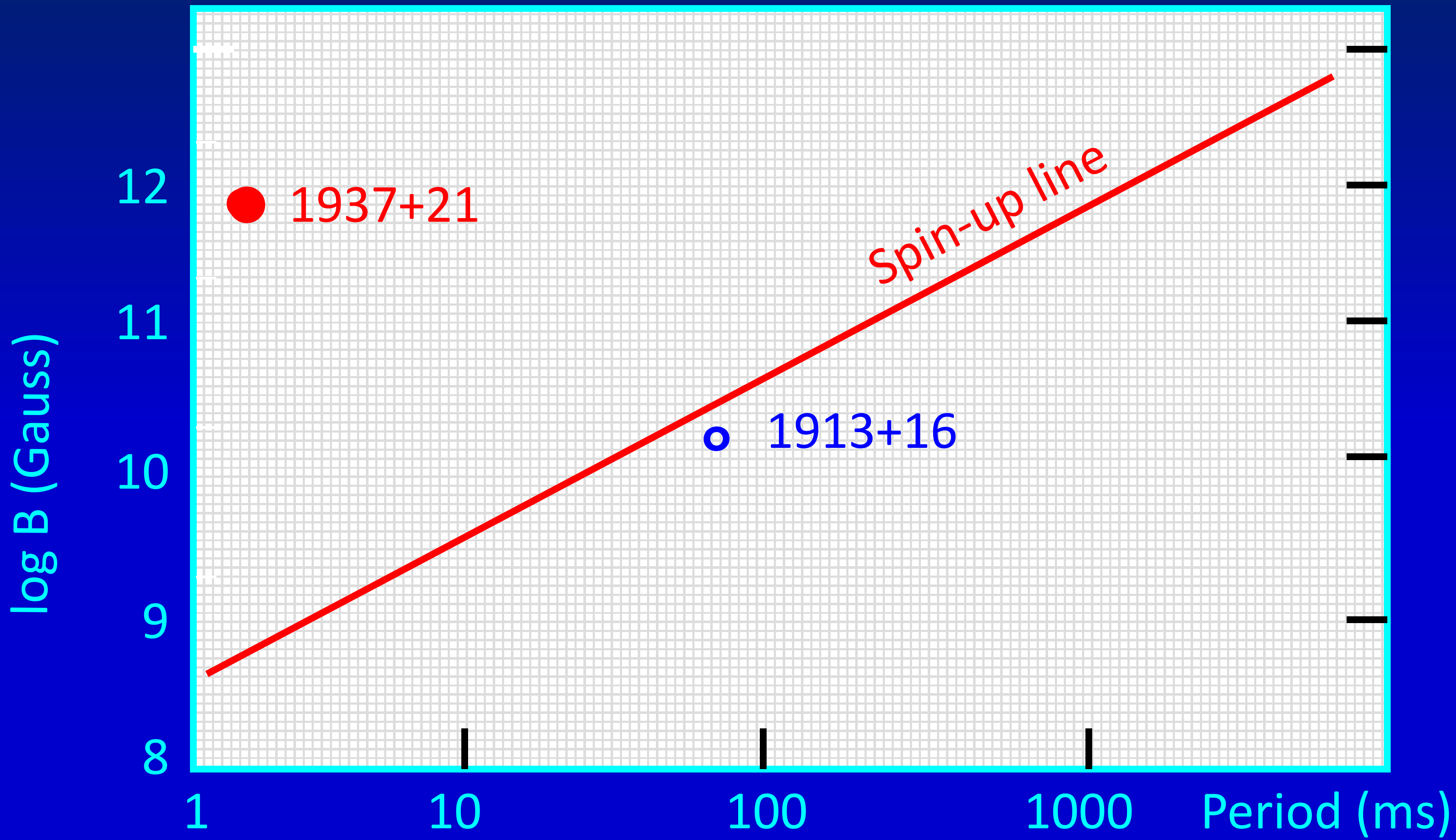
The First Millisecond Pulsar

- Discovered by Backer, Kulkarni, et al. in 1982.
- Spinning 642 times a second! P=1.5 ms.
- Discovery Telegram claimed that it had a canonical high magnetic field: $B \sim 10^{12} G$
- Period derivative ~ $10^{-14} ss^{-1}$
- It was a solitary pulsar.

The Millisecond Pulsar

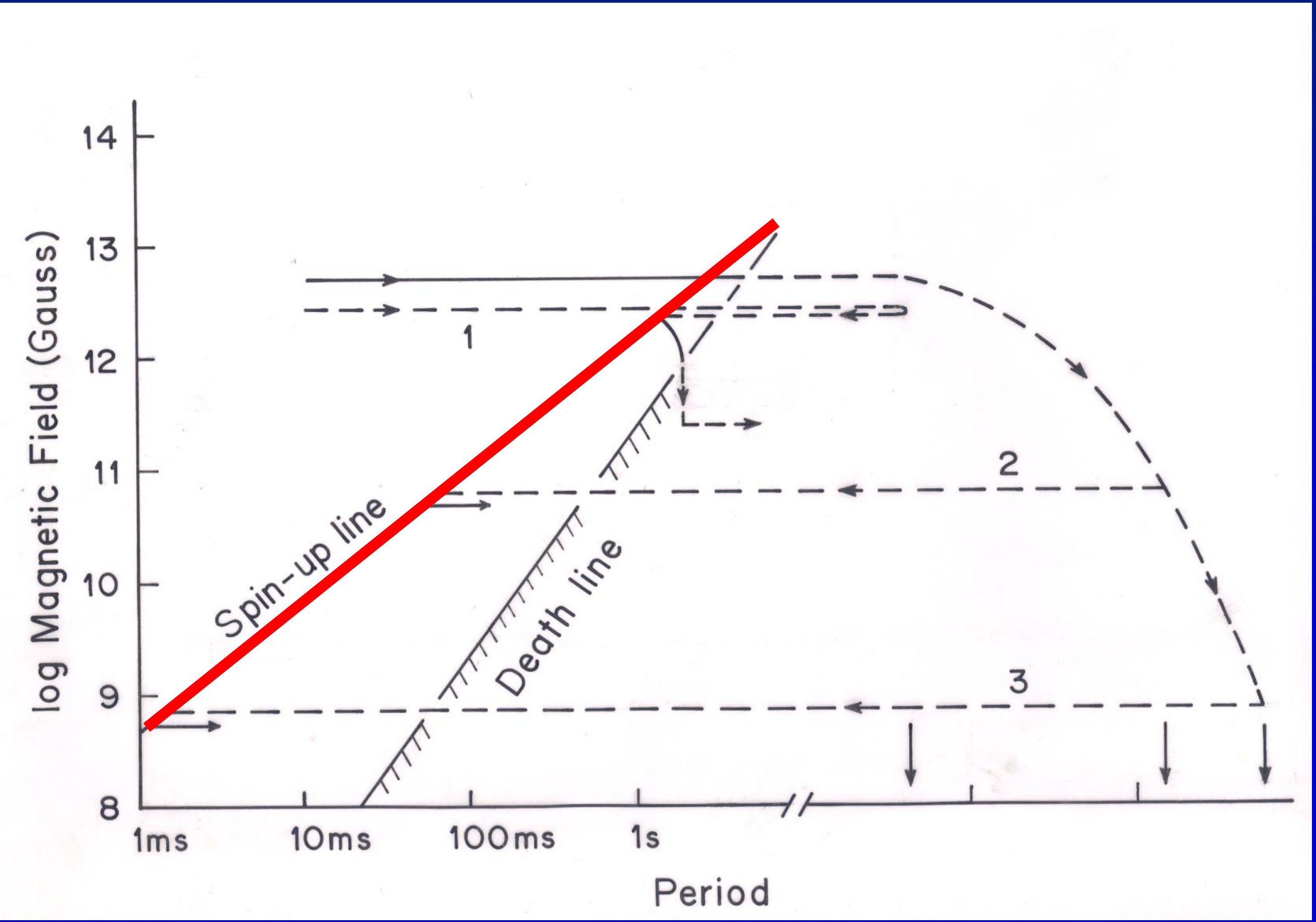
- The ms-PSR must be a RECYCLED PULSAR.
- It is spinning fast because it was spun up in a binary.
- Its magnetic field must be very small.
- Its companion must have been “blown away”

The first Millisecond Pulsar



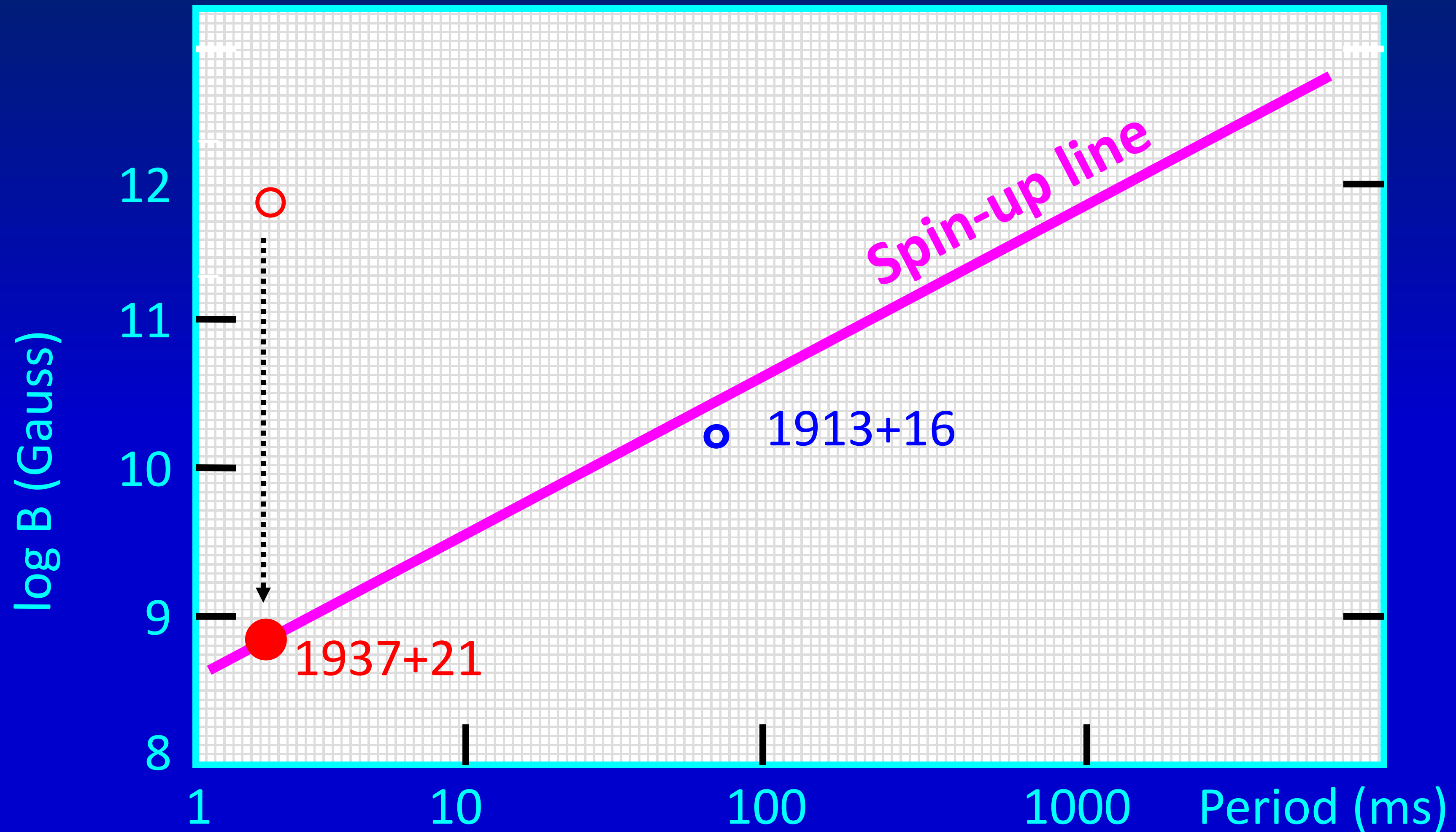
The location of the pulsar as suggested by the early data

Recycled Pulsars



The period to which a recycled pulsar is spun up to will be determined by the strength of its magnetic field. (GS, 1979)

The solitary Millisecond PSR must be a recycled PSR



Prediction: B = 5 x 10^{8} Gauss

Radhakrishnan and Srinivasan, *Current Science, 1982*

The Millisecond Pulsar – 1937+21

- Although 'solitary', the millisecond pulsar must be a 'recycled' pulsar.

- It's magnetic field must be $\sim 5 \times 10^{8} G$

- It's period derivative must be $\dot{P} \approx 10^{-19} s s^{-1}$

Radhakrishnan and Srinivasan, *Current Science, 1982*

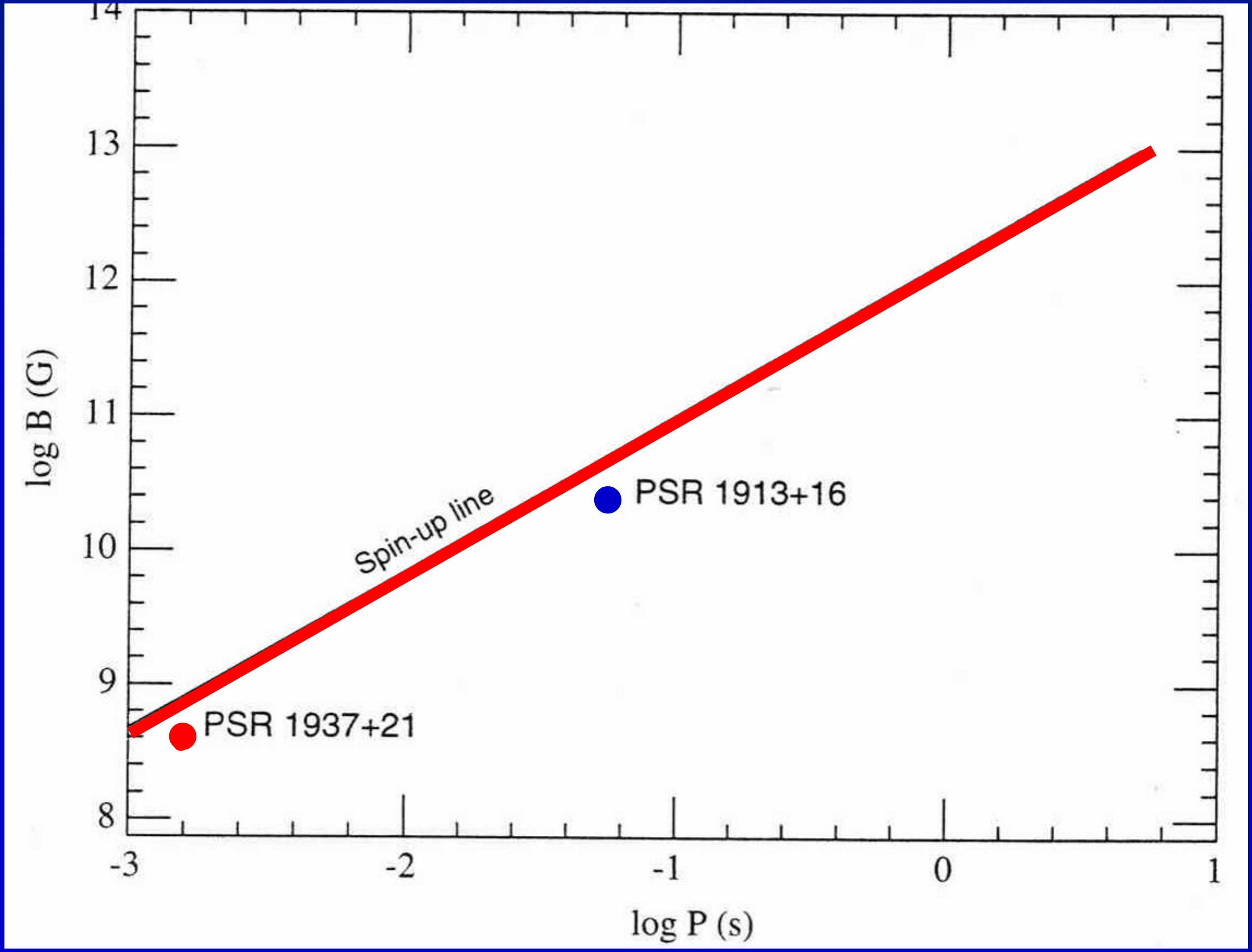
The Millisecond Pulsar 1937+21

- Roughly four months after these predictions were made, Andrew Lyne et al. at Jodrell Bank accurately measured the period derivative of his pulsar.

$$\dot{P} = 1.2 \times 10^{-19} ss^{-1}$$

$$B = 4.7 \times 10^{8} G$$

Prediction: B = 5x10^{8} Gauss   Measured: B = 4.7 x 10^{8} G,



PSR 1937+21

$P = 0.0015578064488727\ (3)\ \mathrm{s}$

$\dot{P} = 1.05105 \times 10^{-19} \pm 10^{-24}\ \ \mathrm{s/s}$

PSR 1937+21

Predicted: $\dot{P} = 1.2 \times 10^{-19}$ s/s

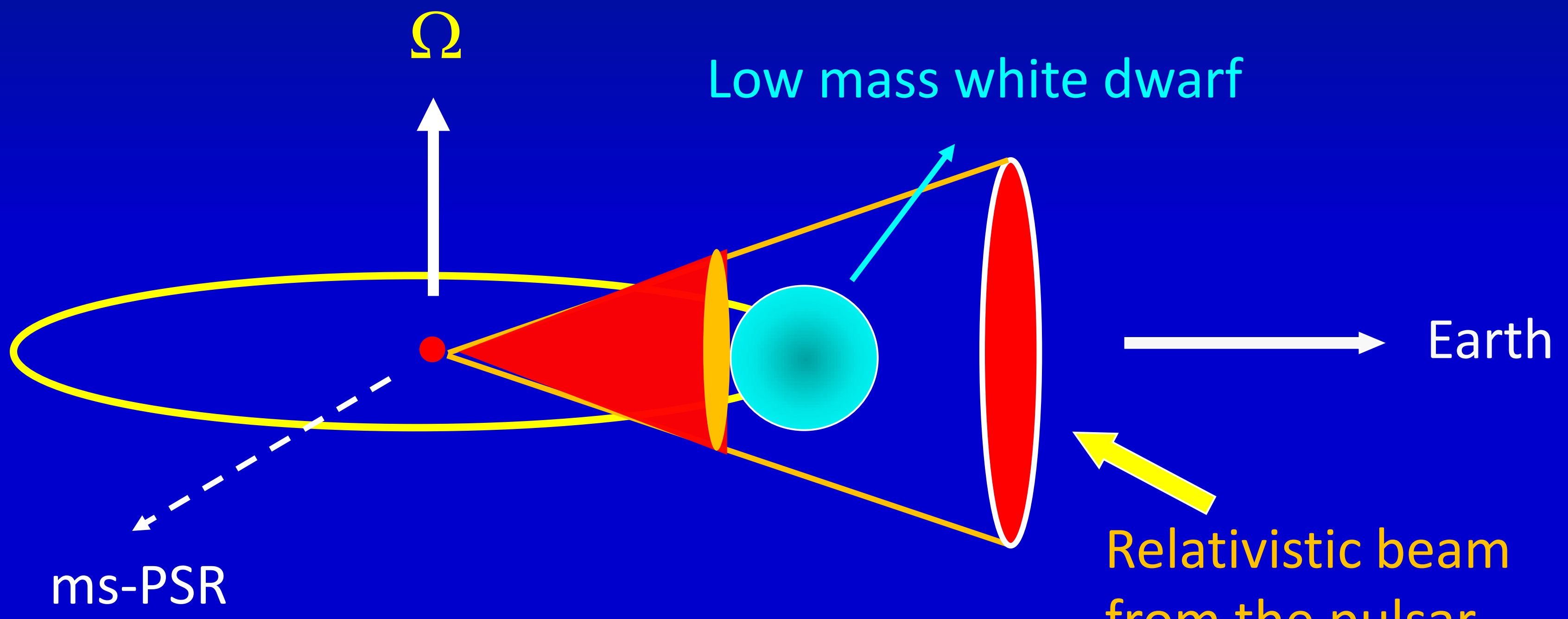
Measured: $\dot{P} = 1.05105 \times 10^{-19}$ s/s

The Missing Companion !

- If the ms-PSR was recycled in a LMXB, where is its companion?

- The companion must have been ‘blown away’!

The ungrateful neutron star!

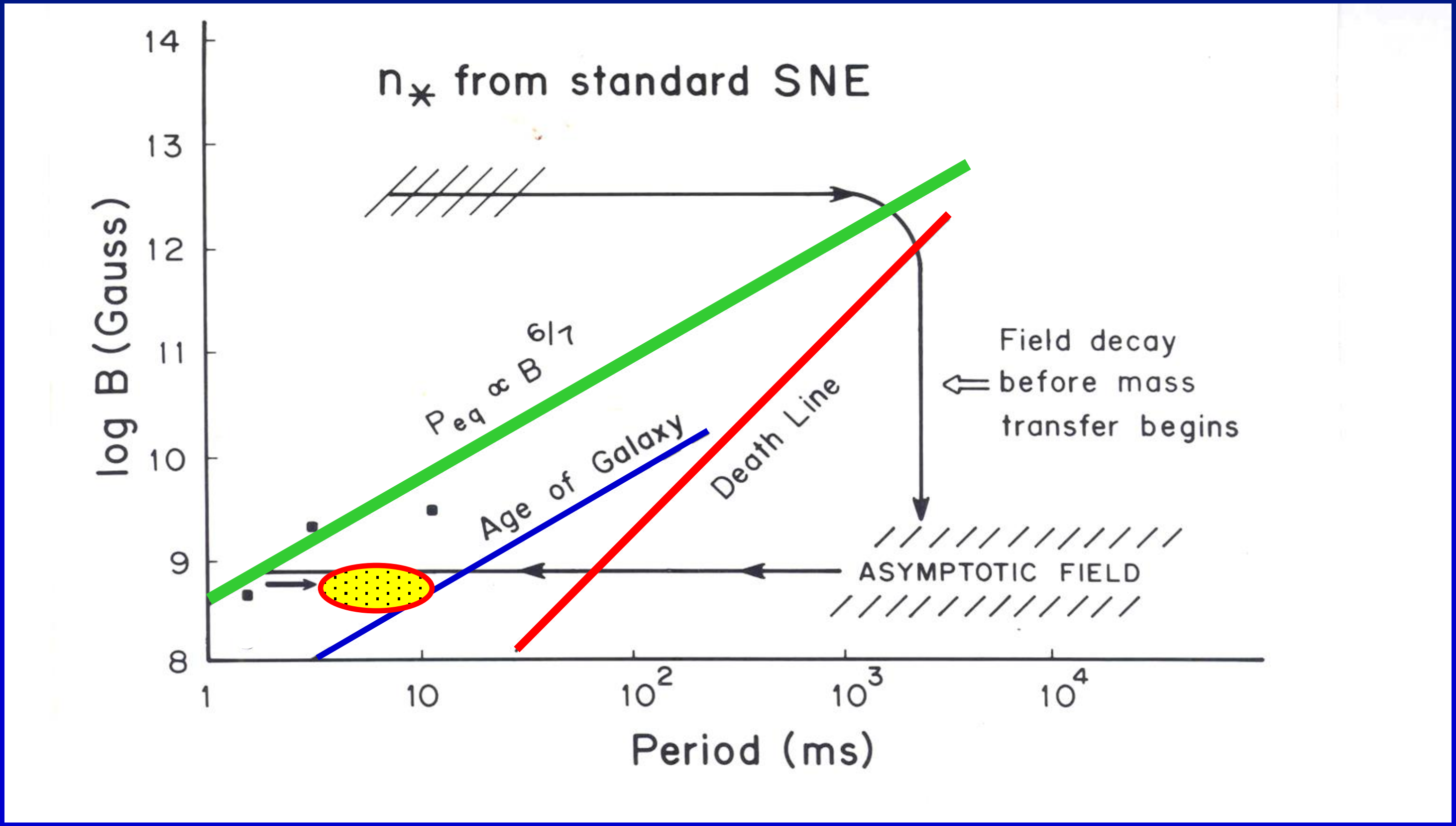


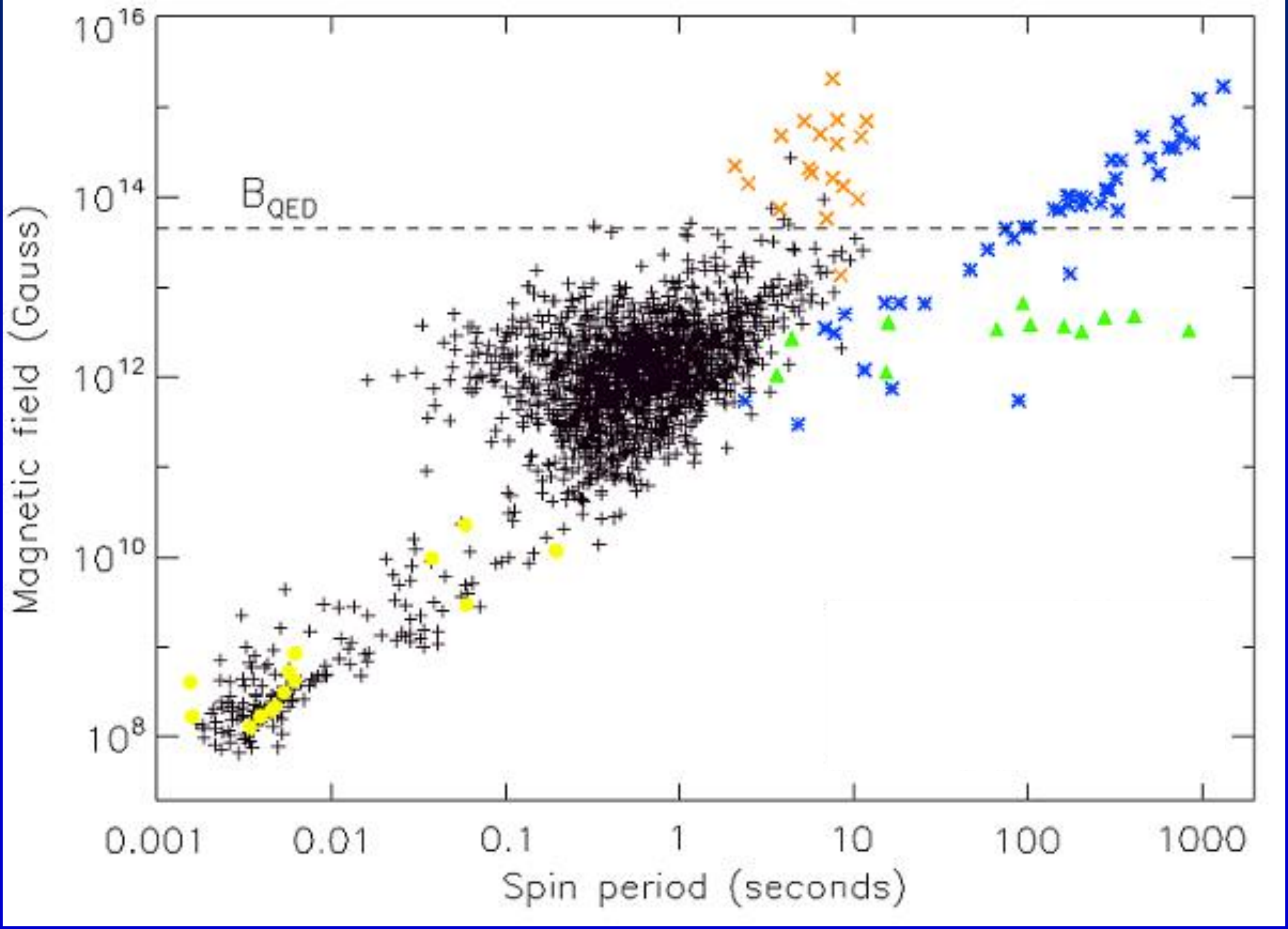
Relativistic beam from the eclipsing millisecond pulsar PSR-1957+20 is blowing away its faithful donor (1988)

Predictions made in 1986

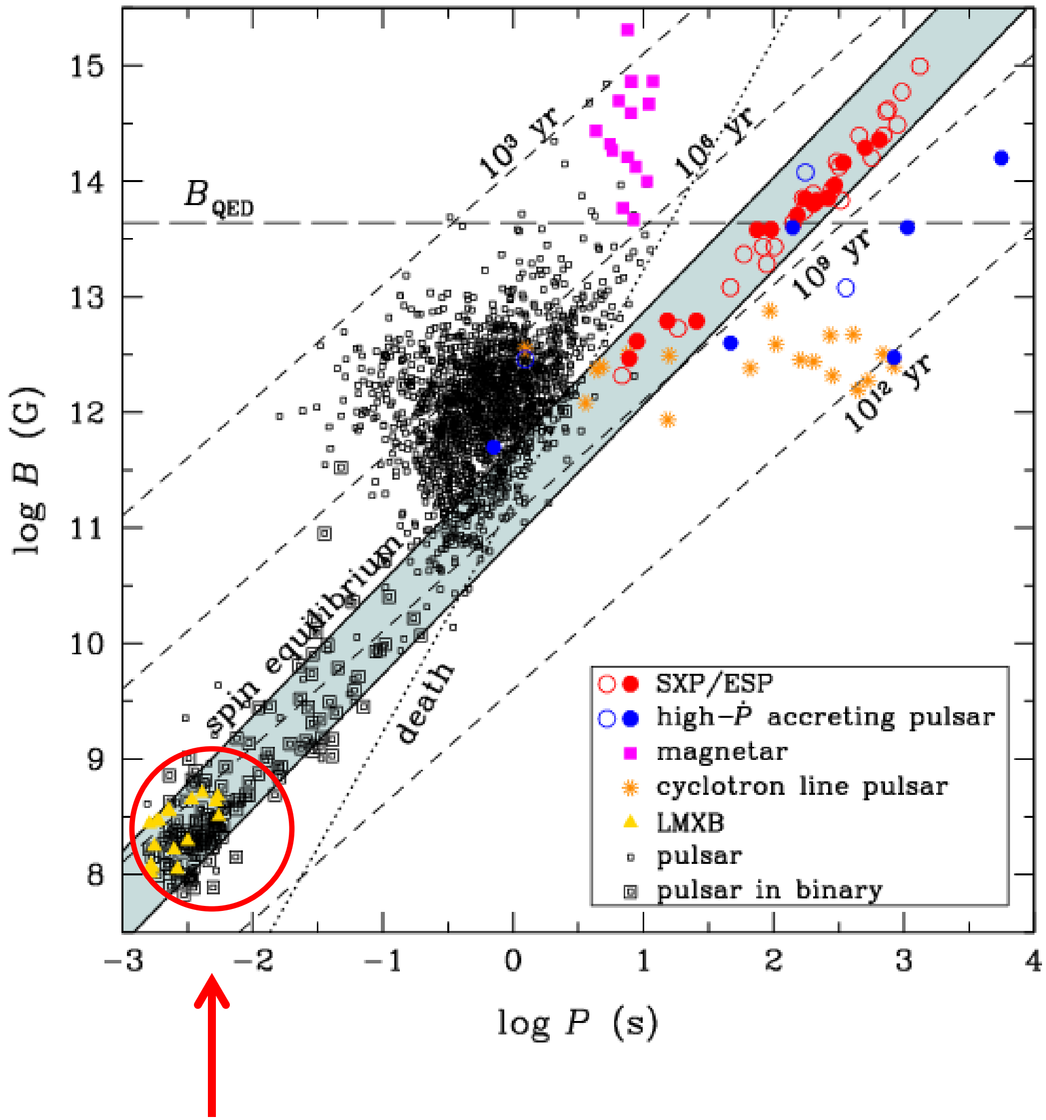
- Majority of millisecond pulsars will be in binaries with low mass white dwarf companions.
- Their orbits will be circular.
- Their magnetic field will be ~ 5 x 10^8 Gauss.
- Their periods will be between 6 – 10 milliseconds.

Millisecond pulsars are for ever!





Magnetic field (Gauss)
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
B_{QED}
0.001
0.01
0.1
1
10
100
1000
Spin period (seconds)



Nearly 100 binary millisecond pulsars discovered so far.

Evolution of magnetic field of neutron stars.

§ Origin of the Magnetic Fields of n_*

- Fossil Field of the Progenitor
  - Flux consv. during core collapse
    → $B \sim 10^{12} - 10^{14}$ G in the n_*

    – Woltjer; Ginzburg 1964
- Field Generation after birth of n_*
  - Thermoelectric Battery in Crust
  - DIFFICULTIES:
    - timescale too long
    - toroidal geometry

- Fossil field hypothesis is best bet!
- For the exterior field to decay the field trapped in the core must be destroyed.

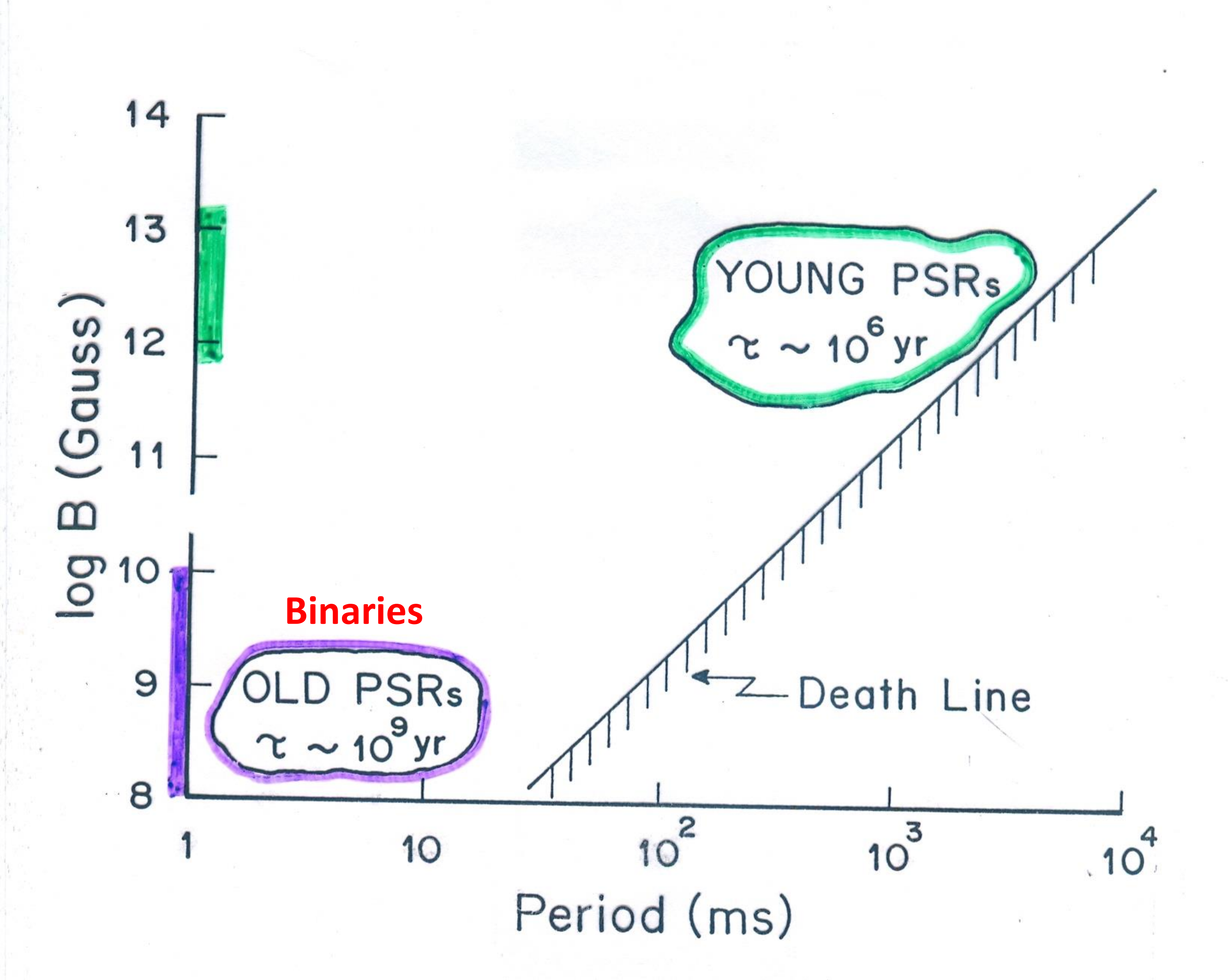
Origin of magnetic field of neutron stars

- Fossil field of the progenitor. Flux conservation. (Woltjer; Ginzburg; 1964).

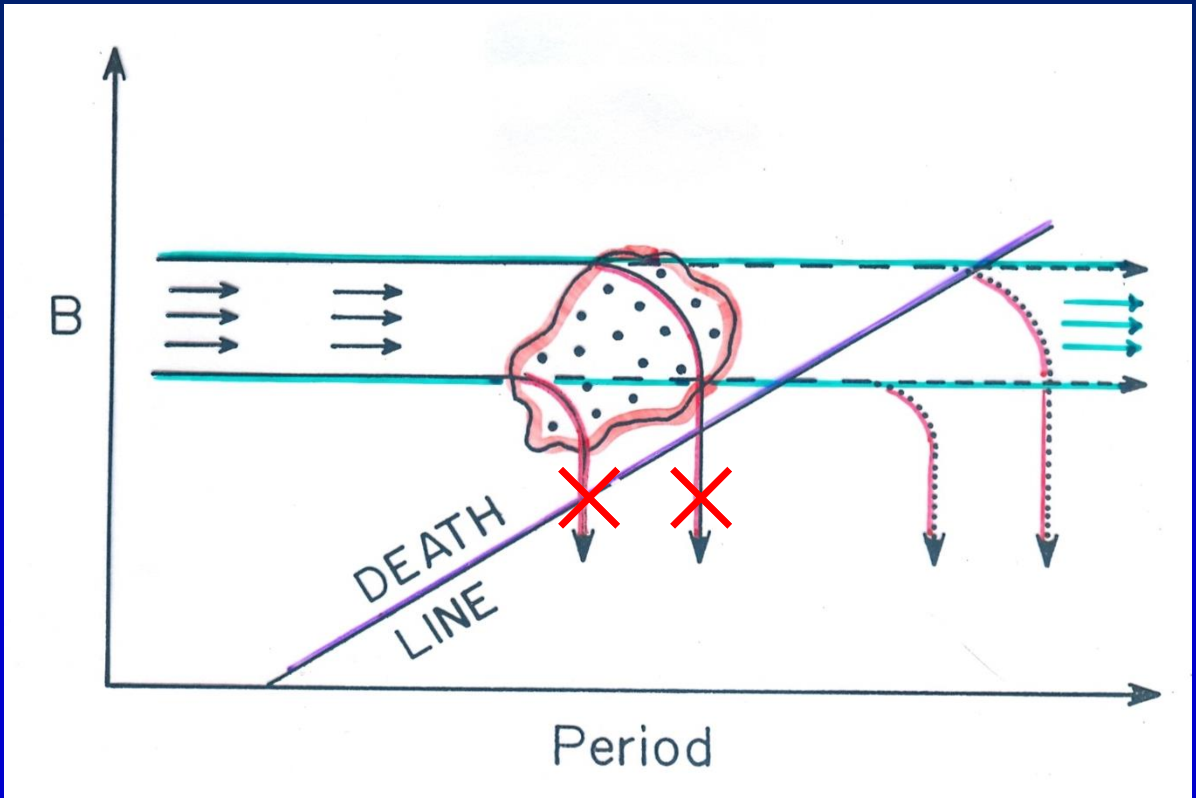
- Field generation after birth. “Battery Effect”.

  Difficulties: Timescale to generate is too long. Also, the field generated will be “toroidal”.

- For the exterior field to decay, the field trapped in the core must be destroyed.

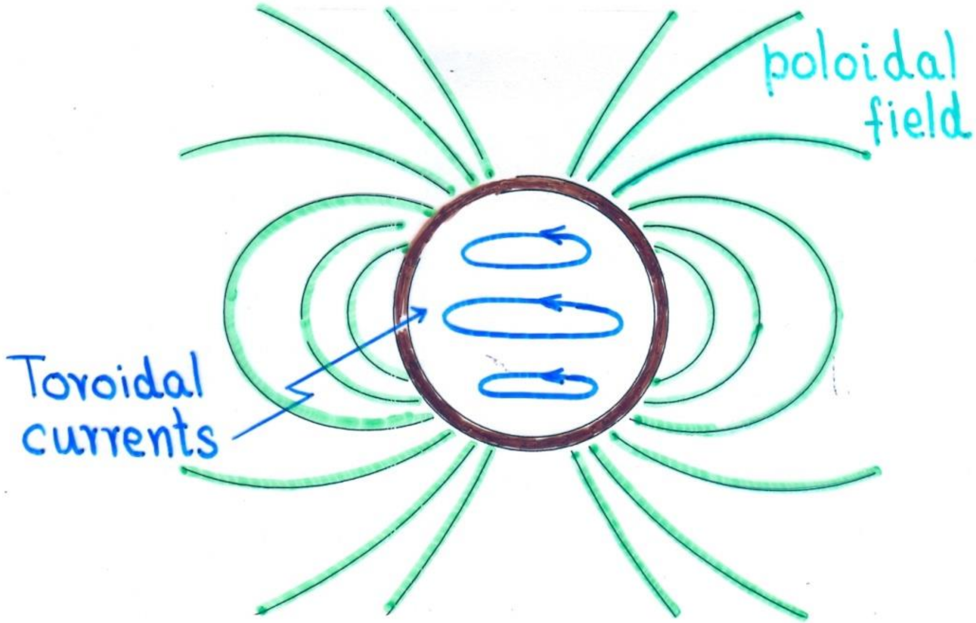


log B (Gauss)
14
13
12
11
10
9
8
YOUNG PSRs
$\tau \sim 10^6$ yr
Binaries
OLD PSRs
$\tau \sim 10^9$ yr
Death Line
1
10
10^2
10^3
10^4
Period (ms)



There is NO evidence of field decay in solitary radio pulsars. Nearly 2500 are known today.

Field decay in Neutron Stars

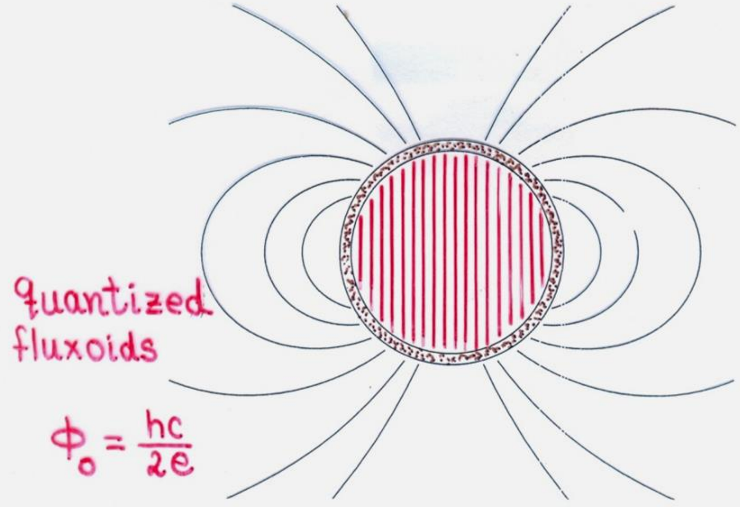


$$\tau_d \sim \frac{4\pi\sigma R^2}{c^2} \sim 10^{13} \text{ years}$$

($\sigma \sim 10^{29}\ s^{-1}$; extreme degeneracy of protons)

- In the classical picture field decay is difficult to understand

§ Proton Superconductivity



- Supercond. will nucleate at constant field: $\tau_{diff} = \frac{4\pi\sigma R^2}{c^2} \sim 10^{13}\,yr$
- protons will form Type II supercond.
  - It will be in a VORTEX STATE
  - # of fluxoids (Abrikosov vortices)

$$N_f = \pi R^2 B/\Phi_0 \sim 10^{31} B_{12}$$

- spontaneous flux expulsion timescale $> t_{UNIVERSE}$

§ Quantized Fluxoids in a Type II Superconductor

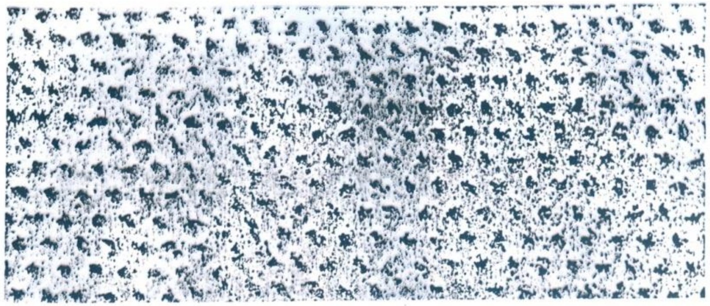
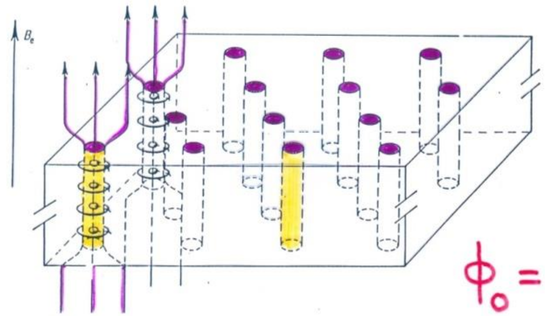


Fig. 79. Electron micrograph of a flux quantum lattice after decoration with colloidal iron. Frozen-in flux with field zero. Material: Pb + 6.3 atom% In, temperature: 1.2 K, sample shape: cylinder 60 mm long, diameter 4 mm, magnetic field B parallel to axis, enlargement: 8300-fold (reproduced by kind permission of Dr. Eβmann).

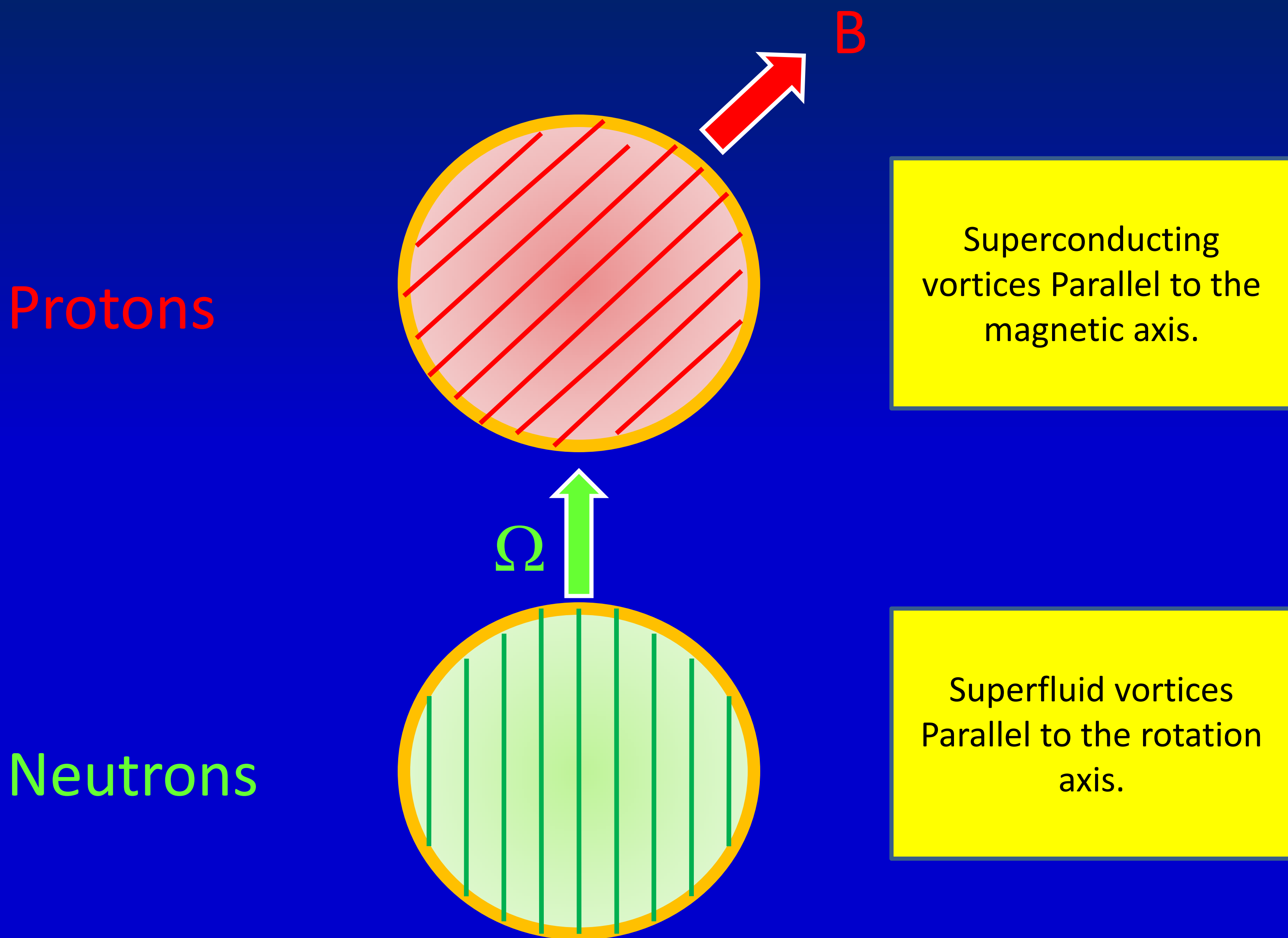


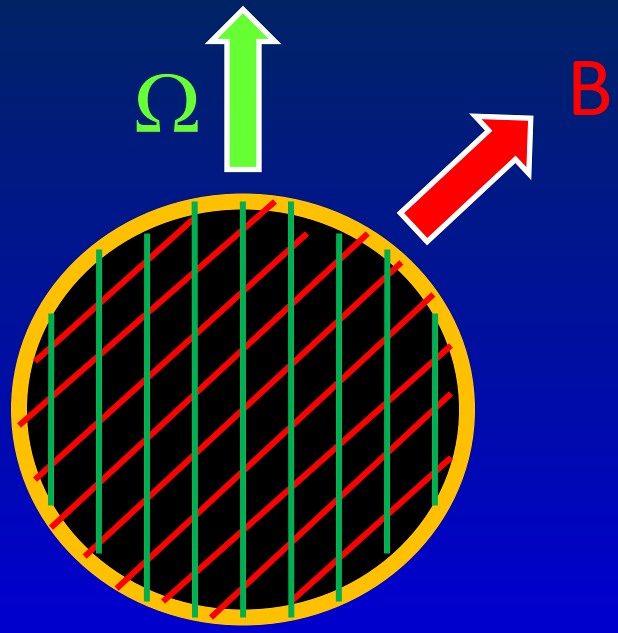
$$\phi_0 = \frac{hc}{2e}$$

Fig. 78. Schematic representation of the Shubnikov phase. Magnetic field and supercurrent are only illustrated for two flux tubes.

Timescale for spontaneous expulsion of flux trapped in a superconductor is ~ 10^{23} years!

- Solitary neutron stars show no evidence of field decay.
- Why do low field pulsars always occur in binaries ?
- Why and how does the field decay ?
- Why is there a residual field ?


B
Protons
Superconducting vortices Parallel to the magnetic axis.
Ω
Neutrons
Superfluid vortices Parallel to the rotation axis.

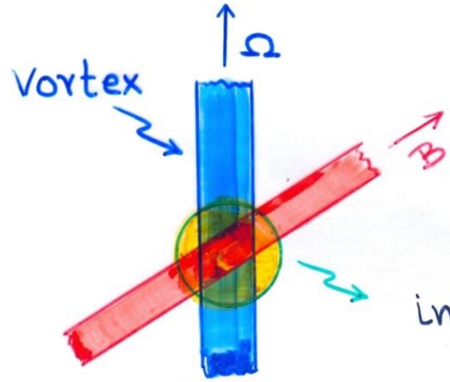


There are two kinds of vortices in a neutron star.

10^{16} Feynman Vortices in the neutron superfluid.

10^{31} Quantized flux tubes in the proton superconductor.

Fluxoid - Vortex Pinning



interaction volume

$\sim \pi \xi_n^2 \xi_p$

$\sim 10^4 \, fm^3$

'Pinning' energy scale

$$\mathcal{E}_p \sim \frac{3}{8} \frac{\Delta_n^2}{E_{Fn}} \cdot n_n \cdot V \qquad (i)$$

(G.S., '89)

$$\mathcal{E}_p \sim n_n \left(\frac{\Delta_n}{E_{Fn}}\right)^2 \cdot \left(\frac{\Delta_p^2}{E_{Fp}}\right) \cdot V \qquad (ii)$$

$\sim (0.1 - 1)$ MeV / connection

(Sauls, 1989)

fluxoid

$$\Delta P \approx \frac{1}{2}\rho_p\left(\frac{h}{2m}\cdot\frac{1}{r}\right)^2 + \frac{B^2}{8\pi}$$

$$\Rightarrow \quad \Delta\rho \sim \Delta P / c_s^2$$

vortex

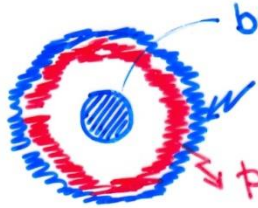
$^3P_2$ neutron vortex : FERROMAGNETIC

$$\langle S_z \rangle = |\Psi_{\uparrow\uparrow}|^2 - |\Psi_{\downarrow\downarrow}|^2$$

$$M_{vortex} \approx (\gamma_N \hbar)\, n_n \left(\frac{\Delta_n}{E_{Fn}}\right)^2$$

$$\sim 10^{11}\ G$$

SUPERFLUID - DRAG



$$\Longrightarrow \quad B_{vortex} \sim 10^{15}\ G\ !$$

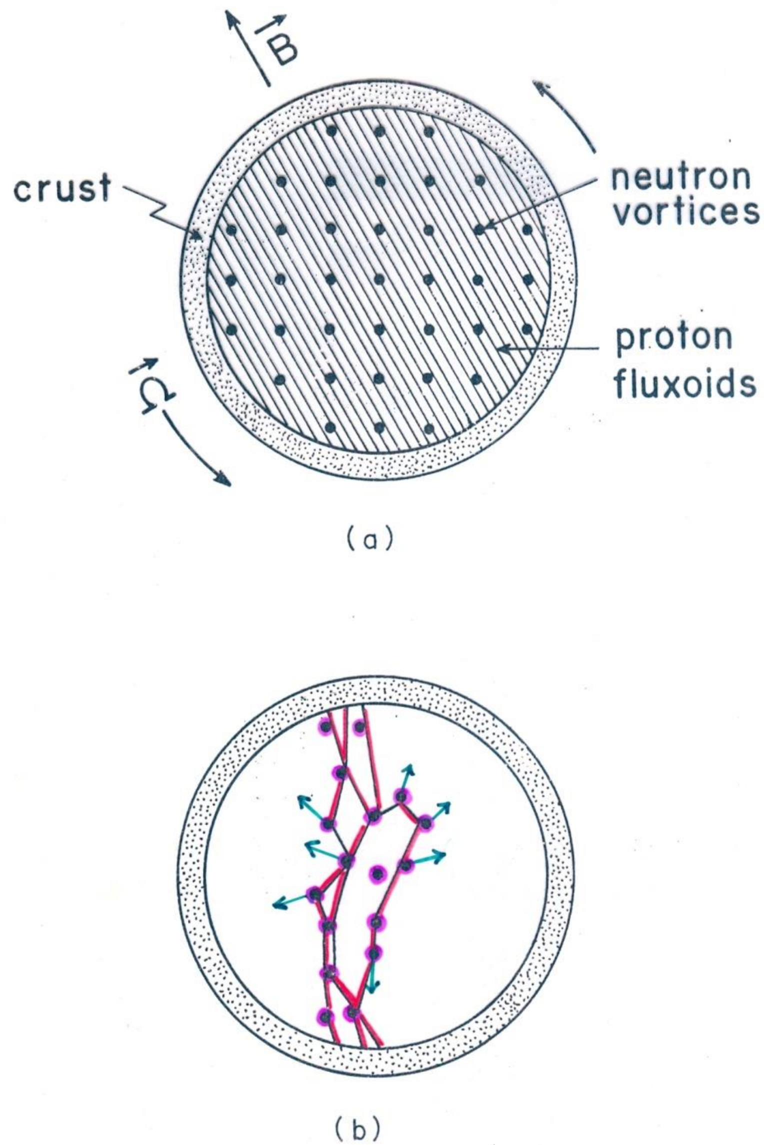
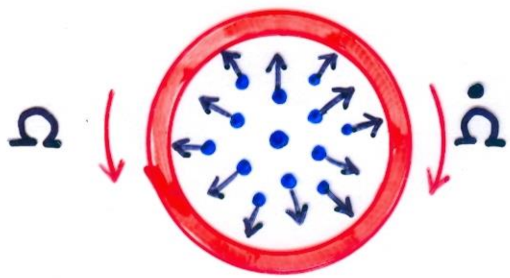


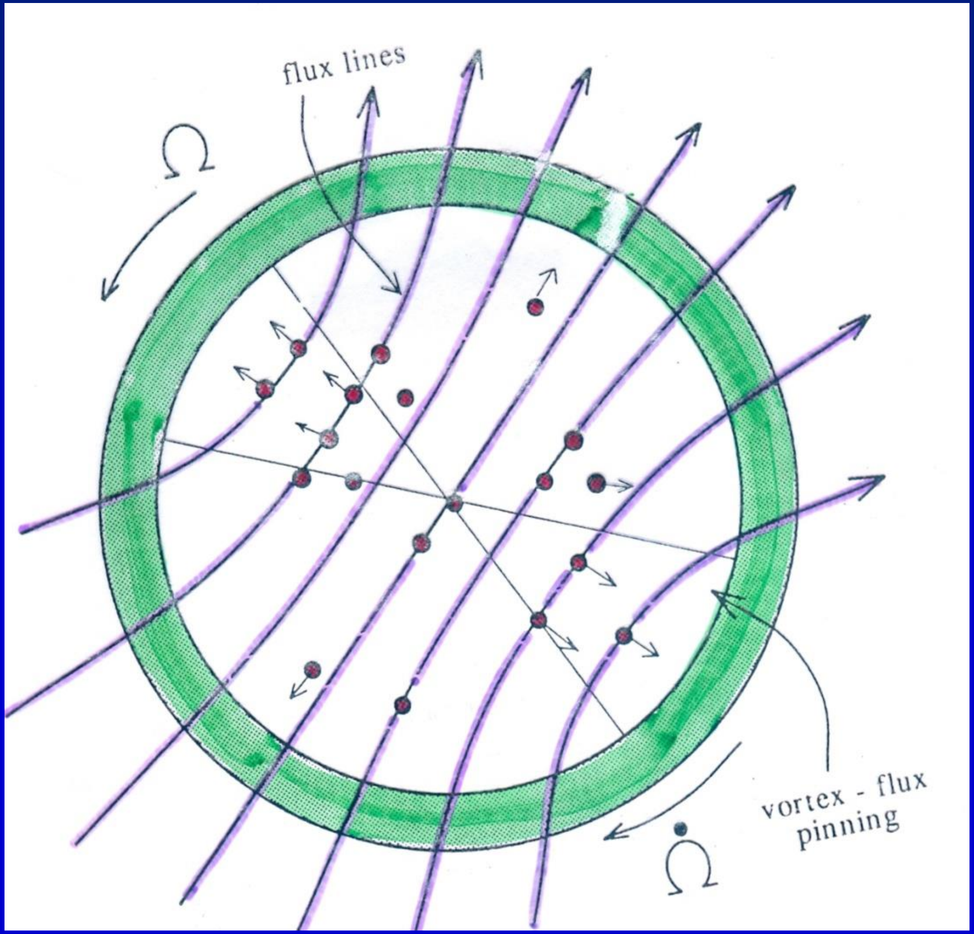
Fig. 1

as the neutron star slows down...



radial outflow of vortices

Because of The strong pinning The vortices will drag The fluxoids with Them!


flux lines
Ω
vortex - flux
pinning
Ω̇

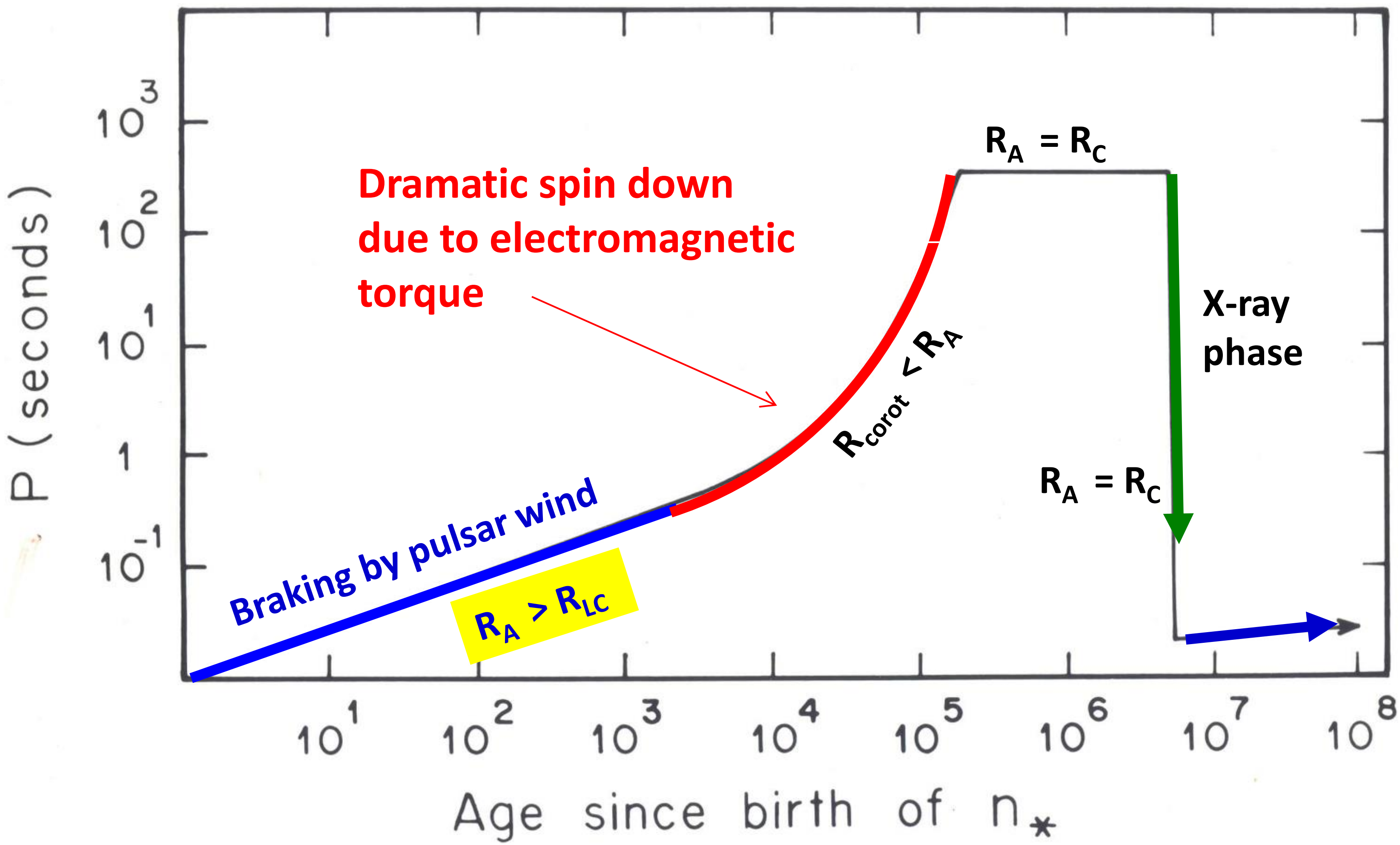
§ ROTATING MAGNETIC DIPOLE

- $-\frac{d}{dt}\left(\frac{1}{2} I \Omega^2\right) = \frac{2}{3c^3} B^2 R^6 \Omega^4$
- $\dot{P} \propto \frac{B^2}{P}$
- $\tau_{sd} = P/2\dot{P}$

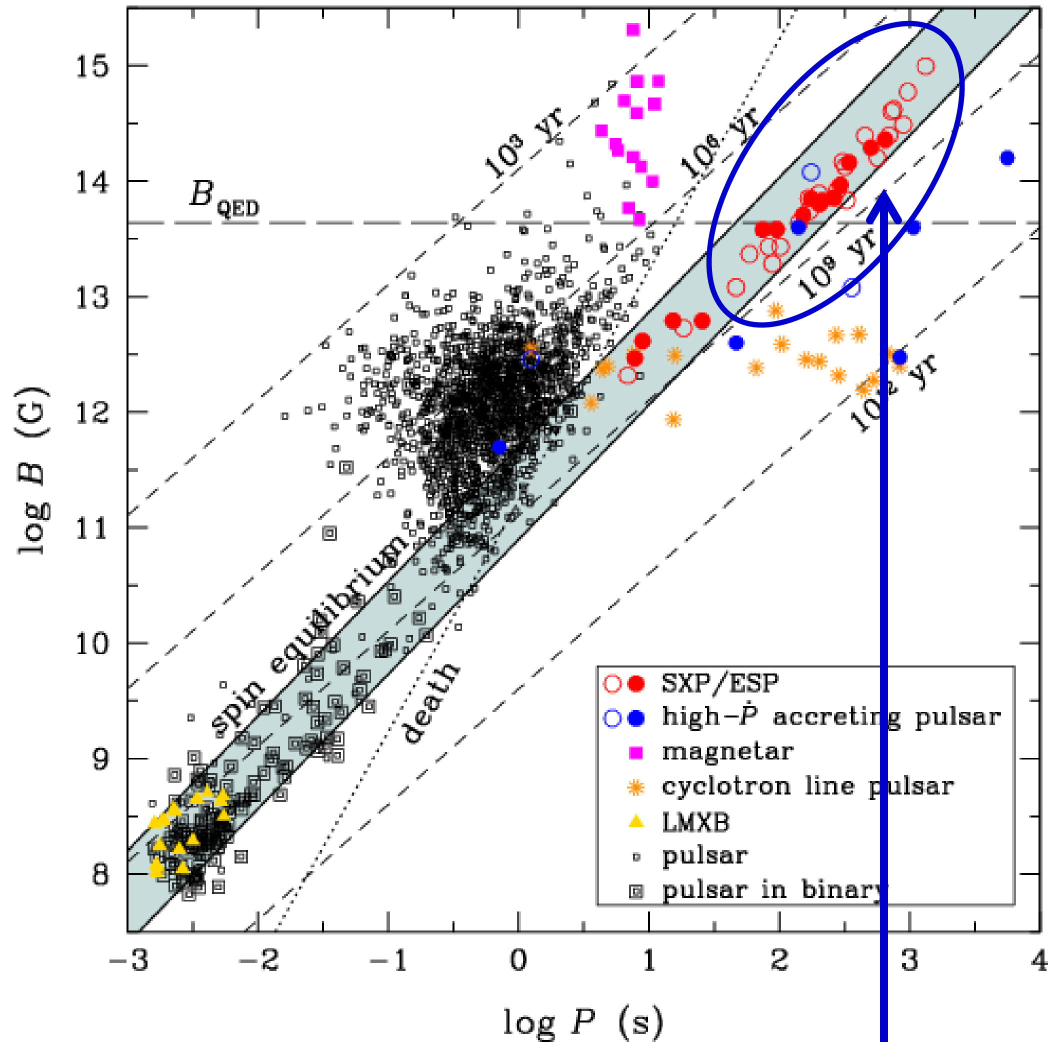
- Two timescales : τ_{sd} vs. τ_{decay}
- at late times,

$$B(t) \propto \left(\frac{t}{\tau_{sd}}\right)^{-1/4}$$ ← very slow!

∴ there will **NOT** be significant flux expulsion from the superconducting cores of solitary neutron stars.

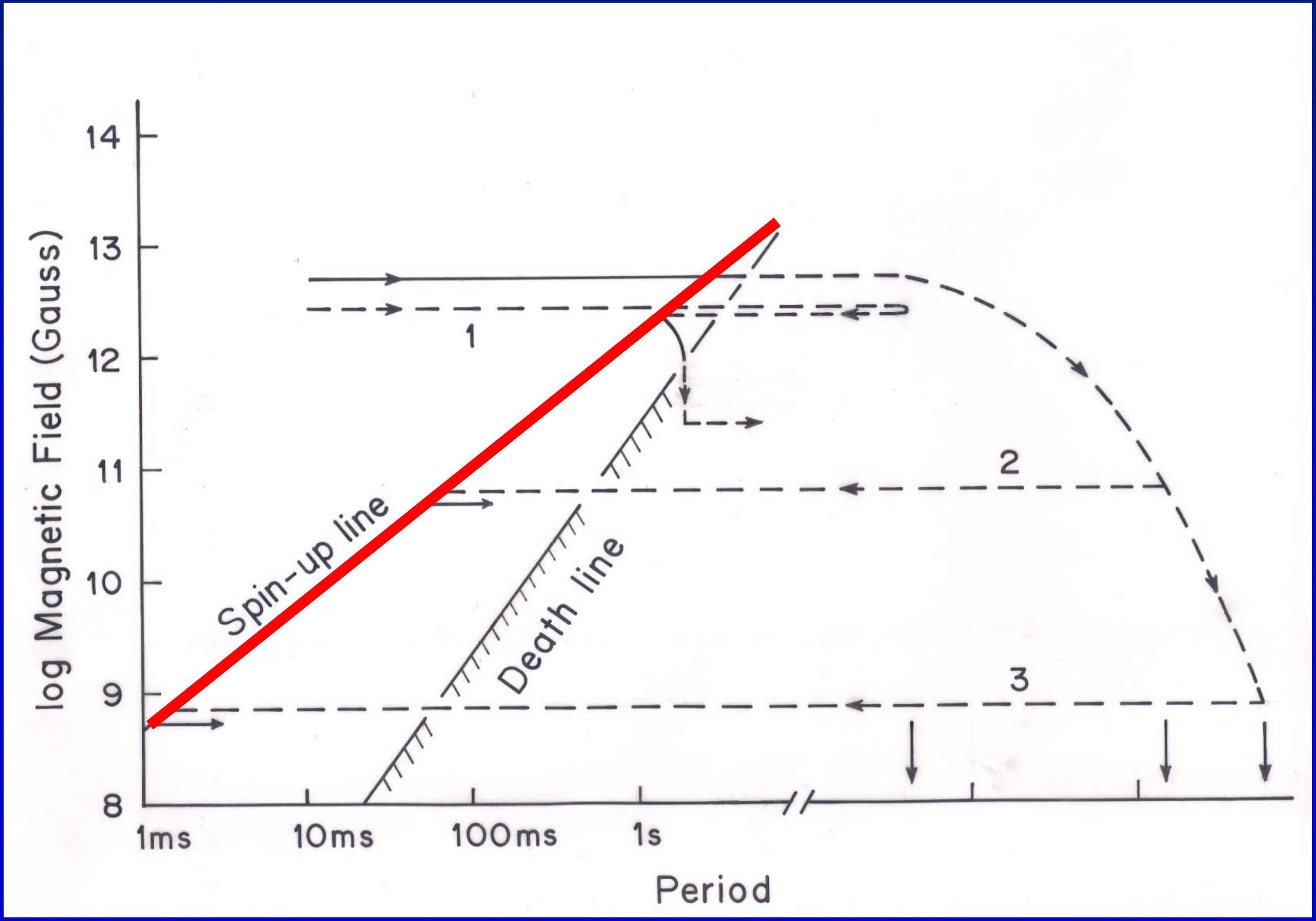


P (seconds)
10^3
10^2
10^1
1
10^-1
Age since birth of n*
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
Dramatic spin down due to electromagnetic torque
Braking by pulsar wind
R_A > R_LC
R_corot < R_A
R_A = R_C
X-ray phase
R_A = R_C

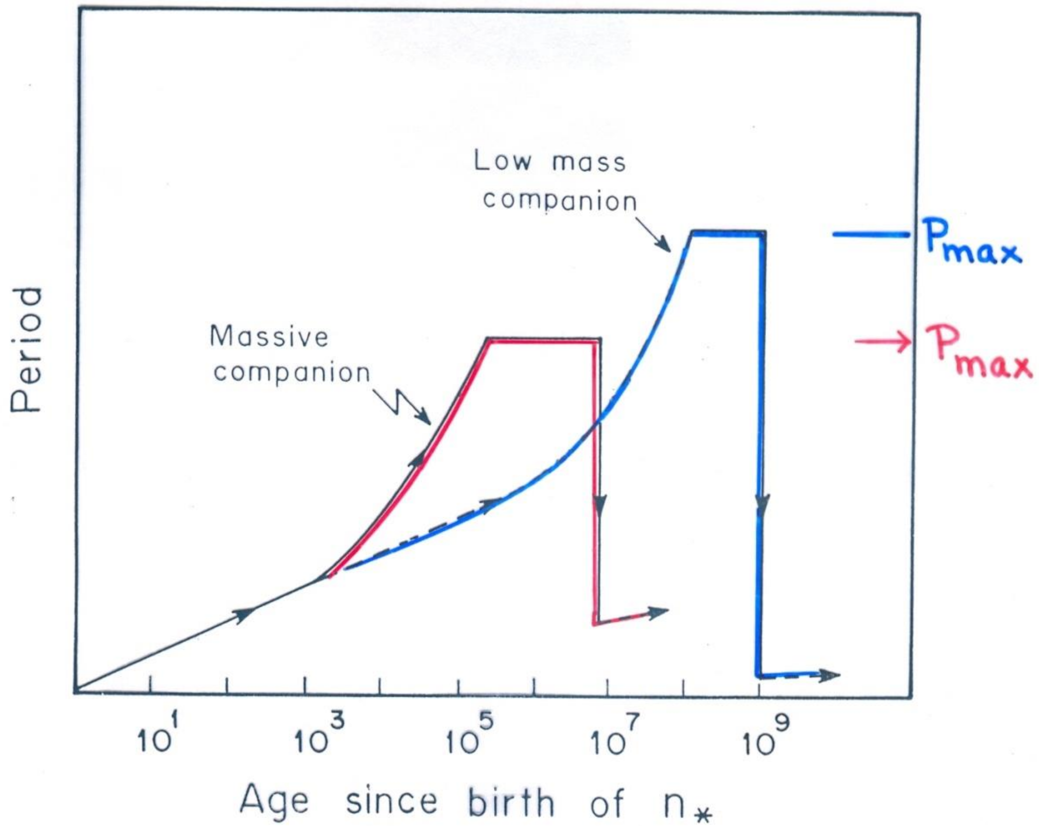


These accreting neutron stars have periods of hundreds to thousands of seconds

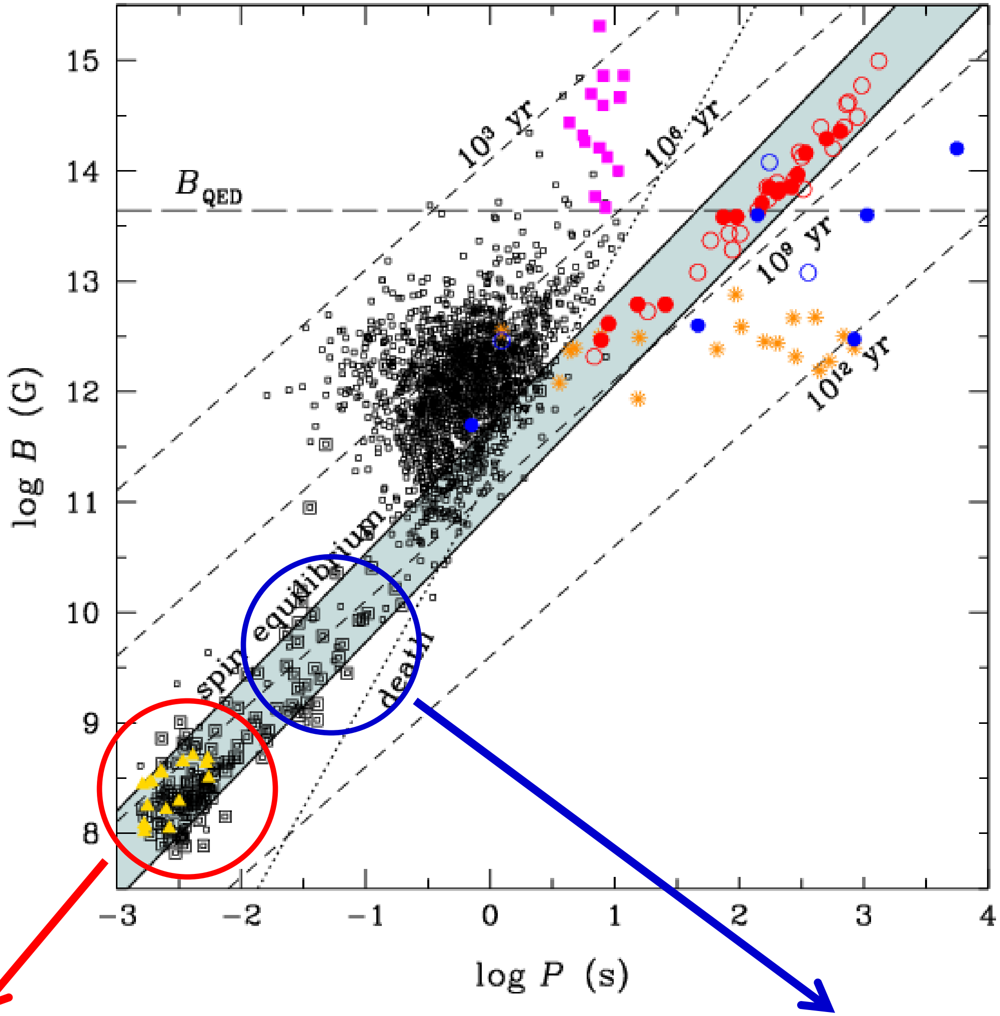
Recycled Pulsars



The period to which a recycled pulsar is spun up to will be determined by the strength of its magnetic field.



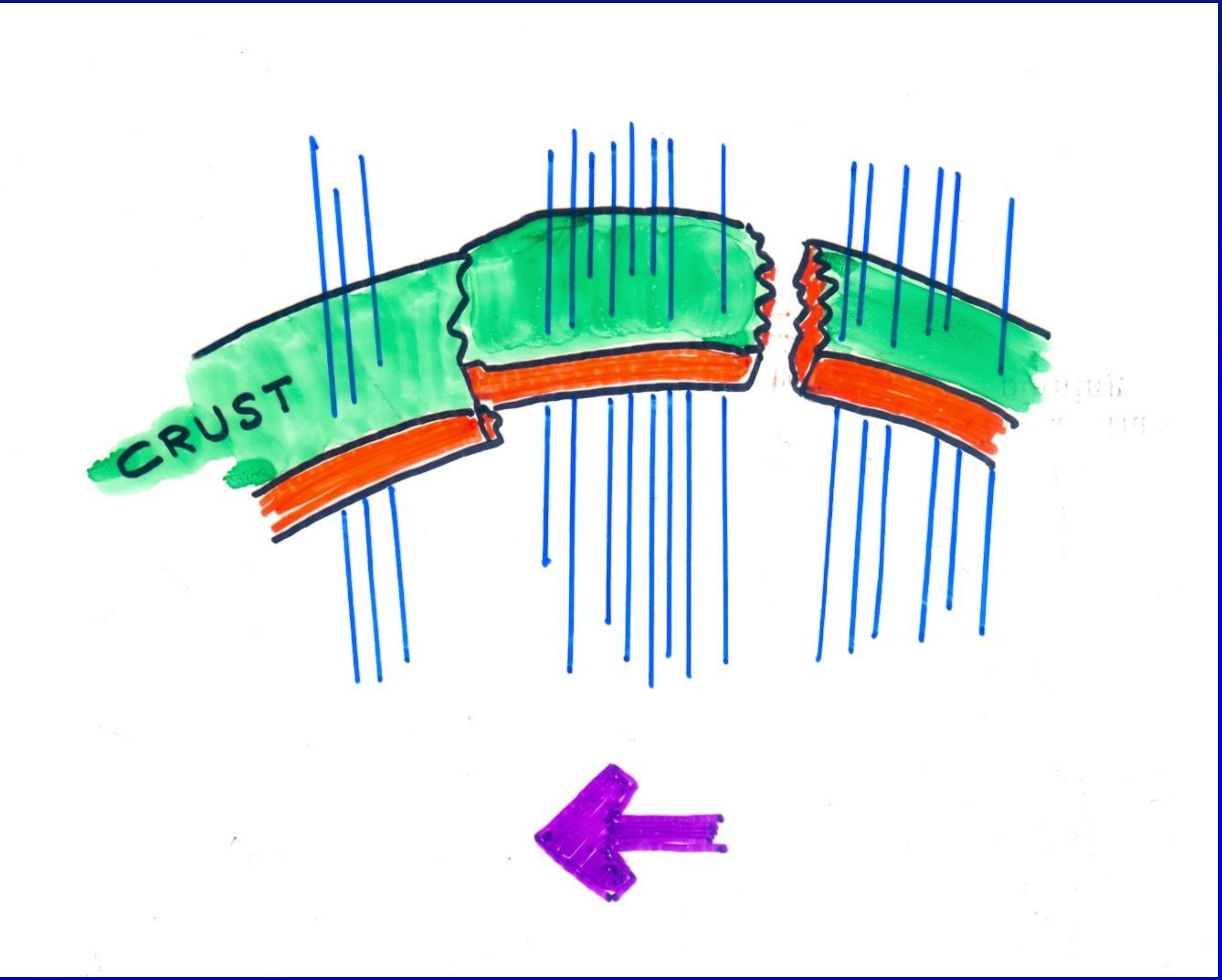
$$B_{asymptotic} \propto \frac{1}{P_{max}}$$

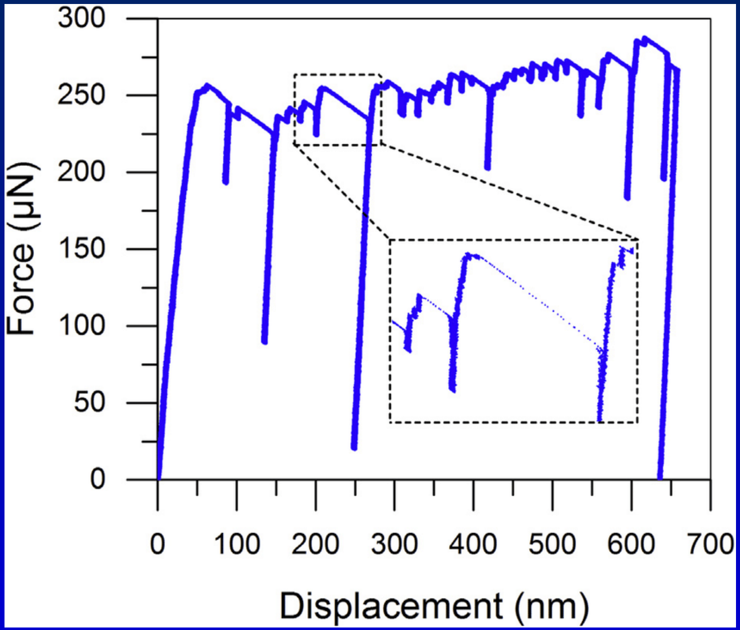


Millisecond pulsars with low mass white dwarf companions.

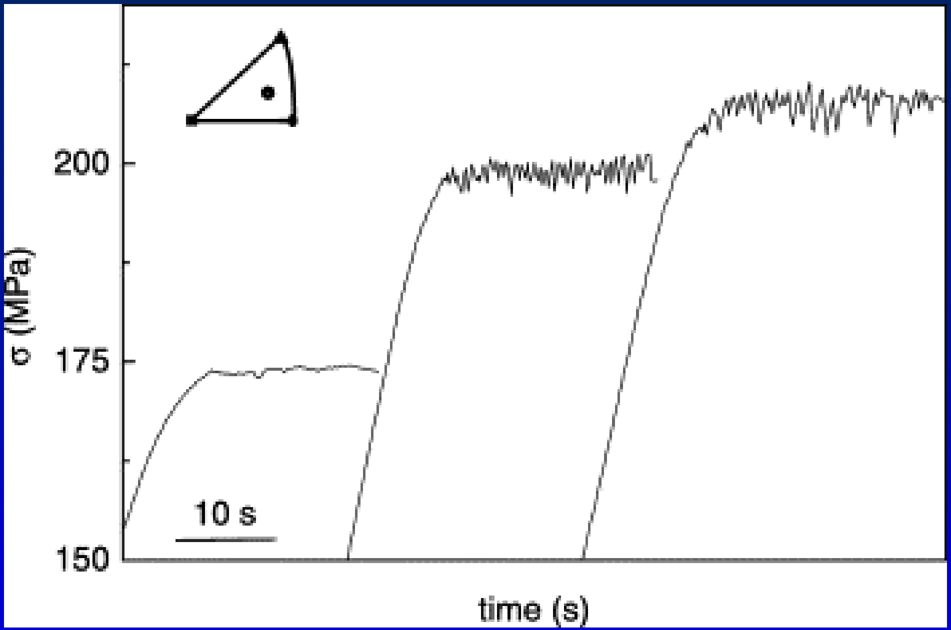
Hulse-Taylor-like pulsars with massive companions; neutron stars or white dwarfs.

Plate Tectonics


CRUST

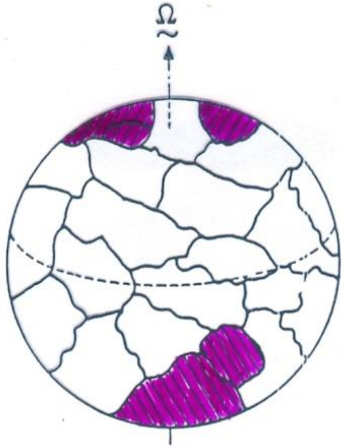


300
250
200
150
100
50
0
Force (μN)
0
100
200
300
400
500
600
700
Displacement (nm)



200
175
150
σ (MPa)
10 s
time (s)

Plate Tectonics



- Under extreme stress the crust may yield through substantial CRACKS.
- over long periods of time PLATES will form
- Plates will flow towards equatorial zone

Plate tectonics may be responsible for

- Observed star quakes (30 on RICHTER Scale!)
- Gamma Ray Bursts
- evolution of magnetic field struct.

Constraints on EOS from rotation rates

- For a n_* of a given mass, the maximum angular velocity is constrained by the EOS.

- In our “spin up” scenario, the minimum period is determined by the maximum possible accretion rate. We took the Eddington Rate as the limiting value.

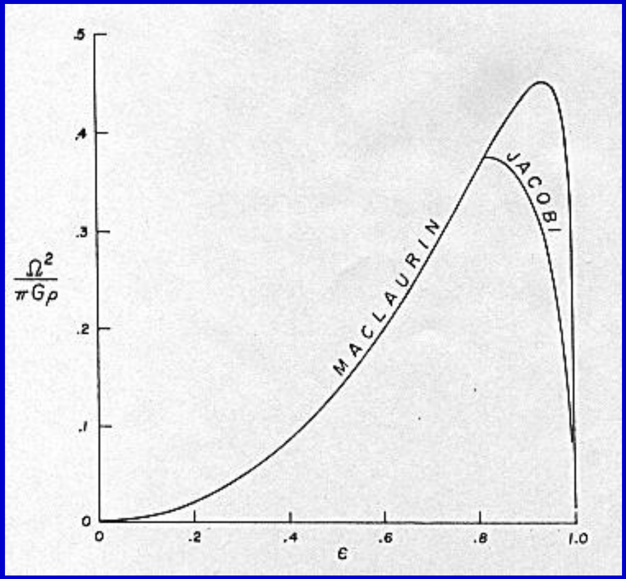
- **But it could also be that the spin up is terminated by instabilities arising due to rotation.**

- Mass shedding at the Kepler frequency is an example.

- But there could be other interesting possibilities.

Nonaxisymmetric Instabilities driven by Gravitational Radiation

- Newtonian stars that rotate sufficiently rapidly are unstable to bar-mode (non-axisymmetric) instability, having an angular dependence cos mϕ for m=2.
- This is the point at which the sequence of **Maclaurin spheroids** bifurcates into the sequence of **Jacobi ellipsoids**.
- In models with viscosity, this happens at a slower rotation rate.



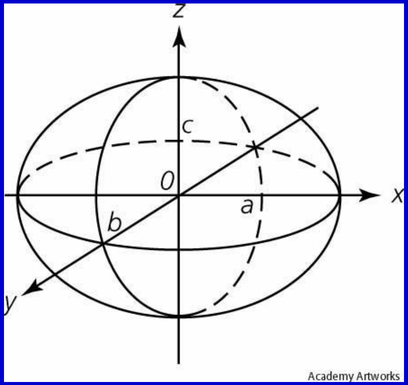
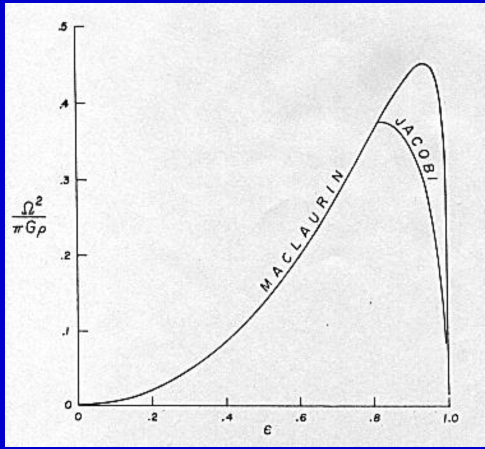
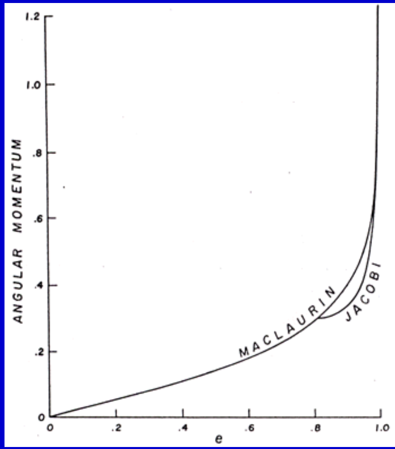


Figure from "Ellipsoidal Figures of Equilibrium" by S. Chandrasekhar

- Jacobi triaxial ellipsoids are static in the rotating frame.
- Chandrasekhar showed in 1969 that this instability is suppressed by radiation reaction to emission of gravitational waves.
- Chandrasekhar discovered that there is an instability at the bifurcation point when one includes gravitation radiation.
- This is the sequence of Dedekind ellipsoids (1860).
- These are stationary in the INERTIAL FRAME!
- Their triaxial figure is due to internal "vortical motion".

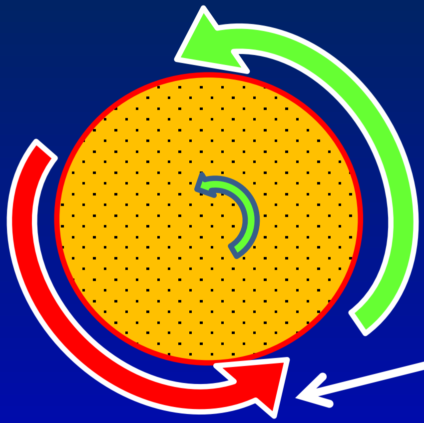




A profound discovery by John Friedman

- In 1978, Friedman and Schutz discovered that nonaxisymmetric instability driven by gravitational radiation is a generic feature of rotating perfect stars in general relativity.

- ALL ROTATING BODIES are unstable in general relativity to nonaxisymmetric perturbations that radiate away its angular momentum.



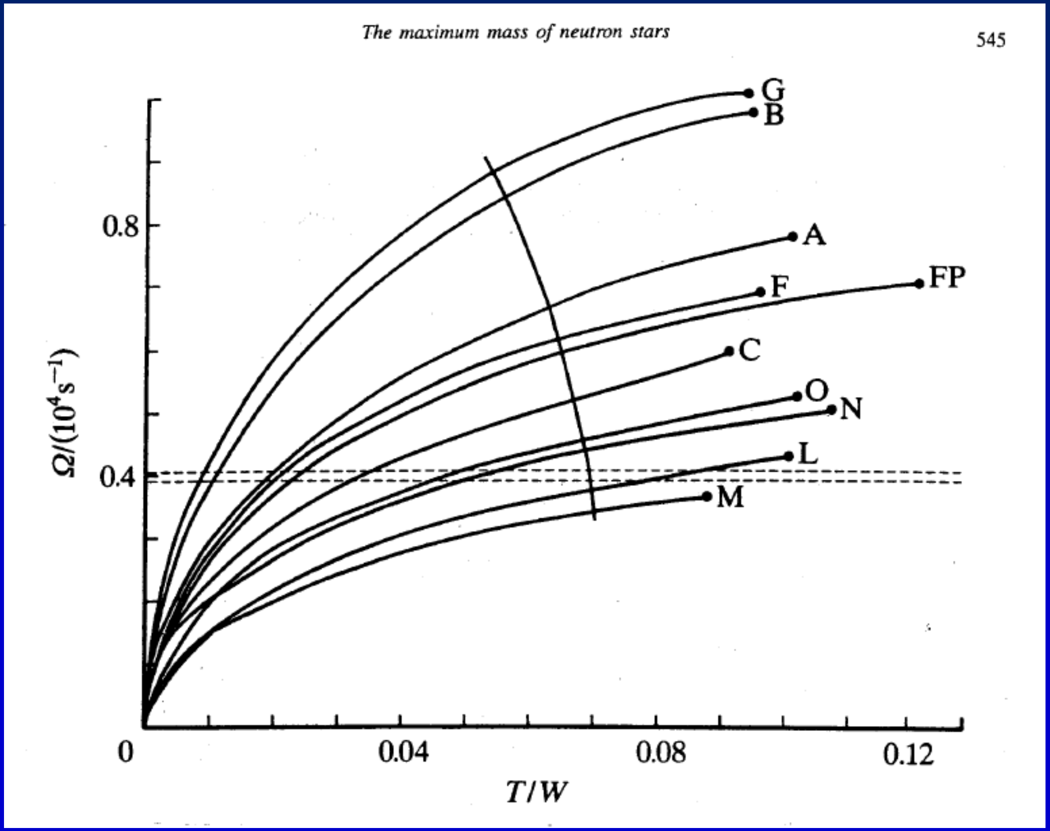
Forward Moving Mode (FMM)
+ve Angular momentum

$e^{\pm im\phi}$

Backward Moving Mode (BMM)
-ve Angular momentum

- Gravitational waves remove + ve angular momentum from FMM and – ve angular momentum from BMM.
- For slowly rotating stars, BOTH modes will be damped.
- In a rapidly rotating star, BMM will be “dragged forward” w.r.t. an inertial observer!
- Grav radn will remove +ve angular momentum from the mode
- But the mode has –ve angular momentum wrt to the fluid.
- Grav radn thus removes +ve angular momentum from a mode whose angular momentum is negative.
- **Thus, gravitational radiation DRIVES THIS MODE!**

- The two fastest ms-PSRs have the same period P=1.5 ms to 3% accuracy.
- Could this be due to the general relativistic instability?
- If so, this would imply a STIFF equation of state.
- This would suggest a MAXIMUM MASS ~ 2 $M_{\odot}$.



$$\frac{T}{W} = \frac{\textbf{Rotational energy}}{\textbf{Modulus of Potential energy}}$$

G : Softest EOS

L and M : Stiffest EOS

Termination points correspond to the Kepler Frequency Ω_K .

Diagonal line is the estimate of the smallest rotation rate at which the gravitational wave instability will set in.

This instability limit is within 15% of the Kepler limit. (Friedman and Ipser, 1992).

- But VISCOSITY can suppress this instability.
- For a perfect fluid, the growth time of an unstable mode is the radiation reaction time.
- If the viscous damping time is less than this then viscosity will stabilize the unstable modes.

- Viscosity of neutron star interior is poorly understood.
- Superfluidity of the interior complicates matter further.
- Paradoxically, the viscosity may be more in the superfluid state! This is because the neutron fluid will be in a vortex state.
- The "core" of the Feynman vortices will be "normal". It has been argued that electron scattering off the vortex cores will enhance the viscosity.