1. Suppose you were to observe the spin frequency of a pulsar with high precision over a long time. If you can measure the "braking index" $\Omega\ddot{\Omega}/\dot{\Omega}^2$, then you can in principle discriminate between spindown due to gravitational radiation and spindown due to magnetic torques. Recall that gravitational radiation gives $\dot{\Omega} \propto \Omega^5$, and magnetic dipole torques give $\dot{\Omega} \propto \Omega^3$. Use these to compute the braking indices for pure gravitational radiation and for pure magnetic dipole torques.

Suppose that a neutron star with zero magnetic field accretes matter from a companion at a rate $\dot{M}$. The rate at which angular momentum is accreted is $\sqrt{12}\dot{M}GM/c$. In the following, assume that the mass M and moment of inertia I of the star do not change significantly in the period of interest. In addition, we are working under the assumption that a lump, rather than a wave, produces gravitational radiation.

2. As we said in the notes, magnetic torques are likely to set the equilibrium spin frequencies of accreting neutron stars. However, just for the sake of argument, suppose that torques from gravitational radiation balance the accretion torques at a frequency $\Omega_{\rm equil}$. As a function of $\dot{M}$, M, I, and $\Omega_{\rm equil}$, calculate the required ellipticity ϵ. Approximately what is the value of ϵ if $\dot{M} = 10^{17}$ g s^{-1}, $M = 3 \times 10^{33}$ g, $I = 10^{45}$ g cm^2, and $\Omega_{\rm equil} = 2\pi \times 300$ Hz?

3. Suppose that an accreting nonmagnetic neutron star has an observed electromagnetic flux F, which we assume is produced by accretion onto the surface: the luminosity is $L = 0.2\dot{M}c^2$, where the 0.2 is the efficiency of energy release. In the same spirit as the previous problem, suppose that the torque from the accreting matter is balanced exactly by gravitational radiation. Given this assumption, show that the flux in gravitational radiation has a simple relation to the electromagnetic flux, and find that relation. Assume isotropic emission of both.

4. One burst source some people have proposed is pulsar glitches. In a glitch, the spin frequency of the pulsar changes suddenly, due (we think) to a sudden coupling between the normal matter and the superfluid matter in the crust. The energy release is $I\Omega\Delta\Omega$, but I is the moment of inertia of the crust, which is perhaps 10^{43} g cm^2, or 1% of the moment of the inertia of the star (because the crust exists only at low densities). In a really big glitch, $\Delta\Omega \sim 10^{-6}\Omega$. Let's say that such a glitch happens to a star with $\Omega = 100$ rad s^{-1}, and that *all* the energy comes out in gravitational waves with frequency 2000 Hz (comparable to double the sound crossing frequency), in a period of only 1 second. If this is a vary close source, at 1 kpc (or about 3×10^{21} cm), could this be seen with Advanced LIGO (sensitivity $\sim 2 \times 10^{-23}$ Hz$^{-1/2}$ at 2000 Hz)?

5. Dr. Sane doesn't understand all this focus on binary compact object mergers. He thinks that direct collisions of single neutron stars in clusters with each other will make wonderful burst sources. He has requested that you work out the numbers for him. Suppose that you consider a dense globular cluster, such that in the center the number density of neutron stars is 10^6 pc^{-3} and there are 1000 total neutron stars per cluster. Suppose that each neutron star has a radius of 10 km and mass of $1.5\,M_\odot = 3\times 10^{33}$ g, and that the typical random speed in the cluster is 10 km s^{-1}. To within an order of magnitude, calculate how often two neutron stars in a given cluster will hit each other. If there are 10^{10} such clusters in the universe, how often will this happen in the universe? **Note:** be careful when you calculate the cross section for collisions, because gravitational focusing is important. Thus your first step will be to determine the *impact parameter* b that results in a collision: if an object's velocity vector would have a closest approach b if it continued in a straight line, then b would be the impact parameter. Then the cross section is πb^2. If gravitational focusing were unimportant then $b \approx R$, where R is the radius of the neutron star, but you should find that $b \gg R$ in this case.

6. Dr. Sane has heard about the possibility of seeing continuous wave emission from lumpy neutron stars using high-frequency ground-based detectors. He suggests that lumpy white dwarfs might be good sources for low-frequency space-based detectors.

To evaluate this suggestion, do the following. Recall that $h \sim (1/r)(\partial^2 I/\partial t^2)$. For a rotating source only the nonaxisymmetric part contributes, so let's say that $I \sim \epsilon M R^2$, where $\epsilon \ll 1$. Also, let $\partial^2/\partial t^2 \sim \Omega^2$, where Ω is the rotation frequency. Supposing that the ellipticity ϵ is the same for neutron stars and white dwarfs, how does the maximum possible h at a given r for white dwarfs compare with the maximum possible h for neutron stars? **Hint:** how fast can they rotate? Combine this with our calculation that gravitational waves can just barely (if at all!) be detected from rotating neutron stars, and the fact that high-frequency detectors have a better h sensitivity than low-frequency detectors, to evaluate Dr. Sane's idea.

7. Darth Sidious has taken an interest in the generation of gravitational waves. He plans to use the Death Star to destroy the planet Alderaan in such a way as to make the resulting gravitational waves visible throughout the universe to detectors that (coincidentally?) have roughly the sensitivity of LIGO-India. He has brought you in to consult on this plan. Can he do it? **Hint:** look at the notes about core-collapse supernovae, and remember that a planet like Earth has $\sim$few$\times 10^{-6}$ times the mass of the core of a massive star. Be careful how you deliver your answer: the Emperor does not like bad news.