

PN theory: Far-zone physics

Recall the wave-zone solution to $\square\psi = -4\pi\mu$

$$\frac{\mu(t - |\mathbf{x} - \mathbf{x}'|/c, \mathbf{y})}{|\mathbf{x} - \mathbf{x}'|} = \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\ell!} x'^L \partial_L \frac{\mu(t - r/c, \mathbf{y})}{r}.$$

$$\psi_{\mathcal{N}}(t, \mathbf{x}) = \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \mu(\tau, \mathbf{x}') x'^L d^3x' \right]$$

Must add the integral over the wave zone $\psi_{\mathcal{W}}$

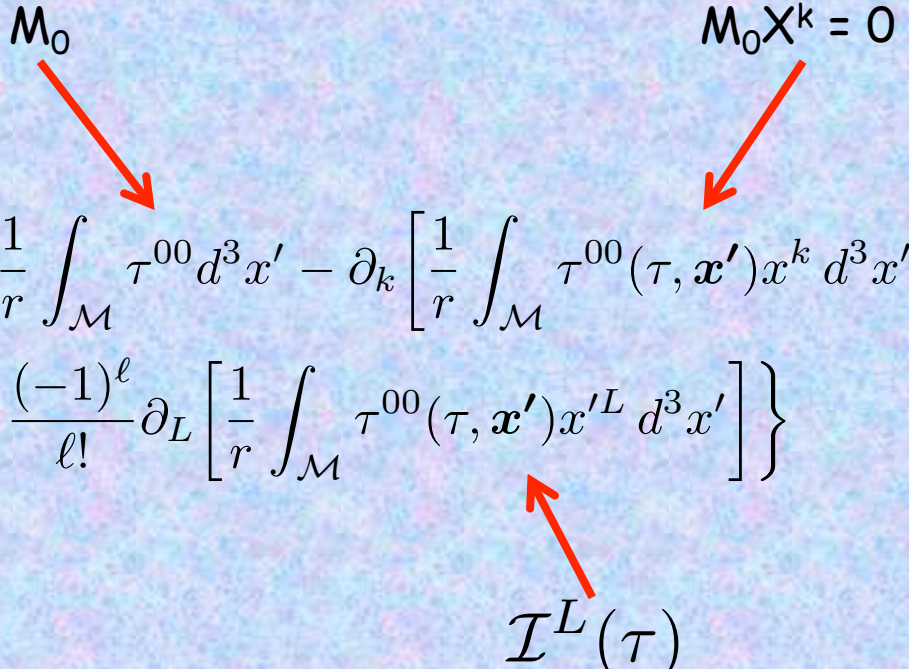
For the field $h^{\alpha\beta}$:

$$h_{\mathcal{N}}^{\alpha\beta}(t, \mathbf{x}) = \frac{4G}{c^4} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{\alpha\beta}(\tau, \mathbf{x}') x'^L d^3x' \right]$$



General form of wave-zone fields

$$h_{\mathcal{N}}^{00}(t, \mathbf{x}) = \frac{4G}{c^4} \left\{ \frac{1}{r} \int_{\mathcal{M}} \tau^{00} d^3 x' - \partial_k \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{00}(\tau, \mathbf{x}') x^k d^3 x' \right] + \sum_{\ell=2}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{00}(\tau, \mathbf{x}') x'^L d^3 x' \right] \right\}$$

M_0 $M_0 X^k = 0$

 $\mathcal{I}^L(\tau)$

$$h_{\mathcal{N}}^{00} = \frac{4GM_0}{c^2 r} + \frac{4G}{c^2} \sum_{\ell=2}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{\mathcal{I}^L(\tau)}{r} \right]$$



General form of wave-zone fields

$$p_j = 0$$

$$\tau^{0j} x^k = \frac{1}{2} \partial_0 (\tau^{00} x^j x^k) + \tau^{0[j} x^{k]} + \frac{1}{2} \partial_p (\tau^{0p} x^j x^k)$$

$$h_{\mathcal{N}}^{0j}(t, \mathbf{x}) = \frac{4G}{c^4} \left\{ \frac{1}{r} \int_{\mathcal{M}} \tau^{0j} d^3 x' - \partial_k \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{0j}(\tau, \mathbf{x}') x^k d^3 x' \right] \right. \\ \left. + \sum_{\ell=2}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{0j}(\tau, \mathbf{x}') x'^L d^3 x' \right] \right\}$$

$$h_{\mathcal{N}}^{0j} = -\frac{2G}{c^3} \frac{(\mathbf{n} \times \mathbf{J}_0)^j}{r^2} - \frac{2G}{c^3} \partial_k \left[\frac{\dot{\mathcal{I}}^{jk}(\tau)}{r} \right] \\ + \frac{4G}{c^4} \sum_{\ell=2}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{0j}(\tau, \mathbf{x}') x'^L d^3 x' \right]$$



General form of wave-zone fields

$$h_{\mathcal{N}}^{jk}(t, \mathbf{x}) = \frac{4G}{c^4} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{jk}(\tau, \mathbf{x}') x'^L d^3 x' \right]$$

$$\tau^{jk} = \frac{1}{2} \partial_{00} (\tau^{00} x^j x^k) + \frac{1}{2} \partial_p (2\tau^{p(j} x^{k)}) - \partial_q \tau^{pq} x^j x^k$$

$$h_{\mathcal{N}}^{jk} = \frac{2G}{c^4} \frac{\ddot{I}^{jk}(\tau)}{r} + \frac{4G}{c^4} \sum_{\ell=1}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{jk}(\tau, \mathbf{x}') x'^L d^3 x' \right]$$



General form of wave-zone fields

In the far-away wave-zone, keep only 1/r terms

$$h_{\mathcal{N}}^{00} = \frac{4GM}{c^2 r} + \frac{4G}{c^2} \sum_{\ell=2}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{\mathcal{I}^L(\tau)}{r} \right]$$

$$\Rightarrow \frac{4GM}{c^2 r} + \frac{4G}{rc^2} \sum_{\ell=2}^{\infty} \frac{n^L}{c^\ell \ell!} \frac{d^\ell}{d\tau^\ell} \mathcal{I}^L(\tau)$$

$$h_{\mathcal{N}}^{0j} = -\frac{2G}{c^3} \frac{(\mathbf{n} \times \mathbf{J}_0)^j}{r^2} - \frac{2G}{c^3} \partial_k \left[\frac{\dot{\mathcal{I}}^{jk}(\tau)}{r} \right] + \frac{4G}{c^4} \sum_{\ell=2}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{0j}(\tau, \mathbf{x}') x'^L d^3 x' \right]$$

$$\Rightarrow \frac{2G}{rc^4} n_k \ddot{\mathcal{I}}^{jk}(\tau) + \frac{4G}{rc^4} \sum_{\ell=2}^{\infty} \frac{n^L}{c^\ell \ell!} \frac{d^\ell}{d\tau^\ell} \mathcal{M}^{0jL}(\tau)$$

$$h_{\mathcal{N}}^{jk} = \frac{2G}{c^4} \frac{\ddot{\mathcal{I}}^{jk}(\tau)}{r} + \frac{4G}{c^4} \sum_{\ell=1}^{\infty} \frac{(-1)^\ell}{\ell!} \partial_L \left[\frac{1}{r} \int_{\mathcal{M}} \tau^{jk}(\tau, \mathbf{x}') x'^L d^3 x' \right]$$

$$\Rightarrow \frac{2G}{rc^4} \ddot{\mathcal{I}}^{jk}(\tau) + \frac{4G}{rc^4} \sum_{\ell=1}^{\infty} \frac{n^L}{c^\ell \ell!} \frac{d^\ell}{d\tau^\ell} \mathcal{M}^{jkL}(\tau)$$

$$\partial_k r = n_k$$

$$\partial_k r^{-1} = -\frac{n_k}{r^2}$$

$$\partial_j n_k = \frac{1}{r} (\delta_{jk} - n_j n_k)$$

$$\partial_k \tau = -\frac{1}{c} n_k$$

$$h^{00} = \frac{4GM}{c^2 R} + \frac{G}{c^4 R} C(\tau, \mathbf{N})$$

$$h^{0j} = \frac{G}{c^4 R} D^j(\tau, \mathbf{N})$$

$$h^{jk} = \frac{G}{c^4 R} A^{jk}(\tau, \mathbf{N})$$



Far wave-zone fields & TT gauge

$$\partial_j h^{\alpha\beta} = -\frac{1}{c} N_j \partial_\tau h^{\alpha\beta} + O(R^{-2})$$

Harmonic gauge

$$\partial_\beta h^{\alpha\beta} = 0 = \frac{1}{c} (\partial_\tau h^{\alpha 0} - N_j \partial_\tau h^{\alpha j}) + O(R^{-2})$$

Integrating:

$$C = N_j D^j = N_j N_k A^{jk}$$

$$D^j = N_k A^{jk}$$

Gauge (coordinate) transformation

$$h^{\alpha\beta} \rightarrow h^{\alpha\beta} - \partial^\alpha \zeta^\beta - \partial^\beta \zeta^\alpha + (\partial_\mu \zeta^\mu) \eta^{\alpha\beta}$$

Stay within harmonic gauge as long as $\square \zeta^\alpha = 0$

$$\zeta^0 = \frac{G}{c^3 R} \alpha(\tau, \mathbf{N}) + O(R^{-2})$$

$$\zeta^j = \frac{G}{c^3 R} \beta^j(\tau, \mathbf{N}) + O(R^{-2})$$

$$h^{00} = \frac{4GM}{c^2 R} + \frac{G}{c^4 R} C(\tau, \mathbf{N})$$

$$h^{0j} = \frac{G}{c^4 R} D^j(\tau, \mathbf{N})$$

$$h^{jk} = \frac{G}{c^4 R} A^{jk}(\tau, \mathbf{N})$$

(from now on, $r \rightarrow R$, $n_j \rightarrow N_j$)



Far wave-zone fields & TT gauge

After a gauge transformation:

$$h^{00} \rightarrow \frac{4GM}{c^2 R} + \frac{G}{c^4 R} (C + \dot{\alpha} + N_j \dot{\beta}^j)$$

$$h^{0j} \rightarrow \frac{G}{c^4 R} (D^j + \dot{\beta}^j + N^j \dot{\alpha})$$

$$h^{jk} \rightarrow \frac{G}{c^4 R} \left[A^{jk} + 2N^{(j} \dot{\beta}^{k)} + \delta^{jk} (\dot{\alpha} - N_m \dot{\beta}^m) \right]$$

Insert β^j

$$h^{jk} \rightarrow \frac{G}{c^4 R} \left[A^{jk} - 2\dot{\alpha} N^j N^k - 2N^{(j} N_n A^{k)n} + \delta^{jk} (2\dot{\alpha} + N_m N_n A^{mn}) \right]$$

Note that $h = h^k_k = \frac{G}{c^4 R} [A + 4\dot{\alpha} + N_k N_n A^{kn}]$

Free to choose α so that $h = 0$: $\dot{\alpha} = -\frac{1}{4}(A + N_k N_n A^{kn})$

Final answer:

$$h^{jk} = \frac{G}{c^4 R} A_{mn} \left[P^{jm} P^{kn} - \frac{1}{2} P^{jk} P^{mn} \right]$$

$$\equiv \frac{G}{c^4 R} A_{\text{TT}}^{jk}$$

To kill the h^{00} & h^{0j} terms:

$$\dot{\beta}^j + N^j \dot{\alpha} = -D^j = -N_m A^{jm}$$

$$N_j \dot{\beta}^j + \dot{\alpha} = -N_j D^j = -C$$

$$N_k h^{jk} = 0$$

$$P^{jm} = \delta^{jm} - N^j N^m$$

$$P^{jm} P^{jn} = P^{mn}$$

$$P^{kk} = 2$$



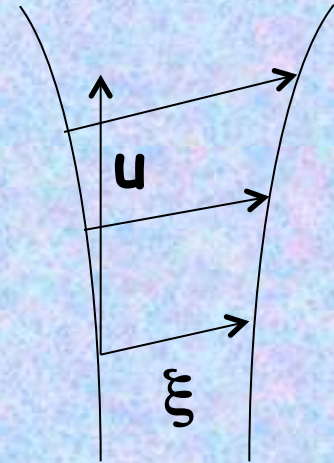
Wave zone physics: Gravitational waves

Geodesic deviation:
$$\frac{D^2 \xi^\alpha}{ds^2} = -R^\alpha_{\beta\gamma\delta} u^\beta \xi^\gamma u^\delta$$

In the rest frame of an observer

$$\begin{aligned} \frac{d^2 \xi^j}{dt^2} &= -c^2 R_{0j0k} \xi^k \\ &= \frac{1}{2} \partial_{\tau\tau} h_{TT}^{jk} \xi^k \\ &= \frac{G}{2c^4 R} \ddot{A}_{TT}^{jk} \xi^k \end{aligned}$$

$$\xi^j(t) = \xi^j(0) + \frac{1}{2} h_{TT}^{jk}(t - R/c) \xi_k(0)$$



Wave zone physics: Gravitational waves

Wave in the z-direction

$$A_{\text{TT}}^{jk} = \begin{pmatrix} A_+ & A_\times & 0 \\ A_\times & -A_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

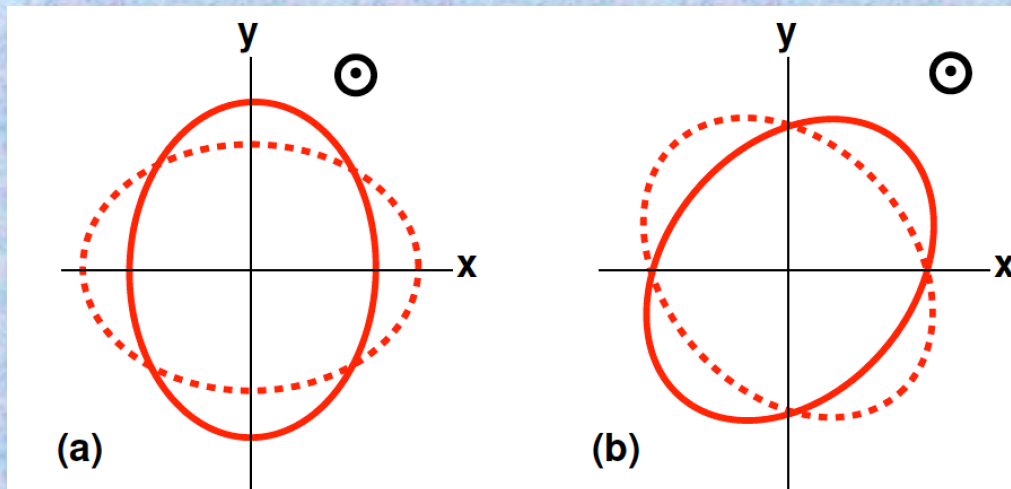
More generally: $e_Z = N$

$$A_{\text{TT}}^{jk} = (e_X^j e_X^k - e_Y^j e_Y^k) A_+ + (e_X^j e_Y^k + e_Y^j e_X^k) A_\times$$

$$x(t) = x_0 + \frac{G}{2c^4 R} (A_+ x_0 + A_\times y_0),$$

$$y(t) = y_0 + \frac{G}{2c^4 R} (A_\times x_0 - A_+ y_0),$$

$$z(t) = z_0$$



Wave zone physics: The quadrupole formula

Requires two iterations of the relaxed Einstein equation:

$$\begin{aligned} h_{2\mathcal{N}}^{ij} &= \frac{4G}{c^4} \int \frac{\tau^{ij}(h_1)(t - |\mathbf{x} - \mathbf{x}'|/c, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' \\ &= \frac{2G}{rc^4} \ddot{\mathcal{I}}^{jk}(h_1)(\tau) + \frac{4G}{rc^4} \sum_{\ell=1}^{\infty} \frac{n^L}{c^\ell \ell!} \frac{d^\ell}{d\tau^\ell} \mathcal{M}^{jkL}(h_1)(\tau) \\ &= \frac{2G}{rc^4} \ddot{\mathcal{I}}^{jk}(h_1)(\tau) + O(c^{-5}) \end{aligned}$$

$$\begin{aligned} \mathcal{I}^{jk}(\tau) &:= \int_{\mathcal{M}} c^{-2} \tau^{00}(\tau, \mathbf{x}') x'^j x'^k d^3x' \\ &= \int_{\mathcal{M}} \rho^*(\tau, \mathbf{x}') x'^j x'^k d^3x' + O(c^{-2}) \end{aligned}$$

For an N-body system

$$\ddot{\mathcal{I}}^{ij} = 2 \sum_A M_A v_A^{ij} - \sum_{A \neq B} \frac{GM_A M_B}{r_{AB}} n_{AB}^{ij}$$

- ❑ By convention, the quadrupole formula is called the ``Newtonian"-order result
- ❑ Higher order PN corrections can be calculated by further iterating the relaxed equations
- ❑ 3 iterations needed for 1 & 1.5 PN order, 4 for 2PN order etc, but full details usually not needed

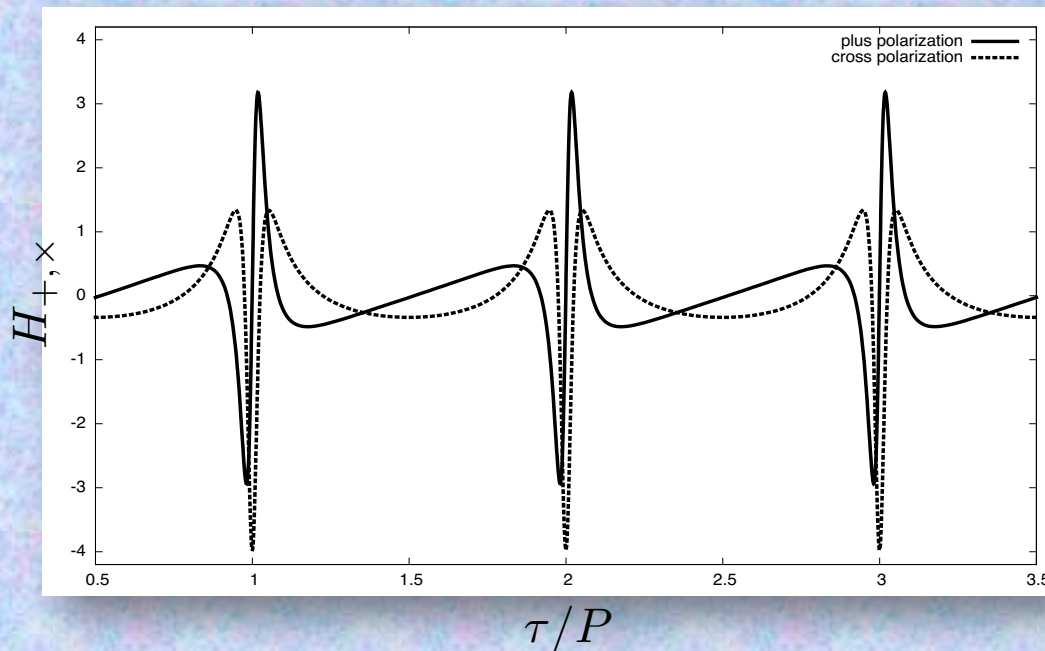
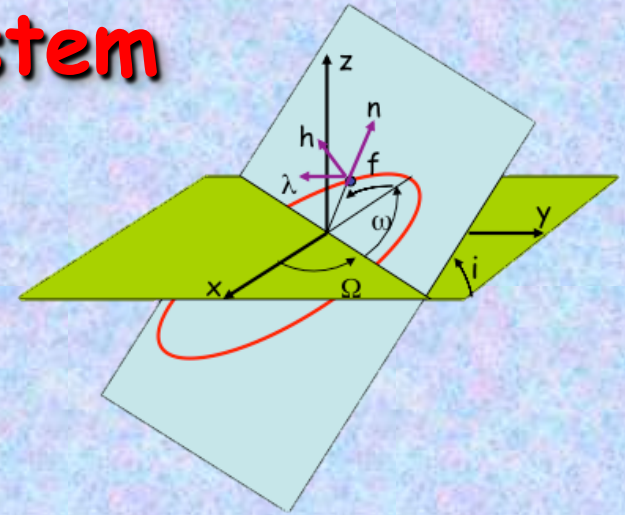


Waves from a binary system

$$h_{+(\times)} = \frac{2Gm\eta}{c^2 R} \left(\frac{Gm}{c^2 p} \right) H_{+(\times)}$$

$$H_+ = -(1 + \cos^2 \iota) \left[\cos(2\phi + 2\omega) + \frac{5}{4}e \cos(\phi + 2\omega) + \frac{1}{4}e \cos(3\phi + 2\omega) + \frac{1}{2}e^2 \cos 2\omega \right] + \frac{1}{2}e \sin^2 \iota (\cos \phi + e),$$

$$H_\times = -2 \cos \iota \left[\sin(2\phi + 2\omega) + \frac{5}{4}e \sin(\phi + 2\omega) + \frac{1}{4}e \sin(3\phi + 2\omega) + \frac{1}{2}e^2 \sin 2\omega \right]$$



$$e = 0.7$$

$$\iota = 30^\circ$$

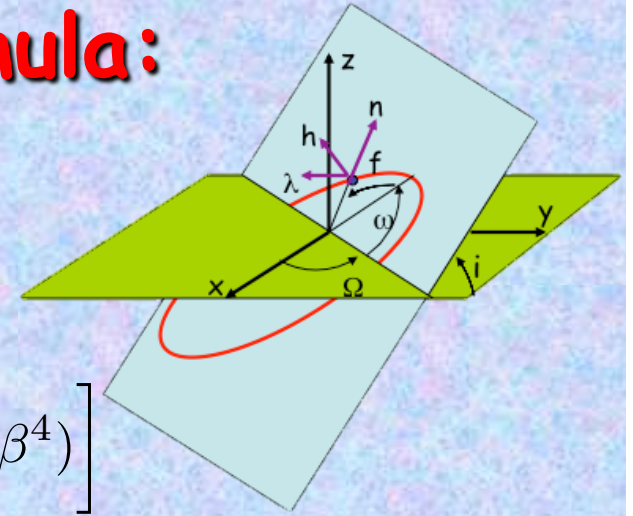
$$\omega = 45^\circ$$



Beyond the quadrupole formula:

For a binary system in a circular orbit:

$$h_{+, \times} = \frac{2\eta Gm}{c^2 R} \beta^2 \left[(1 + 2\pi\beta^3) H_{+, \times}^{[0]} + \Delta\beta H_{+, \times}^{[1/2]} + \beta^2 H_{+, \times}^{[1]} + \Delta\beta^3 H_{+, \times}^{[3/2]} + \beta^3 H_{+, \times}^{\text{tail}} + O(\beta^4) \right]$$



$$H_{\times}^{[0]} = -2C \sin 2\Psi,$$

$$H_{\times}^{[1/2]} = -\frac{3}{4}SC \sin \Psi + \frac{9}{4}SC \sin 3\Psi,$$

$$H_{\times}^{[1]} = \frac{1}{3}C \left[(17 - 4C^2) - (13 - 12C^2)\eta \right] \sin 2\Psi - \frac{8}{3}(1 - 3\eta)S^2C \sin 4\Psi,$$

$$H_{\times}^{[3/2]} = \frac{1}{96}SC \left[(63 - 5C^2) - 2(23 - 5C^2)\eta \right] \sin \Psi$$

$$- \frac{9}{64}SC \left[(67 - 15C^2) - 2(19 - 15C^2)\eta \right] \sin 3\Psi + \frac{625}{192}(1 - 2\eta)S^3C \sin 5\Psi,$$

$$C = \cos \iota$$

$$S = \sin \iota$$

$$\beta = \left(\frac{Gm\Omega}{c^3} \right)^{1/3} \sim \frac{v}{c}, \quad m = M_1 + M_2, \quad \eta = \frac{M_1 M_2}{(M_1 + M_2)^2}, \quad \Delta = \frac{M_1 - M_2}{M_1 + M_2}$$



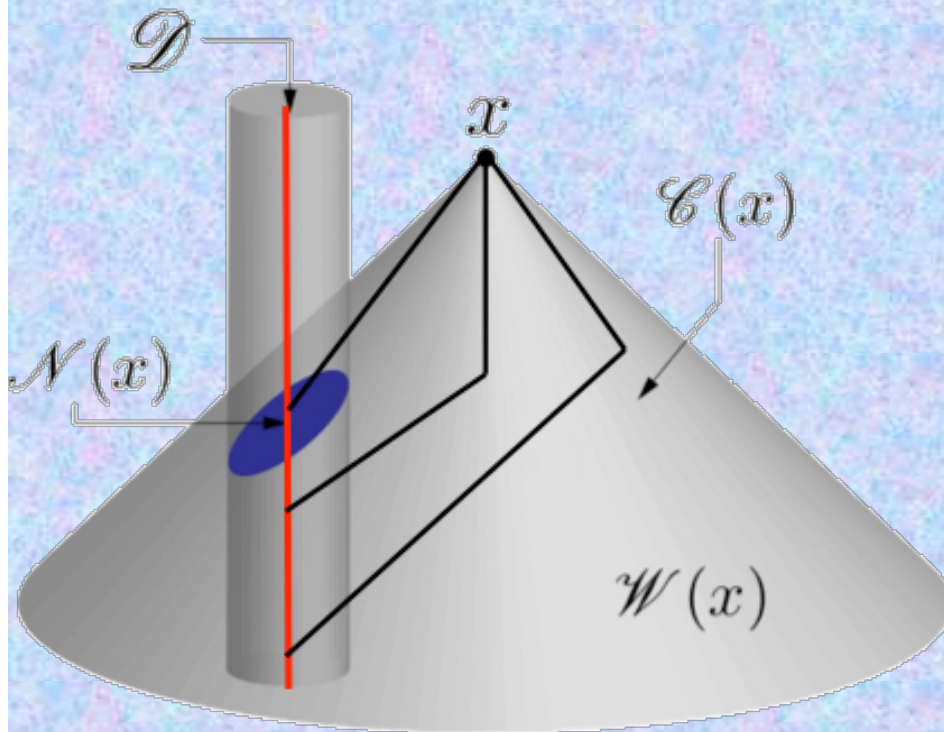
Beyond the quadrupole formula:

$$h_{+, \times} = \frac{2\eta Gm}{c^2 R} \beta^2 \left[(1 + 2\pi\beta^3) H_{+, \times}^{[0]} + \Delta\beta H_{+, \times}^{[1/2]} + \beta^2 H_{+, \times}^{[1]} + \Delta\beta^3 H_{+, \times}^{[3/2]} + \beta^3 H_{+, \times}^{\text{tail}} + O(\beta^4) \right]$$

$$\beta = \left(\frac{Gm\Omega}{c^3} \right)^{1/3}$$

$$H_{\times}^{[0]} = -2C \sin 2\Psi$$

$$H_{\times}^{\text{tail}} = 8C [\gamma + \ln(4\Omega R/c)] \cos 2\Psi$$



Combine with the leading term:

$$\sin 2\Psi - 4 \frac{Gm\Omega}{c^3} \ln(4\Omega R/c) \cos 2\Psi$$

$$= \sin \left[2\Omega \left(t - R/c - 2 \frac{Gm}{c^3} \ln(4\Omega R/c) \right) \right]$$

Shapiro
time delay



Gravitational-wave estimates

$$h_{\text{TT}}^{jk} = \frac{2G}{c^4 R} \ddot{I}_{\text{TT}}^{jk}$$

$$\sim \frac{GM}{c^2 R} (v_c/c)^2 \sim 4.8 \times 10^{-19} \left(\frac{M}{10 M_\odot} \right) \left(\frac{1 \text{ Mpc}}{R} \right) (v_c/c)^2$$

Binary system

$$(v_c/c)^2 \sim GM/c^2 a \sim (2\pi GM/c^3 P)^{2/3}$$

$$h_0 \simeq \frac{3.0 \times 10^{-21}}{1 - e^2} \left(\frac{\mathcal{M}}{20 M_\odot} \right)^{5/3} \left(\frac{10 \text{ ms}}{P} \right)^{2/3} \left(\frac{100 \text{ Mpc}}{R} \right)$$

Chirp mass

$$\mathcal{M} \equiv \eta^{3/5} m = \left(\frac{m_1^3 m_2^3}{m} \right)^{1/5}$$

Deformed neutron star

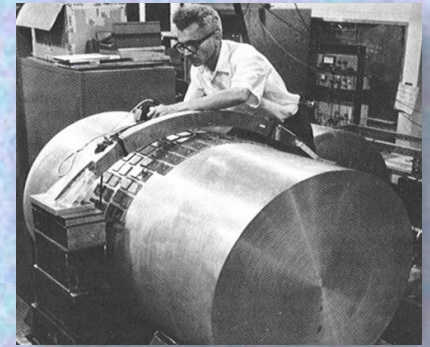
$$h_0 \simeq 6.8 \times 10^{-25} \left(\frac{\varepsilon}{10^{-6}} \right) \left(\frac{I_3}{1.6 \times 10^{38} \text{ kg m}^2} \right) \left(\frac{10 \text{ ms}}{P} \right)^2 \left(\frac{1 \text{ kpc}}{R} \right)$$

Supernova core collapse

$$h_0 \sim 4.8 \times 10^{-21} \left(\frac{\varepsilon}{10^{-3}} \right) \left(\frac{M}{M_\odot} \right) \left(\frac{10 \text{ kpc}}{R} \right)$$



Joseph Weber & Gravitational Waves: Case History in How Science Works



Joe Weber (1919-2000)

- pioneer in electronics, masers and lasers
- 1959 - learned GR, began building GW detectors
- 1969 - reported coincident events in detectors 1000 km apart
- 1971 - more events when bars were perpendicular to galactic center
- strength of signals 1000 times stronger than anybody expected
- 1970s - spurred theoretical work on black holes, neutron stars, supernovas
 - ✓ could not explain Weber's events
- 1970s - spurred other groups to build detectors of better sensitivity
 - ✓ no events detected
- 1980 - consensus developed - no GW seen directly by Weber
- 1980 - 2004 - work on advanced "Weber bars" continued
- mid 1980s - plans developed for large "laser interferometer" observatories
- 1994 - 1999 - construction of LIGO
- 2002 - LIGO's first science run
- 2003 - 2008 - Weber bar detectors phased out
- 2010 - 2015 - upgrade to advLIGO-advVIRGO
- 2015, Sept 14 - detection of waves from binary black hole inspiral

Laser interferometers

Phase difference between 2 arms:

$$\Delta\Phi = 2\pi\nu(2L_1/c - 2L_2/c)$$

Displacement due to the wave:

$$\xi_1^j = L_0 \left(e_1^j + \frac{1}{2} h_{\text{TT}}^{jk} e_1^k \right)$$

$$\xi_2^j = L_0 \left(e_2^j + \frac{1}{2} h_{\text{TT}}^{jk} e_2^k \right)$$

$$\Delta\Phi = \frac{4\pi\nu L_0}{c} \frac{1}{2} (e_1^j e_1^k - e_2^j e_2^k) h_{\text{TT}}^{jk} = \frac{4\pi\nu G L_0}{c^5 R} S(t)$$

$$S(t) = \frac{1}{2} (e_1^j e_1^k - e_2^j e_2^k) A_{\text{TT}}^{jk}(\tau, \mathbf{N})$$

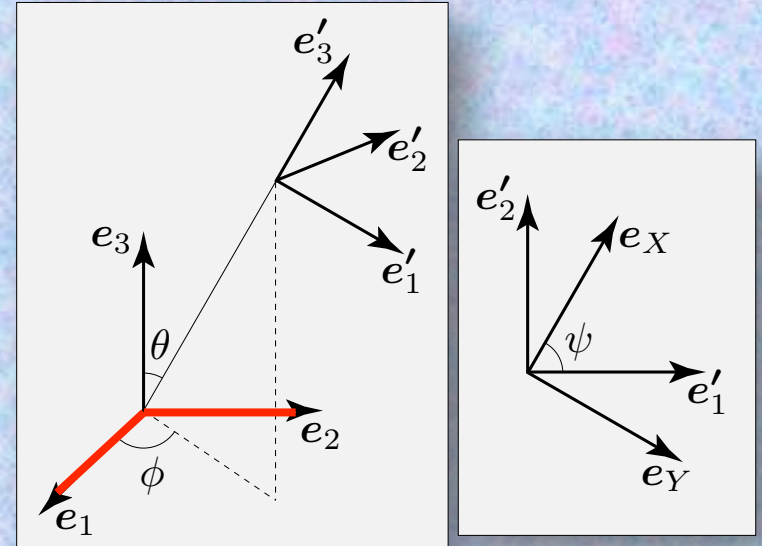
$$= F_+ A_+(t - R/c) + F_\times A_\times(t - R/c)$$

$$A_{\text{TT}}^{jk} = (e_X^j e_X^k - e_Y^j e_Y^k) A_+ + (e_X^j e_Y^k + e_Y^j e_X^k) A_\times$$

Detector pattern functions

$$F_+ := \frac{1}{2} (e_1^j e_1^k - e_2^j e_2^k) (e_X^j e_X^k - e_Y^j e_Y^k) = \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi$$

$$F_\times := \frac{1}{2} (e_1^j e_1^k - e_2^j e_2^k) (e_X^j e_Y^k + e_Y^j e_X^k) = \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi$$



GW interferometers

LIGO Hanford 4&2 km



GEO Hannover 600 m



Kagra Japan 3 km



LIGO Livingston 4 km

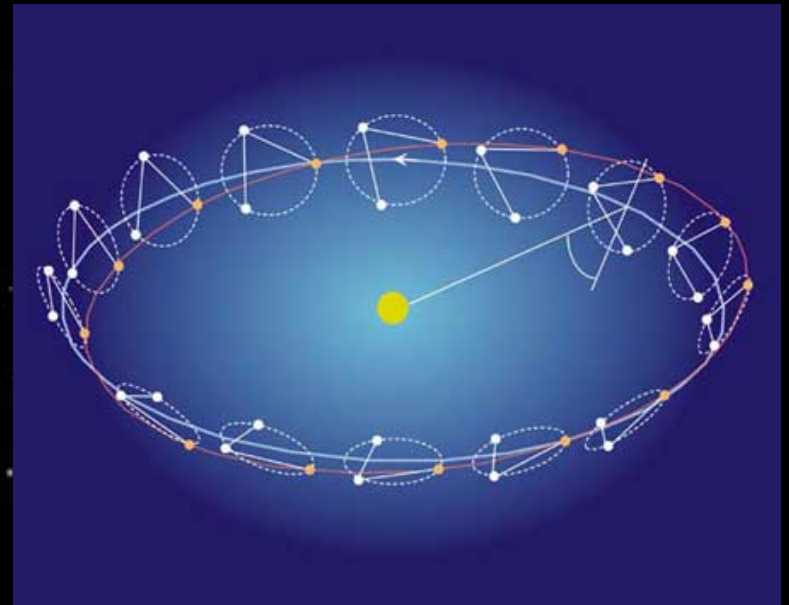
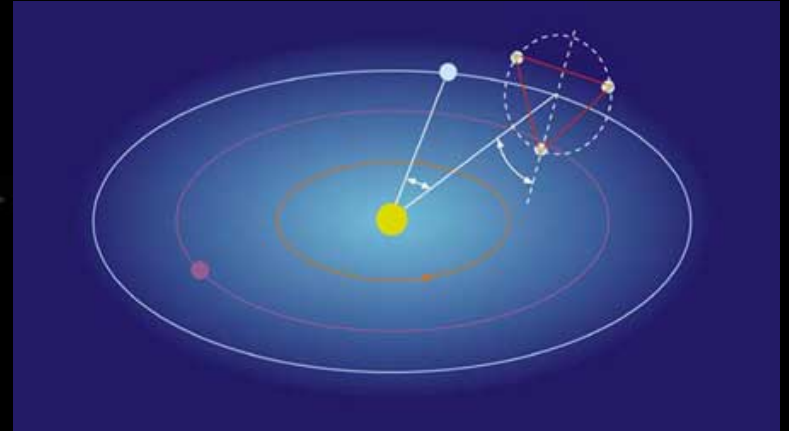
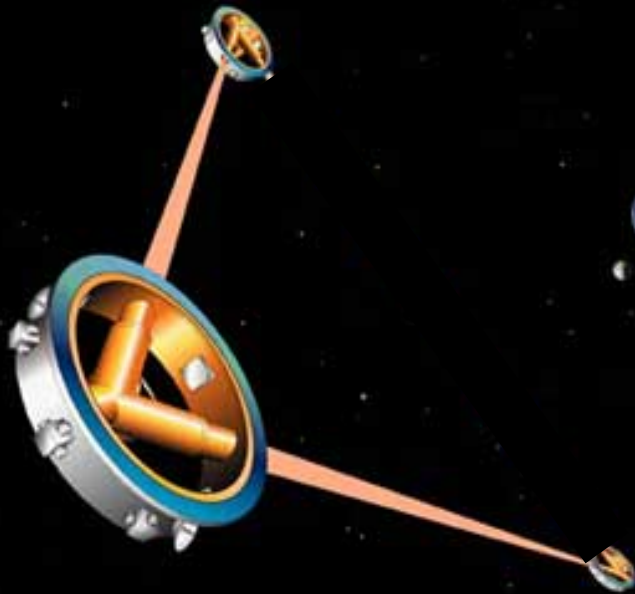


Virgo Cascina 3 km

LIGO South Indigo



eLISA: a European space interferometer



Pulsar Timing Arrays



Green Bank Telescope, WV, US



Arecibo Observatory, PR, US



Nancay Radio Telescope, Nancay, France



Lovell Telescope, Cheshire, UK



Parkes Observatory,
Parkes,
Australia



LOFAR, Exloo, Netherlands



GMRT, Pune, India



WSRT, Westerbork,
Netherlands

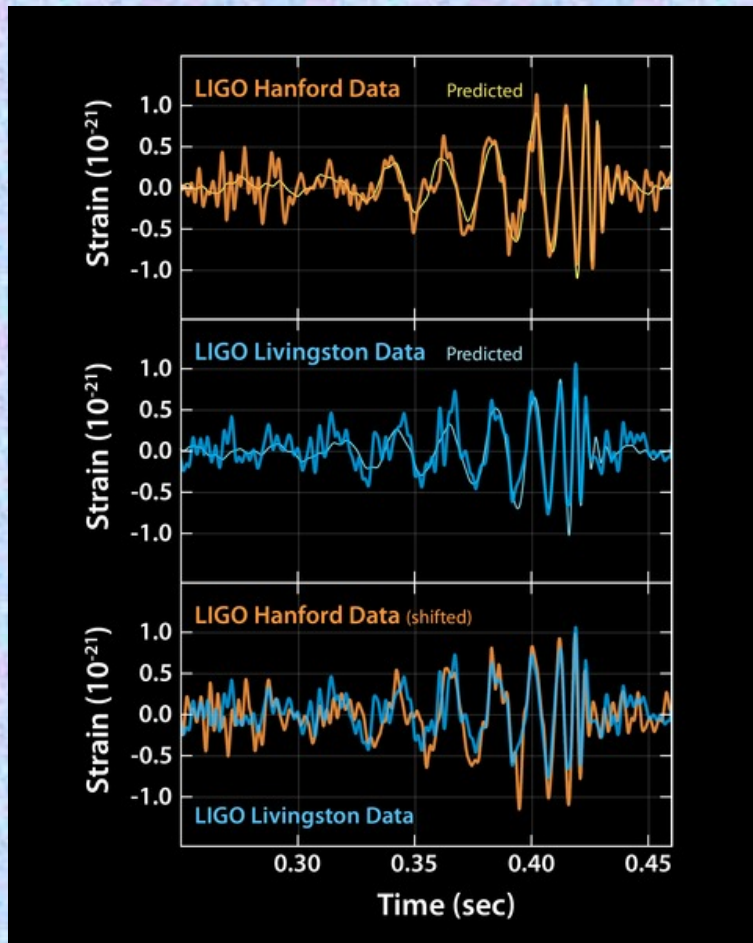


Effelsberg 100-m Radio
Telescope, Effelsberg, Germany



LIGO detects gravitational waves!

- ◆ Advanced LIGO began operations in summer 2015
- ◆ 14 September, near the end of an "engineering run"
- ◆ Signals in both LIGO detectors, 7 milliseconds apart



$$m_1 = 36^{+5}_{-4} M_{\odot}$$

$$m_2 = 29 \pm 4 M_{\odot}$$

$$m_F = 62 \pm 4 M_{\odot}$$

$$S_F / m_F^2 = 0.67^{+0.05}_{-0.07}$$

$$D = 1.3 \text{ bly}$$

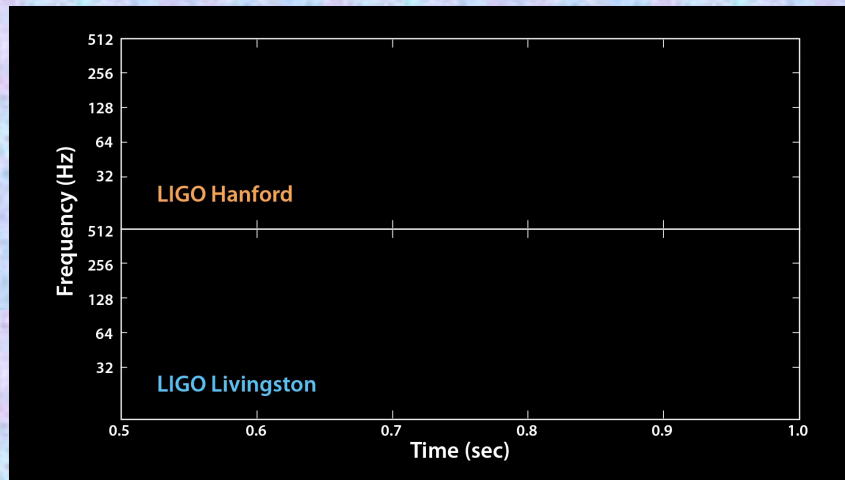
$$L_{\text{max}} = 3.6 \times 10^{56} \text{ erg/s}$$

GW150914: a binary
black hole merger



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Audio Credit: Caltech/MIT/LIGO Lab

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$$S_F / m_F^2 = 0.67^{+0.05}_{-0.07}$$

$$D = 1.3 \text{ bly}$$

$$L_{\text{max}} = 3.6 \times 10^{56} \text{ erg/s}$$

- ◆ SNR ~ 24 , $\sigma \sim 5.1$, false alarm 1/203,000 yr
- ◆ 3 solar masses converted into energy in 0.2 sec
- ◆ More power than all the stars in the universe
- ◆ 1st detection of a BBH, masses larger than expected
- ◆ Revised event rate 2 - 400 /Gpc³/yr
- ◆ Data on GW150914 publicly available

