Stellar Remnants

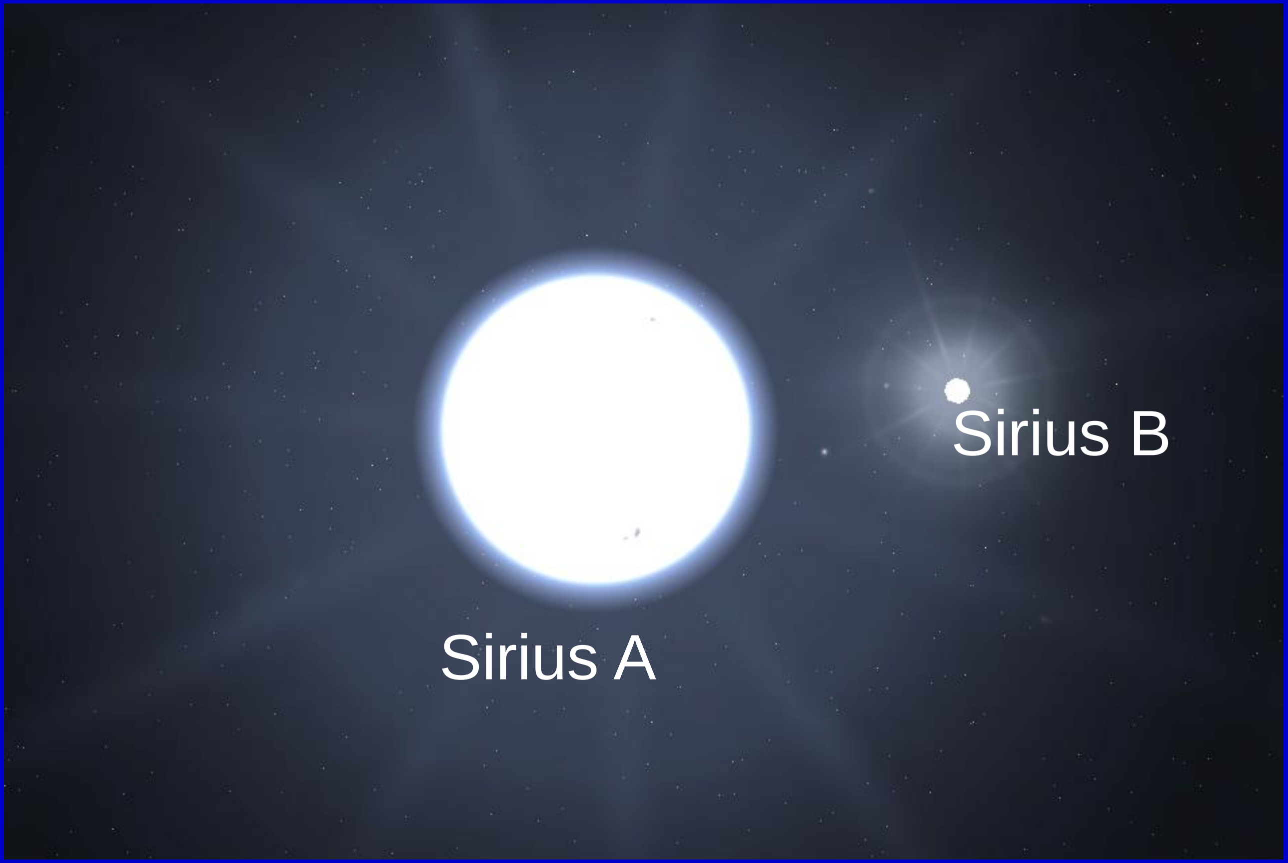
A Historical Perspective

G. Srinivasan

Quantum Stars

White Dwarfs

The strange companion of Sirius!



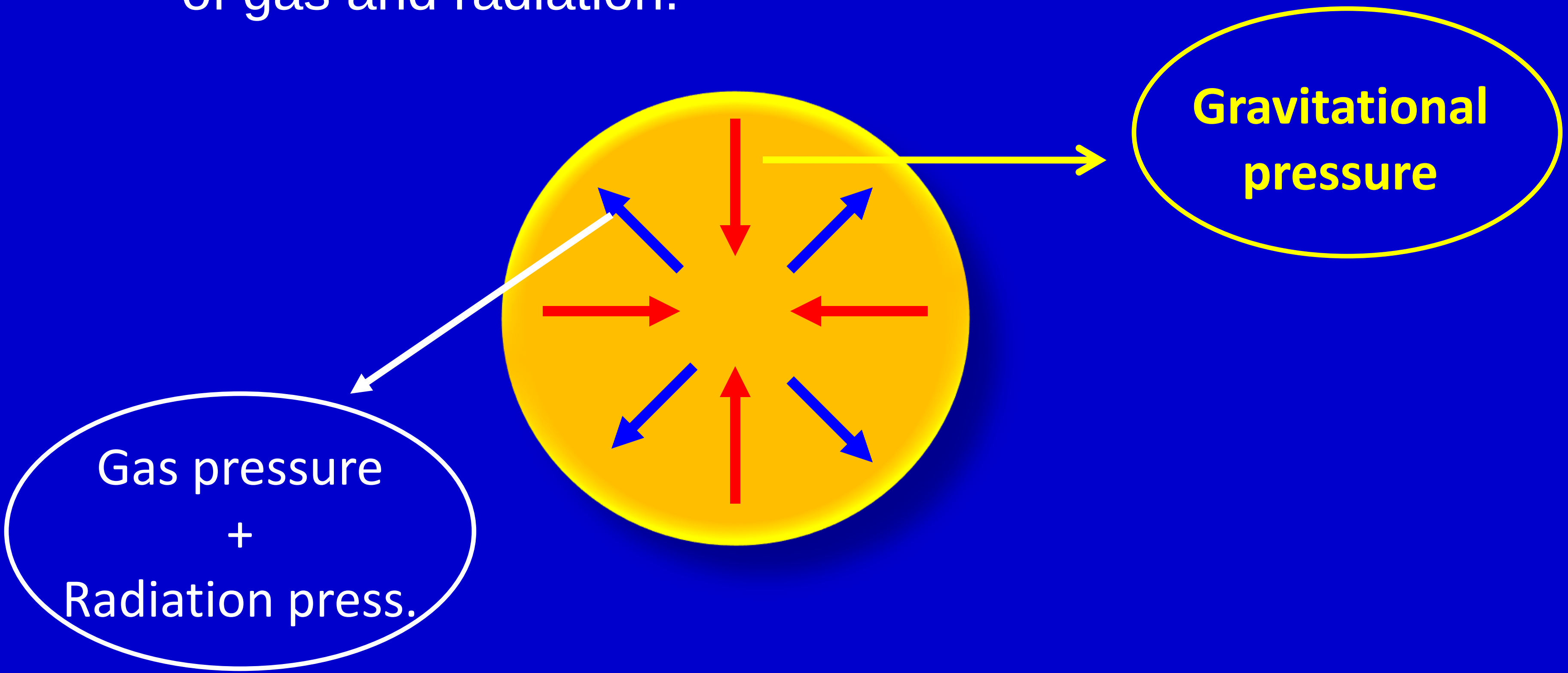
- It all began with Walter Adams measuring the temperature (1914) and the radius (1925) of the companion of SIRIUS.

- With mass roughly equal to the mass of the Sun, but radius roughly equal to that of the Earth, its mean density was staggering:

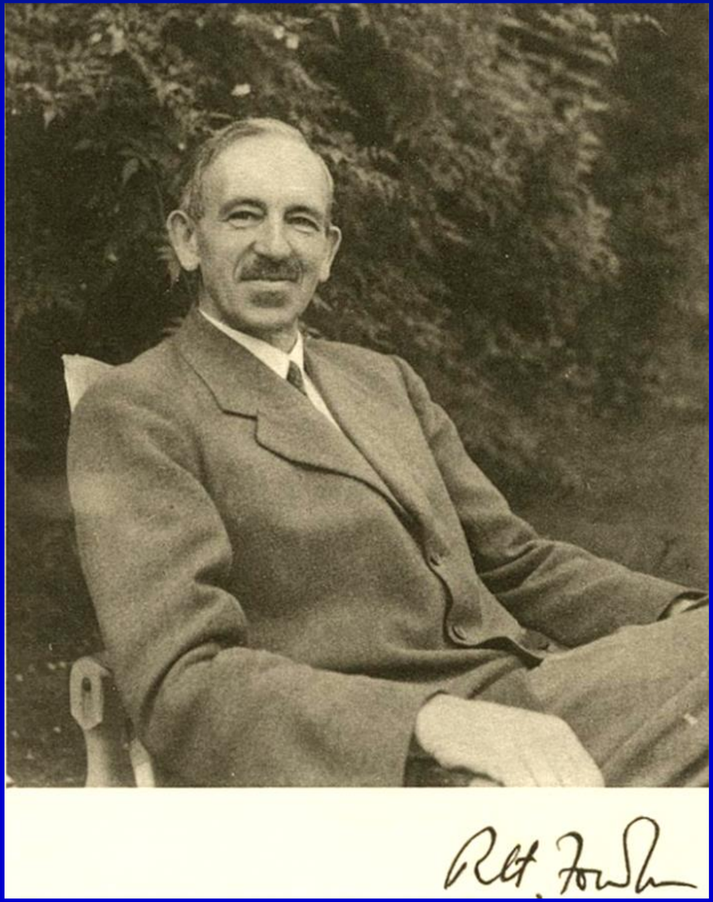
$$\bar{\rho} \sim 10^{6}\, g\ \mathrm{cm}^{-3}\,!$$

Are such super-dense stars doomed?

- Stars are gaseous blobs held together by gravity.
- The inward pull of gravity is balanced by the pressure of gas and radiation.



- What will happen to the star when the nuclear reactor at its centre fails?
- Since there is no further energy generation, the pressure supporting against gravity will decrease, and the star will collapse.
- Will the star collapse to a singularity?!
- This was Eddington's major dilemma in 1925.



Sir Ralph Howard Fowler
1889 - 1944

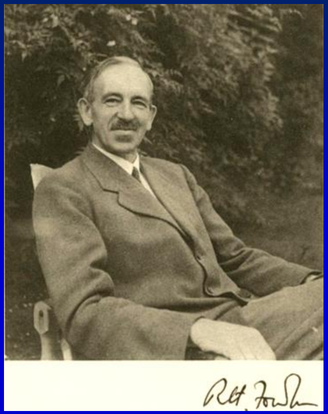
- As the star collapses, and its density increases, at some stage the rules of quantum mechanics will take over from classical physics.

- At high densities, a new pressure arises due to the combined effect of Heisenberg's Uncertainty Principle and Pauli's Exclusion Principle.

- This quantum pressure will stop the collapse at some radius.

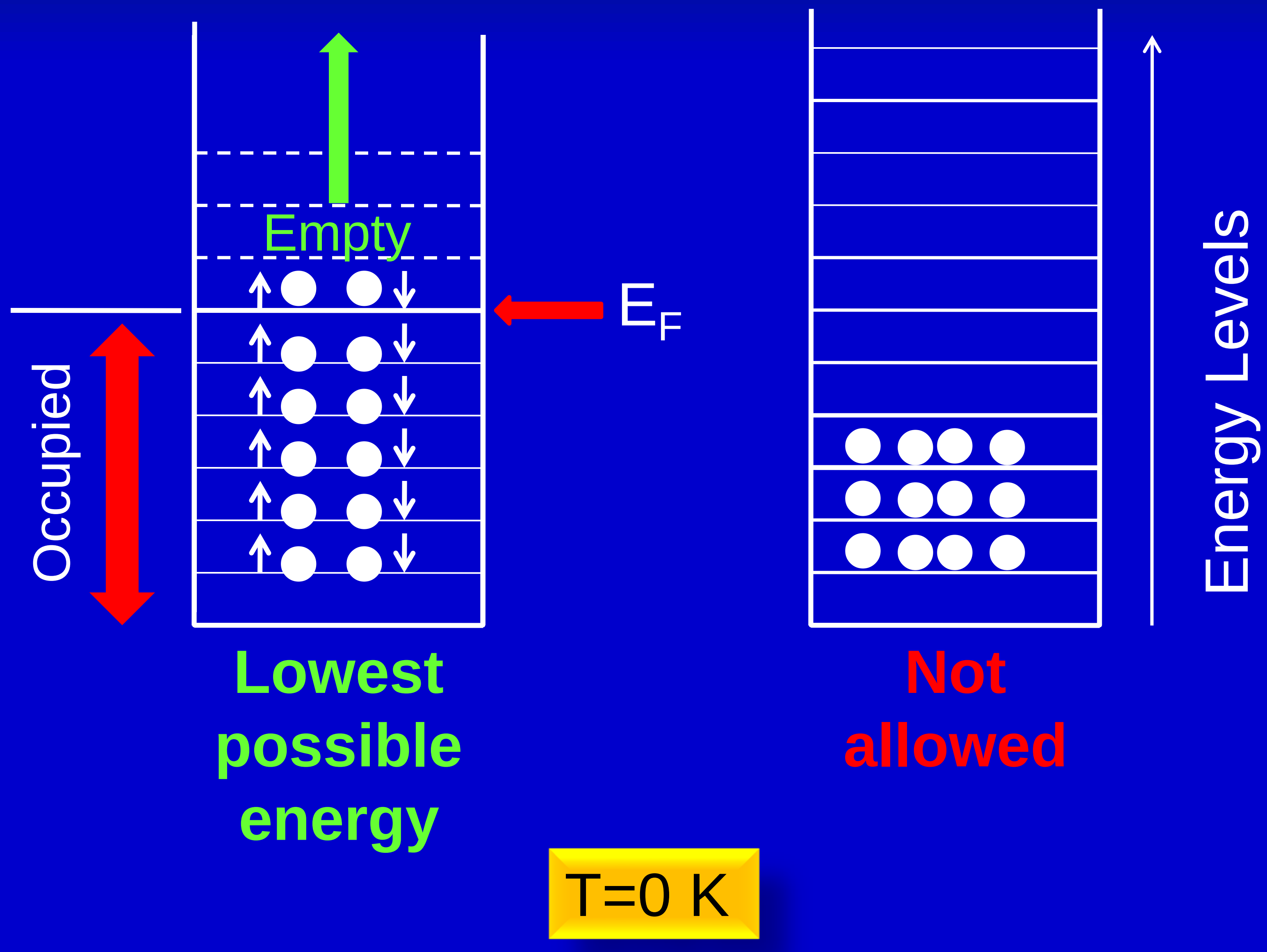
- And we will have a QUANTUM STAR!

Fowler, 1926



Sir Ralph Howard Fowler
1889 - 1944

- DIRAC was Fowler’s student.
- In August 1926, Fowler forwarded Dirac’s paper on the new statistics for publication.
- By November 1926, Fowler had applied these new ideas to save highly condensed stars.
- It is remarkable that this FIRST application of the new rules of quantum physics was to explain the stability of a star!
- Then came Sommerfeld’s application to the free electrons in metals, and Pauli’s application to Paramagnetism of metals.



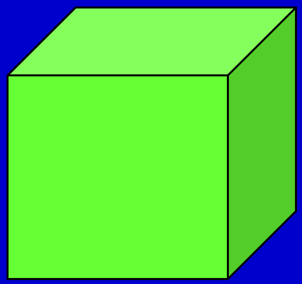
Empty
E_F
Occupied
Energy Levels
Lowest possible energy
Not allowed
T=0 K

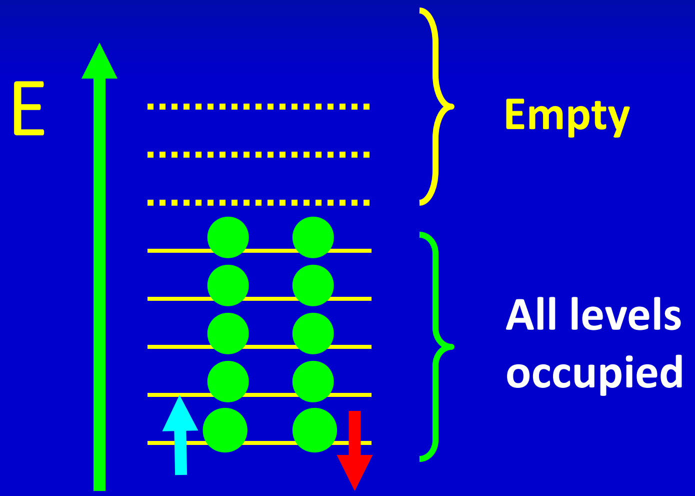
White Dwarfs are Quantum Stars

N electrons

Volume V

T = 0 K





- In quantum physics, energy levels are discrete.
- According to Pauli's Exclusion Principle, one can put only two electrons (↑ ↓) in each level.
- Therefore, an electron gas has energy even at T = 0 K! This zero-point energy will become large as the density of electrons increases.

R.H. Fowler, 1926

White Dwarfs are for ever!

- Once the quantum mechanical pressure of the electrons stops the collapse, the star would have found its ultimate peace.

- Nothing can happen to it. There is no lurking danger.

- What stops gravity is the zero point energy of the electrons. This energy has nothing to do with **heat.**

- Gravity has been stopped by zero-point motion.

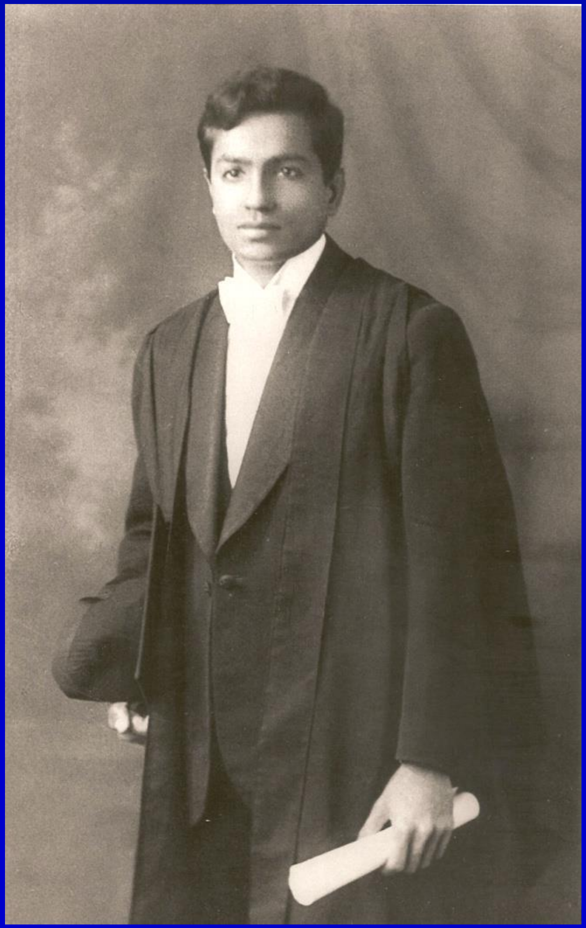
White Dwarfs are for ever!

- The energy of the electron gas at T=0 K is similar to the energy of the electron in the n=1 level of the hydrogen atom.

- Although the electron has a lot of energy, it cannot spend it, because there are no levels with lower energy!

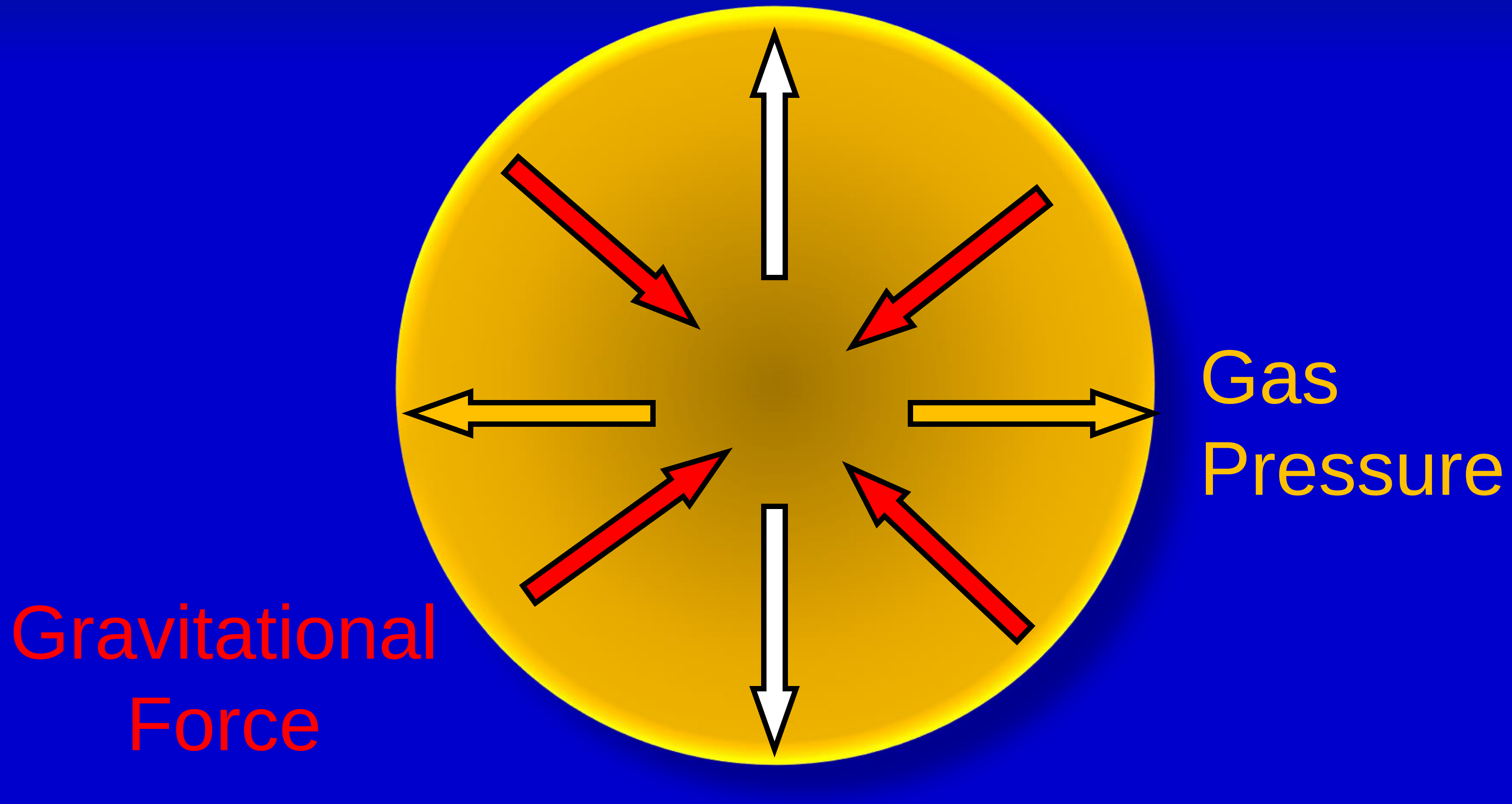
- Same is true of the electron gas in a dense star.

- Therefore, white Dwarfs have found their graveyard. They are for ever!



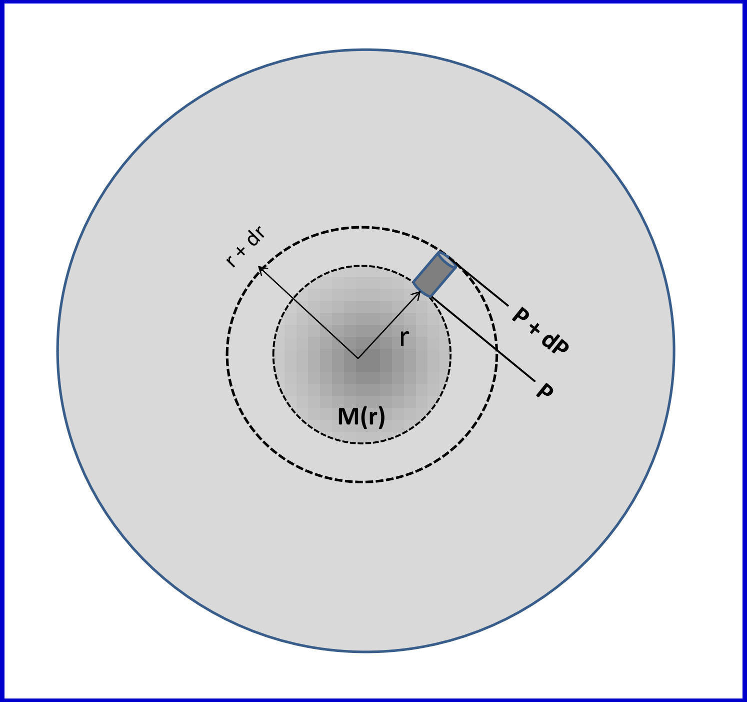
Enter Chandra: 1928

Radiation Pressure



A star like the Sun is stable because the inward pull of gravity is precisely balanced by the combined pressure of the **gas** and **radiation**.

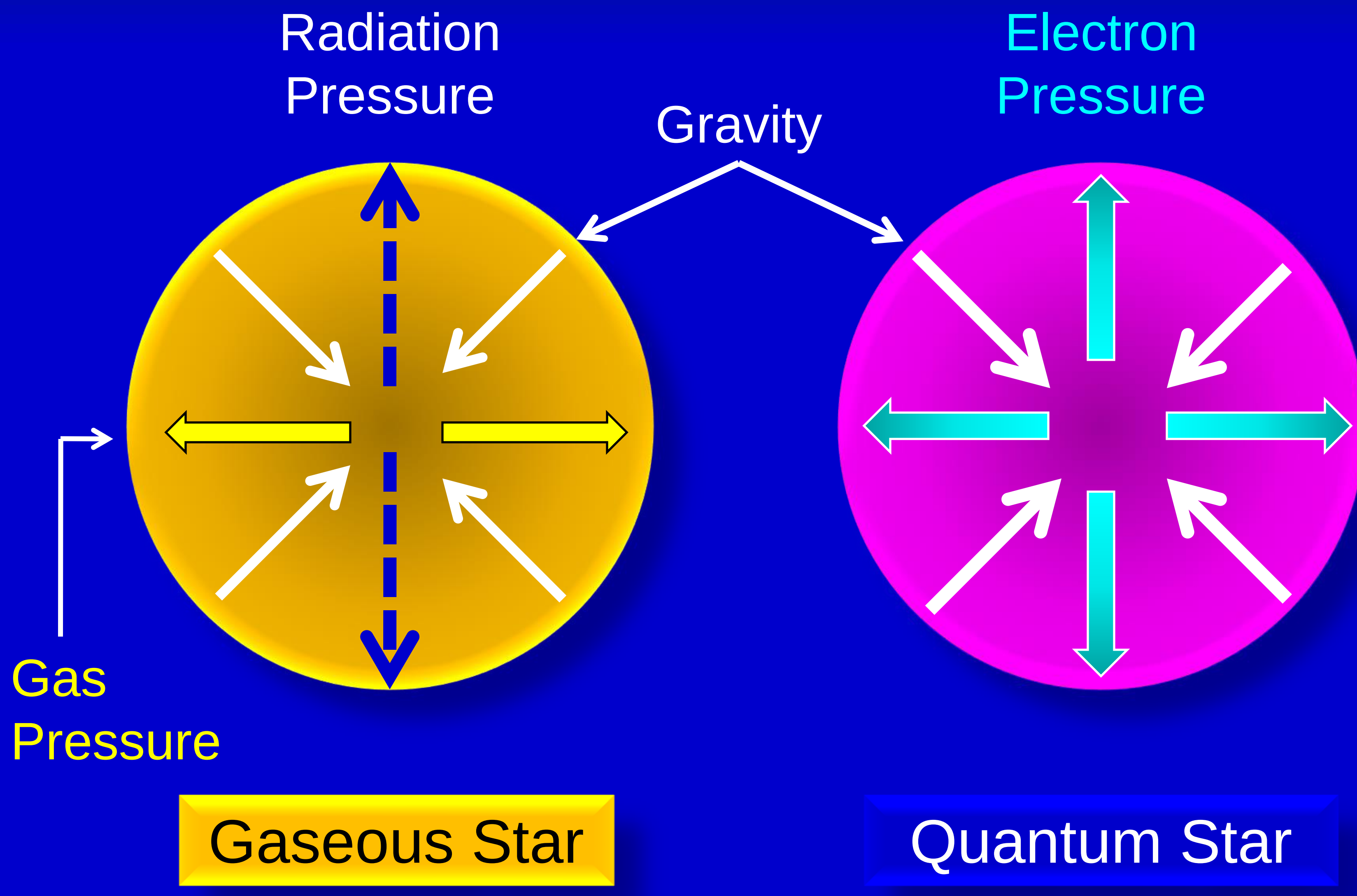
Hydrostatic Equilibrium

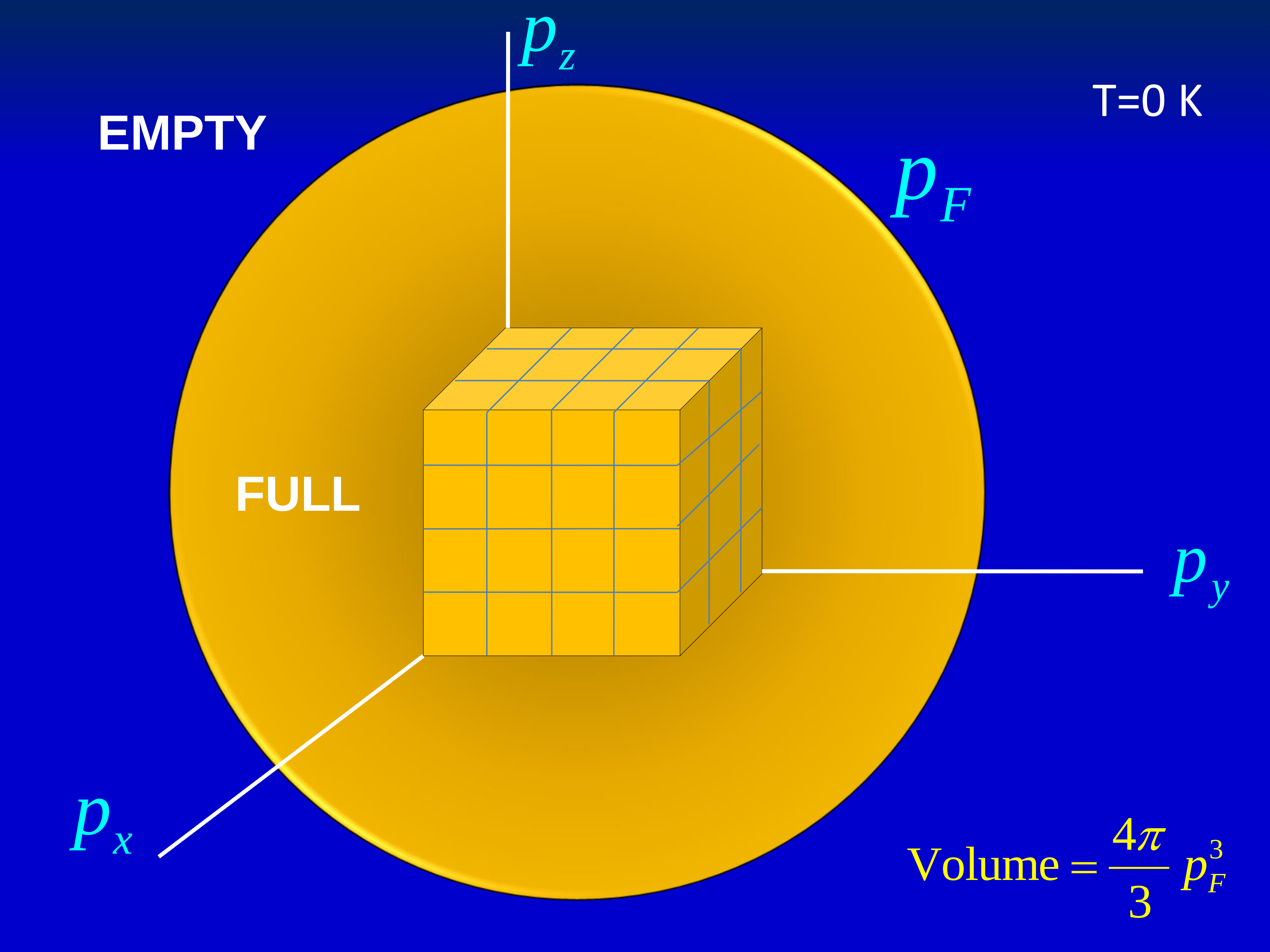


$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

$$P = p_{\text{gas}} + p_{\text{radiation}}$$

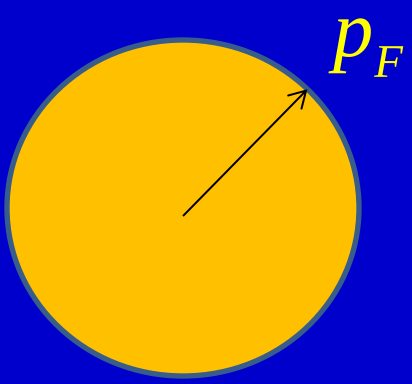
$$p_{\text{gas}} = nk_BT; \qquad p_{radiation} = \frac{1}{3}aT^4$$


Radiation
Pressure
Gravity
Electron
Pressure
Gas
Pressure
Gaseous Star
Quantum Star



p_z
T=0 K
EMPTY
p_F
FULL
p_y
p_x
$\text{Volume} = \frac{4\pi}{3} p_F^3$

Fermi Momentum



At T=0 K, all states of the FERMI SPHERE are occupied. All states outside are empty.

$$\text{Number of occupied cells } = \frac{\left(\frac{4\pi}{3} p_F^3\right)}{\left(\frac{h^3}{V}\right)}$$

Number of particles inside the Fermi sphere is given by

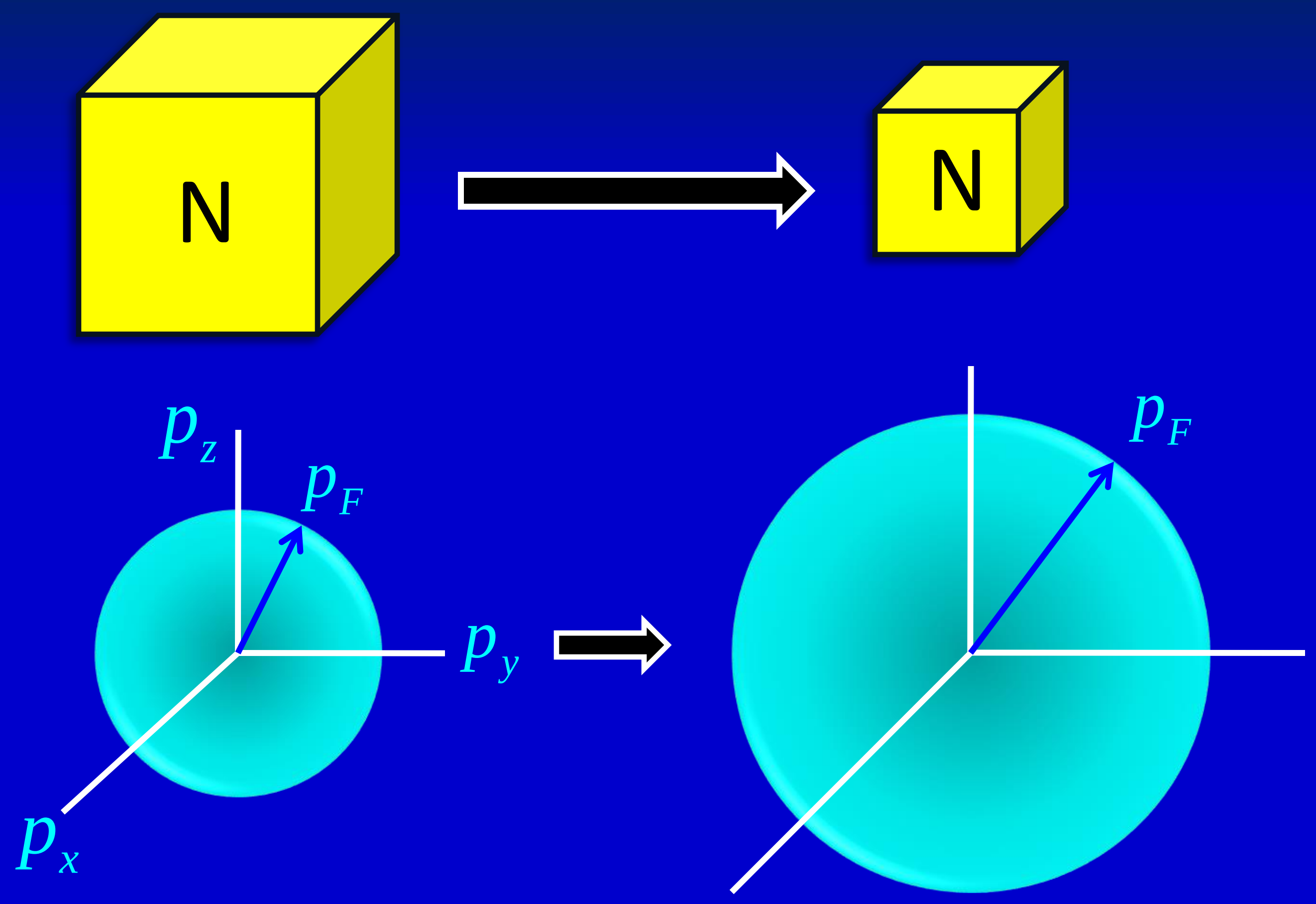
$$N = 2 \times \text{number of occupied cells} = \frac{2V}{h^3}\left(\frac{4\pi}{3} p_F^3\right)$$

$$p_F = \left(\frac{3}{8\pi}\right)^{1/3} h\left(\frac{N}{V}\right)^{1/3}$$

$$p_F \propto \left(\frac{N}{V}\right)^{1/3}$$

Notice that the ‘mass’ of the particle does NOT enter.

Therefore, this expression holds in relativity as well.



$$p_F \propto \left(\frac{N}{V}\right)^{1/3}$$

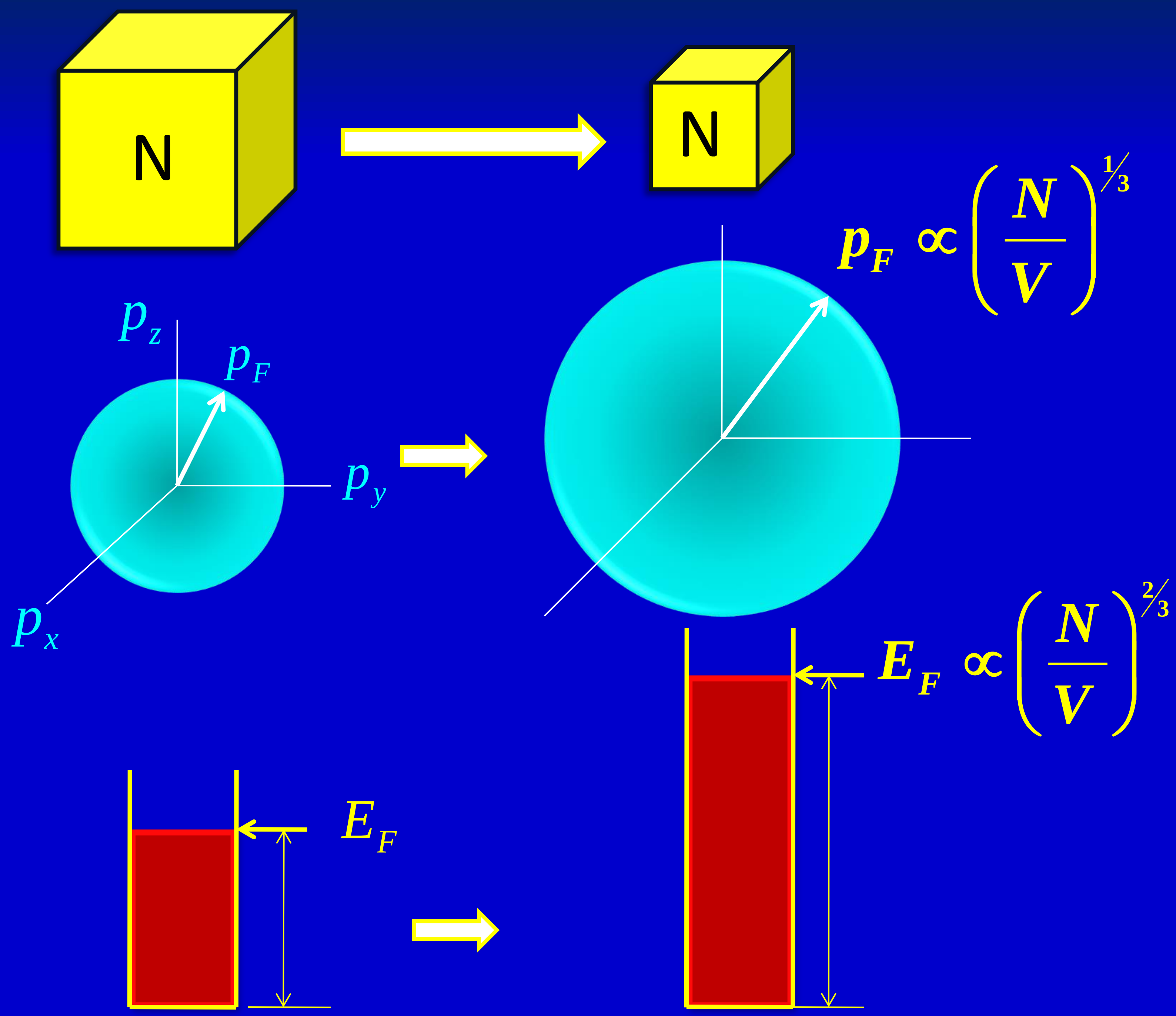
As the density increases, the maximum momentum will increase.

Fermi Energy

$$E_F = \frac{p_F^2}{2m} = \left(\frac{3}{8\pi}\right)^{2/3} \frac{h^2}{2m}\left(\frac{N}{V}\right)^{2/3}$$

$$E_F = \frac{p_F^2}{2m} \propto \left(\frac{N}{V}\right)^{2/3}$$

In non-relativistic mechanics, the mass of the particle comes in the denominator.


N
N
p_z
p_F
p_y
p_x
$p_F \propto \left(\frac{N}{V}\right)^{1/3}$
$E_F \propto \left(\frac{N}{V}\right)^{2/3}$
E_F

Pressure of an ideal classical gas

$$P = \frac{2}{3}\frac{E_{\text{int}}}{V}$$

$$E_{\text{int}} = N \times \text{average energy of particles} = N \times \frac{3}{2}kT$$

$$\text{Pressure} = \frac{2}{3}\frac{E_{\text{int}}}{V} = nkT$$

This is Boyle's Law. Pressure = 0 at absolute zero.

Pressure of a Fermi gas at T=0 K

$$P = \frac{2}{3}\frac{E_{\text{int}}}{V}$$

$$\mathbf{E_{int} = N \times \text{average energy of particles}}$$

$$= (\ldots) N \times E_F = (\ldots) N \times \left(\frac{N}{V}\right)^{\frac{2}{3}}$$

$$\text{Pressure} = \frac{2}{3}\frac{E_{\text{int}}}{V} \propto n^{\frac{5}{3}}$$

This is known as "degeneracy pressure"

Chandrasekhar's Theory of White Dwarfs

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

$$P_{\text{electrons}} = K_1 \rho^{\frac{5}{3}}$$

Chandrasekhar's Theory of White Dwarfs (1930)

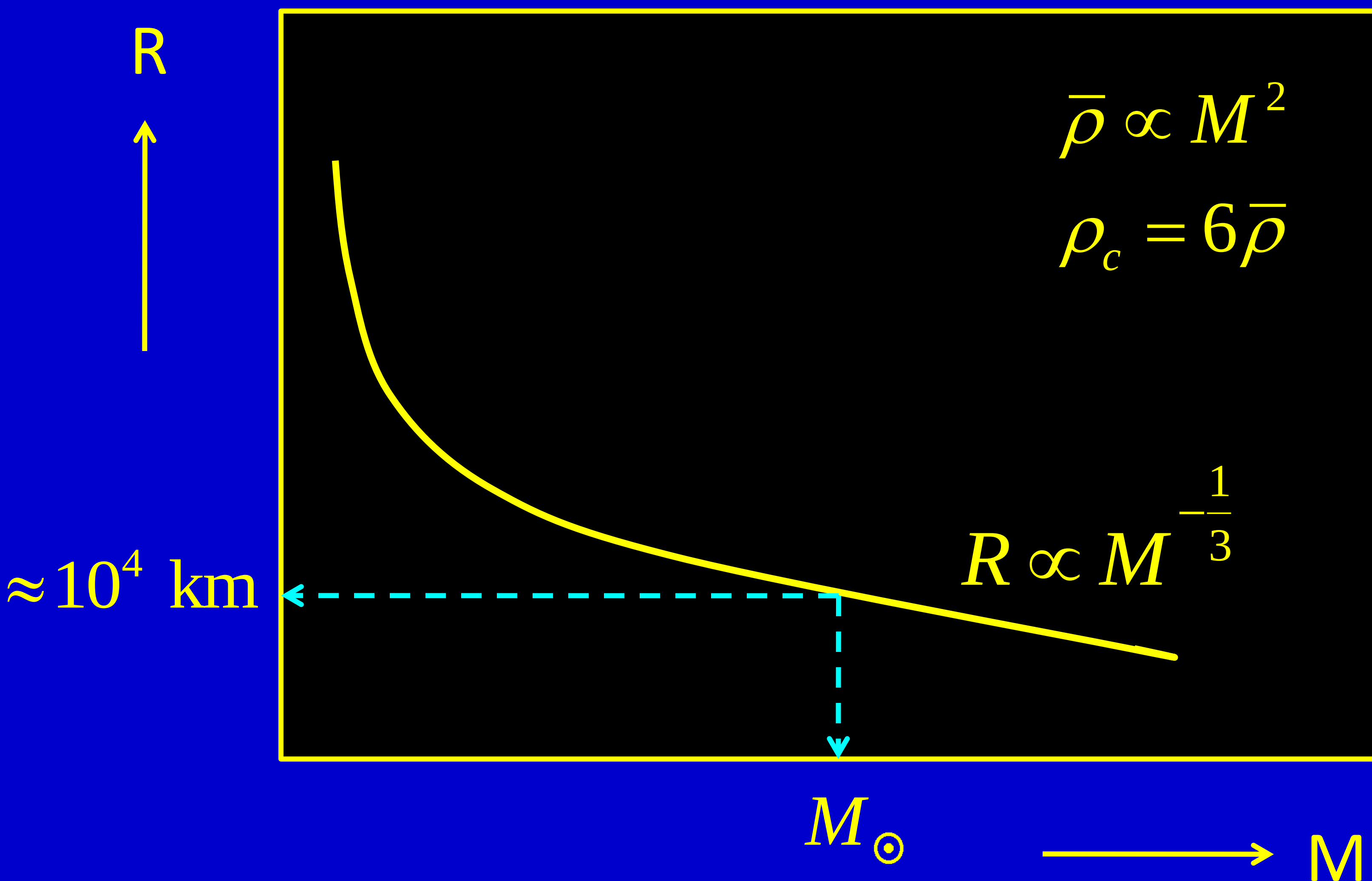
Mass – Radius relation:

$$R = \left(\frac{K_1}{0.424G} \right) \frac{1}{M^{\frac{1}{3}}}$$

$$R \propto M^{-\frac{1}{3}}$$

Mass – Radius Relation for White Dwarfs

S. Chandrasekhar, 1930



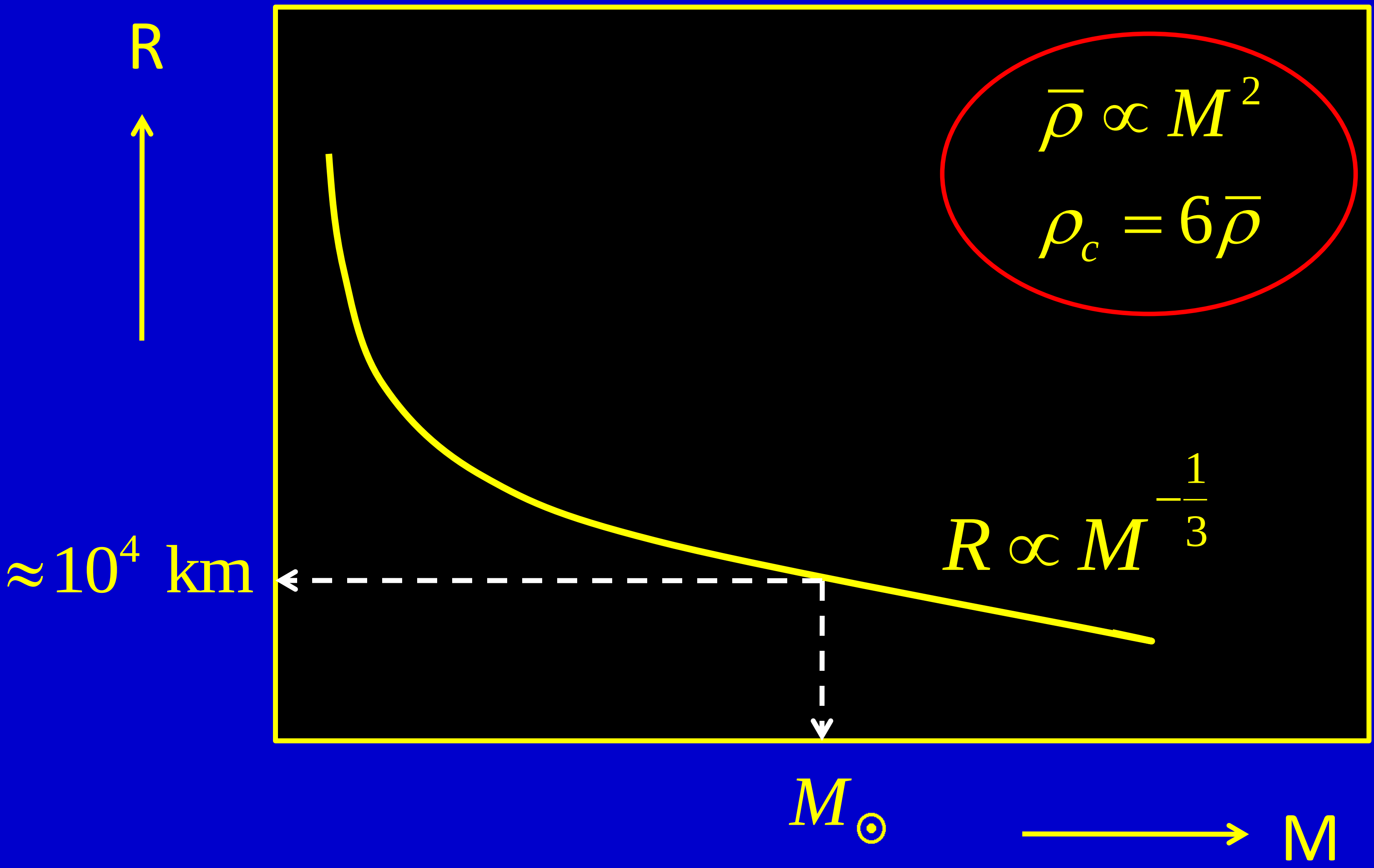
Stars of all mass
will find their ultimate peace as
Quantum Stars supported by the
pressure of electrons.

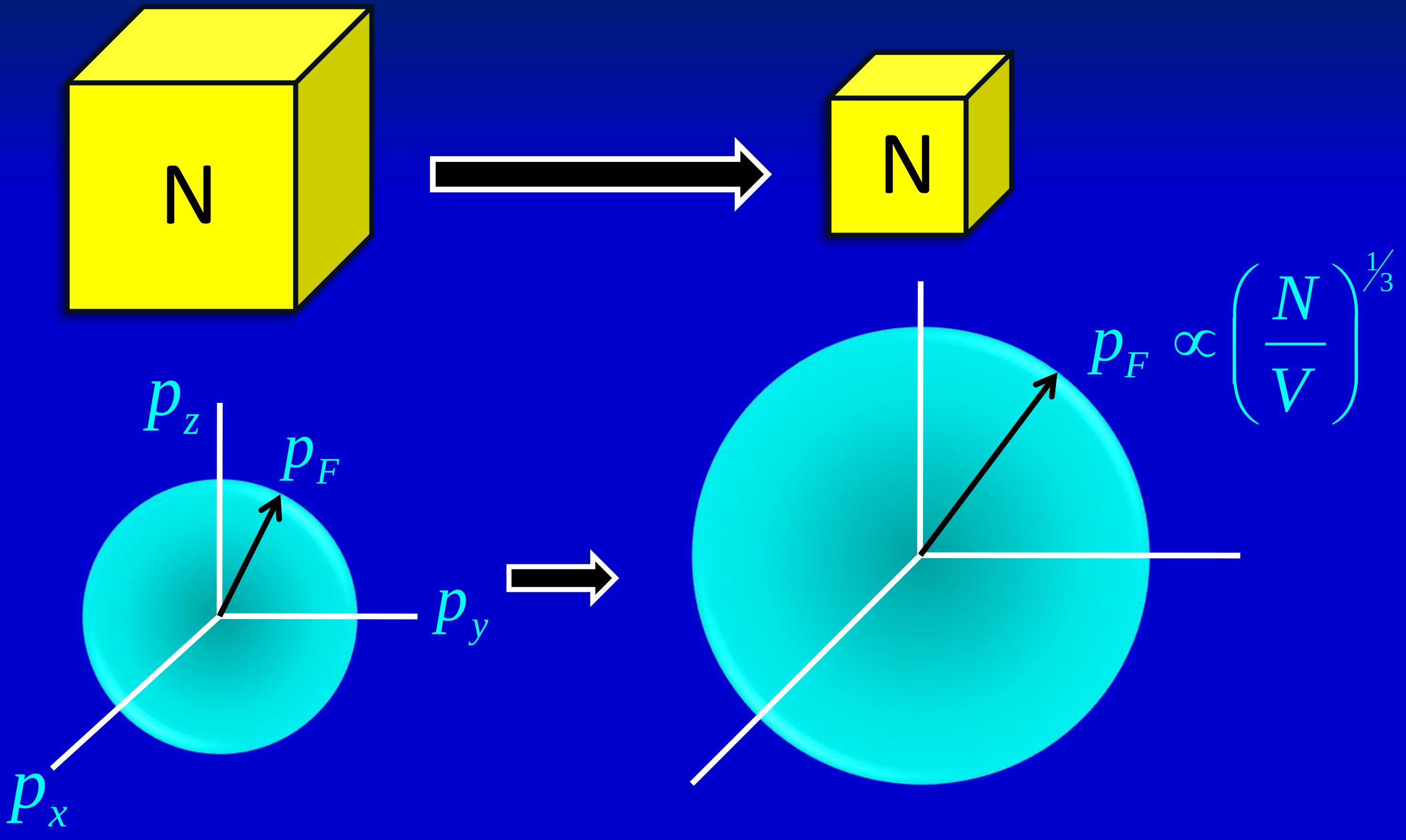
Diamonds in the sky!

Gravitational Collapse of massive white dwarfs

Mass – Radius Relation for White Dwarfs

S. Chandrasekhar, 1930

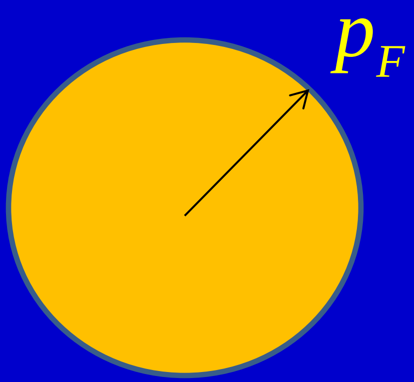




As we go to more massive WD, mean density will increase, the Fermi momentum will increase. At some stage **$p_F \sim mc$** and Special Relativistic effect of variation of mass with velocity has to be taken into account.

Relativistic Fermi Gas

Fermi Momentum



$$p_F = \left(\frac{3}{8\pi}\right)^{1/3} h \left(\frac{N}{V}\right)^{1/3}$$

$$p_F \propto \left(\frac{N}{V}\right)^{1/3}$$

Notice that the ‘mass’ of the particle does NOT enter.

Therefore, this formula holds good even for relativistic particles.

Fermi Energy

$$E_F = \frac{p_F^2}{2m} \propto \left(\frac{N}{V}\right)^{2/3}$$

This relation between energy and momentum is valid only in Newtonian mechanics.

In relativity, relation between energy and momentum is

$$E^2 = m_0^2 c^4 + p^2 c^2$$

For ultrarelativistic particles

$$E \sim pc$$

$$m = \frac{m_o}{\left(1 - v^2 / c^2\right)^{\frac{1}{2}}}$$

$$p = mv = \frac{m_o v}{\left(1 - v^2 / c^2\right)^{\frac{1}{2}}}$$

$$T_{kin} = (m - m_o)c^2 = m_o c^2 \left(\frac{1}{\sqrt{1 - v^2 / c^2}} - 1 \right)$$

$$E = mc^2 = \frac{m_o c^2}{\left(1 - v^2 / c^2\right)^{1/2}}$$

$$m = \frac{E}{c^2}, \qquad p = mv = \frac{Ev}{c^2}.$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

Fermi Energy: Ultra relativistic particles

$$E_F = p_F c = \left(\frac{3}{8\pi}\right)^{1/3} h \left(\frac{N}{V}\right)^{1/3} c \propto \left(\frac{N}{V}\right)^{\frac{1}{3}}$$

In the extreme relativistic case, the mass of the particle does not enter.

This is as it should be. The kinetic energy is >> the rest mass energy.

Degeneracy pressure of an ultra relativistic gas

$$E \approx pc$$

$$E_{Total} = \int_0^{\infty} Ef(E)g(E)dE$$

$$E_{Total} = \int_0^{E_F} Eg(E)dE$$

$$g(p)dp = \frac{8\pi V}{h^3} p^2 dp$$

$$g(E)dE = \frac{8\pi V}{c^3 h^3} E^2 dE$$

$$E_{Total} = \frac{2\pi V}{c^3 h^3} E_F^4$$

$$E_F = p_F c = \left(\frac{3}{8\pi}\right)^{1/3} hc \left(\frac{N}{V}\right)^{1/3}$$

$$E_{Total} = V \frac{3}{4} \left(\frac{3}{8\pi}\right)^{\frac{1}{3}} hc \left(\frac{N}{V}\right)^{\frac{4}{3}}$$

$$P = \frac{1}{3} \frac{E_{Total}}{V}$$

$$P_{rel} = \frac{1}{3} \frac{E_{Total}}{V} = \frac{1}{8} \left(\frac{3}{\pi}\right)^{\frac{1}{3}} hc\, n^{\frac{4}{3}} = (\ldots) n^{\frac{4}{3}}$$

$$P_{\deg} = \frac{2}{3}\frac{E_{Total}}{V} = \frac{1}{5}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}}\frac{h^2}{m}\left(\frac{N}{V}\right)^{\frac{5}{3}} \propto \left(\frac{N}{V}\right)^{\frac{5}{3}}$$

$$P_{rel} = \frac{1}{3}\frac{E_{Total}}{V} = \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} hc \left(\frac{N}{V}\right)^{\frac{4}{3}} \propto \left(\frac{N}{V}\right)^{\frac{4}{3}}$$

$$P_{\text{deg}} \propto \left(\frac{N}{V}\right)^{\frac{5}{3}}$$

$$P_{rel} \propto \left(\frac{N}{V}\right)^{\frac{4}{3}}$$

Chandrasekhar's Theory of White Dwarfs

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

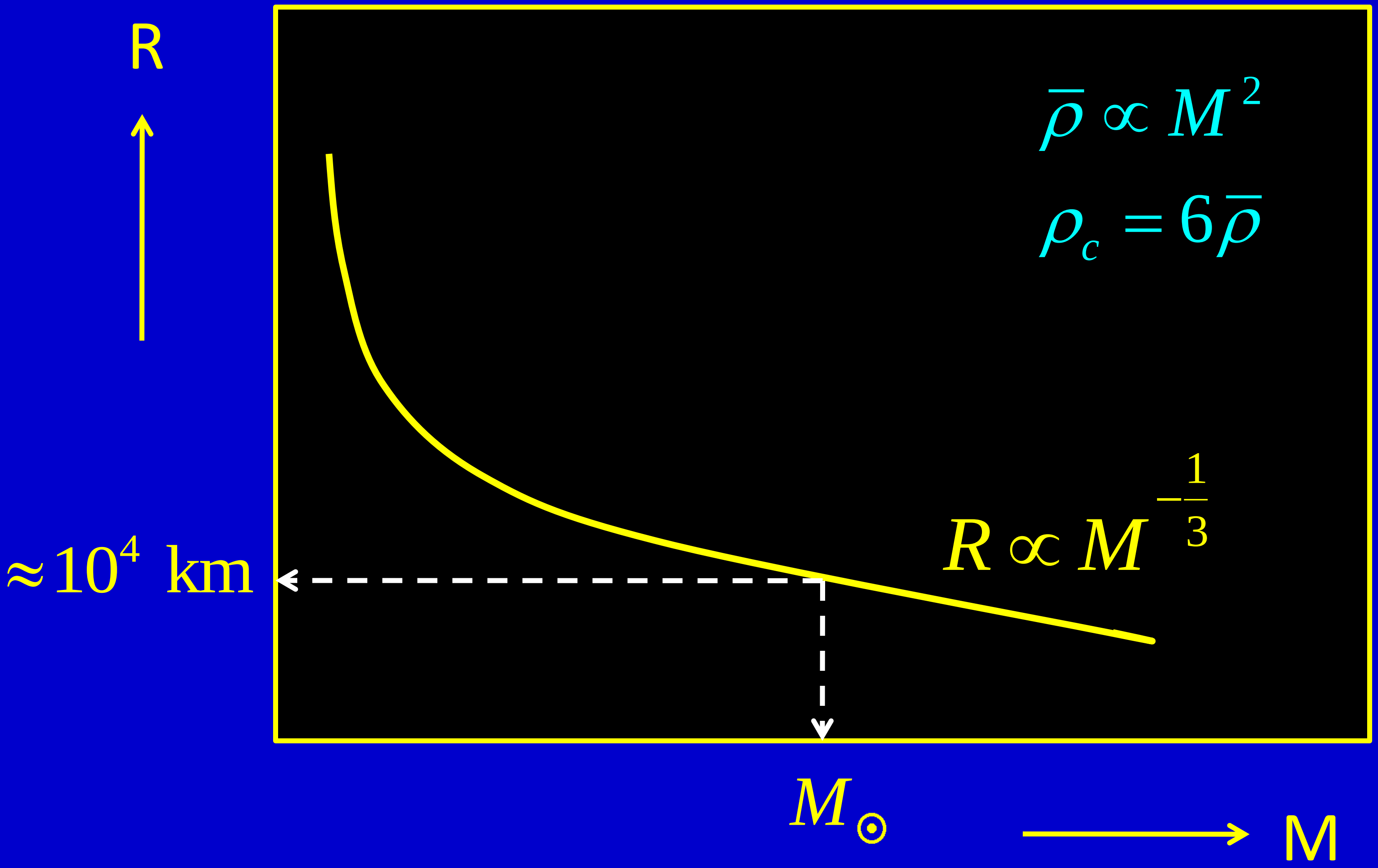
$$E = \frac{p^2}{2m}$$

$$P_{\text{electrons}} = K_1 \rho^{\frac{5}{3}}$$

Quantum pressure of a NONRELATIVISTIC electron gas.

Mass – Radius Relation for White Dwarfs

S. Chandrasekhar, 1930



Relativistic White Dwarfs

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

$$E = pc$$

$$P_{\text{relativistic}} = K_2 \rho^{\frac{4}{3}}$$

Quantum pressure of a RELATIVISTIC electron gas.

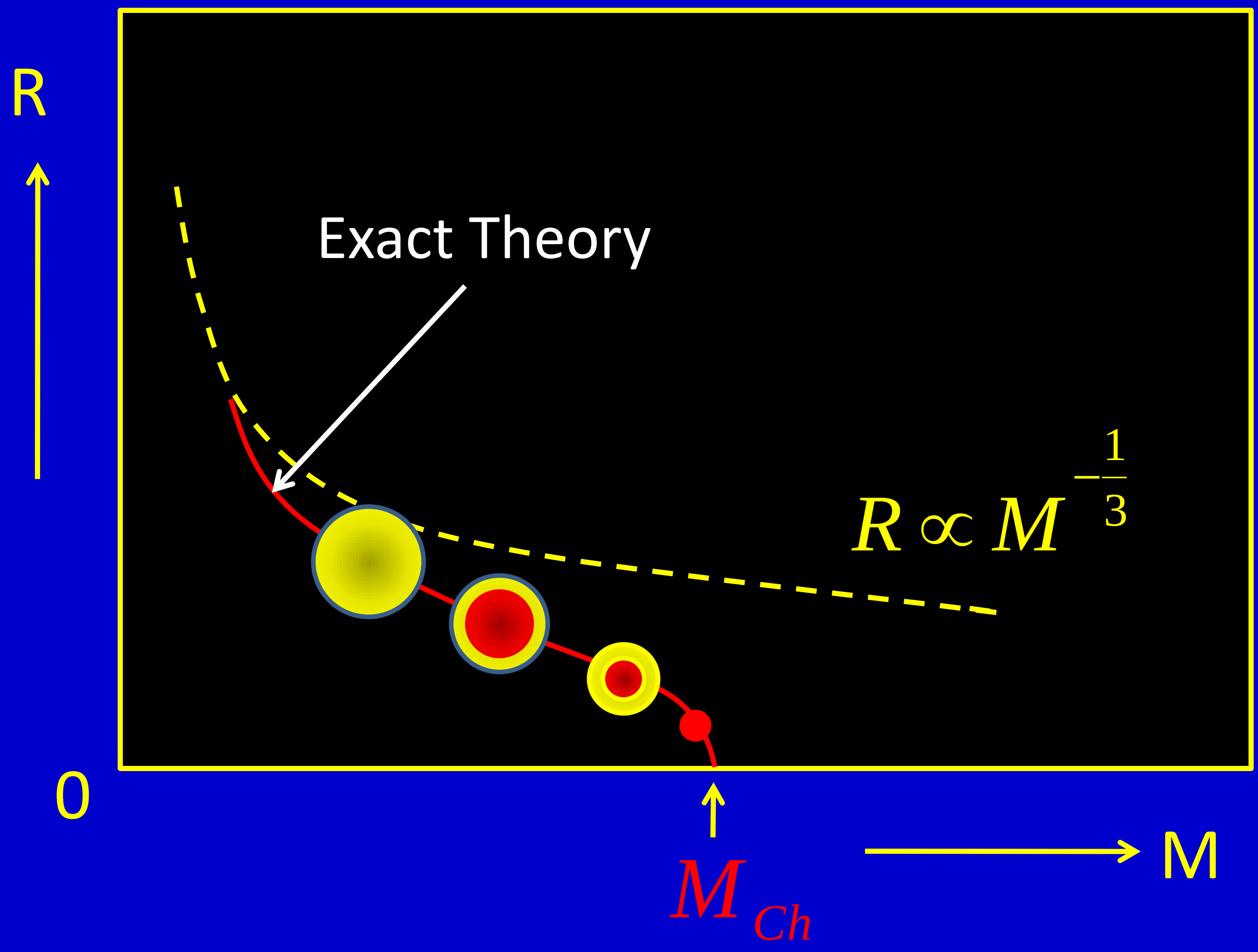
A fully relativistic WD has no radius! But it has a unique mass!

$$M_{Ch} = 0.197\left[\left(\frac{hc}{G}\right)^{\frac{3}{2}}\frac{1}{m_p^2}\right]\times\frac{1}{\mu_e^2} = 1.4M_{\odot}$$

μ_e is the "Mean molecular weight". ($\mu_e m_p$) is the mass per electron. Except for Hydrogen, μ_e is approximately equal to 2.

Mass – Radius Relation for White Dwarfs

S. Chandrasekhar, 1934



Chandrasekhar Limit

$$M_{Ch} = 0.197\left[\left(\frac{hc}{G}\right)^{\frac{3}{2}}\frac{1}{m_p^2}\right]\times\frac{1}{\mu_e^2} = 1.4M_{\odot}$$

Stars more massive than 1.4 solar mass cannot be supported by the quantum pressure of the electrons.

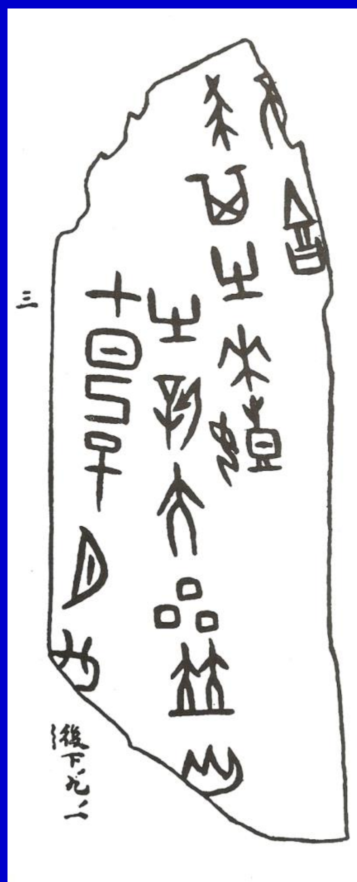
They will COLLAPSE!

Chandrasekhar Limit

- This extraordinary discovery revolutionized our view of the heavens.
- This signalled Gravitational collapse of massive stars.
- Soon, this led to the notion of Neutron stars, Supernova explosions and Black Holes.
- This great discovery marked the beginning of the era of High Energy Astrophysics – of Neutron stars, Supernovae and Black Holes.

Supernovae
and
Neutron Stars

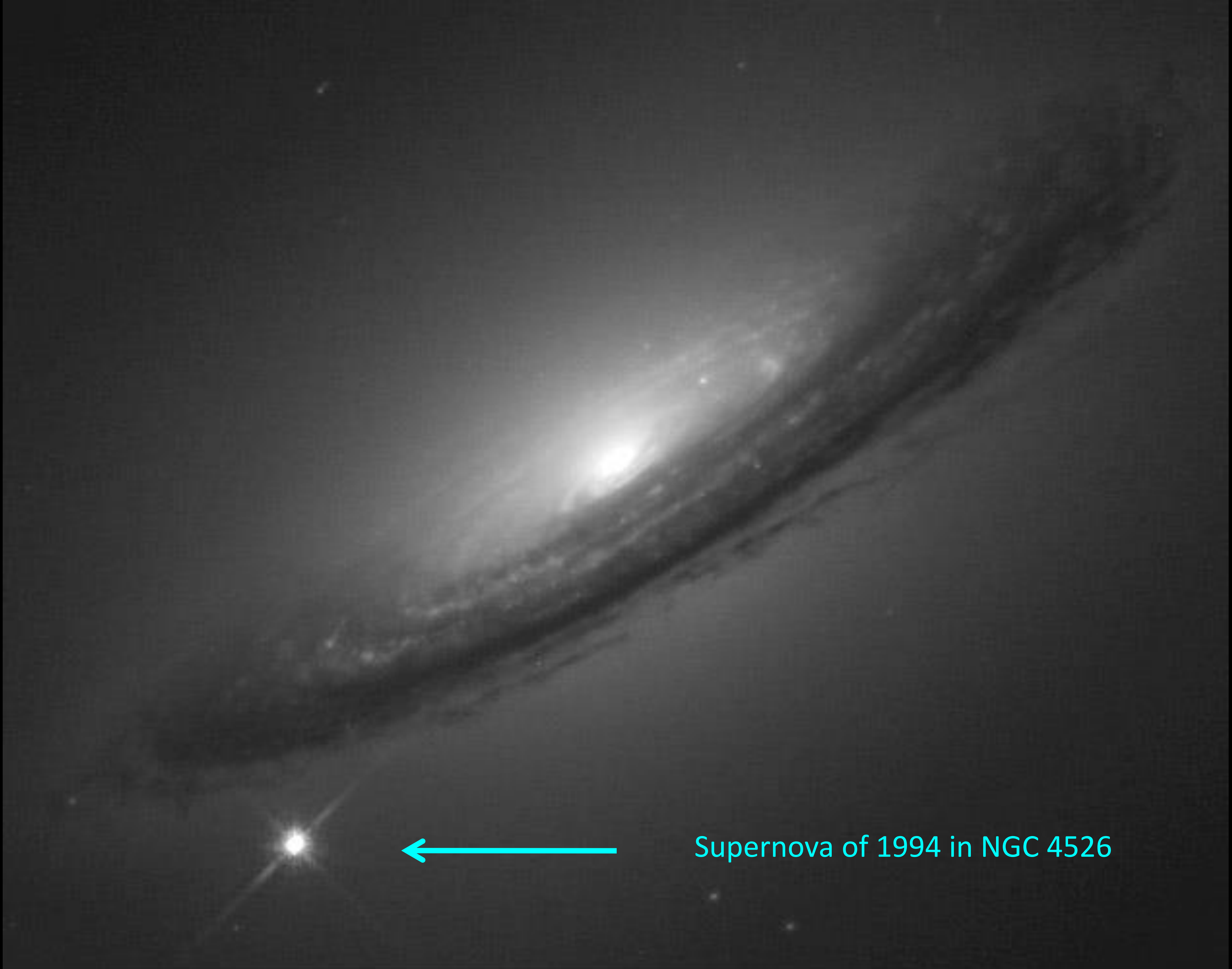
Guest Stars



‘On the 7th day of the month a great new star appeared in the company of Antares.’

Chinese oracle bone. 1300 BC

A Guest Star appeared in the Andromeda Nebula in 1885

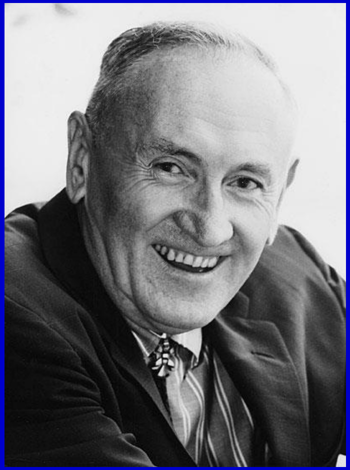

Supernova of 1994 in NGC 4526

JANUARY 15, 1934 PHYSICAL REVIEW VOLUME 45

Supernovae and Cosmic Rays

by

W. Baade and F. Zwicky

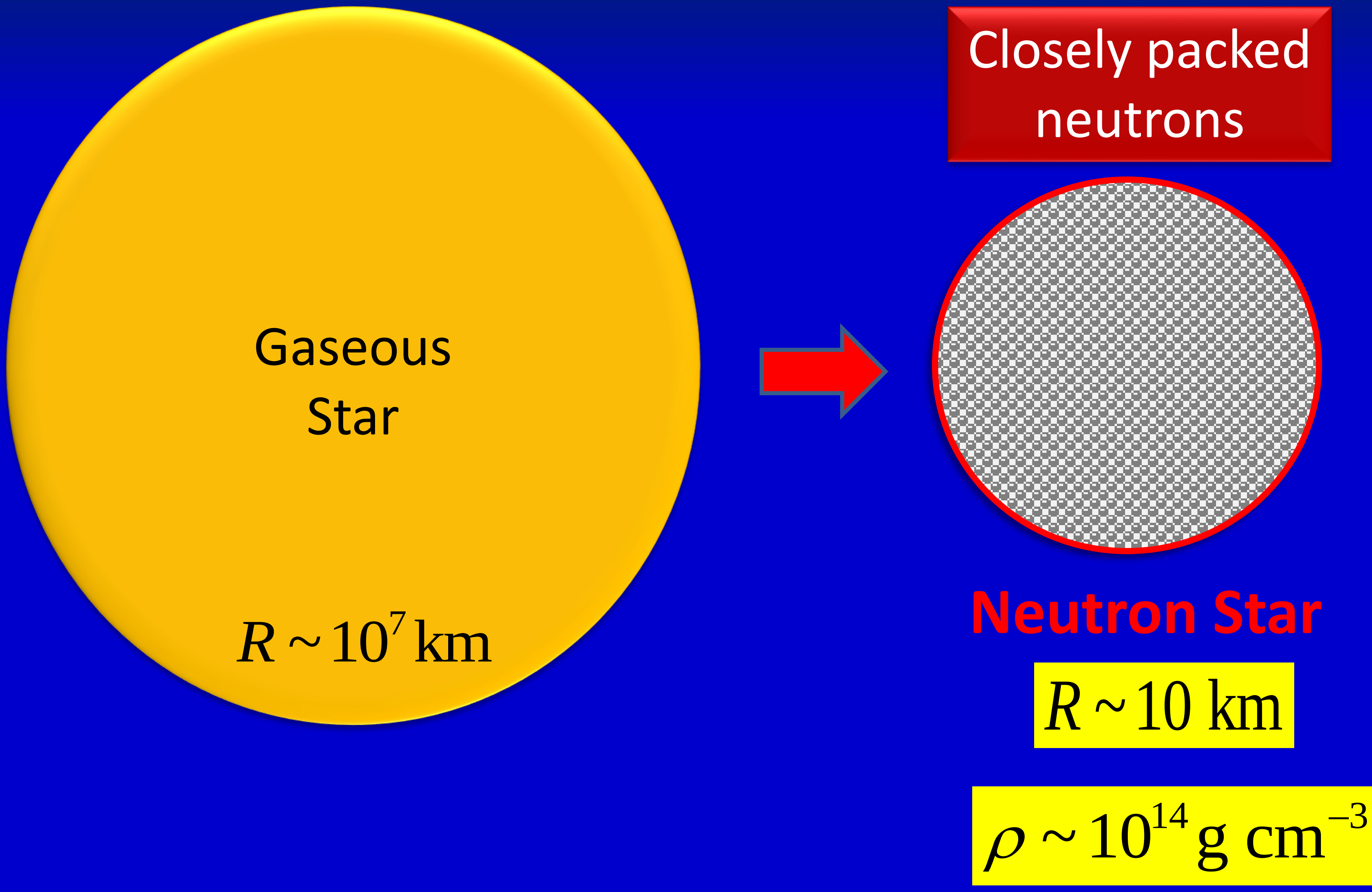


$E_T \geq 10^5 L_T = 3.78 \times 10^{53}$ ergs
E_T / c^2

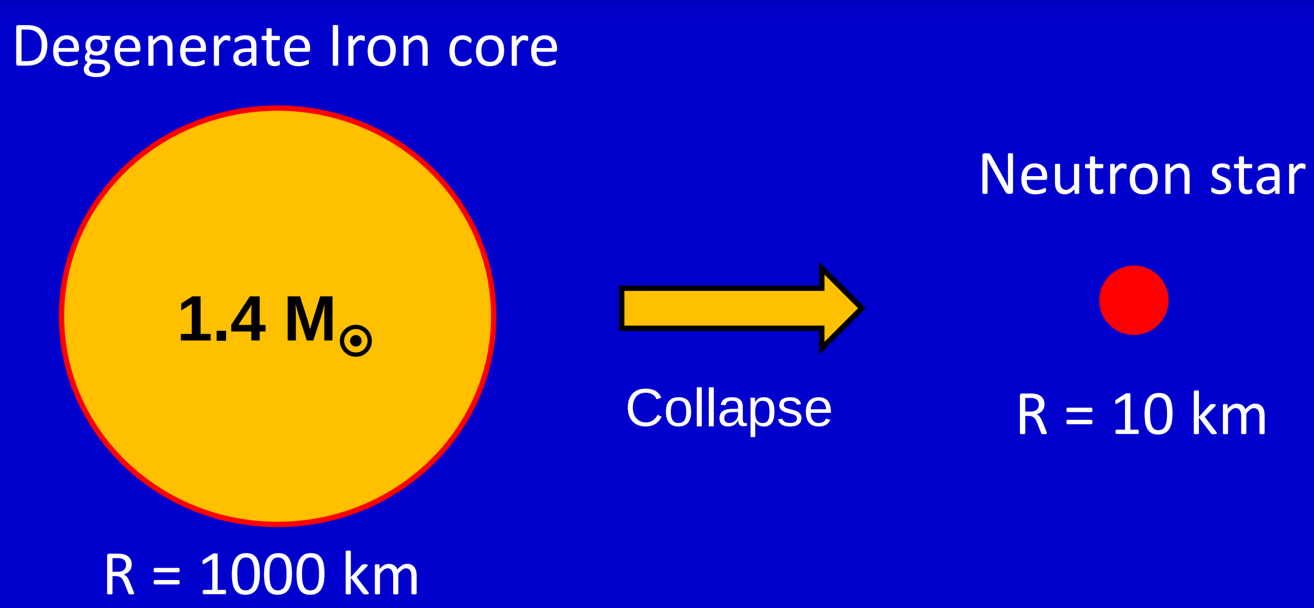
"If supernova initially are quite ordinary stars of mass M<10^{34} g, the total energy released E_T / c^2 is of the same order as M itself".

"In the *supernova process mass in bulk is annihilated*. In addition, the hypothesis suggests itself that *cosmic rays are produced by supernovae"*.

"With all reserve we advance the view that supernovae represent the transitions from ordinary stars into *neutron stars*, which in their final stages consist of extremely closely packed neutrons."


Closely packed neutrons
Gaseous Star
$R \sim 10^7$ km
Neutron Star
$R \sim 10$ km
$\rho \sim 10^{14}$ g cm^{-3}

Supernova Explosion



$$\text{Energy released} = \left\{ \left(-\frac{GM^2}{10^3\,\text{km}} \right) - \left(-\frac{GM^2}{10\text{km}} \right) \right\} \sim \frac{GM^2}{10\text{km}} \sim 10^{53}\,\text{erg}$$

Between 1932 and 1937:

- Chadwick discovered the ‘Neutron’ in 1932.
- In 1934, Enrico Fermi discovered the Theory of Beta Decay.

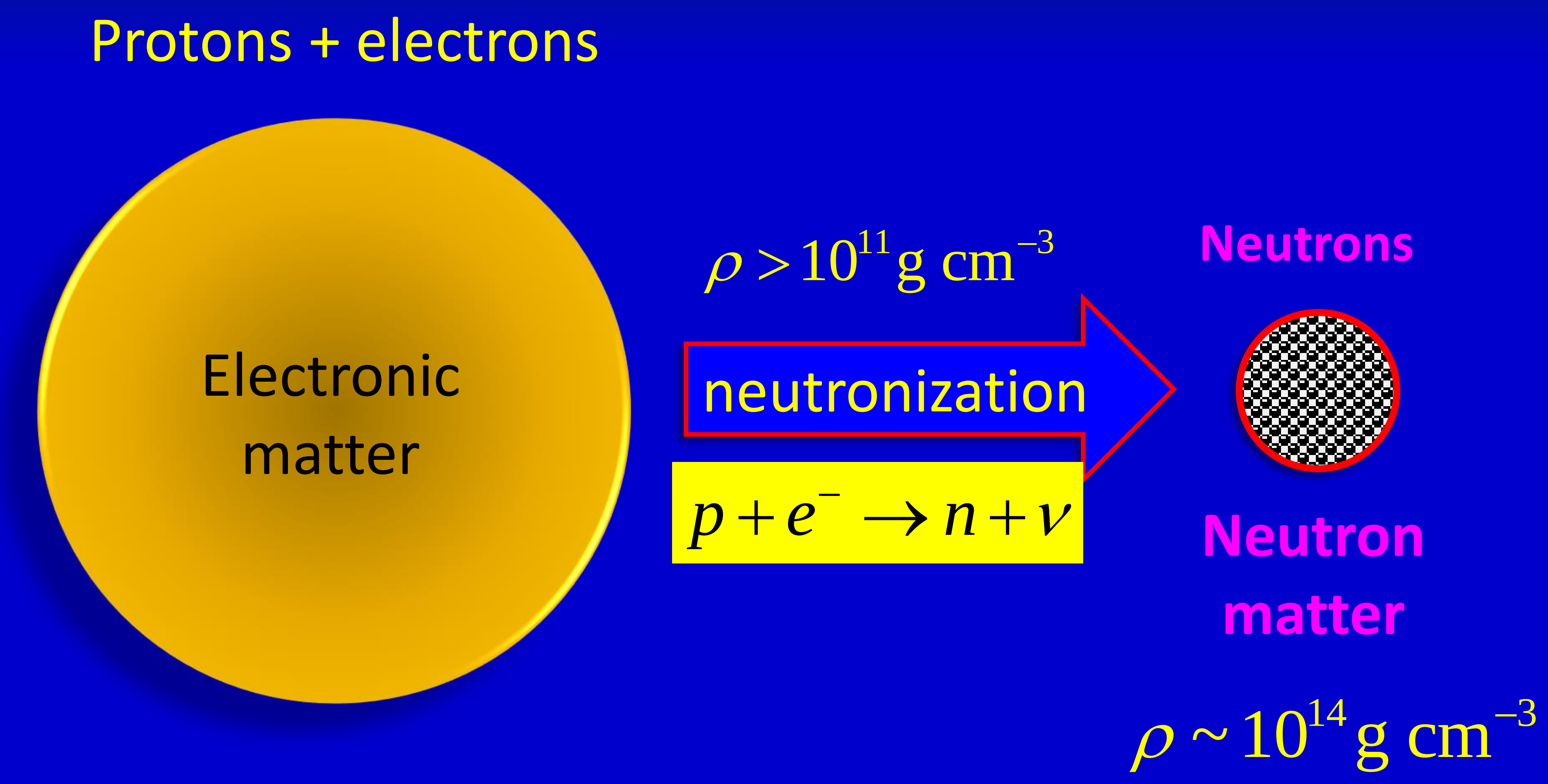
$$n \rightarrow p + e^{-} + neutrino$$

- Today, we know that this should be an antineutrino.

Lev Landau

Landau invents neutron stars in 1938

- The idea of 'neutron matter' – the state where all the nuclei and electrons have combined to form neutrons – was suggested first by F. Hund in 1936.
- The reaction $p + e^{-} \rightarrow n + \nu$ is strongly endothermic.
- To transform one gram of matter to neutrons will cost 7×10^{18} erg.
- Landau built on Hund's idea and argued that neutron matter could be stable if the body was sufficiently massive. The gain in the gravitational energy can compensate for the loss in internal energy.



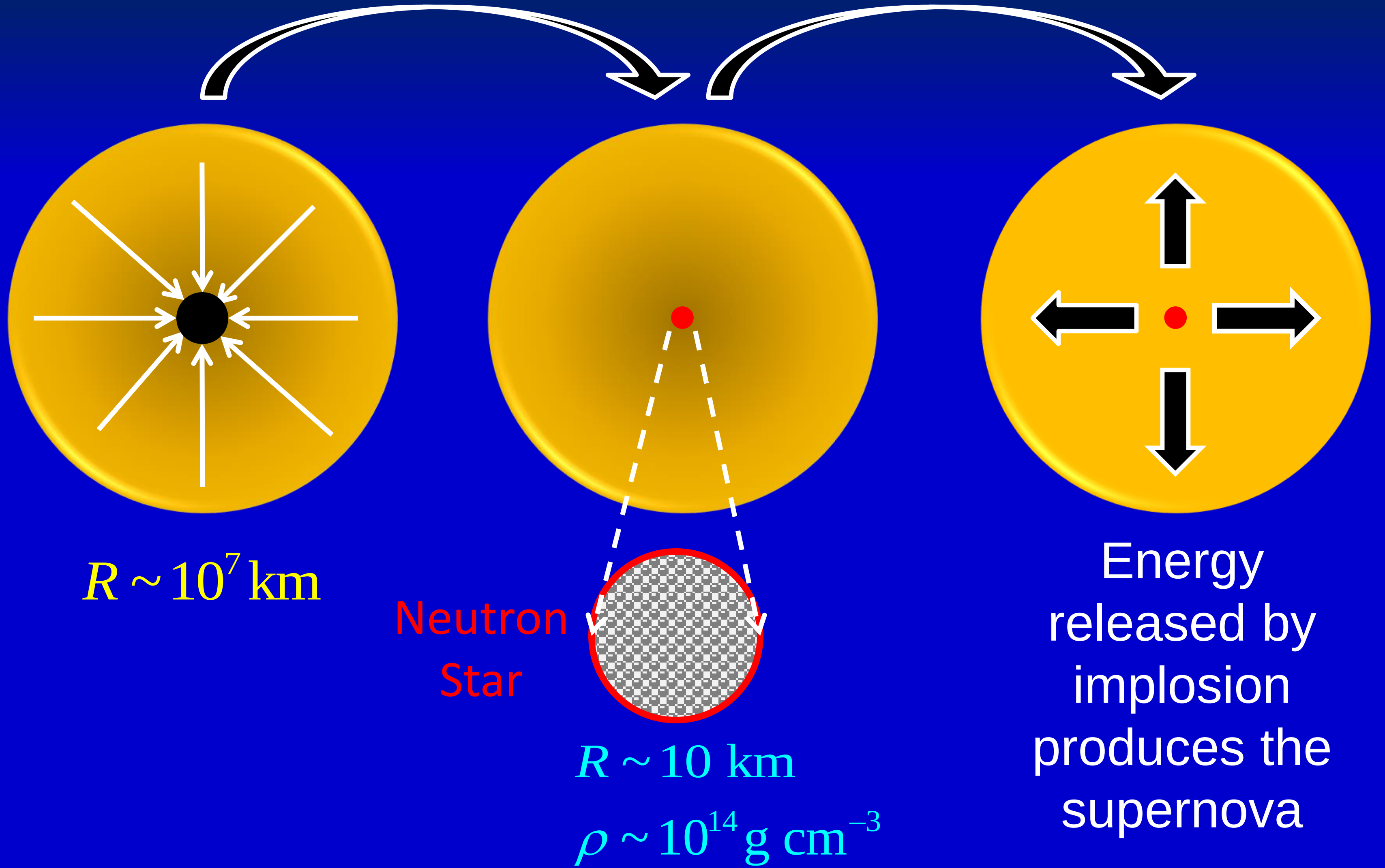
Neutronization of matter at high density

Lev Landau

- Landau invented neutron stars as a source of stellar energy.
- He wanted all stars to have a tiny “neutron core”.
- His idea was that when matter “rained” on the neutron core, gravitational energy will be released.
- According to him, this was the source of the energy radiated by the stars!
- This paper appeared in Nature just one year before Bethe worked out the details of the transmutation of hydrogen into helium in the Sun.

Dynamical instability

- According to the modern picture, a white dwarf forms at the centre.
- Over time, the white dwarf's mass will increase.
- As the mass increases, the central density will increase. When the central density reaches 10^{11} g cm^{-3} , neutronization will begin.
- As electrons disappear, the degeneracy pressure will decrease because this depends on the number density of electrons.
- This will lead to a contraction of the core, resulting in an acceleration of neutronization of matter.
- This is a "positive feedback" resulting in a dynamical collapse.



$$\text{Energy released} = \left\{ \left(-\frac{GM^2}{10^7\,\text{km}} \right) - \left(-\frac{GM^2}{10\text{km}} \right) \right\} \sim \frac{GM^2}{10\text{km}} \sim 0.1Mc^2$$

J. Robert Oppenheimer

THE MAXIMUM MASS OF NEUTRON STARS

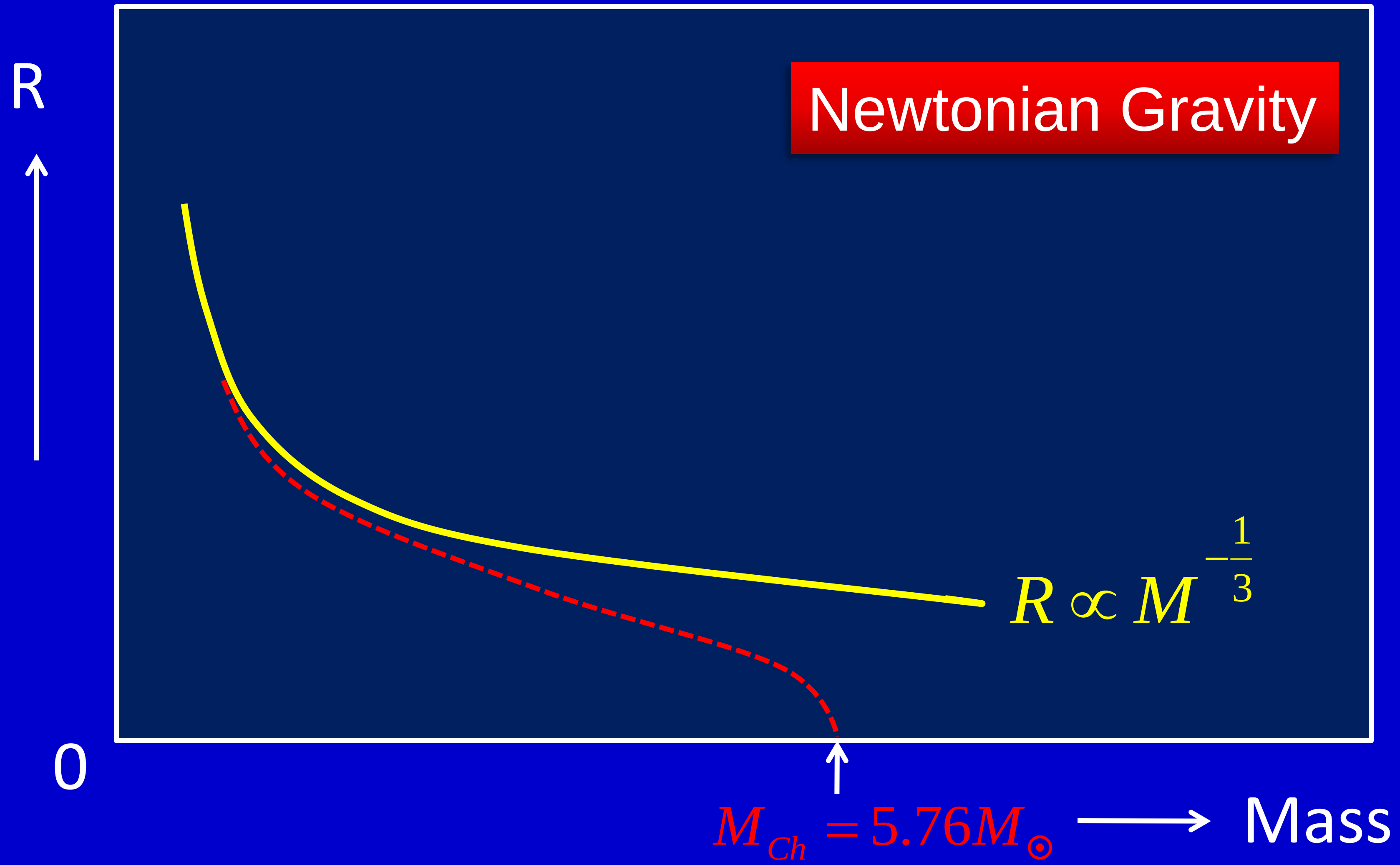
Oppenheimer and Volkoff, 1938

- Chandrasekhar limiting mass for White Dwarfs is 1.4 solar mass.
- To find the maximum mass of Neutron Stars, they followed the same procedure as Chandra, but with two differences:

1. They assumed the neutron gas to be an ideal Fermi gas, with Chandrasekhar's equation of state.
2. They used Einstein's gravity, instead of Newtonian gravity.

The Maximum Mass for Neutron Stars

Oppenheimer & Volkoff, 1938



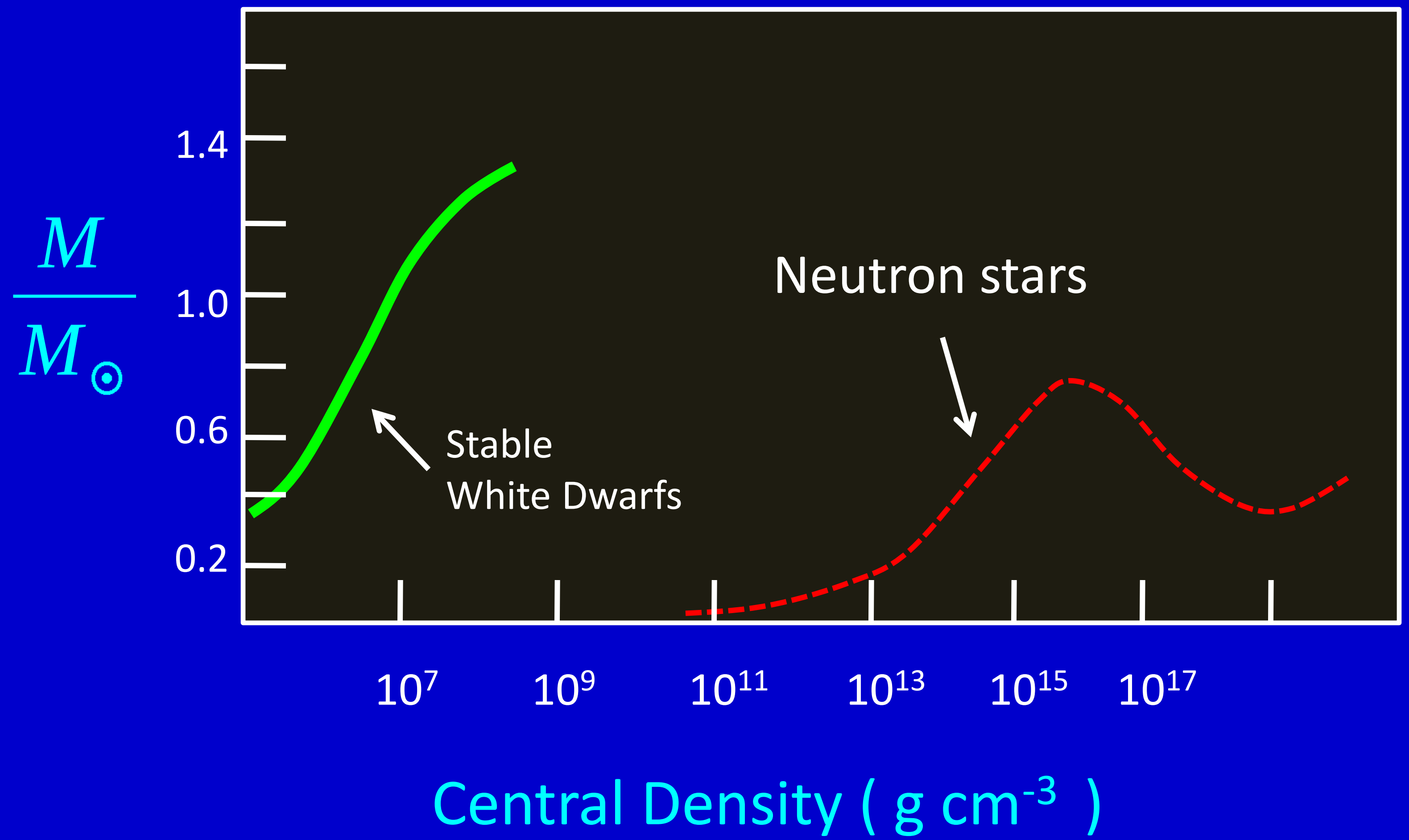
Tolman, Oppenheimer, Volkoff equation

$$\frac{dP}{dr} = -\frac{G\left[m(r) + 4\pi r^3 P(r)/c^2\right]\left[\rho(r) + P(r)/c^2\right]}{r^2\left[1 - \frac{2Gm(r)}{rc^2}\right]}$$

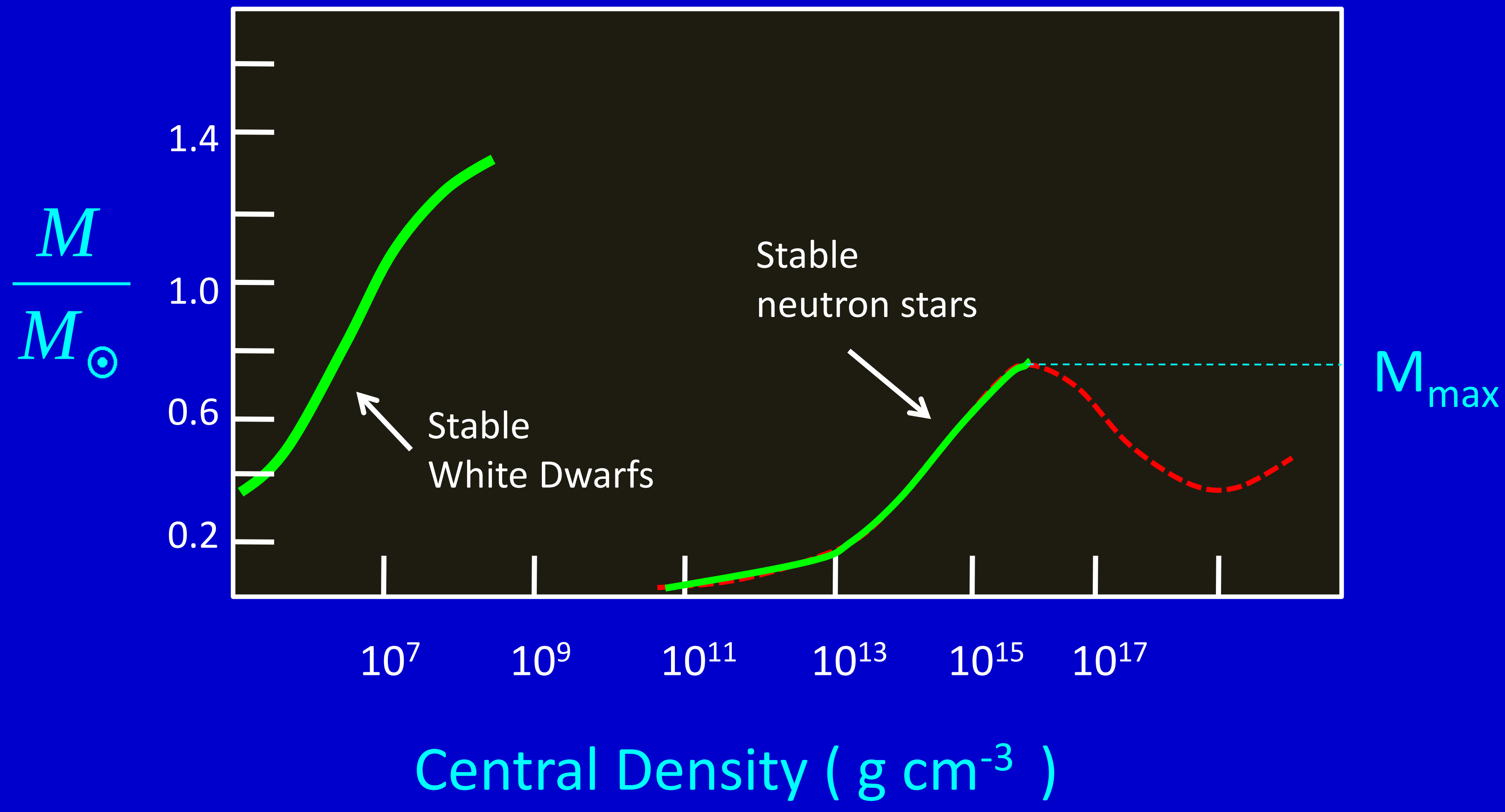
This is the modified Equation of Hydrostatic Equilibrium in Einstein's theory of gravity. This reduces to our old eqn. when c $\rightarrow$ ∞. Oppenheimer and Volkoff used this equation for determining the maximum mass of neutron stars.

For the 'pressure', they assumed that neutrons formed an ideal Fermi gas, and that the pressure is given by the formula derived earlier by Chandrasekhar.

OV: Oppenheimer – Volkoff Equation of State



OV: Oppenheimer – Volkoff Equation of State



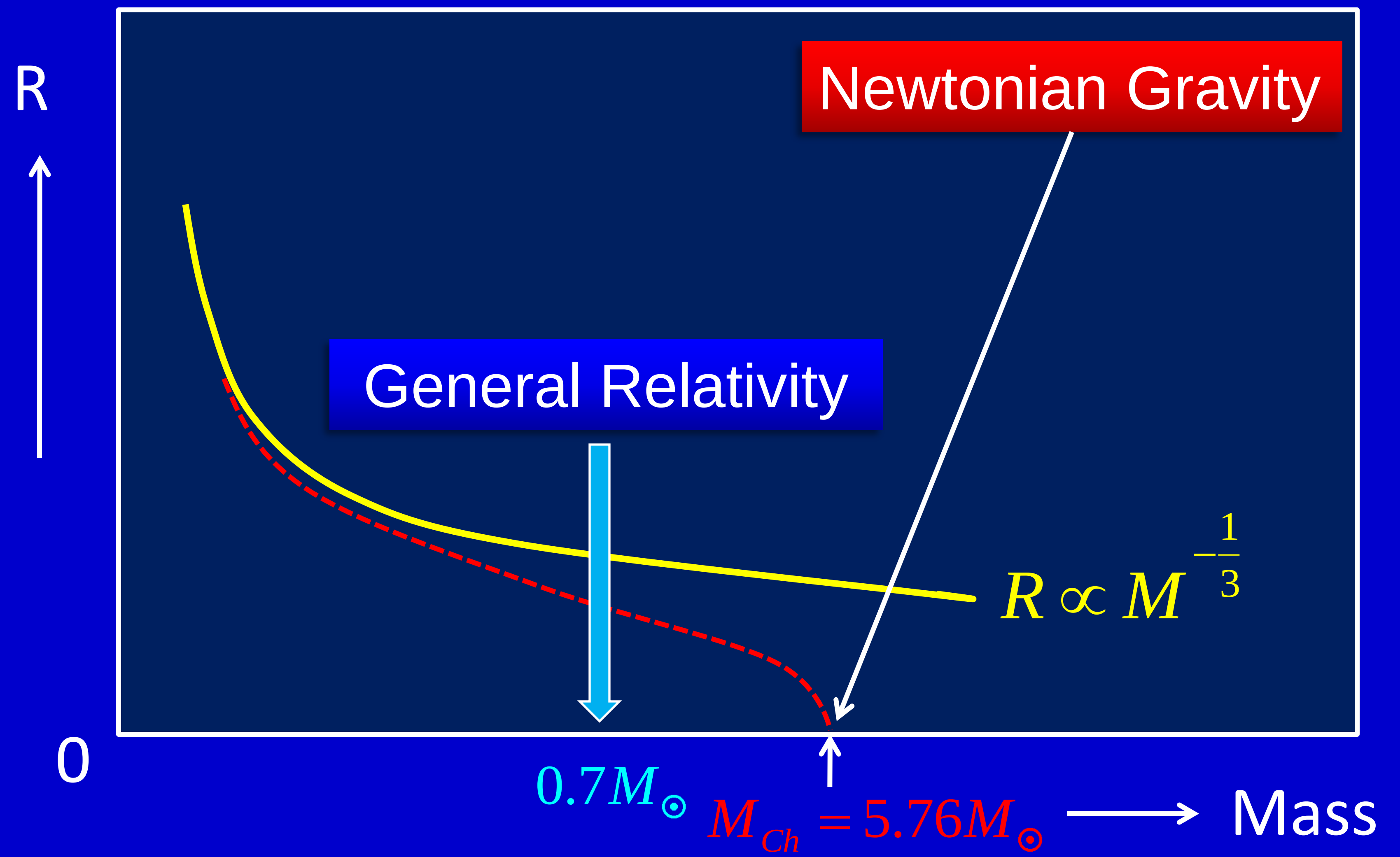
Oppenheimer and Volkoff, 1938

- The maximum mass of neutron stars is $0.7M_{\odot}$.
- The radius of the neutron star of this mass would be about 10 km, and
- The central density of a neutron star of maximum mass would be ~ 5×10^{15} g cm^{-3}.

Oppenheimer conjectured that 'nuclear repulsion' would increase the maximum mass to a "few solar mass"

Limiting Mass for Neutron Stars

Oppenheimer & Volkoff, 1938

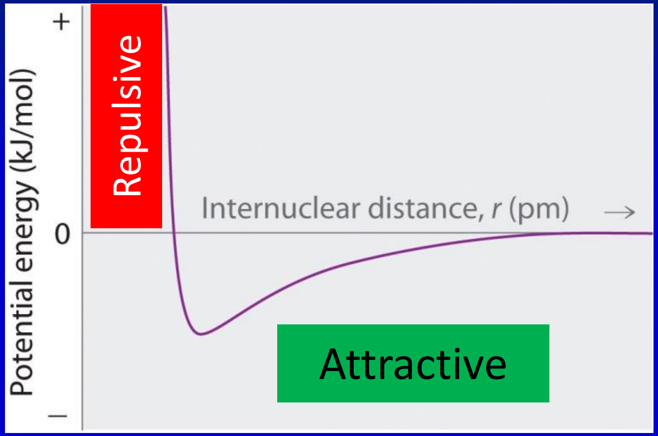


Oppenheimer and Volkoff, 1938

- The O-V maximum mass is much smaller than the "Chandrasekhar limiting mass" of 5.73 $M_{\odot}$.

- The radius of the neutron star of this mass would be about 10 km, and not zero!

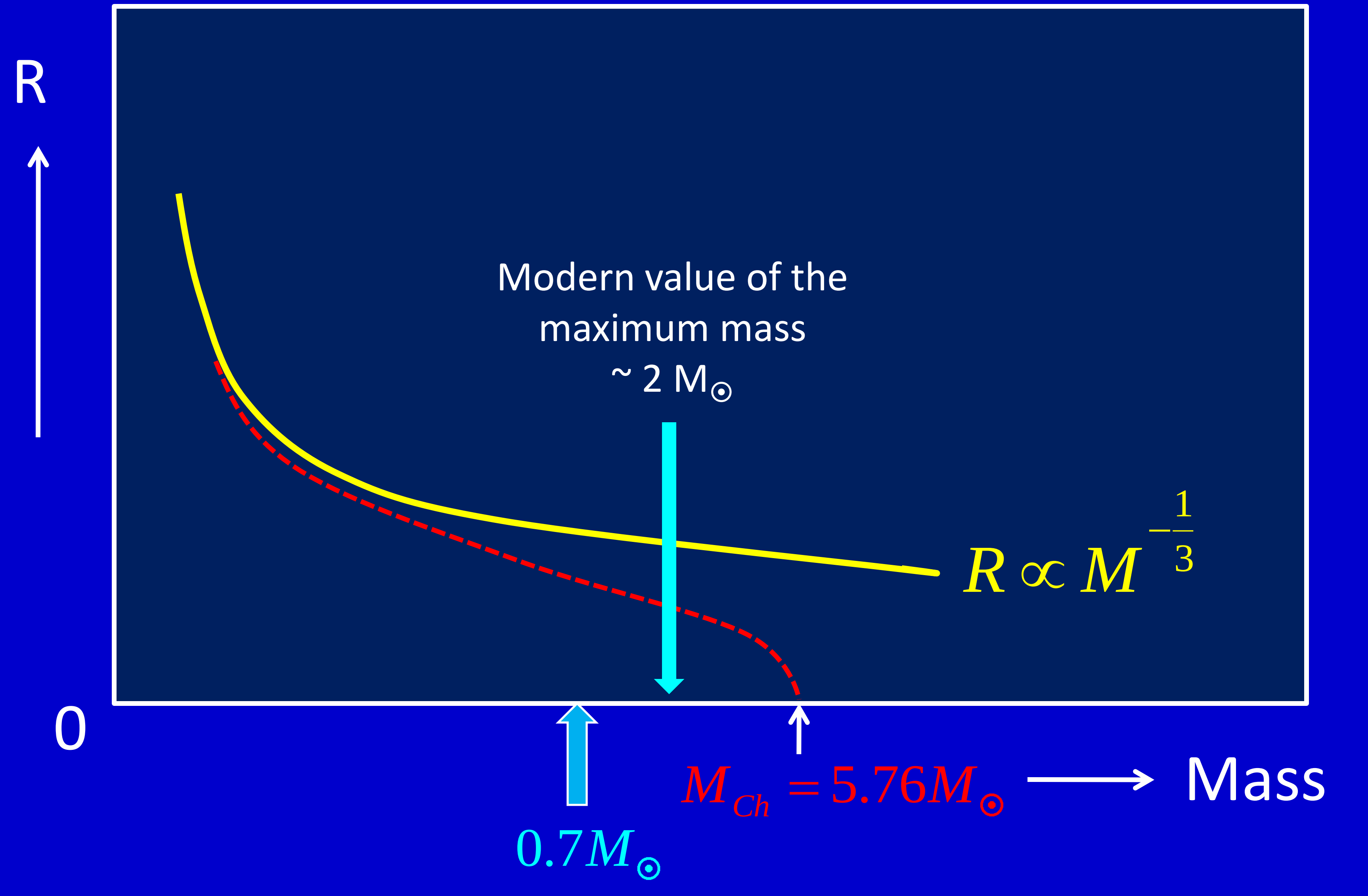
- This is because, at the maximum mass of the sequence of neutron stars, the neutrons are only mildly relativistic but not ultrarelativistic, as in the case of a white dwarf with limiting mass.



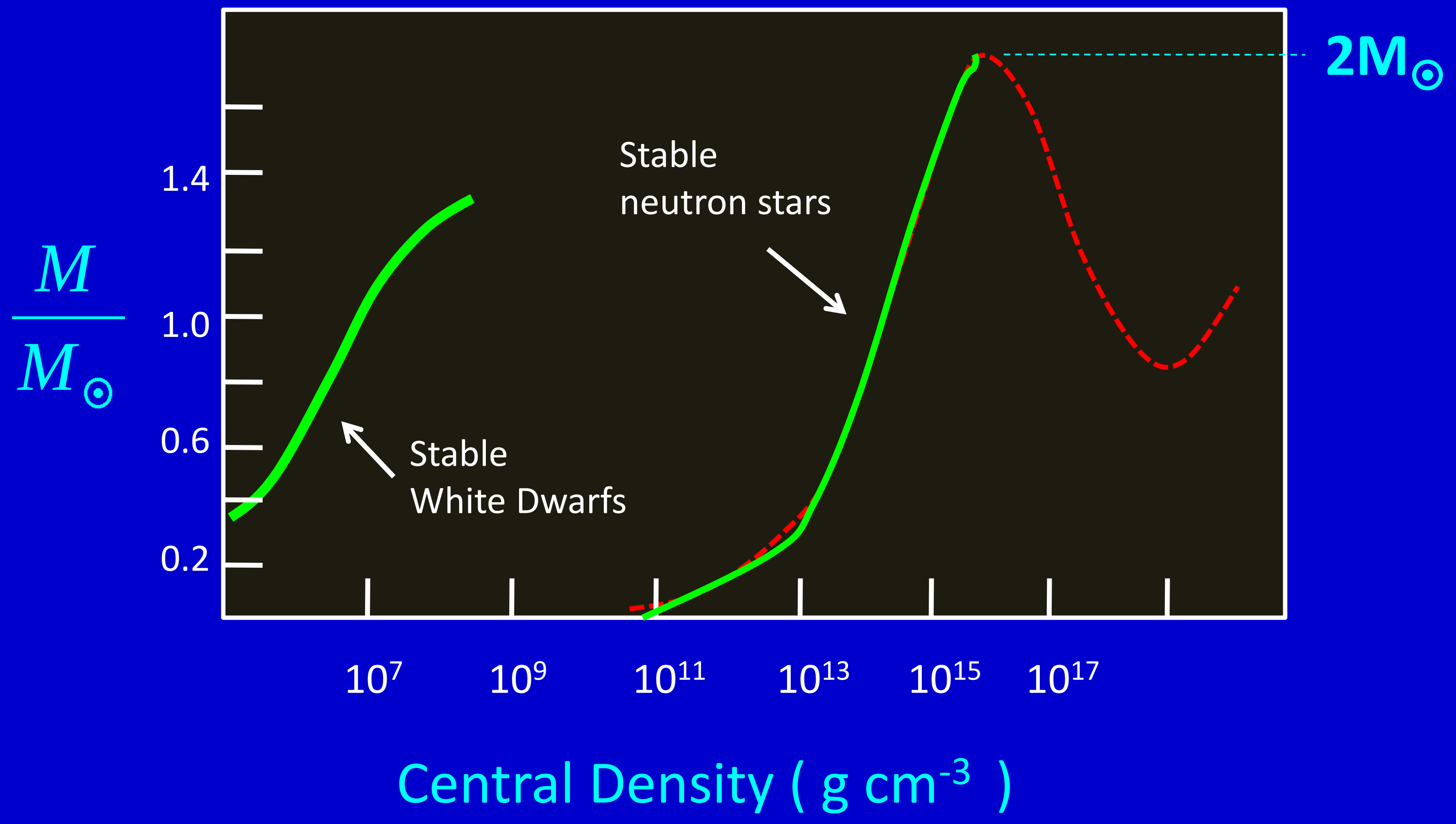
Inter nuclear potential

- O&V supported a neutron star against gravity with the degeneracy pressure of neutrons.
- As may be seen in the figure, the neutron-neutron potential energy is repulsive at short distances.
- This repulsion will help to 'support' the star. The details of the nuclear potential were not yet known in 1938.
- Nevertheless, Oppenheimer conjectured that 'nuclear repulsion' would increase the maximum mass to a "few solar mass"

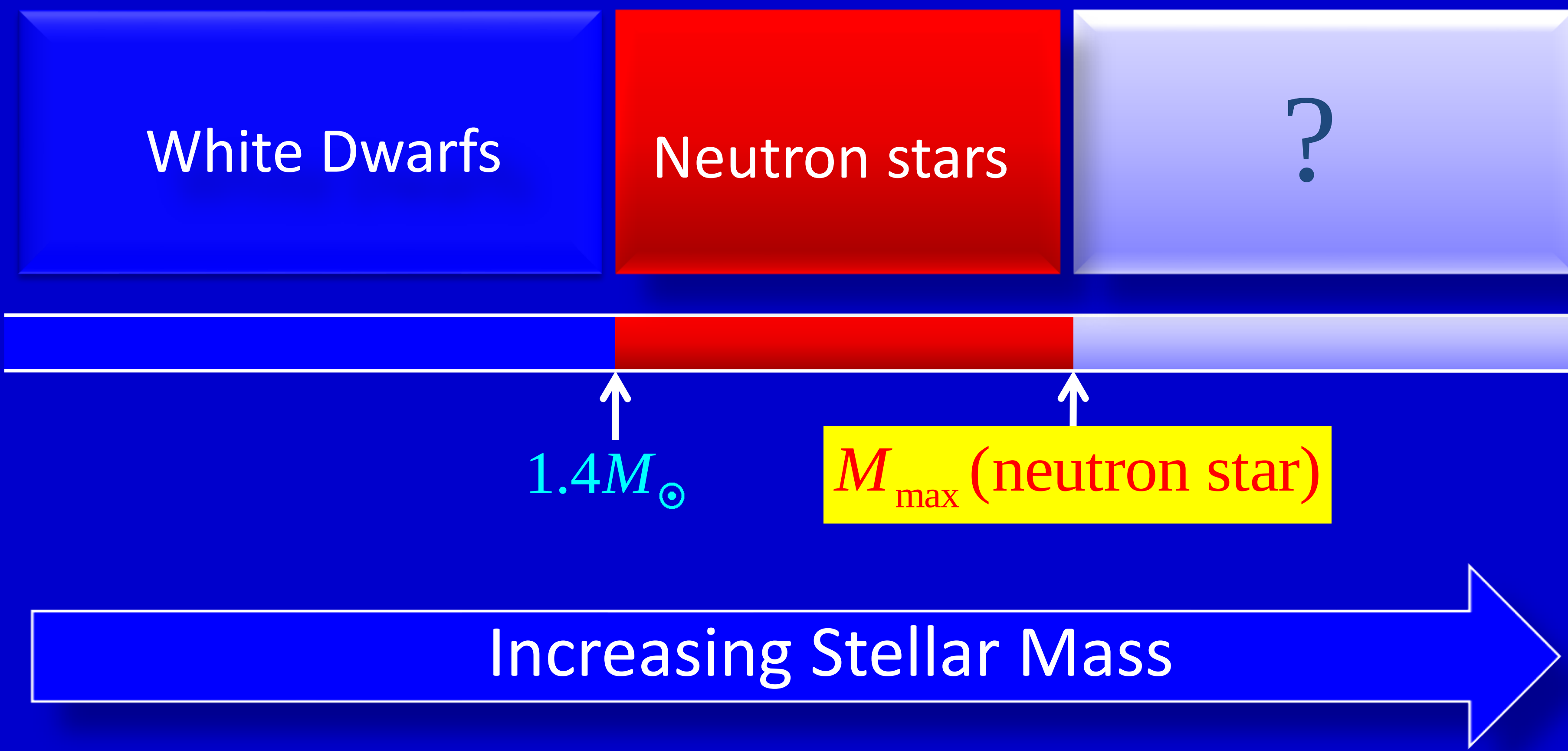
The Maximum Mass for Neutron Stars



The modern picture



Oppenheimer and Volkoff, 1938



Why are there only two classes of “Cold Stars”?

White Dwarfs and Neutron Stars

Stability of Matter

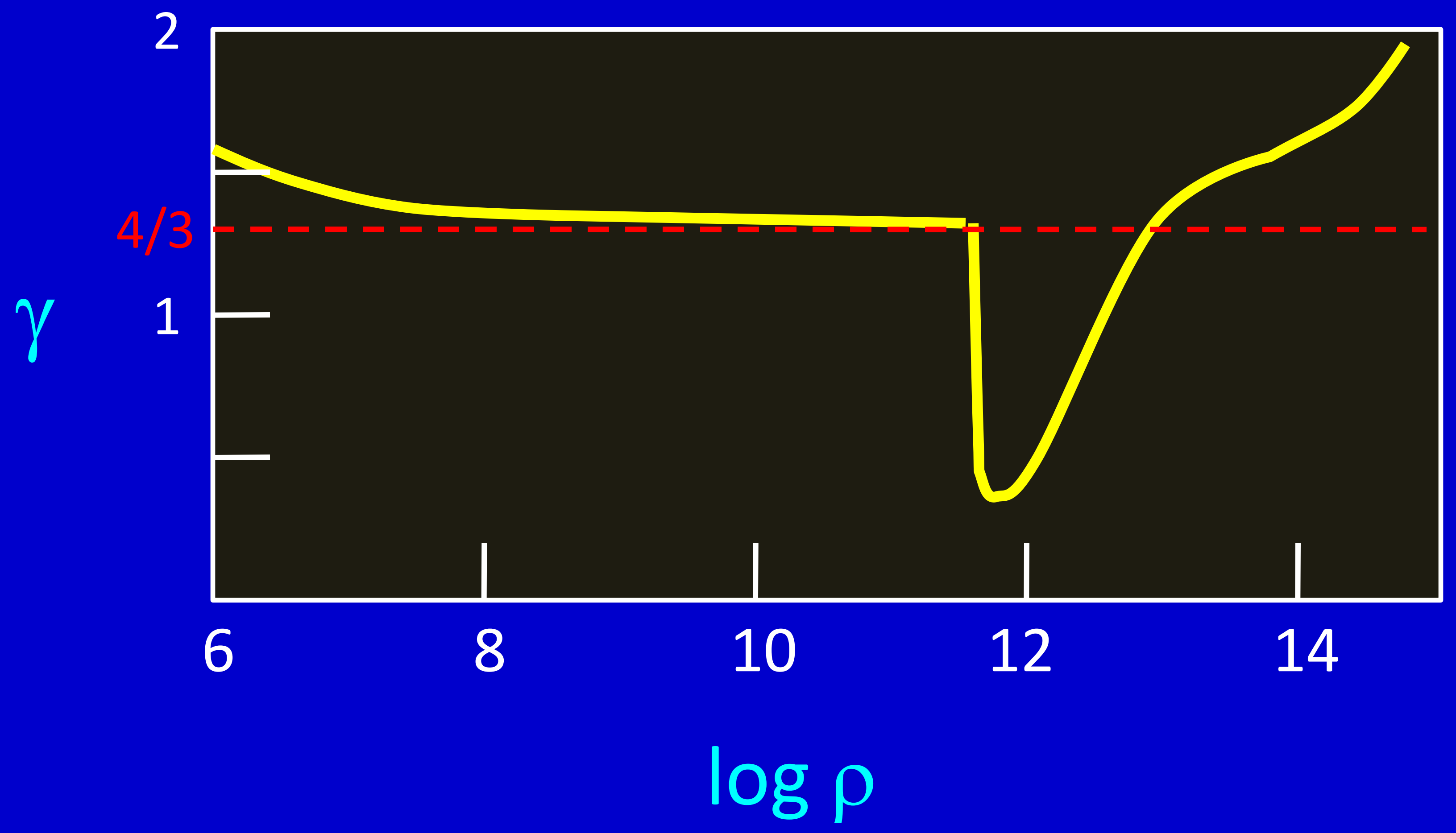
- For a star to be stable, γ, the ratio of specific heats, should be greater than 4/3.
- If $\gamma < 4/3$, the compressibility would be negative.
- Since the pressure varies over the star, the relevant parameter is the pressure-averaged value of γ over the star.

Stability of Matter

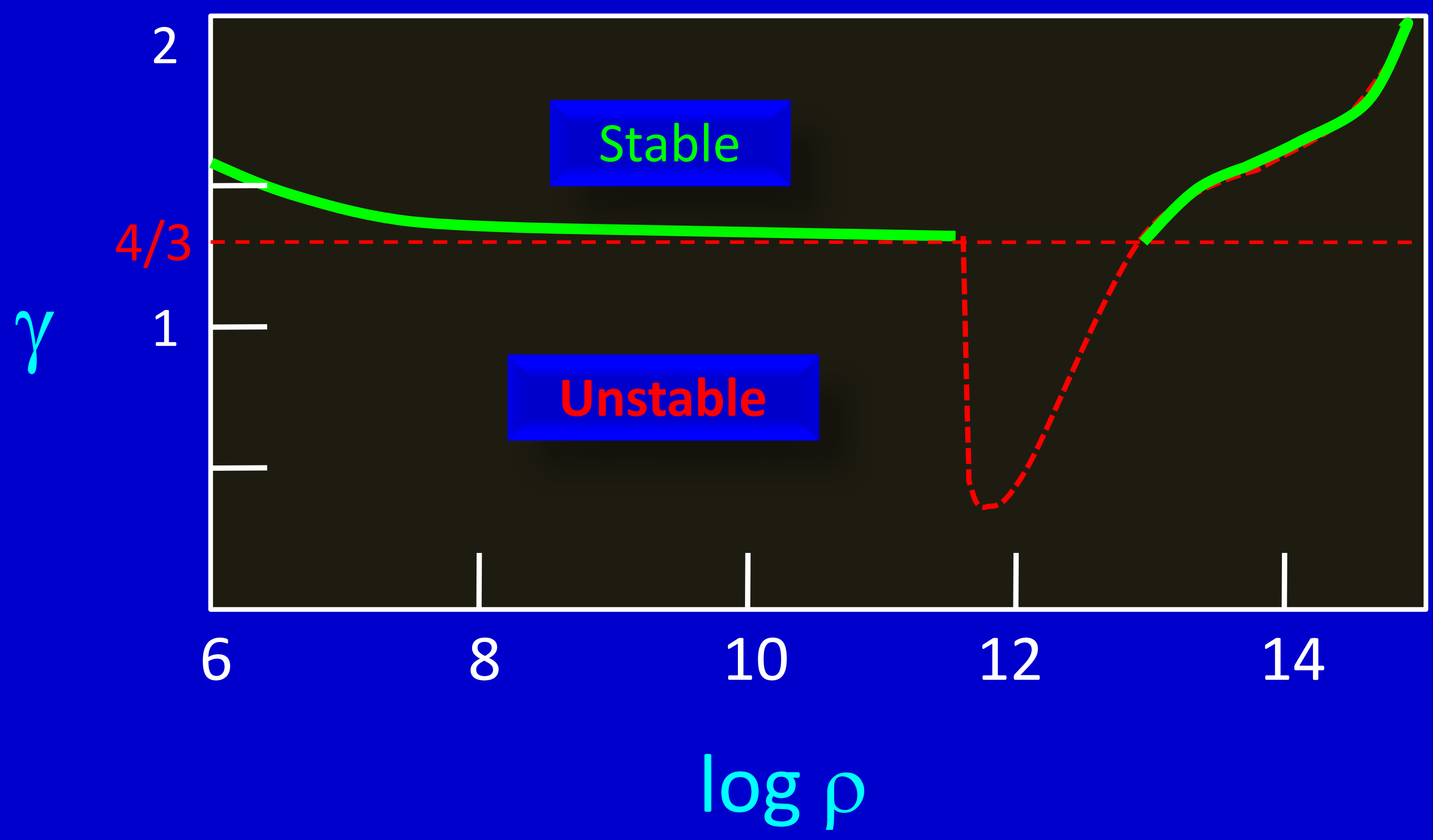
- If one has an "equation of state" – pressure as a function of density – the one can calculate γ as a function of density.

- We will come to this in the next lecture.

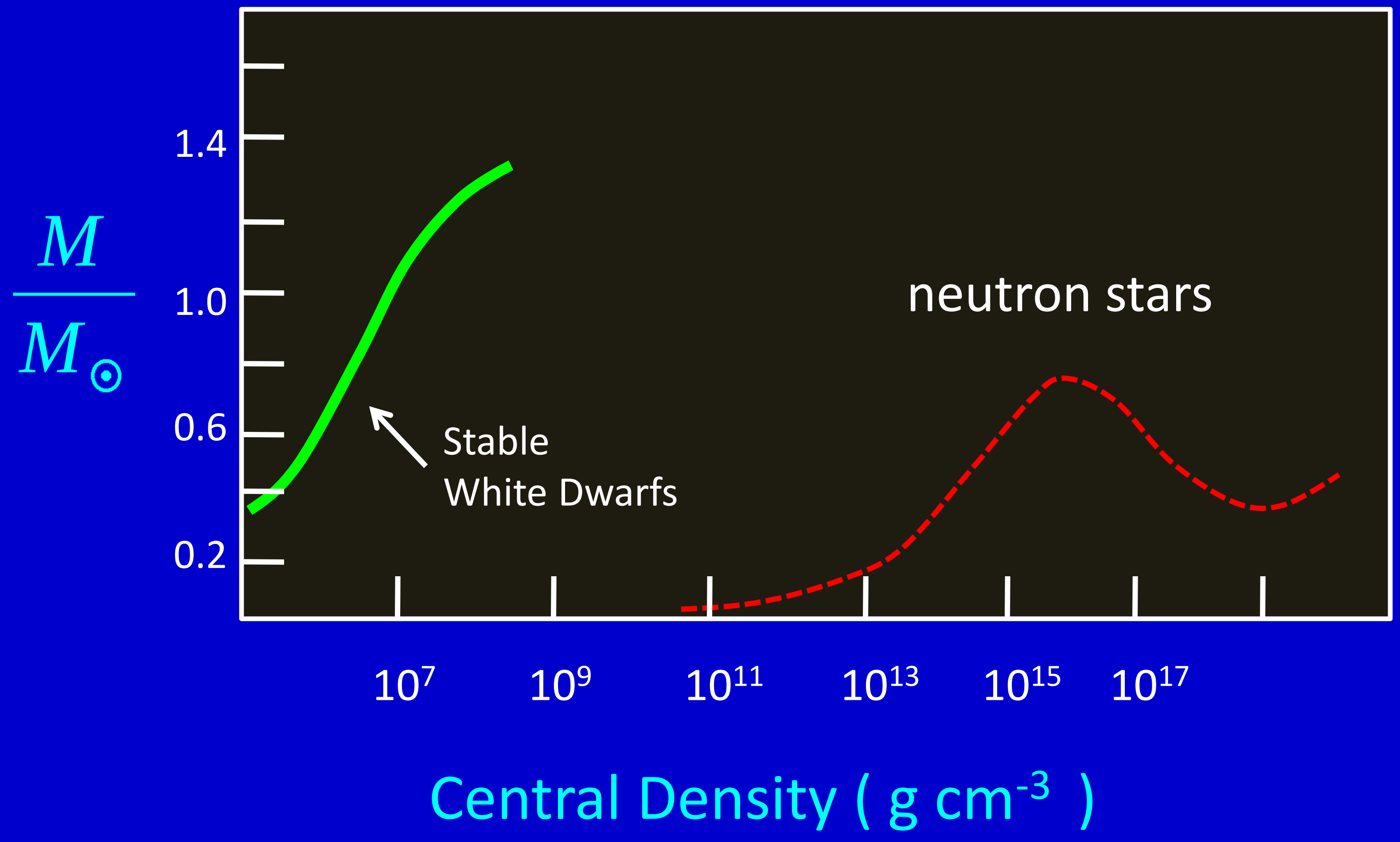
Stability of Matter



Stability of Matter

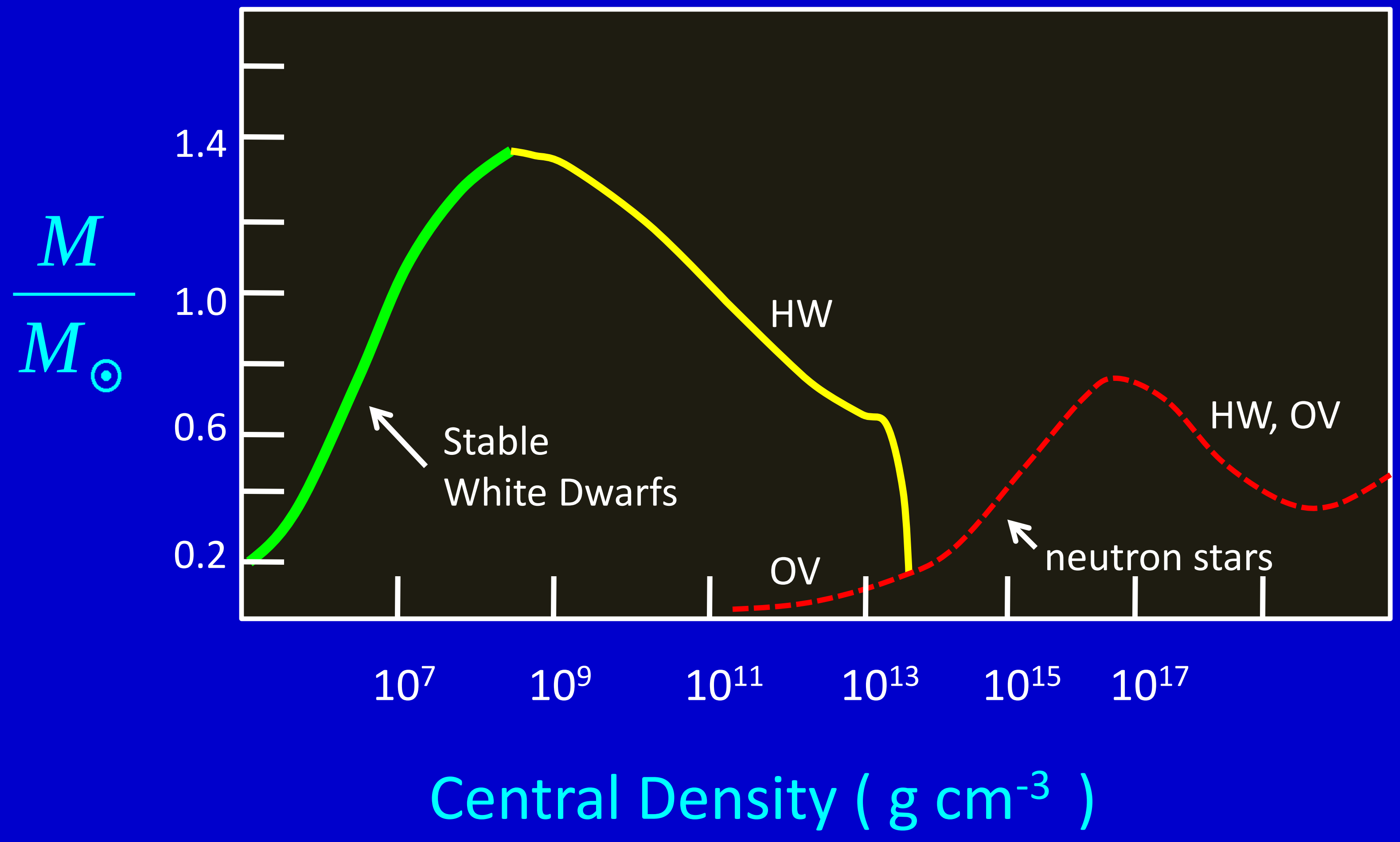


Oppenheimer – Volkoff Equation of State

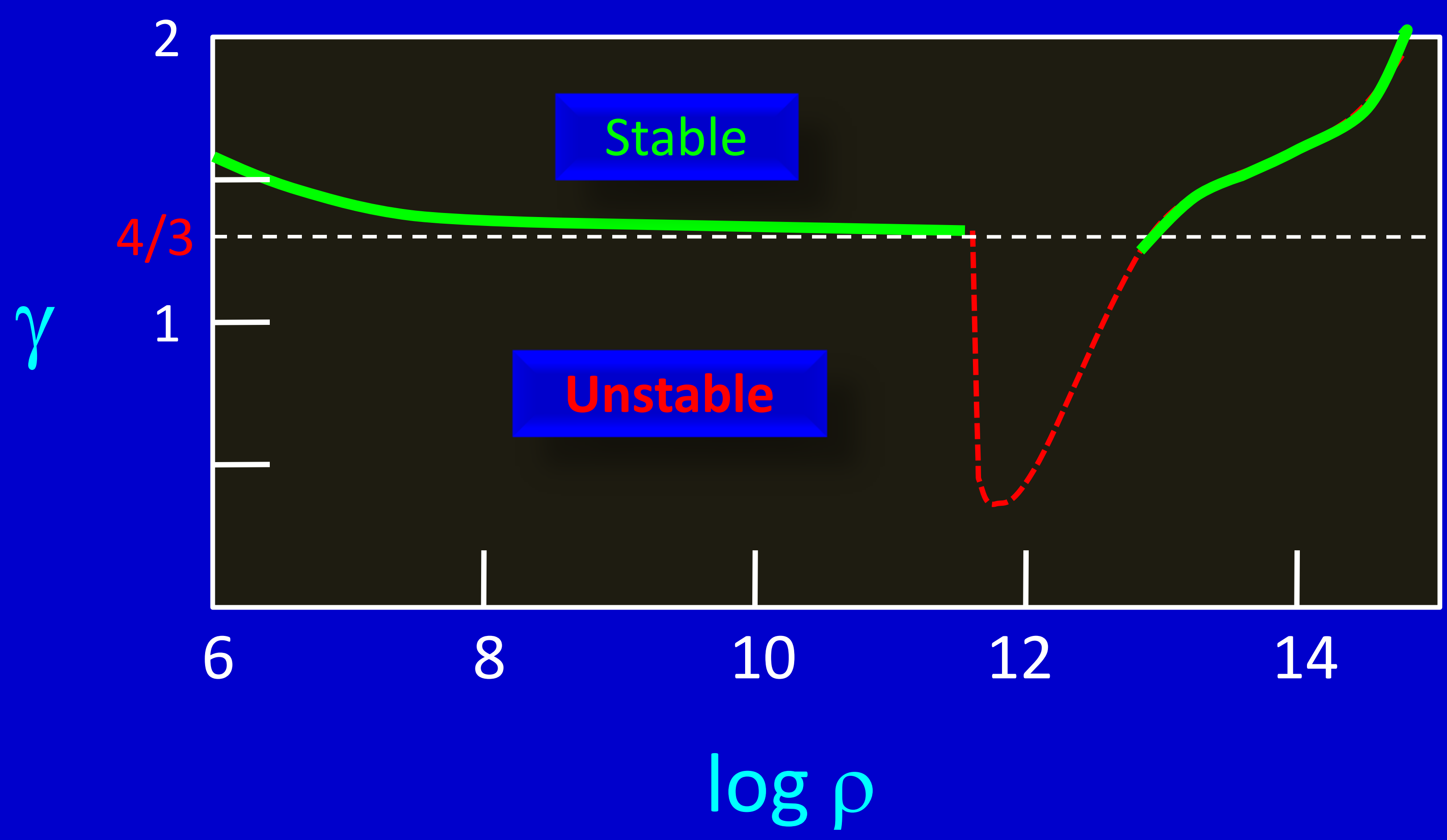


HW: Harrison – Wheeler Equation of State

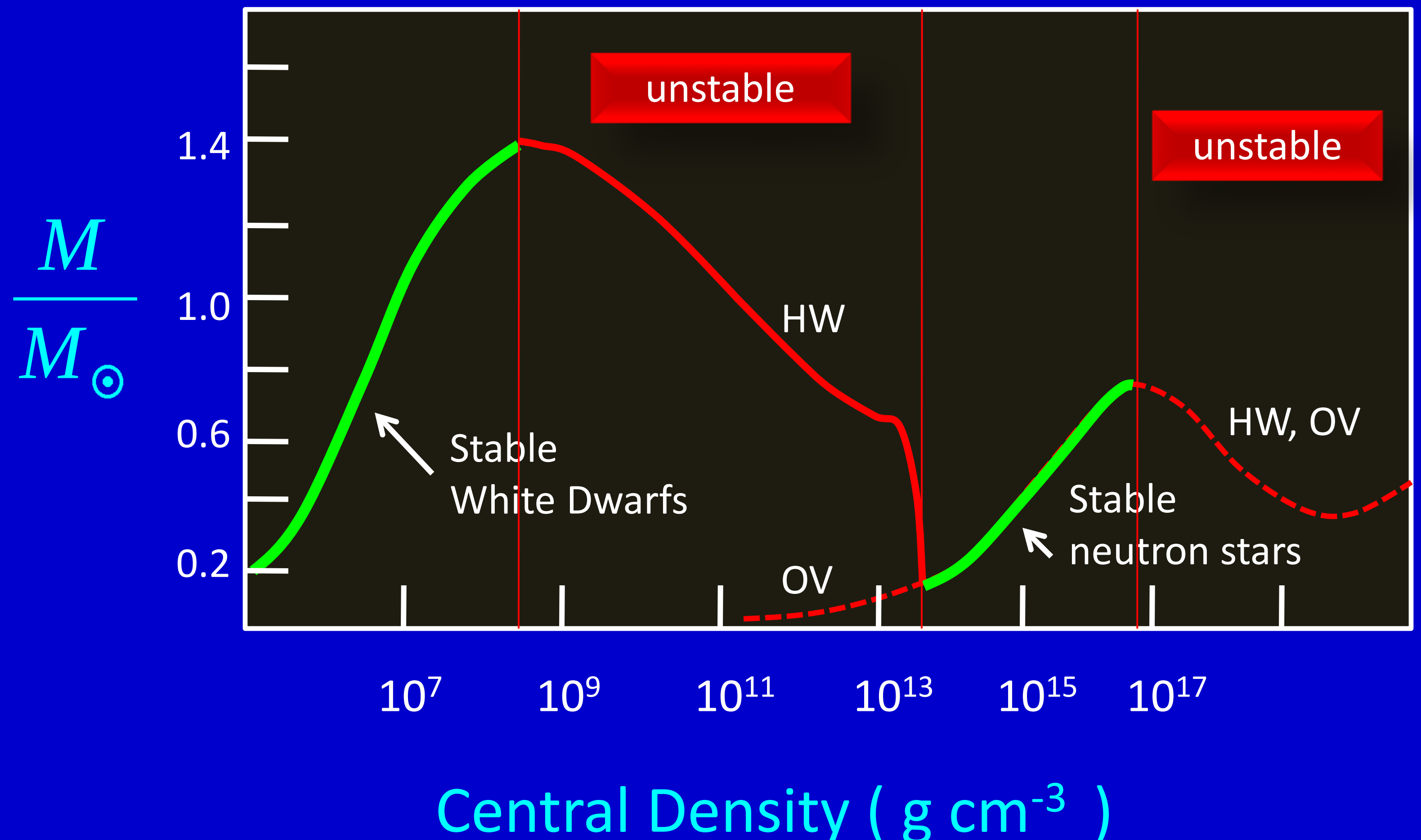
OV: Oppenheimer – Volkoff Equation of State



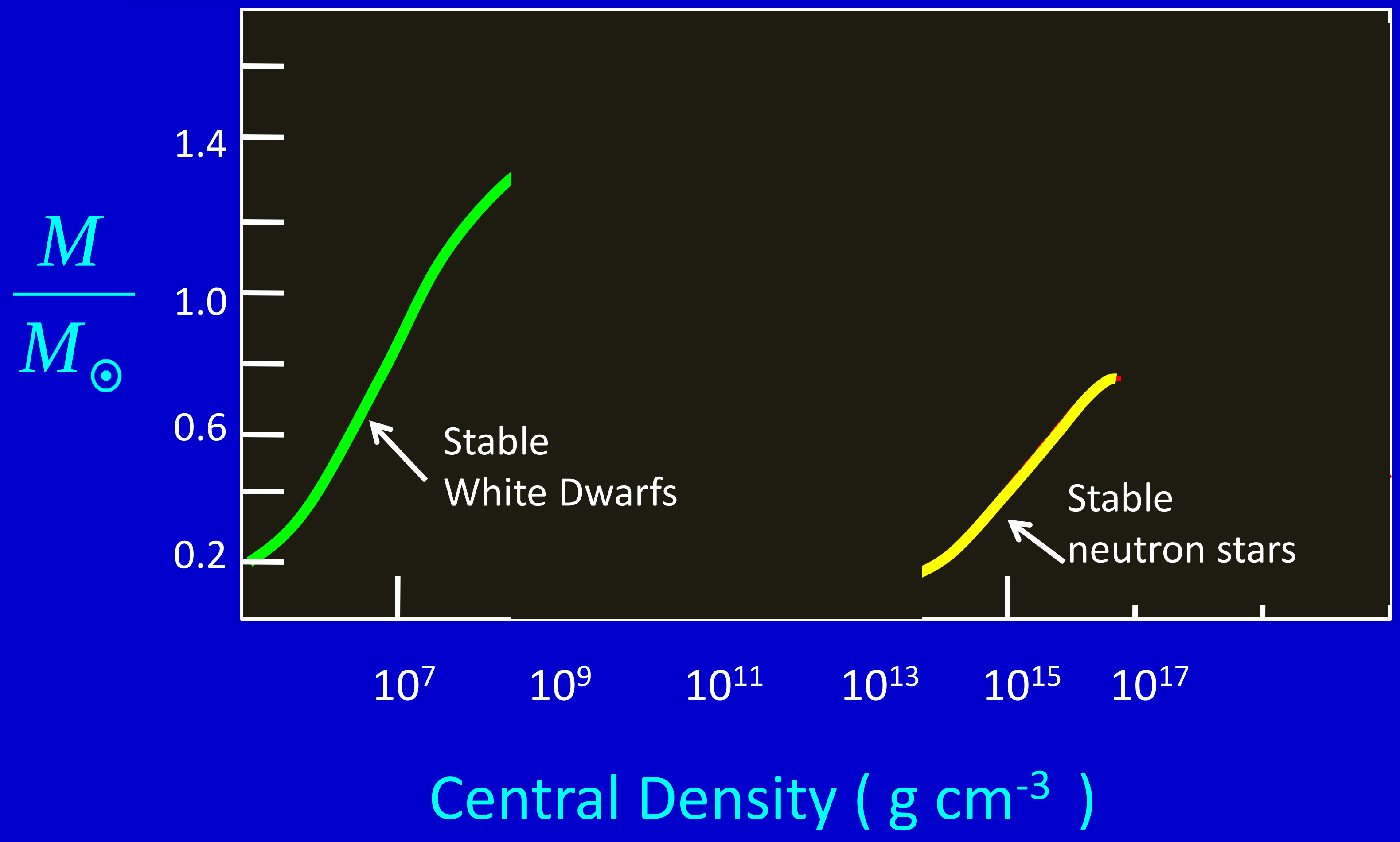
Stability of Matter



HW: Harrison – Wheeler Equation of State
OV: Oppenheimer – Volkoff Equation of State



There are only two types of cold stars in nature!



What is the fate of massive stars?

The fate of massive stars

- The fate of massive stars is very different.
- This is essentially due to the role of Radiation Pressure

- One of Eddington's great insights was that the importance of radiation pressure will increase with the mass of the star.

- In 1932, Chandrasekhar made an important discovery which is at the base of our present understanding of the fate of massive stars.

If radiation pressure exceeds 9.2% of the TOTAL pressure then matter will never become degenerate no matter what the density is.

S. Chandrasekhar, 1932

Condition for degeneracy

- A gas will obey the ideal gas law if

$$k_B T >> E_F$$

- Or, equivalently

$$p_{\mathrm{ideal}} >> p_{\mathrm{degeneracy}}$$

$$P_{Total} = \frac{1}{\beta}\left(\frac{\rho k T}{\mu m_p}\right) = \frac{1}{1-\beta}\left(\frac{1}{3} a T^4\right)$$

$$P = \frac{1}{\beta}\left(\frac{\rho k T}{\mu m_p}\right) = \frac{1}{1-\beta}\left(\frac{1}{3} a T^4\right)$$

$$\frac{1}{\beta}\frac{\rho k T}{\mu m_p} = \frac{1}{1-\beta}\frac{1}{3} a T^4$$

$$T = \left[\frac{3}{a}\frac{k}{\mu m_p}\frac{1-\beta}{\beta}\right]^{\frac{1}{3}} \rho^{\frac{1}{3}}$$

$$P = \frac{1}{\beta}\left(\frac{\rho k T}{\mu m_p}\right) = \frac{1}{1-\beta}\left(\frac{1}{3} a T^4\right)$$

$$\frac{1}{\beta}\frac{\rho k T}{\mu m_p} = \frac{1}{1-\beta}\frac{1}{3} a T^4$$

$$T = \left[\frac{3}{a}\frac{k}{\mu m_p}\frac{1-\beta}{\beta}\right]^{\frac{1}{3}} \rho^{\frac{1}{3}}$$

$$p_{gas} = \frac{\rho k T}{\mu m_p} = \left[\frac{3}{a}\left(\frac{k}{\mu m_p}\right)^4 \frac{1-\beta}{\beta}\right]^{\frac{1}{3}} \rho^{\frac{4}{3}} = C(\beta)\rho^{\frac{4}{3}}$$

Radiation pressure and Degeneracy

- If classical pressure of electrons is greater than degeneracy pressure, then gas will remain classical.

$$p_{gas} = \frac{\rho k T}{\mu m_p} = \left[\frac{3}{a} \left(\frac{k}{\mu m_p} \right)^4 \frac{1-\beta}{\beta} \right]^{\frac{1}{3}} \rho^{\frac{4}{3}} = C(\beta)\rho^{\frac{4}{3}}$$

$$p_{\text{degeneracy}} = \mathrm{K}_2 \rho^{\frac{4}{3}}$$

Radiation pressure and Degeneracy

- The condition that classical pressure is greater than degeneracy pressure implies:

$$\left[\frac{3}{a}\left(\frac{k}{\mu m_p}\right)^4 \frac{1-\beta}{\beta}\right]^{\frac{1}{3}} > \mathrm{K}_2$$

- This can be simplified as $1-\beta > 0.092$.

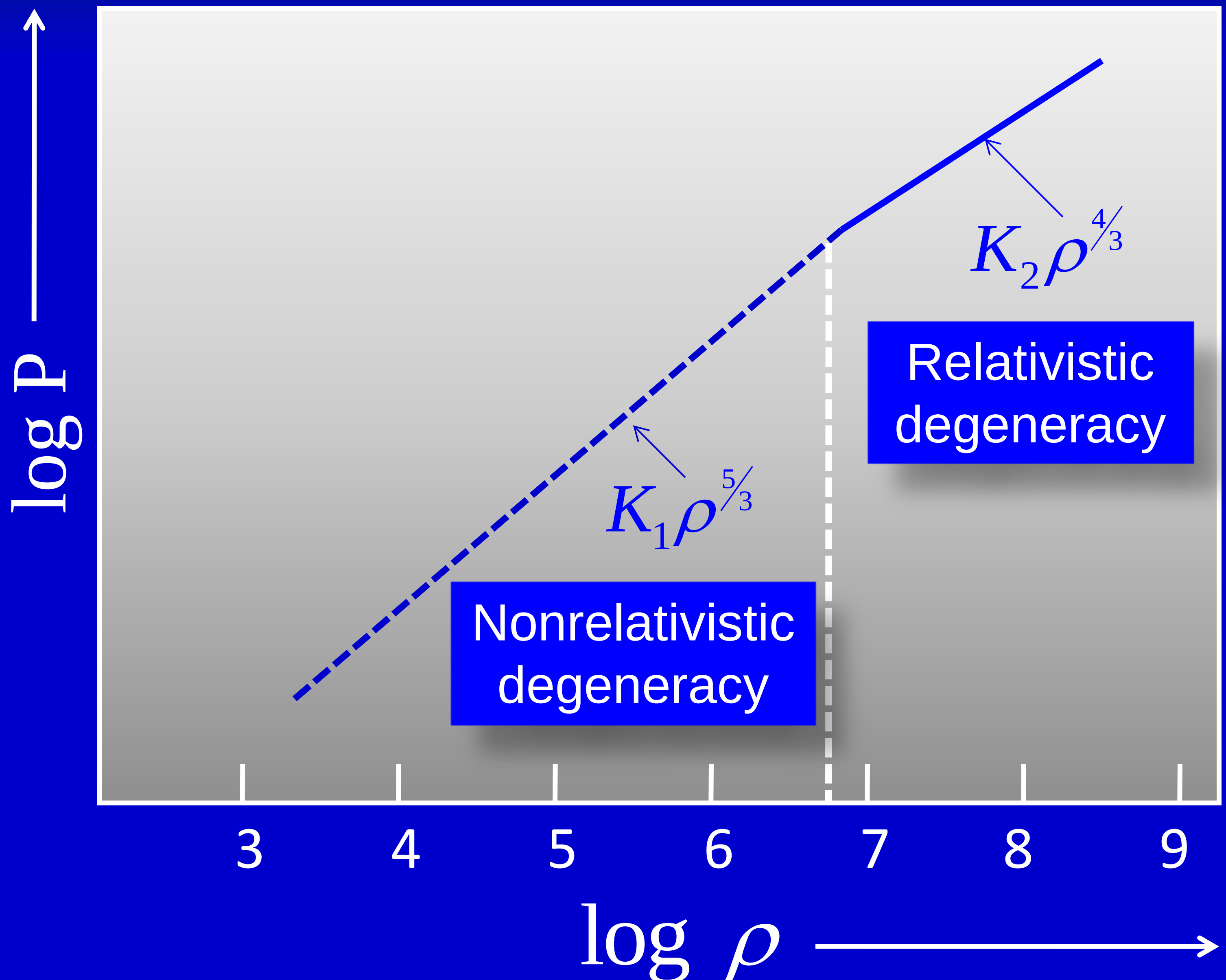
- Thus, if radiation pressure is greater than 9.2% of the TOTAL PRESSURE then degeneracy will not set in.

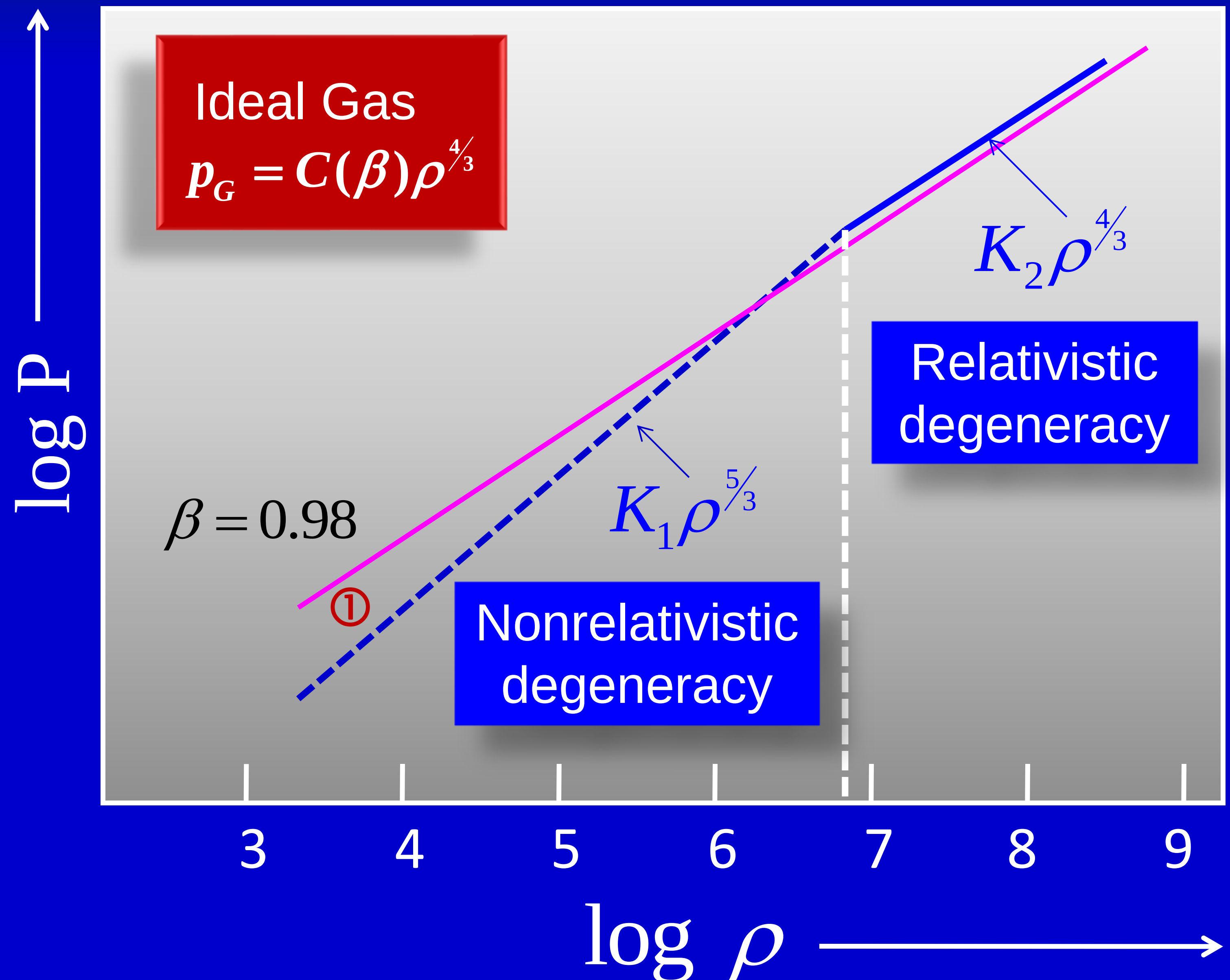
If radiation pressure exceeds 9.2% of the TOTAL pressure then matter will never become degenerate no matter what the density is.

S. Chandrasekhar, 1932

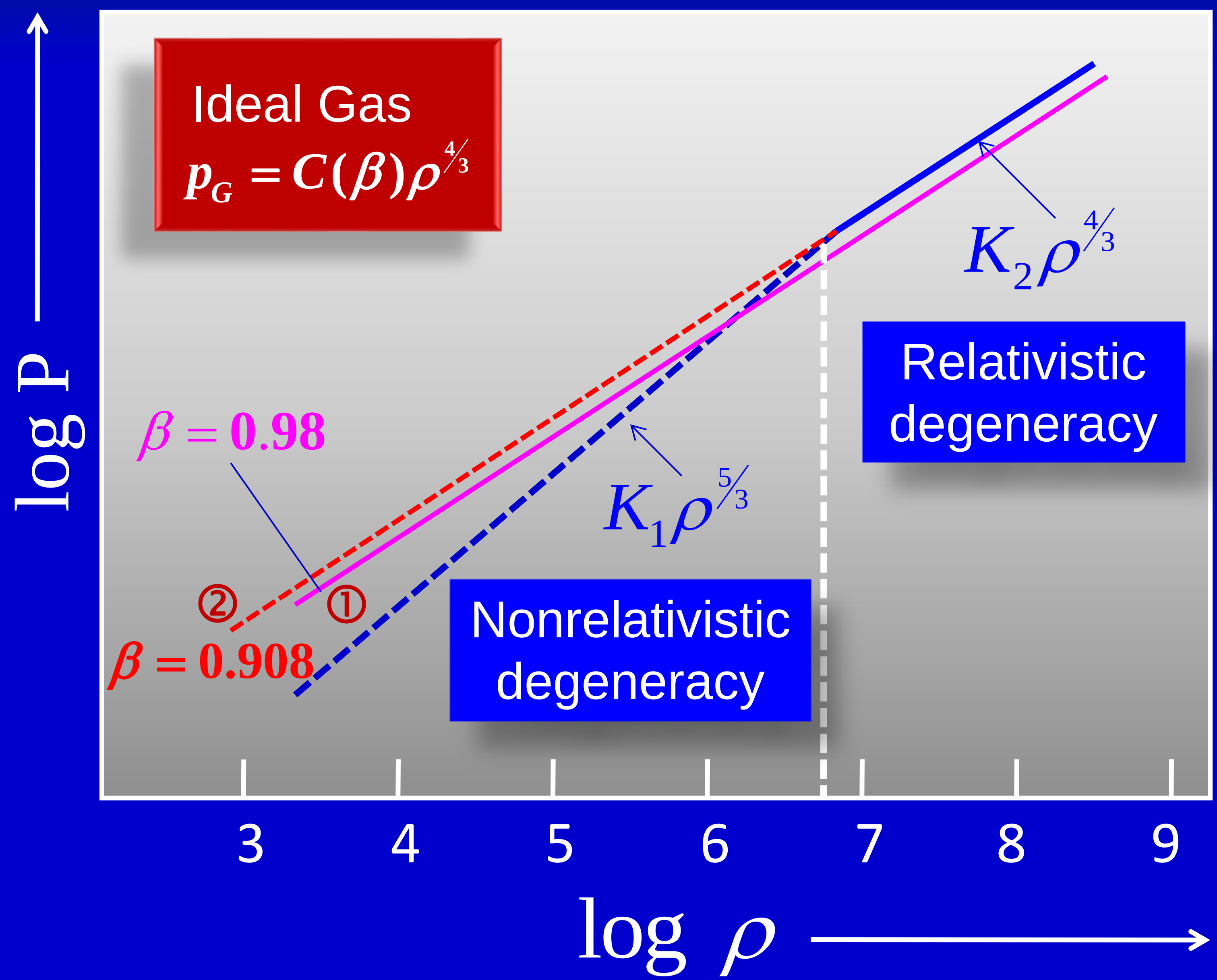
Therefore, stars above a critical mass will never become degenerate.

- If degeneracy does set in at high densities, the gas will be ‘relativistic’.
- “Boyle’s Law is valid regardless of Relativity” (Chandrasekhar, 1932)!

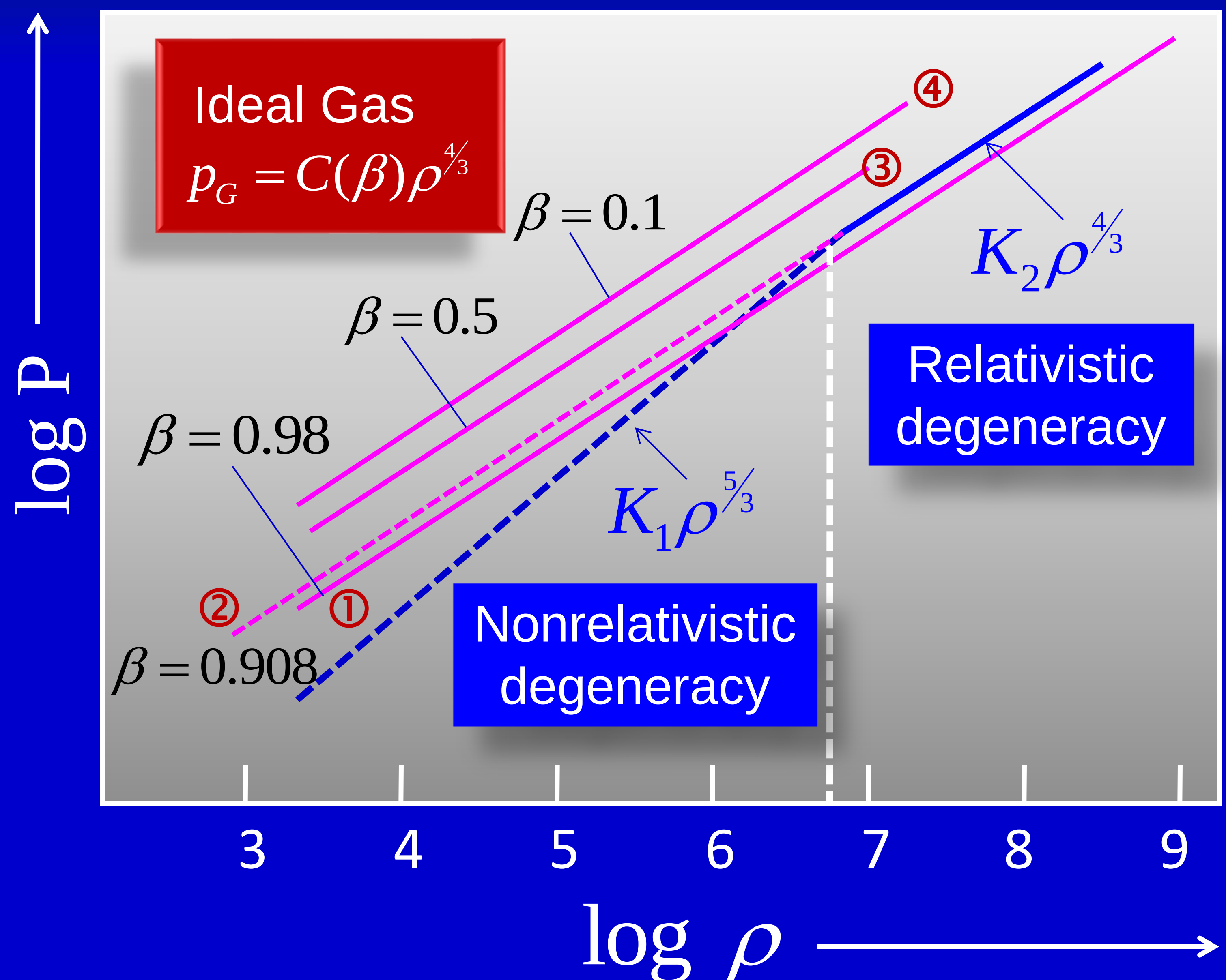

log P
$K_2\rho^{4/3}$
Relativistic degeneracy
$K_1\rho^{5/3}$
Nonrelativistic degeneracy
3
4
5
6
7
8
9
log ρ



Ideal Gas
$p_G = C(\beta)\rho^{4/3}$
$K_2\rho^{4/3}$
Relativistic degeneracy
$K_1\rho^{5/3}$
$\beta = 0.98$
①
Nonrelativistic degeneracy
3
4
5
6
7
8
9
log P
log ρ



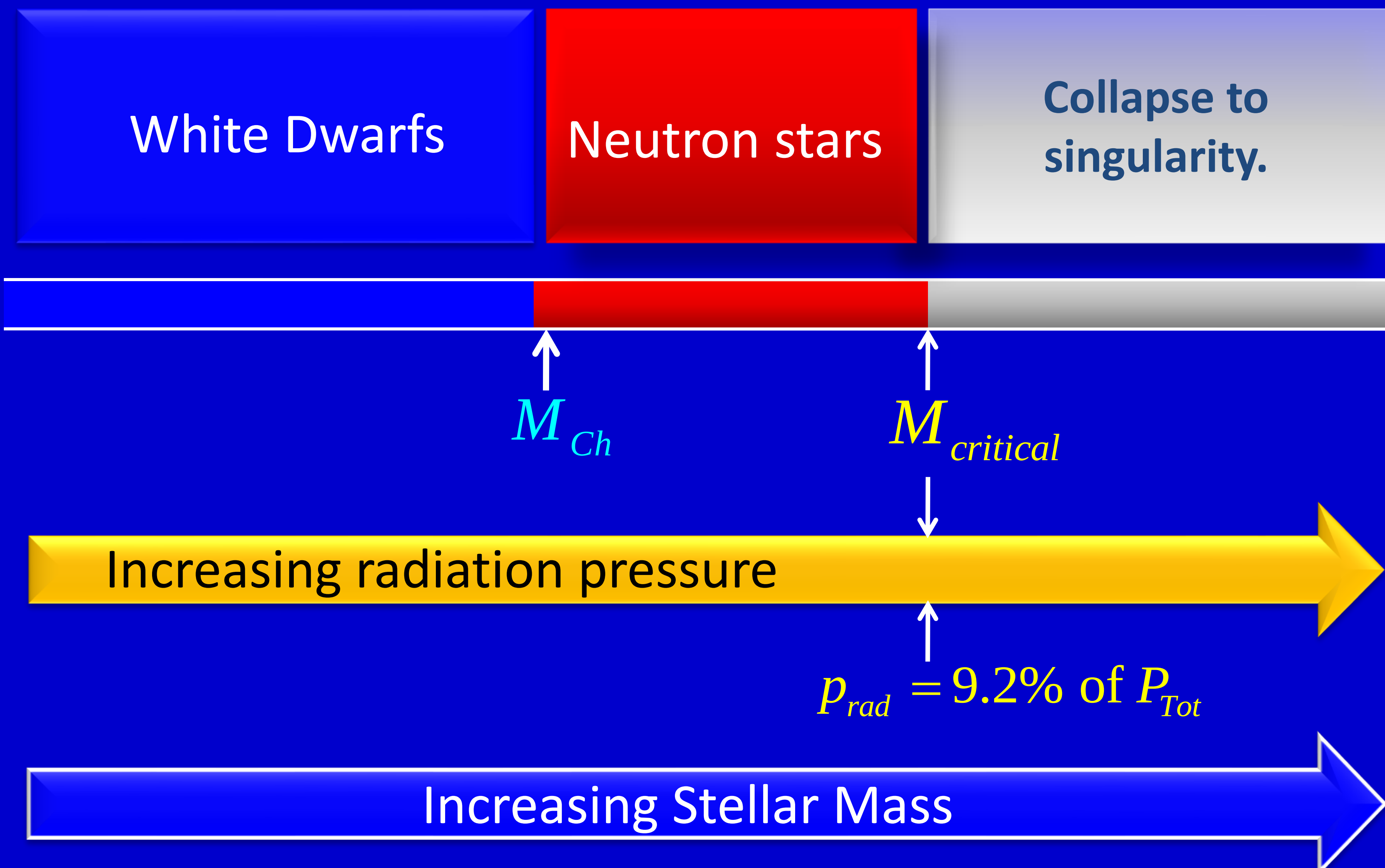
Ideal Gas
$p_G = C(\beta)\rho^{4/3}$
$K_2\rho^{4/3}$
Relativistic degeneracy
$\beta = 0.98$
$K_1\rho^{5/3}$
②
①
$\beta = 0.908$
Nonrelativistic degeneracy
log P
3
4
5
6
7
8
9
log ρ



Ideal Gas
$p_G = C(\beta)\rho^{4/3}$
$\beta = 0.1$
$\beta = 0.5$
$\beta = 0.98$
$\beta = 0.908$
④
③
②
①
$K_2\rho^{4/3}$
$K_1\rho^{5/3}$
Relativistic degeneracy
Nonrelativistic degeneracy
3
4
5
6
7
8
9
log P
log ρ

"For all stars above a critical mass, the perfect gas equation of state does not break down, however high the density may become, and matter does not become degenerate. An appeal to Fermi-Dirac statistics to avoid the central singularity cannot be made."

Chandrasekhar, 1932


White Dwarfs
Neutron stars
Collapse to singularity.
M_{Ch}
$M_{critical}$
Increasing radiation pressure
$p_{rad} = 9.2\%$ of P_{Tot}
Increasing Stellar Mass

J. Robert Oppenheimer

Having discovered the Maximum Mass of Neutron Stars, Oppenheimer set out to investigate the fate of massive stars. He knew that this would have to be done within the premise of General Relativity. He picked Hartland Snyder to work with him.

Hartland Snyder – a very special student!

He was a truck driver, but decided to switch to physics!

He was very different from Oppie's other students: he couldn't care less about BACH and MOZART, String Quartets, fine food and wine or liberal politics!

But he was extremely talented in mathematics, and extremely careful with his calculations.

So Oppie picked Snyder to study the implosion of a star in General Relativity.

The instructions to Snyder were the following.

Assume a spherically symmetric star, with no internal pressure, energy generation, etc.

Having assumed this, do the calculation EXACTLY within GTR.

Snyder succeeded in this. And the results were stunning!

The implosion of a star

- As the star nears the critical radius, its shrinkage slows down, till it becomes frozen precisely at the critical radius.

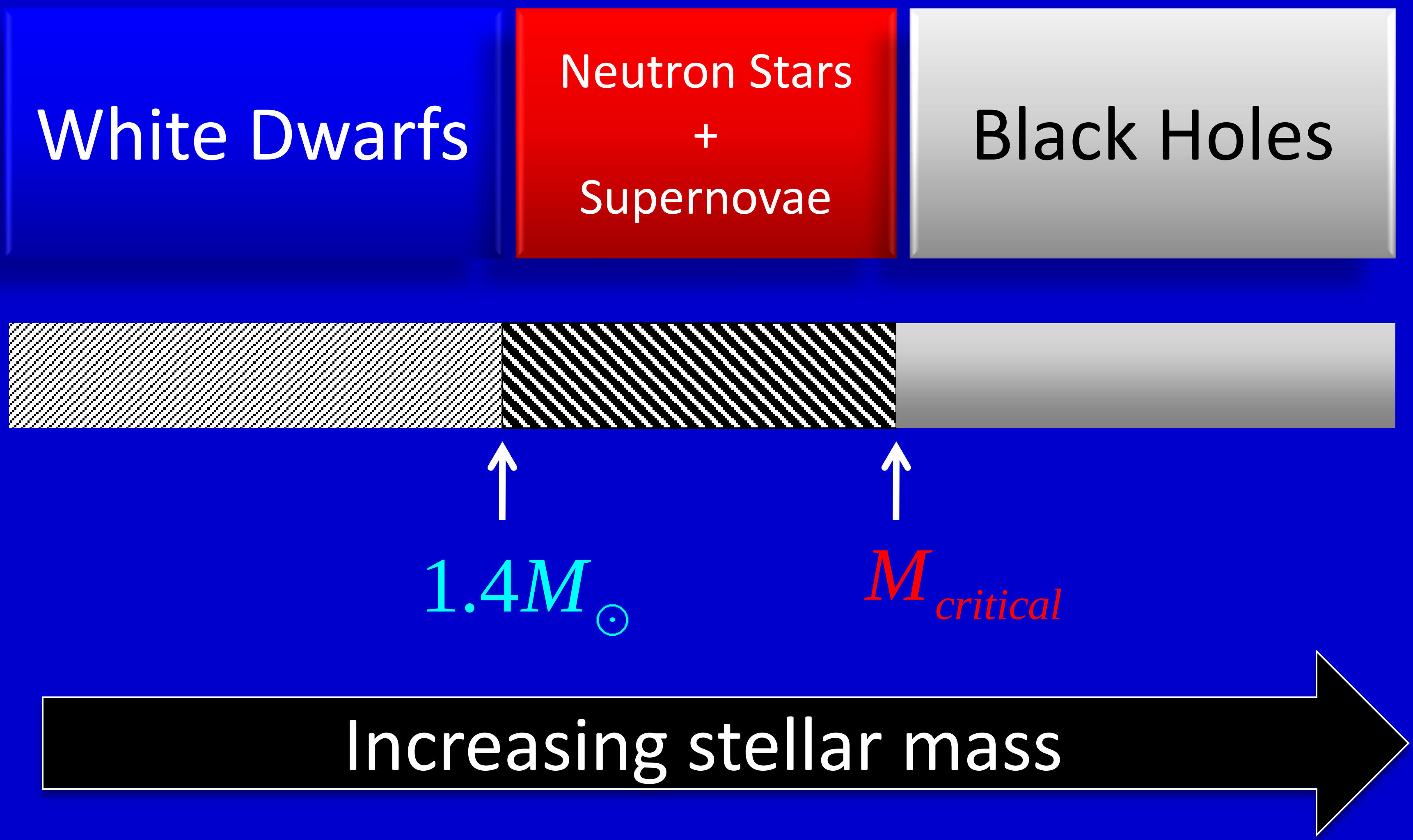
- However, as seen by an observer riding on the surface of the star, the implosion does not freeze at all!

Oppenheimer and Snyder, 1939

When all thermonuclear sources of energy are exhausted a sufficiently massive star will collapse…… The radius of the star approaches asymptotically its gravitational radius; light from the surface of the star is progressively reddened, and can escape over a progressively narrower range of angles…….”

“The star thus tends to close itself off from any communication with a distant observer; only its gravitational field persists.”

Oppenheimer and Snyder,1939.


White Dwarfs
Neutron Stars
+
Supernovae
Black Holes
$1.4M_{\odot}$
$M_{critical}$
Increasing stellar mass

Aristotle
384 BC – 322 BC

- The matter of which the heavens are made is imperishable, and thus not subject to generation or corruption.

- In contrast, the earth is made of the classical elements (earth, water, air, fire) and is perishable.

- The motions in the heavens are eternal and perfect; the heavens are the realm of peace and perfection.

- This was the Aristotelian view of the Universe that dominated astronomy for more than 2000 years.

- Indeed, the serenity and permanence of the night sky was a source of awe and security for mankind from time immemorial.

'For all stars of mass greater than $\mathcal{M}$ the perfect gas equation of state does not break down, however high the density may become, and the matter does not become degenerate. An appeal to the Fermi-Dirac statistics to avoid the central singularity cannot be made'.

'Great progress in the analysis of stellar structure is not possible before we can answer the following fundamental question: Given an enclosure containing electrons and atomic nuclei (total charge zero) what happens if we go on compressing the material indefinitely?'

S.Chandrasekhar
(1932)

‘Finally, it is necessary to emphasize one major result of the whole investigation, namely, that it must be taken as well established that the life history of a star of small mass must be essentially different from the life history of a star of large mass. For a star of small mass the natural white dwarf stage is an initial step towards complete extinction. A star of large mass ($>\mathcal{M}$) cannot pass into the white dwarf stage, and one is left speculating on other possibilities’.

S. Chandrasekhar (1934)

Sir Arthur Eddington, at the meeting of the Royal Astronomical Society in London in 1935.

'I do not know whether I shall escape from this meeting alive, but the point of my paper is that there is no such thing as relativistic degeneracy!....

'Chandrasekhar, using the relativistic formula which has been accepted for the last five years, shows that a star of mass greater than a certain limit $\mathcal{M}$ remains a perfect gas and can never cool down.

The star has to go on radiating and radiating, and contracting and contracting until, I suppose, it gets down to a few km radius, when gravity becomes strong enough to hold in the radiation, and the star can at last find peace.

'... Dr. Chandrasekhar had got this result before, but he has rubbed it in in his last paper; and, when discussing with him, I felt driven to the conclusion that this was almost a reductio ad absurdum of the relativistic degeneracy formula.

Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way'.

"An essential result of this investigation is a clear understanding as to why the 'Schwarzschild Singularities' do not exist in physical reality".

- Albert Einstein, 1939

And so Aristotle continued to prevail.

Until.....

One day in August 1968, when young Jocelyn Bell made a stunning discovery that heralded the end of the Aristotelian view of the Universe.