Gravitational waves and their interaction with detectors

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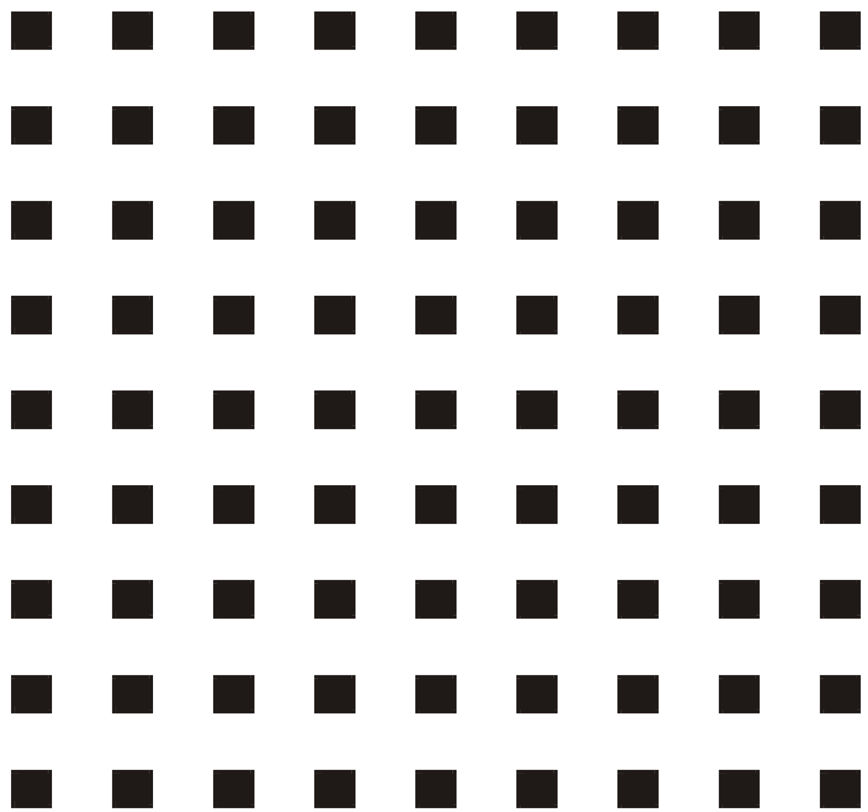
Outline

- Goals and methods of these lectures
- What is a gravitational wave?
- How does a gravitational wave interact with an ideal detector?
- Why is gravitational wave detection hard?

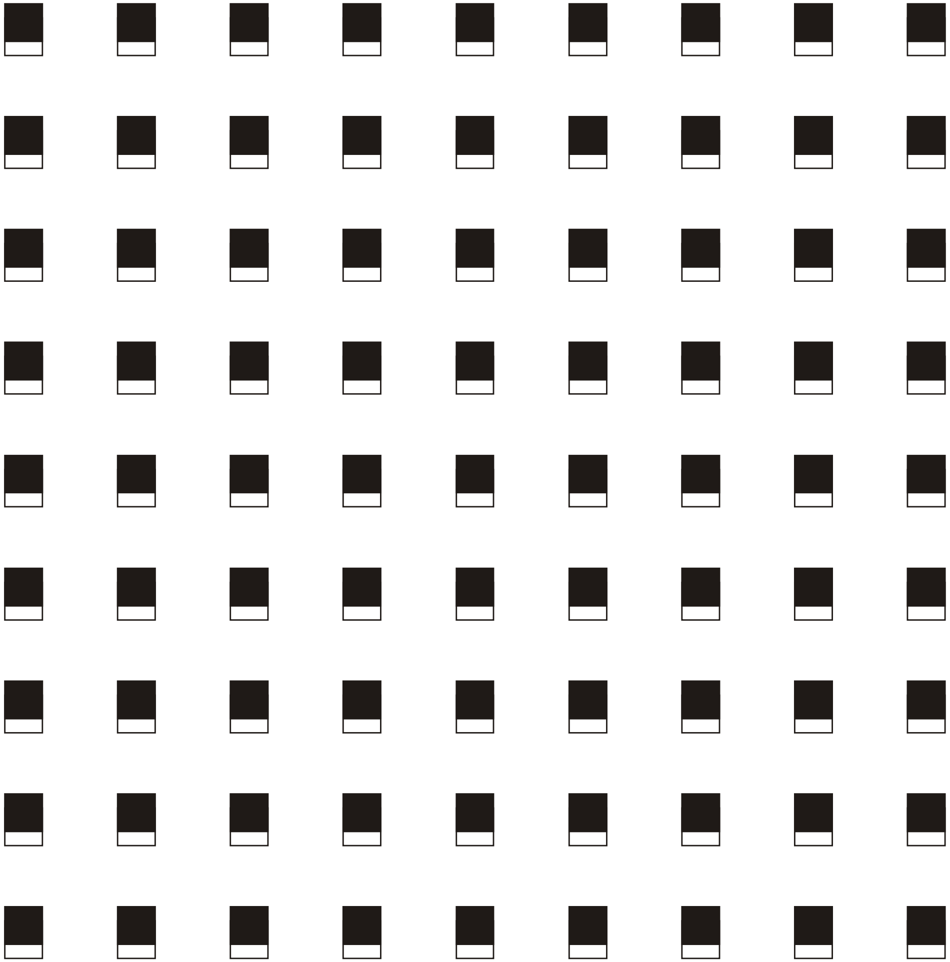
Goals and methods of this set of lectures

1. I want to build concrete (and intuitive) physical understanding of how to detect gravitational waves.
2. Beautiful mathematics has its place, but not here. Here, math is in service to goal #1.
3. Numbers are important.
4. Because of small numbers, many physical phenomena come into play in gravitational wave detection. The field is rich!
5. Experimental physics is a noble calling, and nowhere nobler than in gravitational wave detection.

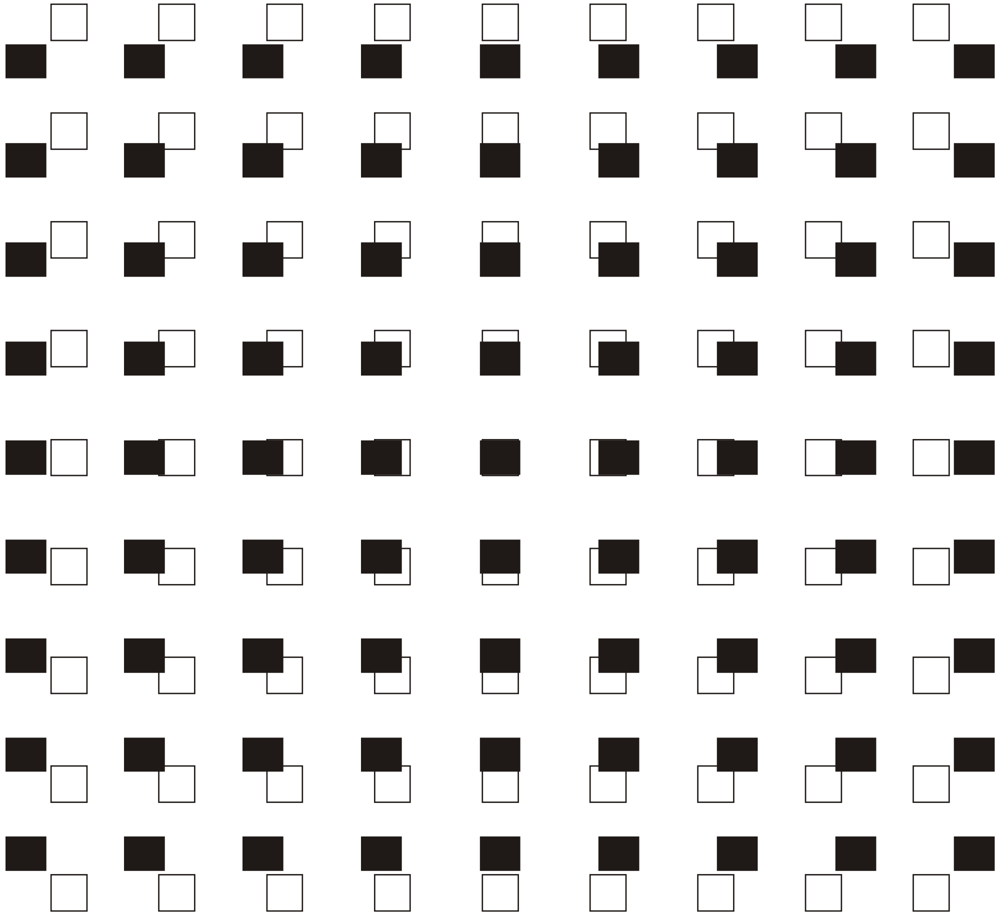
A set of freely-falling test particles



Electromagnetic wave moves charged test bodies

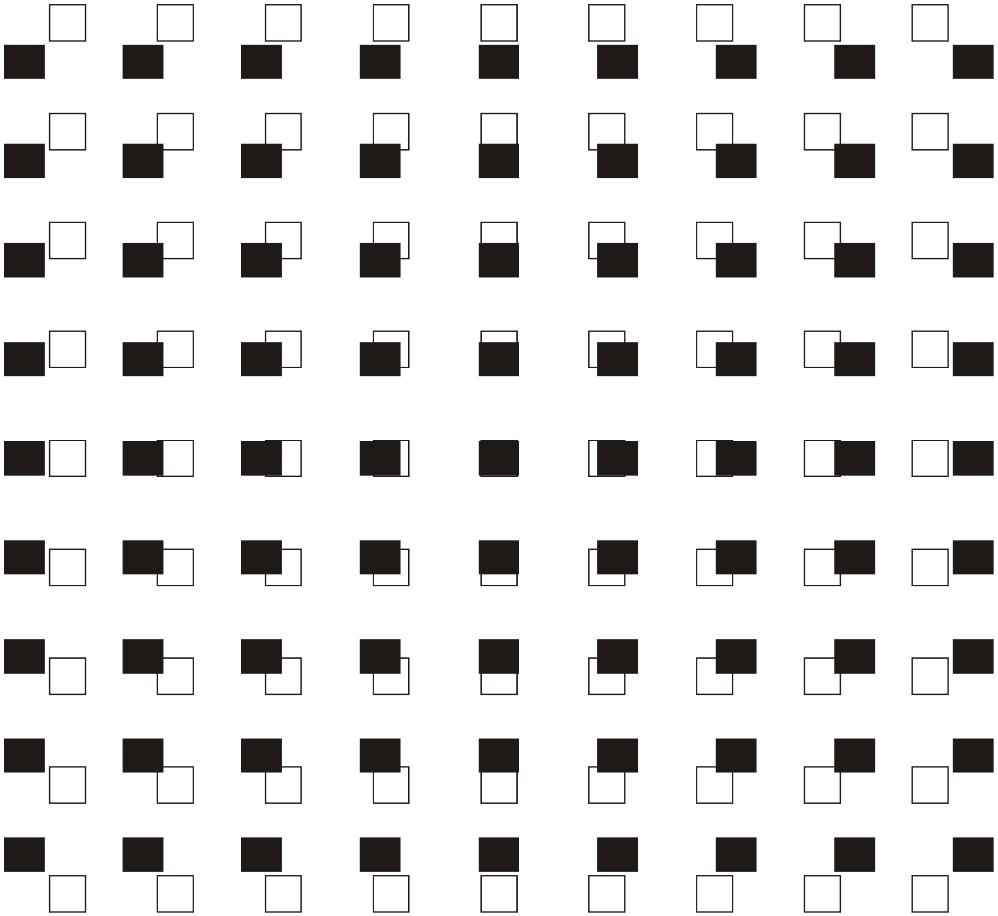


Gravity wave: distorts set of test masses in transverse directions



Gravitational wave:
a transverse quadrupolar strain

strain amplitude:
$h = 2\Delta L/L$



Gravitational waveform lets you read out source dynamics

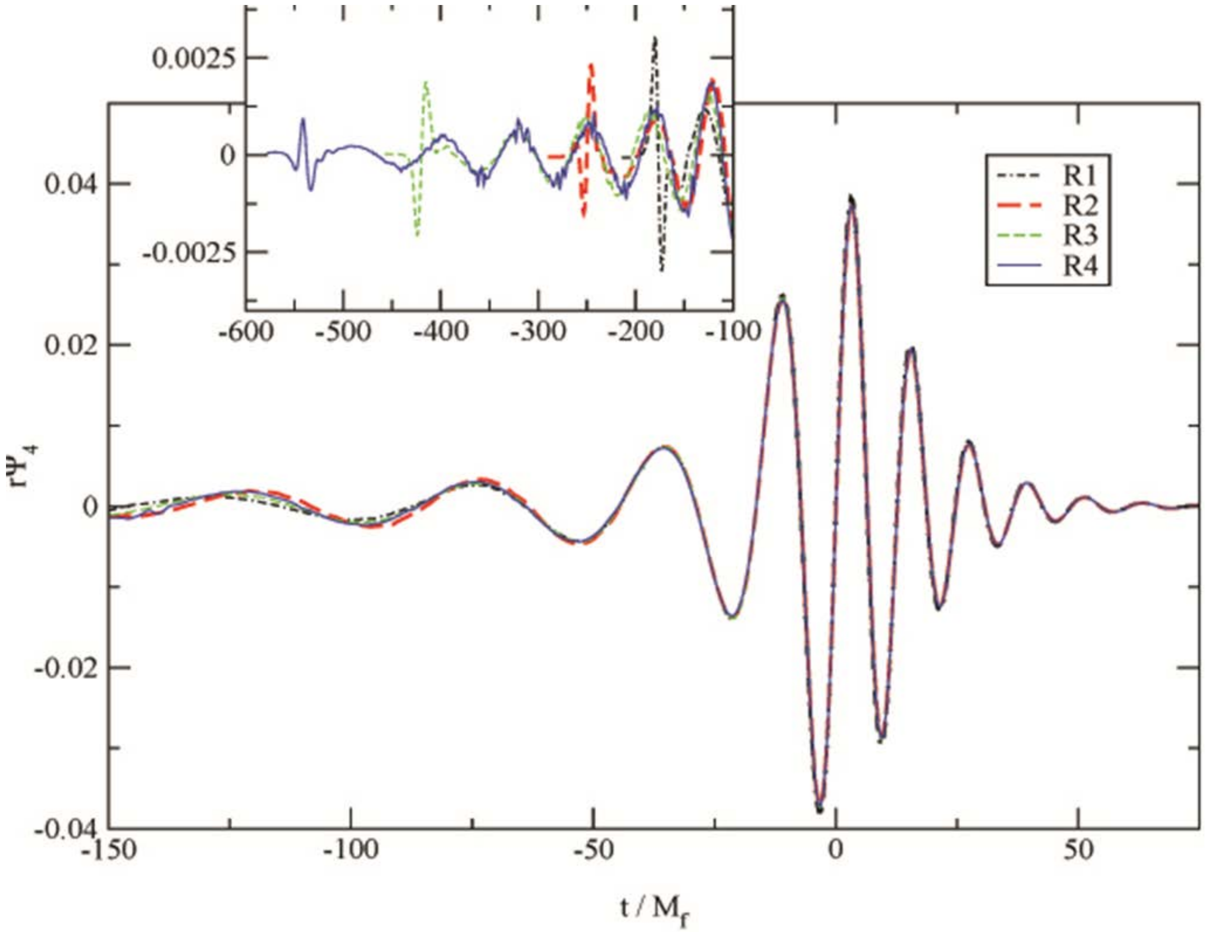
The evolution of the mass distribution can be read out from the gravitational waveform:

$$h_{\mu\nu}(t) = \frac{1}{R}\frac{2G}{c^4}\ddot{I}_{\mu\nu}(t - R/c)$$

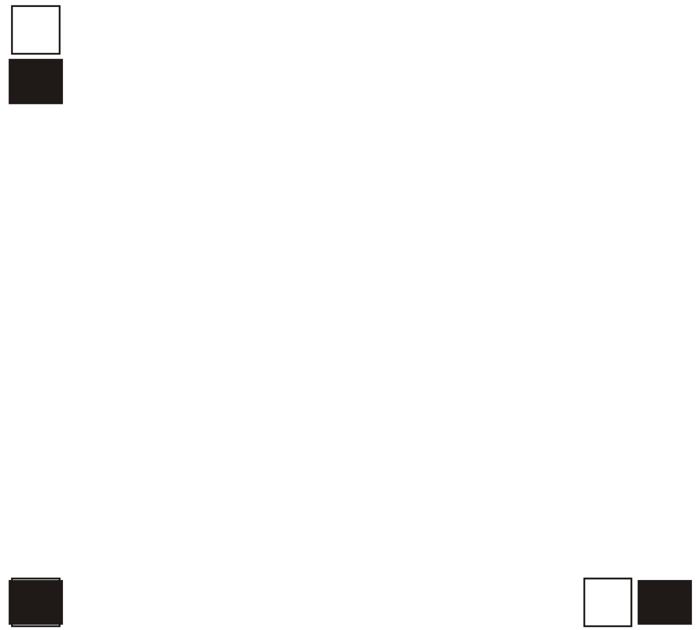
Coherent relativistic motion of large masses can be directly observed from the waveform!

$$I_{\mu\nu} \equiv \int dV\left(x_\mu x_\nu - \delta_{\mu\nu} r^2/3\right)\rho(r).$$

Gravitational waveform = oscillation pattern of test masses



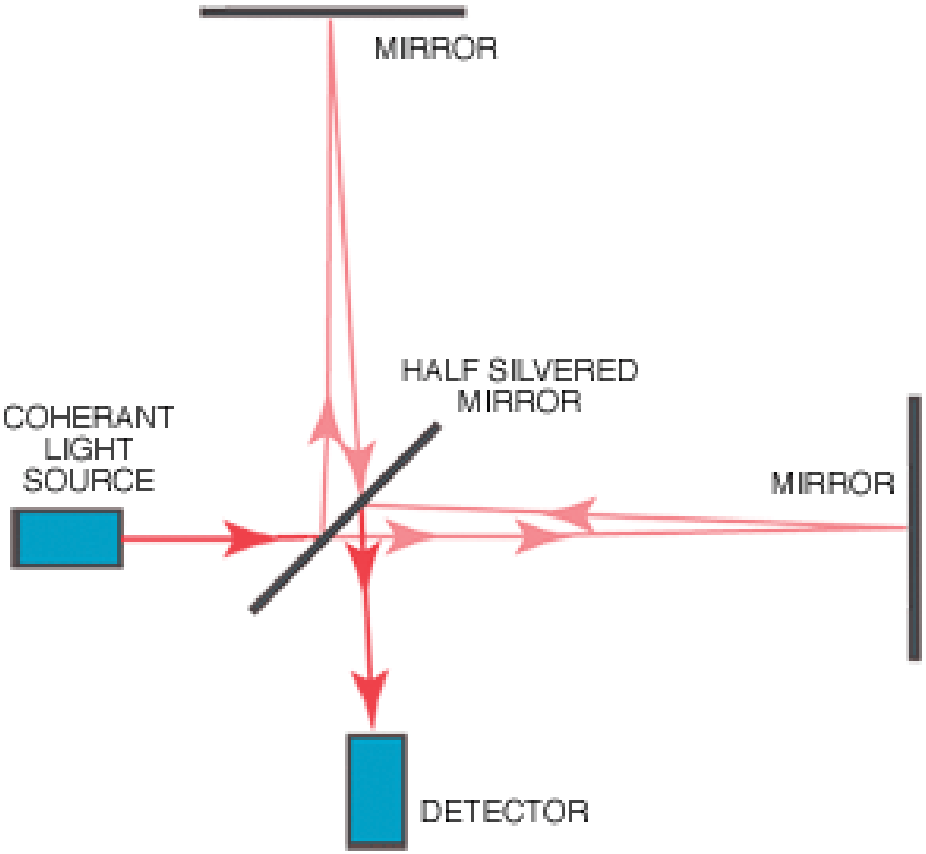
A more modern detection strategy



Tidal character of wave argues for test masses as far apart as practicable. Perhaps masses hung as pendulums, kilometers apart.

Sensing relative motions of distant free masses

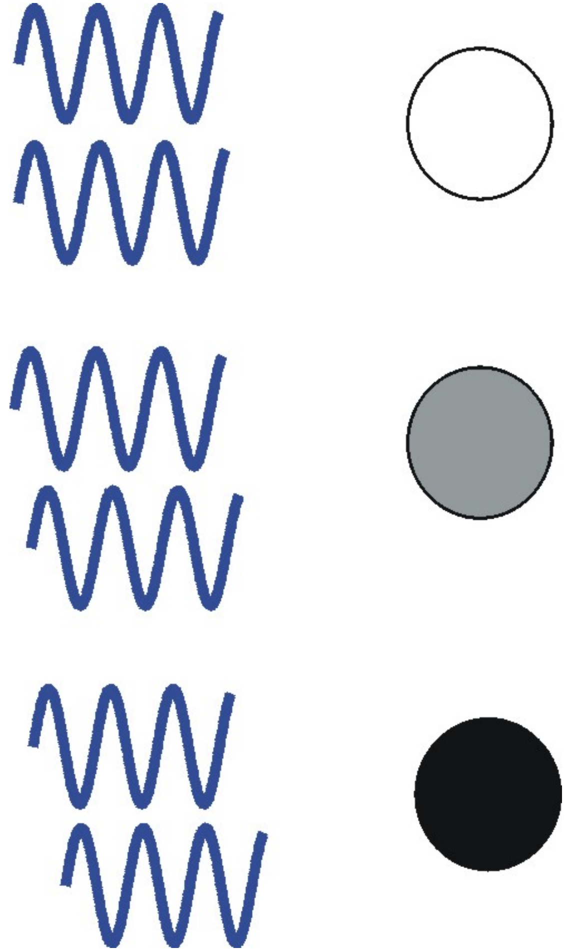
Michelson interferometer



A length-difference-to-brightness transducer

Wave from x arm.

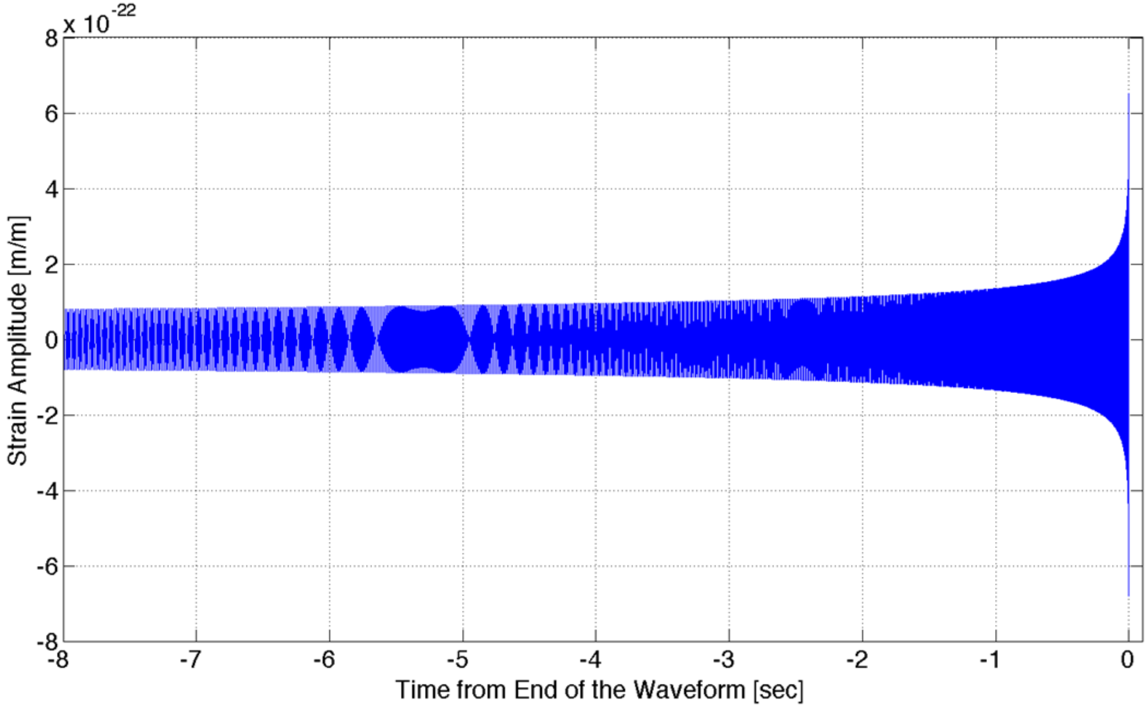
Wave from y arm.



Light exiting from beam splitter.

As relative arm lengths change, interference causes change in brightness at output.

Since we understand general relativity, we can calculate waveforms



Stellar-mass objects give signals in the audio band. (!)

Distance measurement in relativity…

… is done most straightforwardly by measuring the light travel time along a round-trip path from one point to another. (Felix Pirani, 1956)

- Because the speed of light is the same for all observers.

Examples:

- light clock
- Einstein's train *gedanken* experiment

The *space-time interval* in special relativity

Special relativity says that the interval

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

between two events is *invariant* (and thus worth paying attention to.)

In shorthand, we write it as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ with the Minkowski *metric* given as

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Generalize a little

General relativity says almost the same thing, except the metric can be different.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The trick is to find a metric $g_{\mu\nu}$ that describes a particular physical situation.

The metric carries the information on the space-time curvature that, in GR, embodies gravitational effects.

Gravitational waves

Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

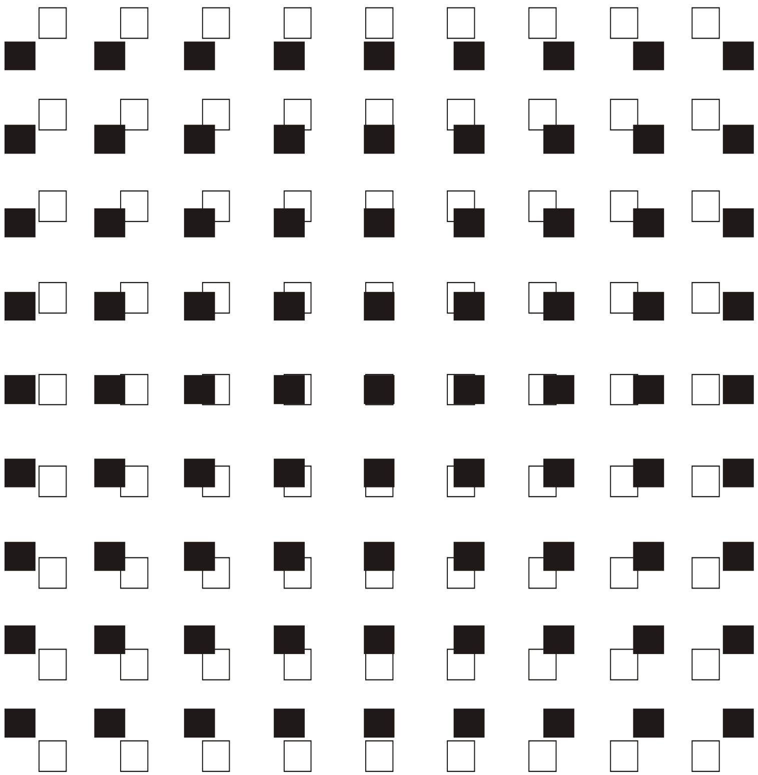
A wave propagating in the *z*-direction is described by

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Two free parameters implies two polarizations

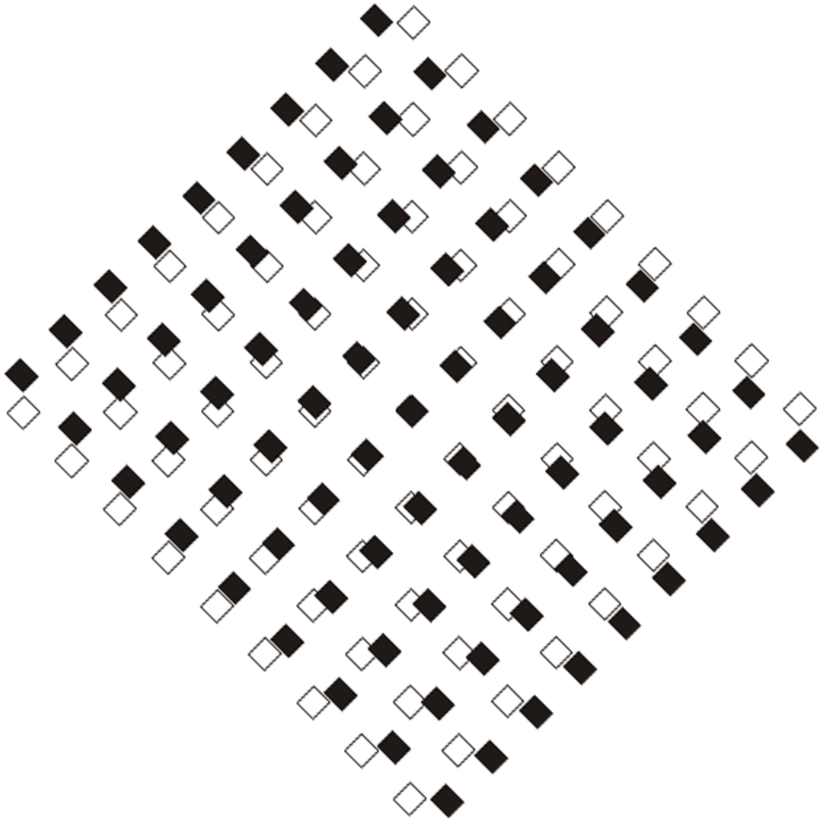
Plus polarization

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

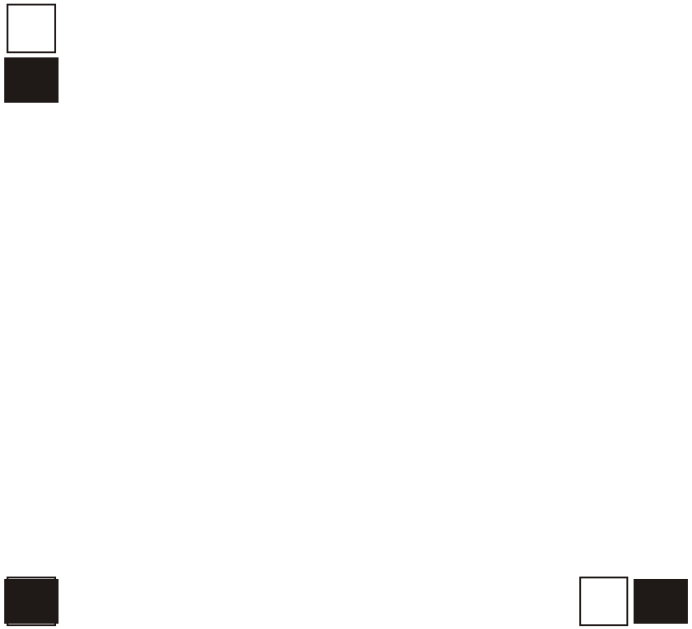


Cross polarization

$$\hat{h}_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Three test masses



Solving for variation in light travel time

For light moving along the x axis, we are interested in the interval between points with non-zero *dx* and *dt*, but with *dy* = *dz* = 0:

$$ds^2 = 0 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2$$

Gravity wave detectors

Need:

- A set of test masses,
- Instrumentation sufficient to see tiny motions,
- Isolation from other causes of motions.

Challenge:

Best astrophysical estimates predict fractional separation changes of only 1 part in 10^{21}, or less.

Laser Interferometer Gravitational Wave Observatory

4-km Michelson interferometers, with mirrors on pendulum suspensions, at Livingston LA and Hanford WA.

Initial LIGO had $h_{rms} \sim 10^{-21}$.

Advanced LIGO will be 10x more sensitive.

Other large interferometers

- Virgo (Italy, France, etc.) 3 km

  Advanced Virgo upgrade now under way; will be almost as sensitive as Advanced LIGO, almost as soon.

- GEO (Germany, Britain), 600 m

  Now studying squeezing and doing some "astrowatch" observing. Will continue upgrades through the advanced detector era.

- KAGRA (Japan), 3 km

  Underground cryogenic detector, now under construction.

- and, LIGO-India, 4 km !

Gravity wave detection: challenge and promise

Challenges of gravity wave detection appear so great as to make success seem almost impossible.

The challenges are real, but are being overcome.

Gravitational wave detection is almost impossible

What is required for LIGO to succeed:

- interferometry with free masses,
- with strain sensitivity of 10^{-21} (or better!),
- (which is equivalent to ultra-subnuclear position sensitivity),
- in the presence of much larger noise.

Interferometry with free masses

What's "impossible": everything!

Mirrors need to be very accurately aligned (so that beams overlap and interfere) and held very close to an operating point (so that output is a linear function of input.)

Otherwise, interferometer is dead or swinging through fringes.

Michelson bolted everything down.

Strain sensitivity of 10^{-21}

Why it is "impossible":

Sensitivity h_{rms} can be expressed as

$$h_{rms} \sim \frac{\text{precision to which we can compare arm lengths}}{\text{length of arms}}.$$

Natural "tick mark" on interferometric ruler is one wavelength.

Michelson could read a fringe to $\lambda/20$, yielding h_{rms} of a few times 10^{-9}.

Ultra-subnuclear position sensitivity

Why people thought it was impossible:

- Mirrors made of atoms, 10^{-10} m.
- Mirror surfaces rough on atomic scale.
- Atoms jitter by large amounts.

Large mechanical noise

How large?

Seismic: $x_{rms} \sim 1\ \mu$m.

Can you filter it enough?

Thermal:

- mirror's CM: ~ 3 x 10^{-12} m.
- mirror's surface: ~ 3 x 10^{-16} m.

No filtering is possible. Can lower the temperature, but by enough?

Gravitational wave detection will succeed very soon

All of these challenges sound impossible.

And yet, all of them can be met.

Detectors of 10^{-21} have been built and run.

Detectors 10 or more times better will start operating in a few years, including in India.

With them, we are just about certain to detect gravitational waves.

This week's goal is to know why we should be confident that this is true.

Tutorial exercises

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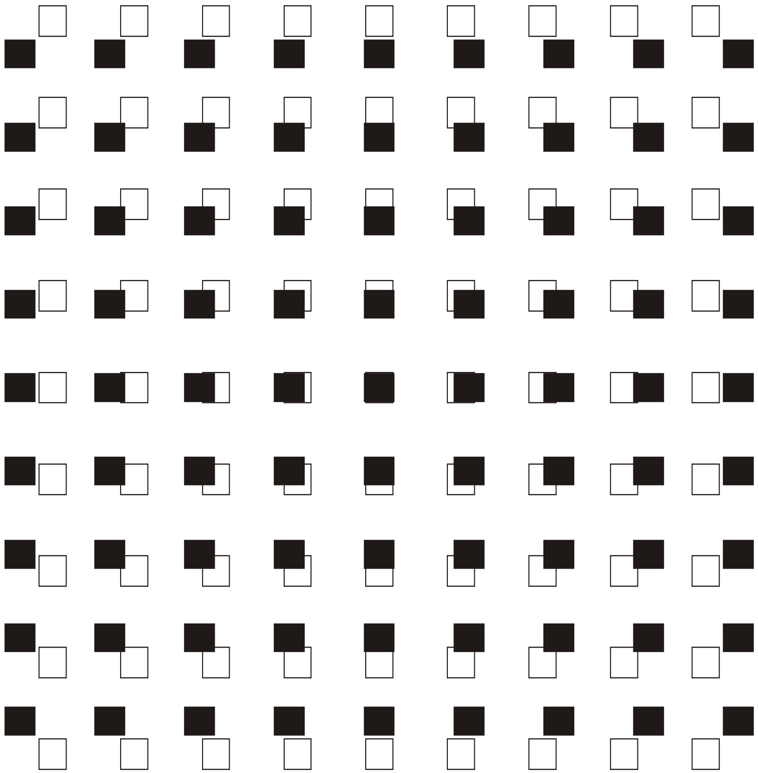

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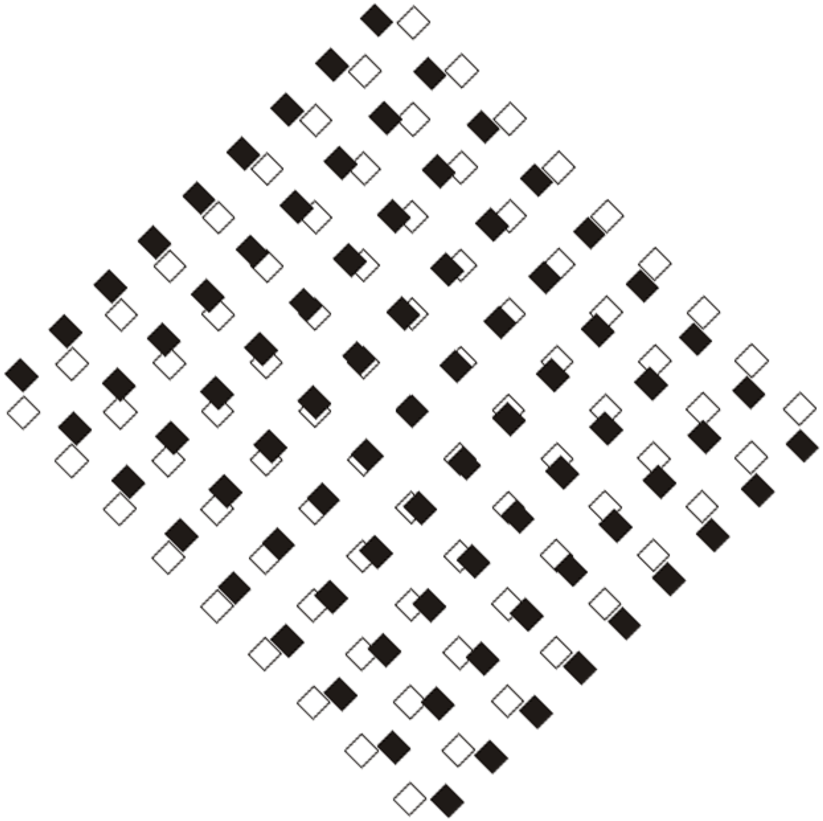

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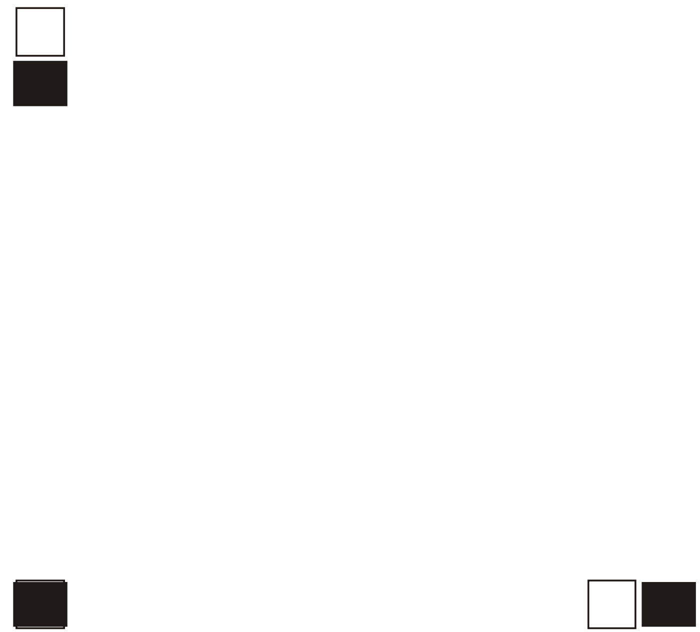

Solving for variation in light travel time

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$$ds^2 = 0 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2$$

Solving for variation in light travel time: start with x arm

$$ds^2 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2 = 0$$

- *h(t)* can have any time dependence, but for now assume that *h(t)* is constant during light's travel through ifo.
- Rearrange, take square root, and replace square root with 1st two terms of binomial expansion

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{11}\right) dx$$

- then integrate from $x = 0$ to $x = L$:

$$\Delta t = h_{11} L / 2c$$

Solving for variation in light travel time (II)

In doing this calculation, we choose coordinates that are marked by free masses.

"Transverse-traceless (TT) gauge"

Thus, end mirror is always at $x = L$.

Round trip back to beam-splitter:

$$\Delta t = h_{11} L / c$$

y-arm ($h_{22} = - h_{11} = -h$):

$$\Delta t_y = -hL / c$$

Difference between x and y round-trip times:

$$\Delta \tau = 2hL / c$$

Multipass, phase diff

To make the signal larger, we can arrange for N round trips through the arm instead of 1.

More on this in a later lecture.

$$\Delta\tau = h\frac{2NL}{c} \equiv h\tau_{stor}$$

It is useful to express this as a <u>phase</u> difference, dividing time difference by radian period of light in the ifo:

$$\Delta\phi = h\tau_{stor}\frac{2\pi c}{\lambda}$$

How do we make the travel-time difference visible?

In an ifo, we get a change in output power as a function of phase difference.

At beamsplitter, light beams from the two arms are superposed. Thus, at the port away from laser (XX true?)

$$\left|E_{out}\right| = E_0 \cos \Delta\phi$$

and at the port through which light enters

$$\left|E_{refl}\right| = E_0 \sin \Delta\phi$$

Output power

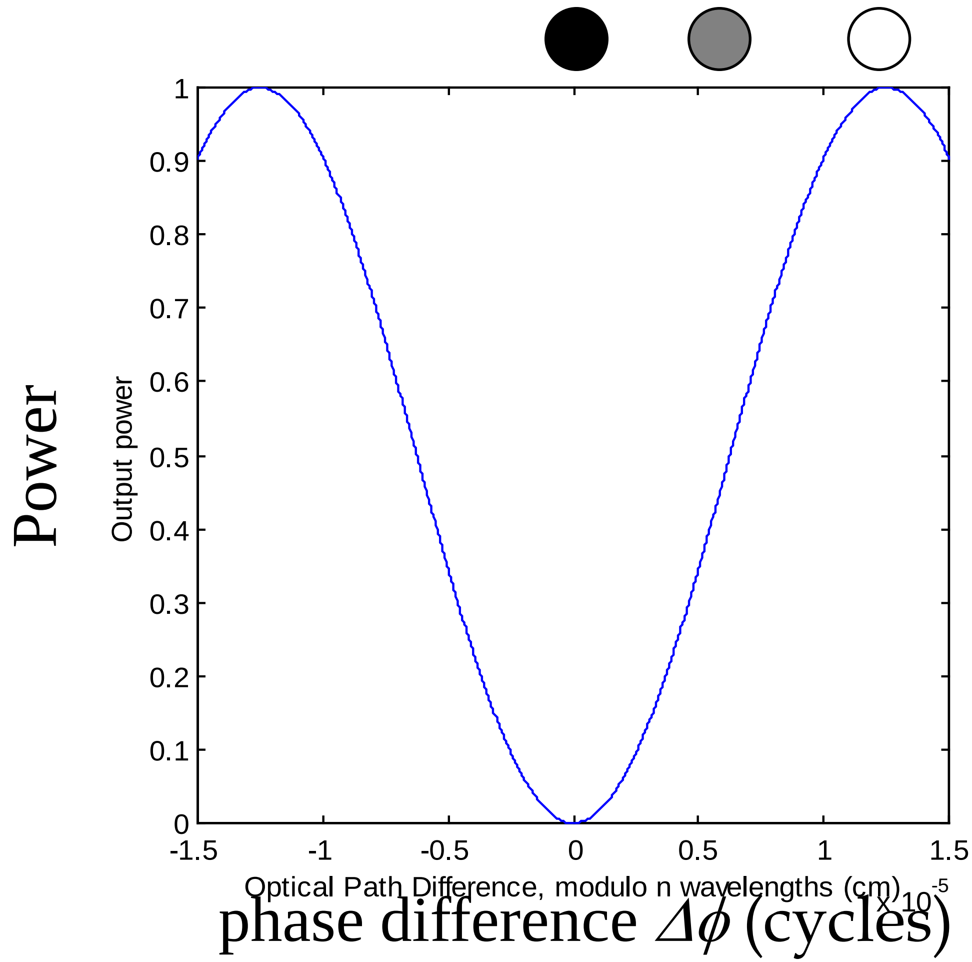
We actually measure the optical power (not the electric field) at the output port (recall $P \propto E^2$)

$$P_{out} = \frac{P_{in}}{2}\left(1 + \cos\Delta\phi\right)$$

Note that energy is conserved:

$$P_{out} + P_{refl} = P_{in}\left(\cos^2\Delta\phi + \sin^2\Delta\phi\right) = P_{in}$$

Interferometer output vs. arm length difference



Ifo response to *h(t)*

Free masses are free to track time-varying *h*.

As long as τ_{stor} is short compared to time scale of *h(t)*, then output tracks *h(t)* faithfully.

- If not, then put time-dependent *h* into integral of slide 13 before carrying out the integral.
- Response "rolls off" for fast signals.

This is what is meant by interferometers being *broad-band* detectors.

But, noise is stronger at some frequencies than others. (More on this later.) This means some frequency bands have good sensitivity, others not.