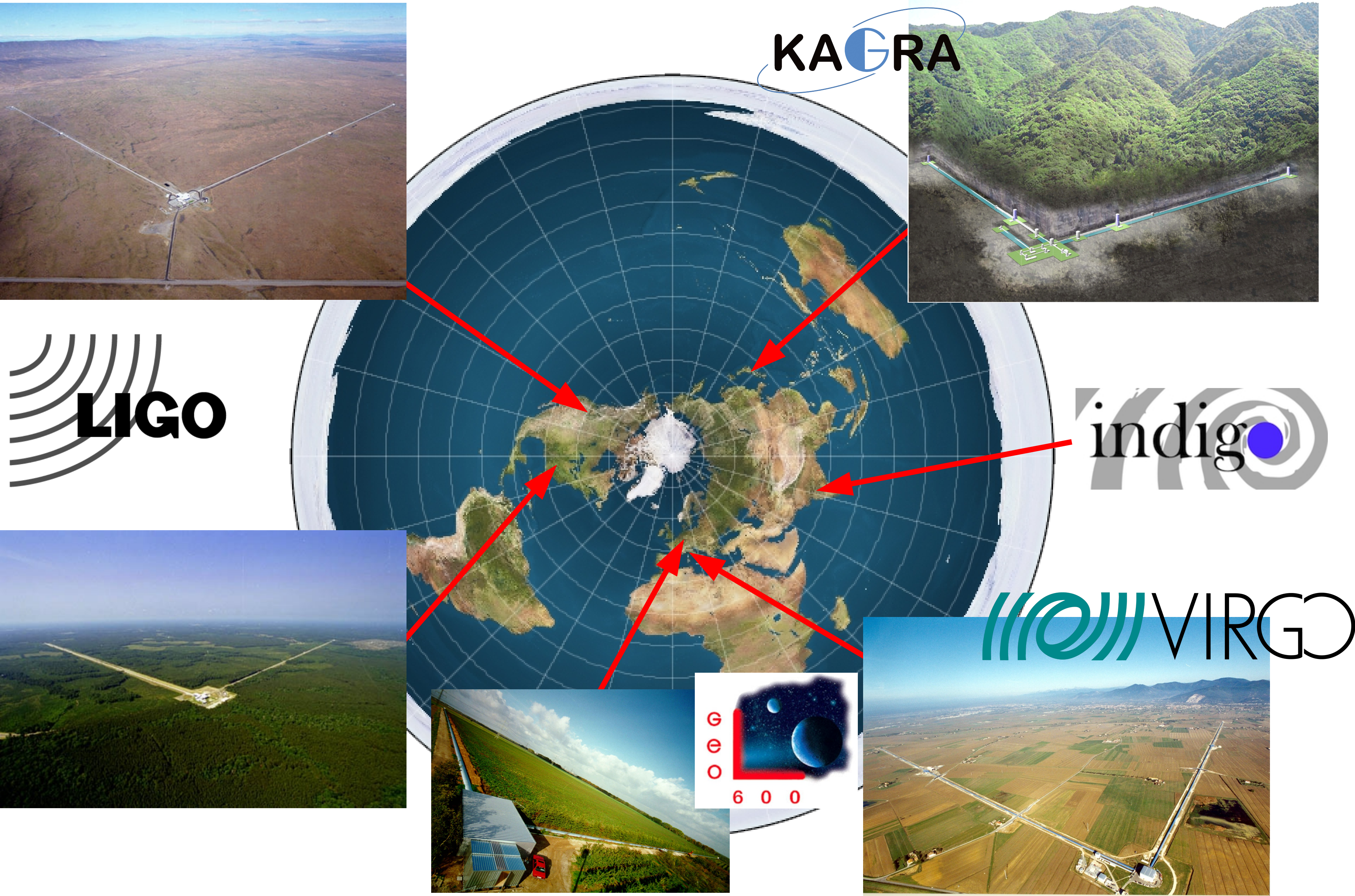
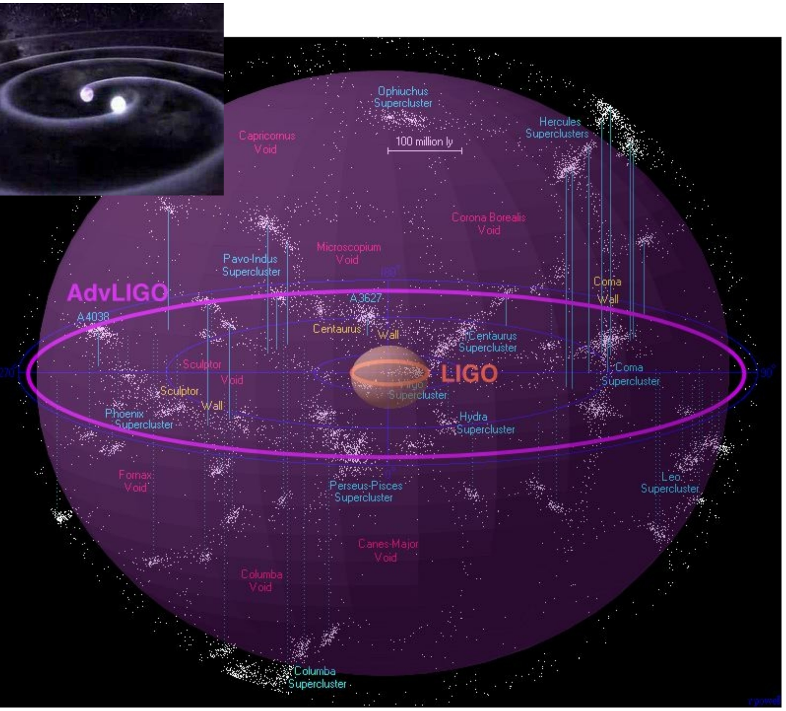
The detection and interpretation of gravitational waves: Introduction

The advanced detector network



From initial to advanced detectors

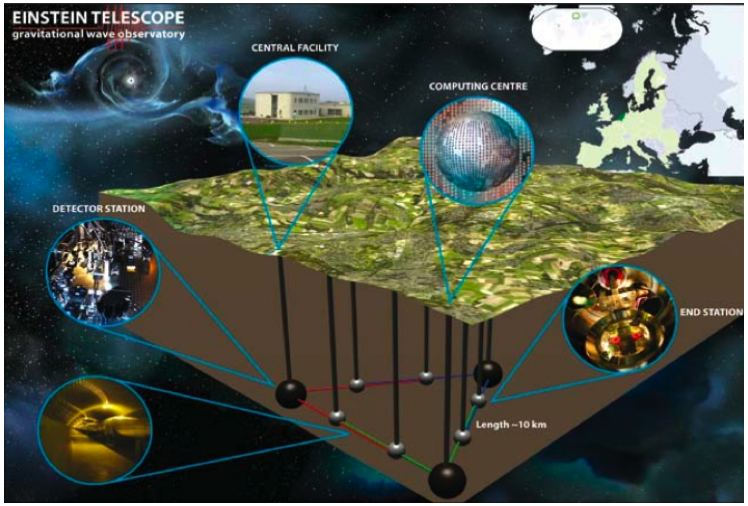


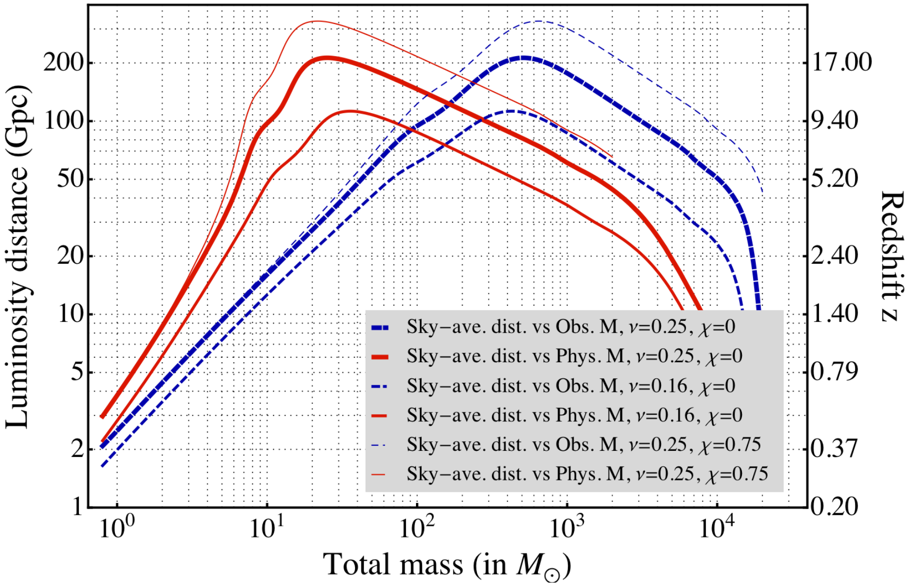
Network	Source	$\dot{N}_{\rm low}$ (yr^{-1})	$\dot{N}_{\rm re}$ (yr^{-1})	$\dot{N}_{\rm high}$ (yr^{-1})
	NS-NS	2×10^{-4}	0.02	0.2
Initial	NS-BH	7×10^{-5}	0.0004	0.1
	BH-BH	2×10^{-4}	0.007	0.5
	NS-NS	0.4	40	400
Advanced	NS-BH	0.2	10	300
	BH-BH	0.4	20	1000

- Initial LIGO, Initial Virgo
  - Active 2002-2011
  - No detections
  - Design sensitivity reached
    - Proof of the technology!
- Advanced detectors
  - **2015**: Advanced LIGO, limited sensitivity
  - 2016-2018: Advanced LIGO + Virgo (+ KAGRA)
  - 2019+: Design sensitivity
  - 2022: LIGO-India

Einstein Telescope (2030?)

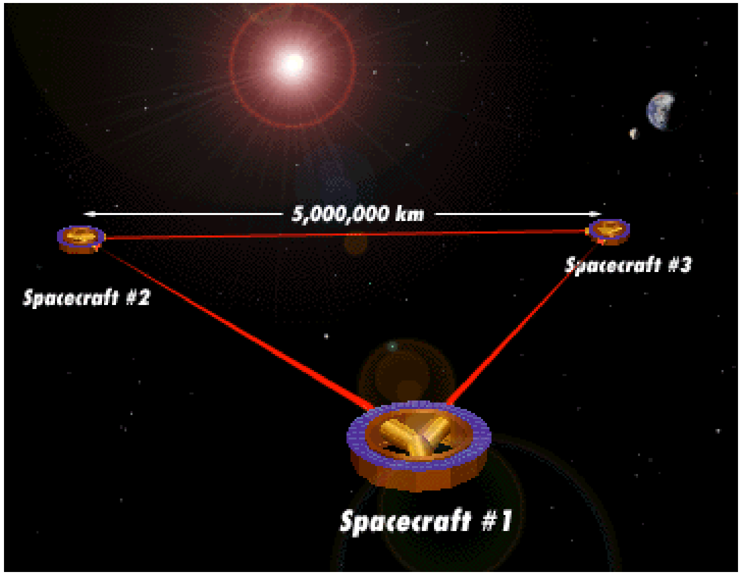
- Underground, cryogenic, 6 interferometers, 10 km arm length
- Design sensitivity: factor ~10 better than 2nd generation detectors

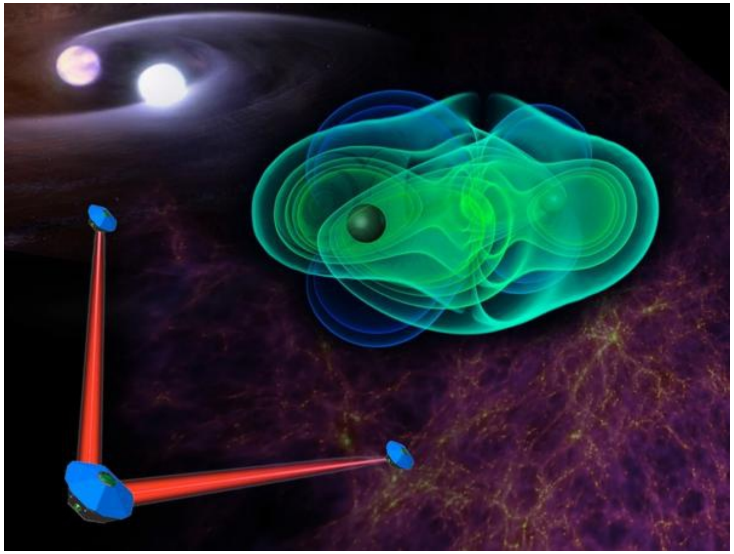




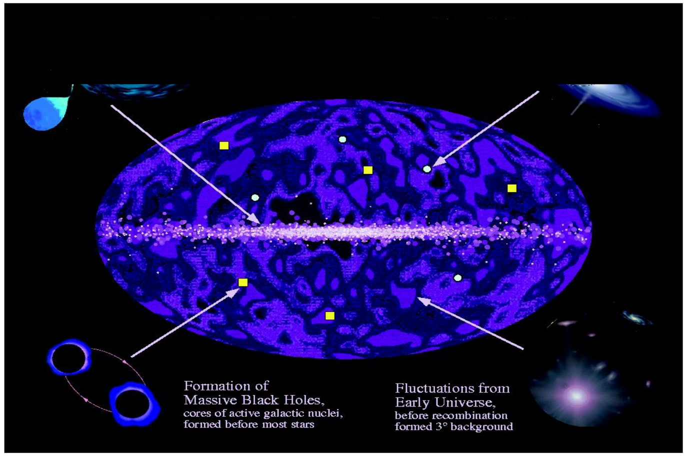
Source	BNS	NS-BH	BBH
Rate (Mpc^{-1} Myr^{-1})	0.1–6	0.01–0.3	2×10^{-3}–0.04
Event Rate (yr^{-1}) in aLIGO	0.4–400	0.2–300	2–4000
Event Rate (yr^{-1}) in ET	$\mathcal{O}(10^3$–$10^7)$	$\mathcal{O}(10^3$–$10^7)$	$\mathcal{O}(10^4$–$10^8)$

(e)LISA





- Project of the European Space Agency
  - Launch in 2034
- 3 probes in orbit around the Sun
- Orbits chosen so as to retain triangle configuration
- Probes exchange laser beams: interferometry
- Sensitive to much lower frequencies



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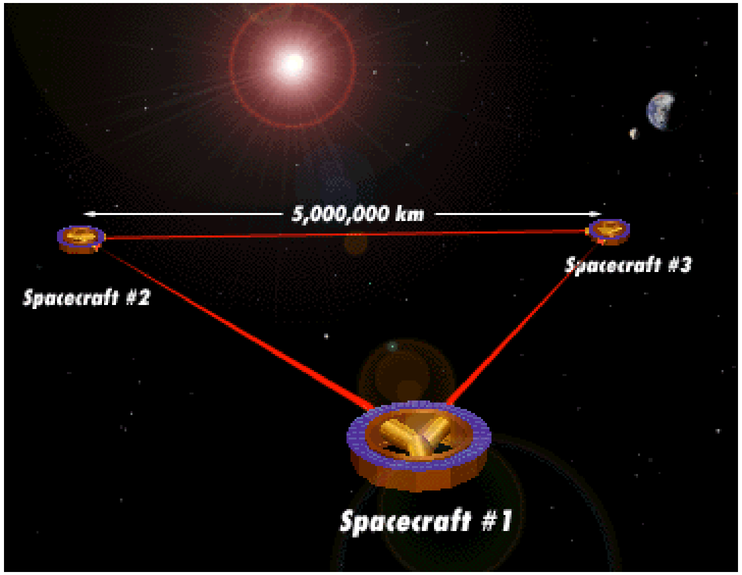

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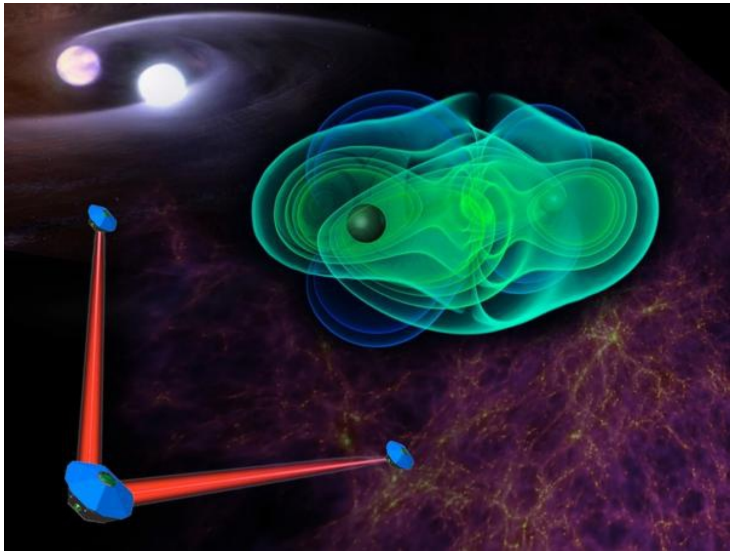

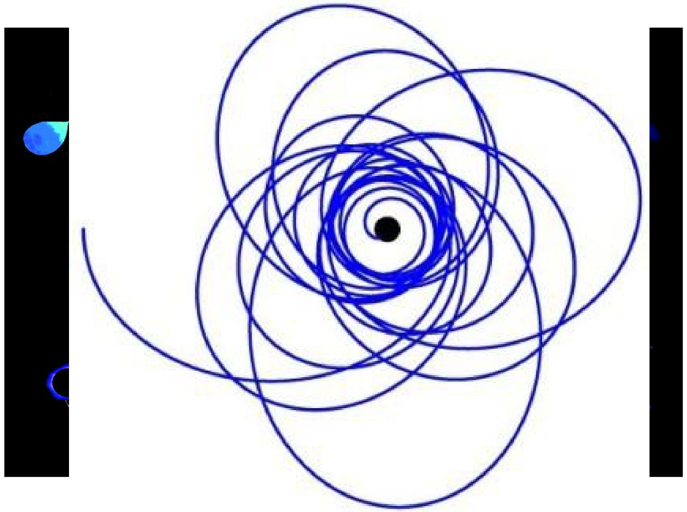

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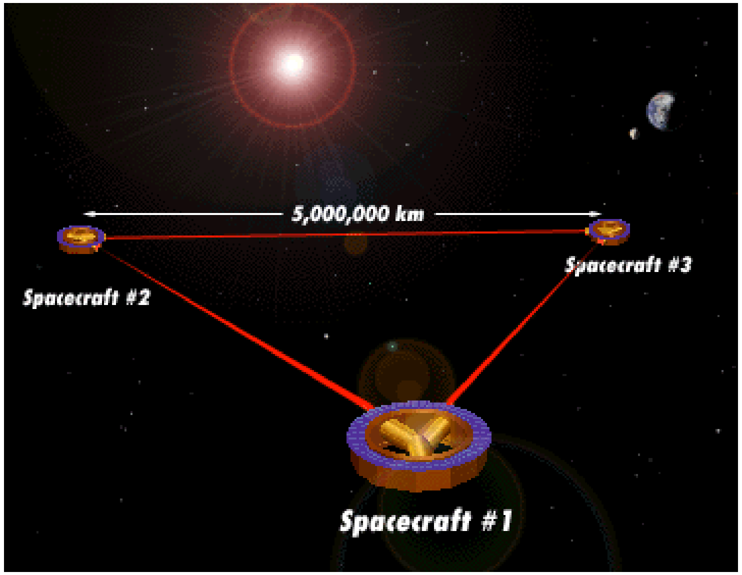

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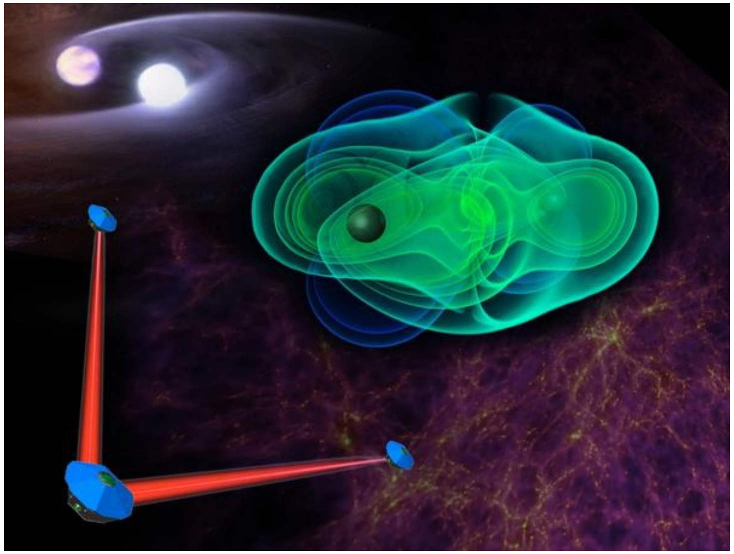
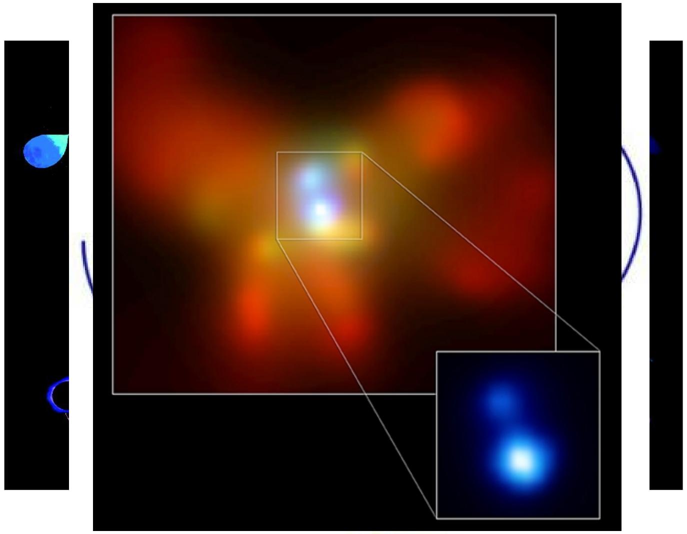




Strength of gravitational waves

- Gravitational wave strain measured on Earth:

$$h \propto \frac{M}{D} \frac{M}{R} \left(\frac{v}{c}\right)^{\alpha} \epsilon$$

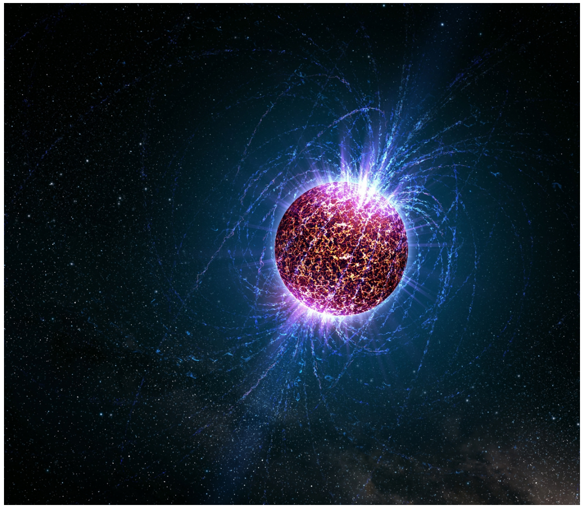
  - M/R: compactness
  - D: distance to the Earth
  - v/c: characteristic speeds inside the source
  - ε: asymmetry

⇒ Need sources that are highly compact, rapidly varying, asymmetric

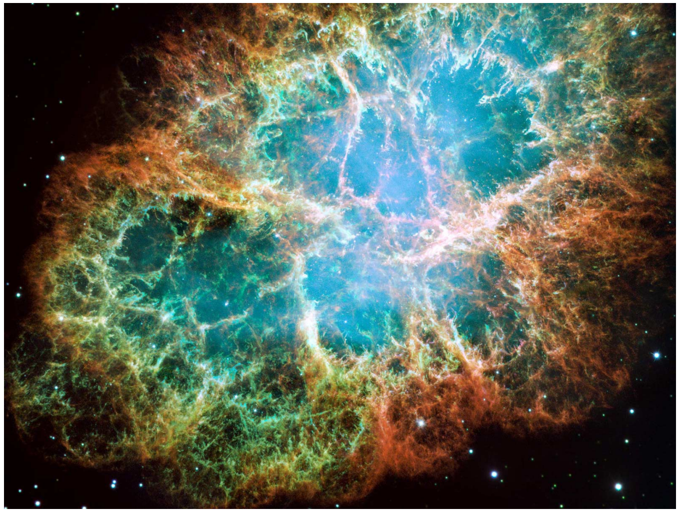
Sources of gravitational waves

Coalescing binary neutron stars and black holes

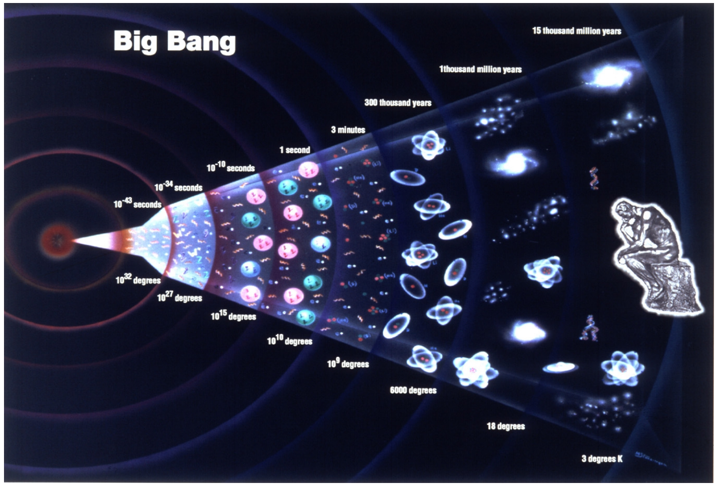
Fast-spinning neutron stars



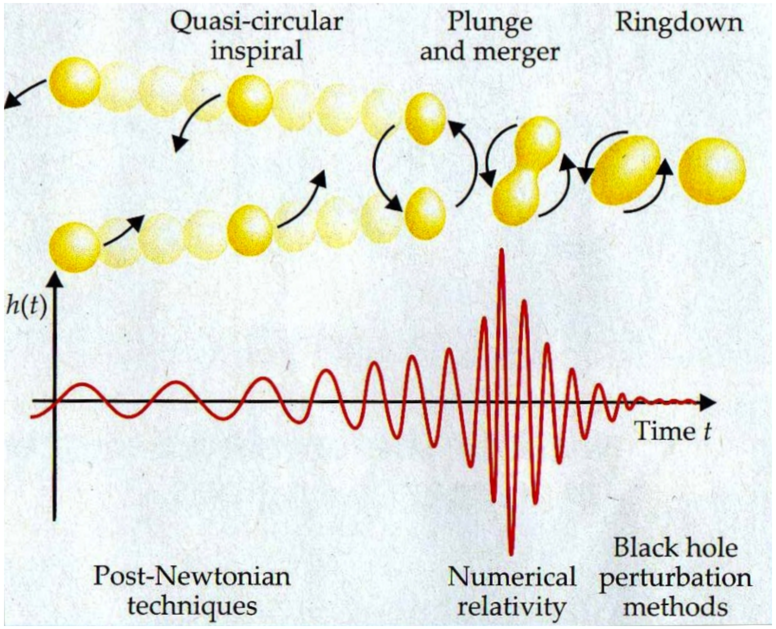
Bursts (e.g. supernovae)

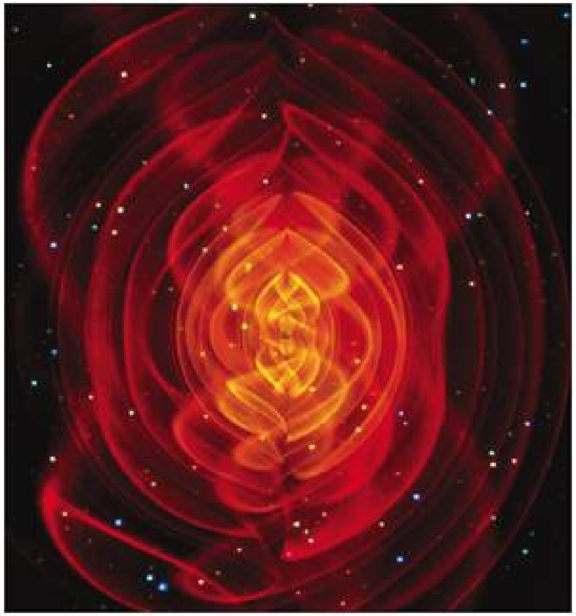


“Stochastic” gravitational waves



Coalescence of compact binaries

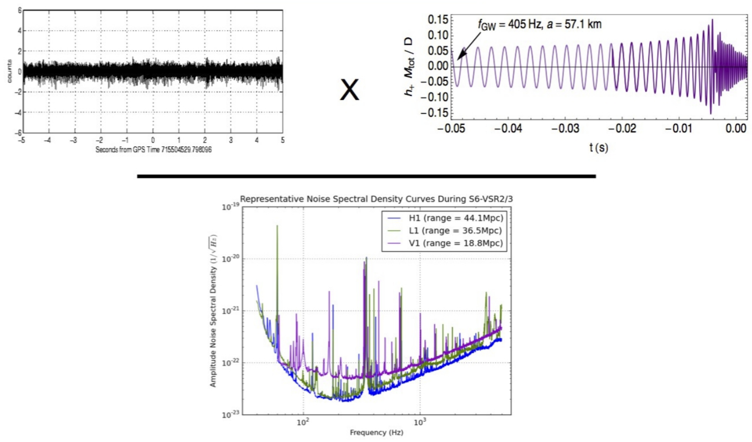




- NS-NS, NS-BH, BH-BH
  - Possibly intermediate mass black holes
- Signal is determined by

  at least 15 parameters
  - Masses, spins
  - Distance, sky position, orientation
  - Time and phase at arrival
- Search by “matched filtering”

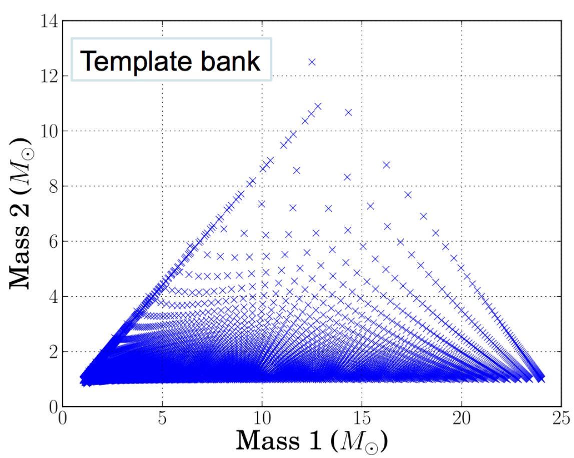
Coalescence of compact binaries



- Search by “matched filtering”
  - Integrate waveform against data
    - “Extrinsic” parameters (sky position, orientation, distance) absorbed into overall amplitude
  - Fold in known properties of the noise (“spectral density”)

  ⇒ “Signal-to-noise ratio”

- Do this for large number of mass choices
  - “Template bank”



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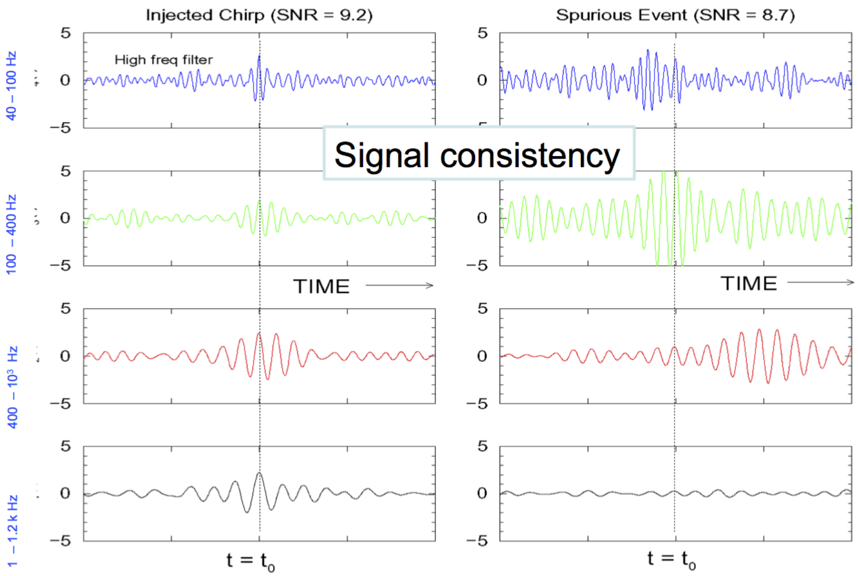
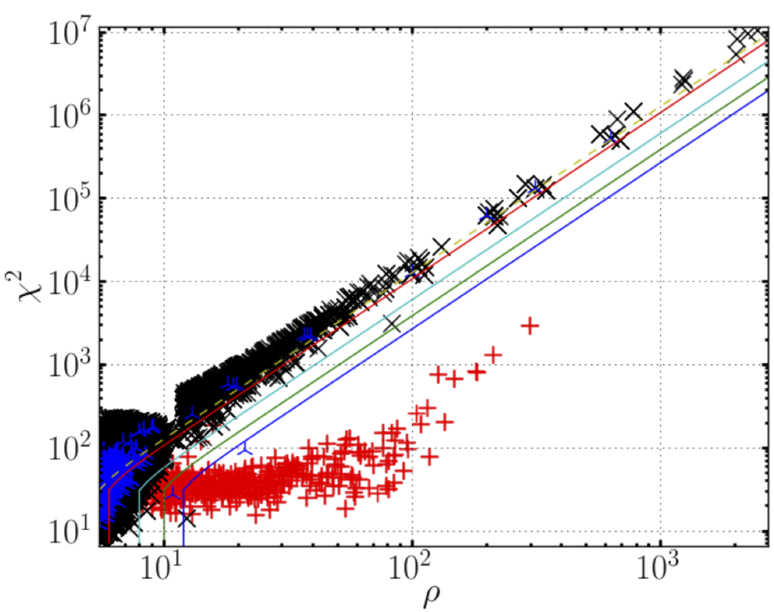
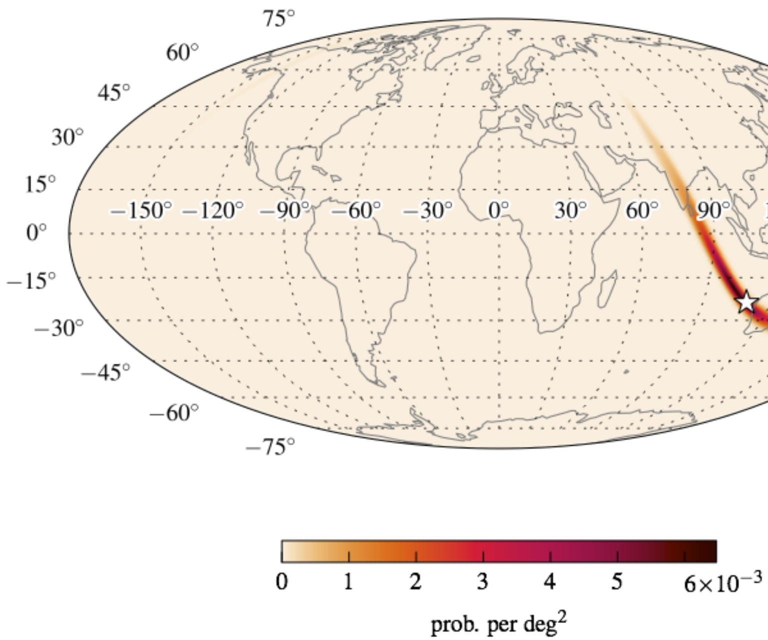
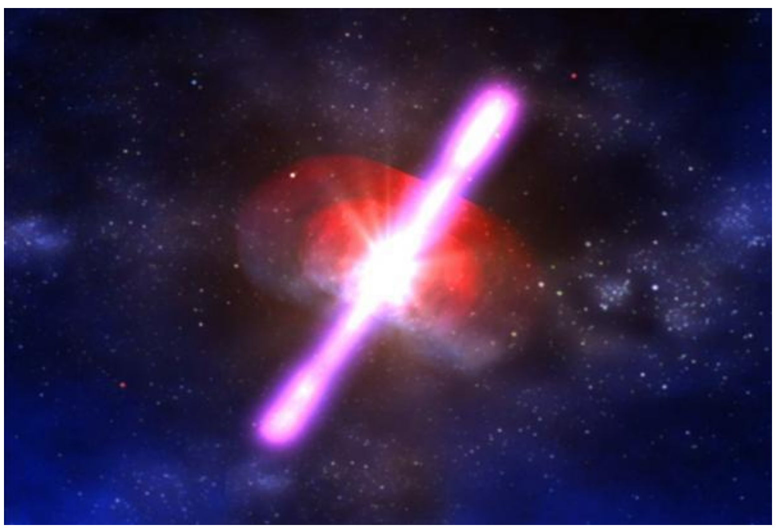




- Determine consistency of data with real signal being present
  - Check for consistency between detectors
    - In time, and in parameter space
- Time-slide data streams with respect to each other and analyze
  - Arrive at "background distribution" of spurious events (because not coincident in wall clock time)
  - Separate genuine or simulated events from background in terms of signal-to-noise ratio ρ, and χ^2
  - Assign significance to candidate events
    - False alarm probability

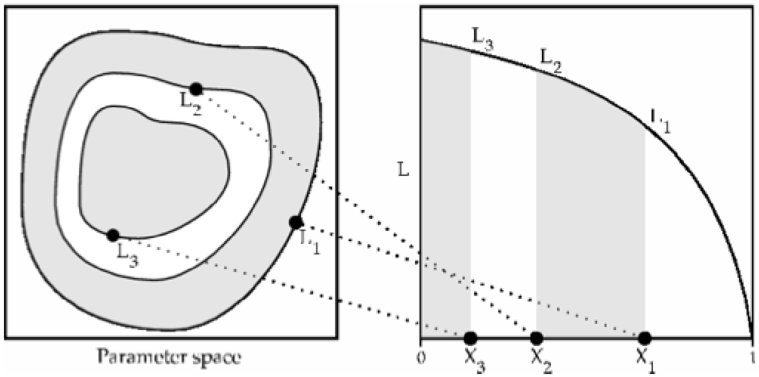
Coalescence of compact binaries

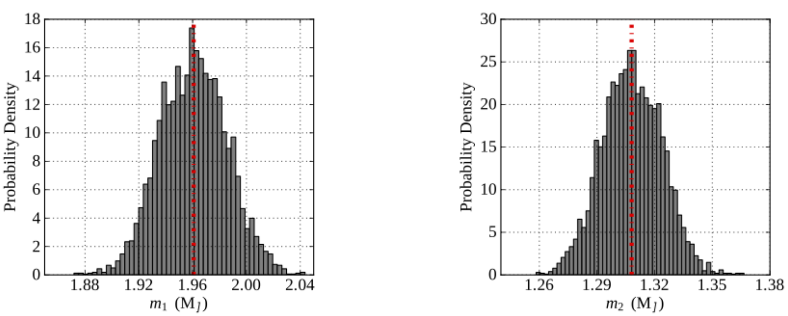


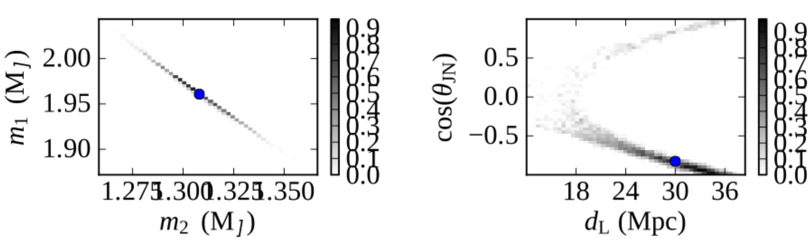


- All-sky, all-time searches
  - "Low latency"
    - Triggers to EM partners in minutes
    - Sky maps from detector network
  - "High latency"
    - Better characterization of instrument
    - Better estimate of significance of candidate events
- Triggered searches
  - Short, hard gamma rays bursts believed to be caused by NS-NS or NS-BH mergers

Coalescence of compact binaries



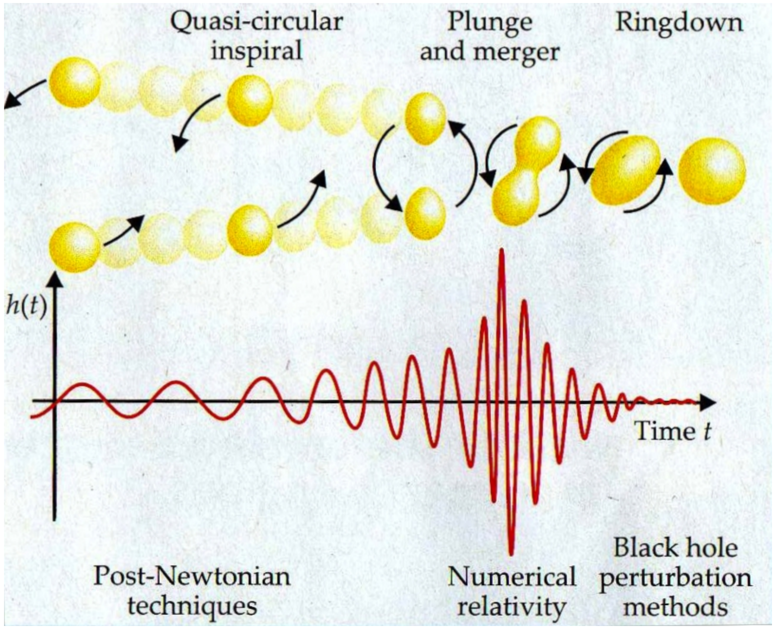




After confident detection: source reconstruction

- Measure masses, spins, distance, sky location, orientation, …
- Techniques to explore 15-dimensional parameter space and find most likely values
- Example: nested sampling
  - Identify nested hypervolumes in parameter space bounded by increasingly larger values of likelihood
- Arrive at probability distributions for parameters
- Scientific pay-off:
  - Astrophysical mass and spin distributions of black holes
  - Equation of state of neutron stars
  - Tests of the genuinely strong-field dynamics of GR
  - Cosmology without a cosmic distance ladder
  - ...

Was Einstein right?

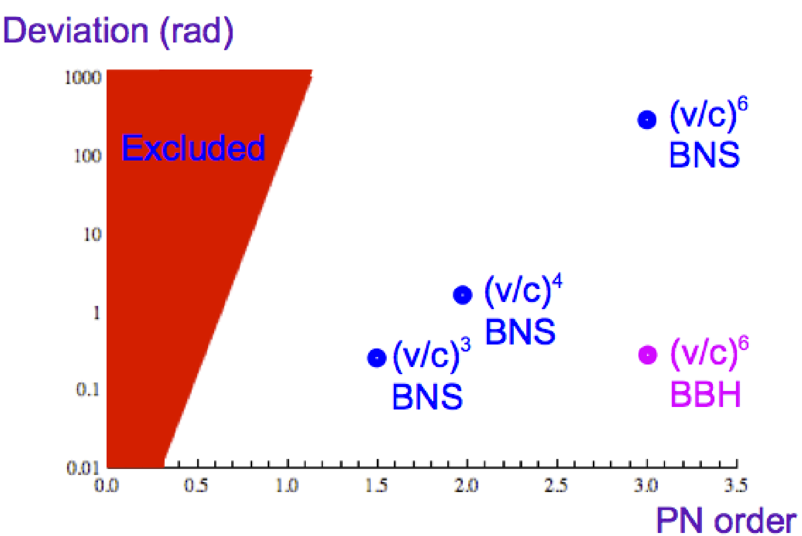


Inspiral-merger-ringdown:

- Inspiral regime well-understood in terms of post-Newtonian approximation
- Increasing analytic insight into (pre-)merger phase with the help of numerical simulations
- Ringdown regime well-understood in terms of black hole perturbation theory

Most existing tests of general relativity:

- Small spacetime curvature
- Dynamics of spacetime not very important



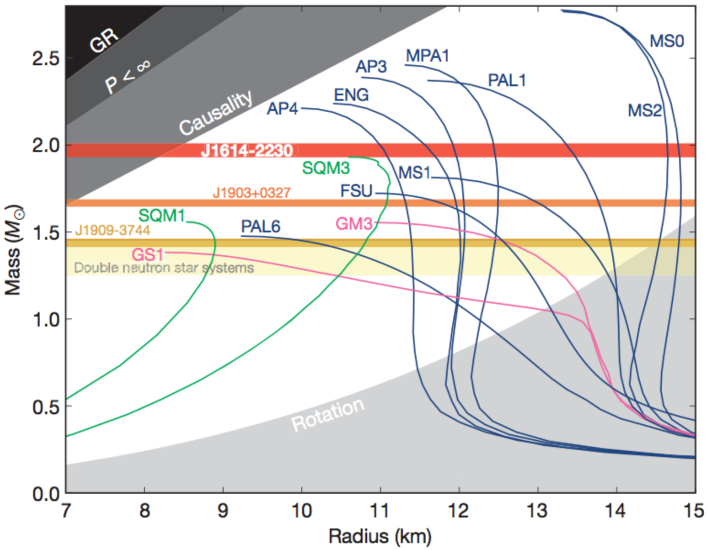
EM observations of binary neutron stars:

- Mostly weak-field tests: $GM/c^2R = O(10^{-6})$
- Not very relativistic: $v/c = O(10^{-3})$

Compact binary coalescences:

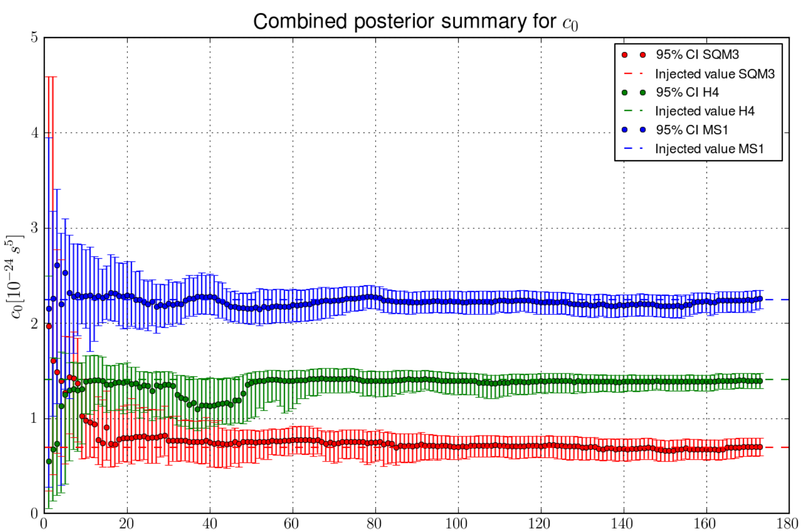
- $GM/c^2R > 0.2$
- $v/c > 0.5$

What is the equation of state of neutron stars?

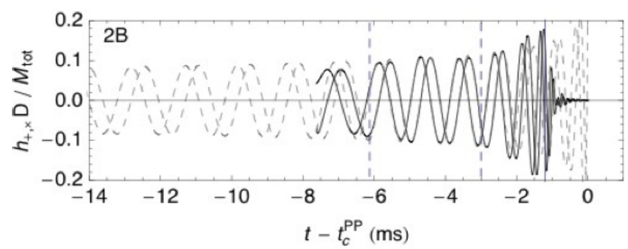


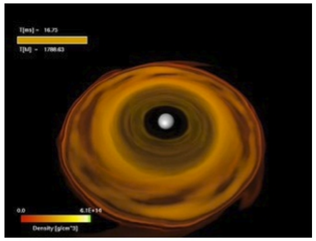
Equation of state determines deformability of neutron stars

- Visible in late inspiral regime
  - Neutron stars get deformed by each other's tidal field
  - This deformation affects orbital motion
  - Imprinted upon gravitational waveform
- Affects the merger waveform

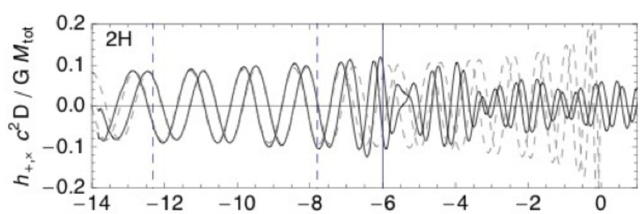


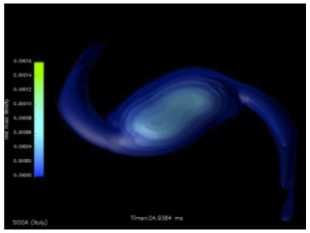
- "Soft" EOS: prompt collapse to a black hole



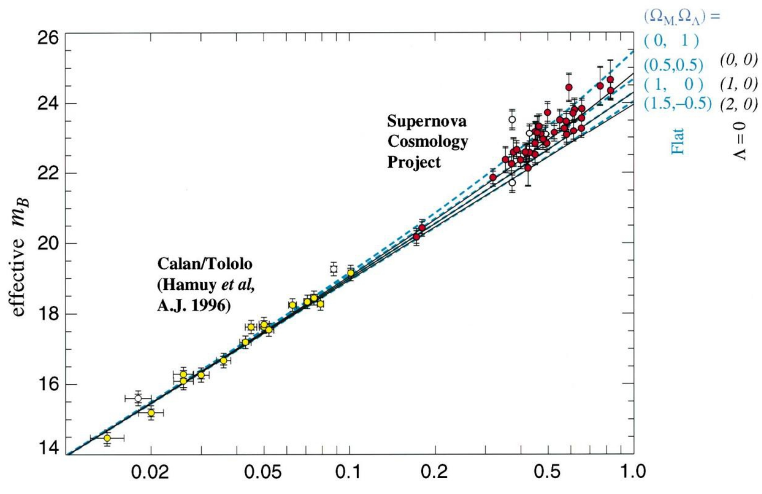


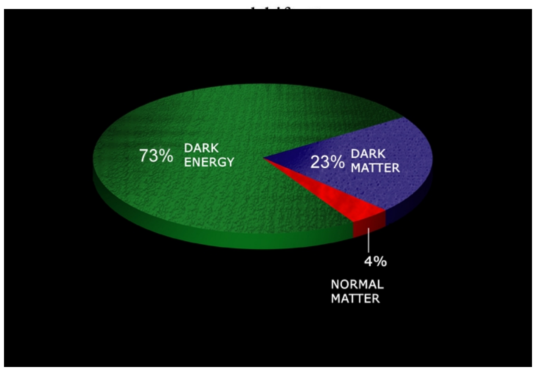
- "Hard" EOS: unstable bar mode, eventually black hole

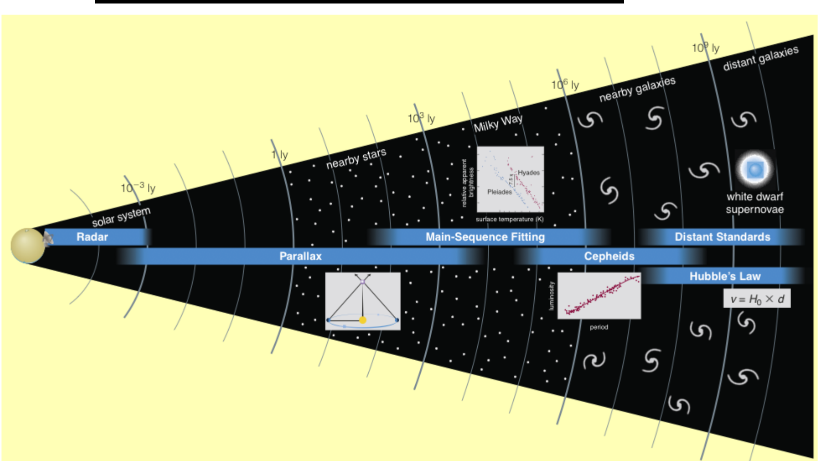




Probing the large-scale evolution of the Universe







"Standard candles" in cosmology:

- Source for which intrinsic luminosity known
- Infer distance
- If redshift also known:
  Make a fit of distance *D* versus redshift *z*
- This relationship depends on
  - Present-day expansion rate
  - Density of matter
  - Density of dark energy
  - Equation of state of dark energy

  Knowledge of D(z) ⇒ knowledge of these quantities

- Problem:

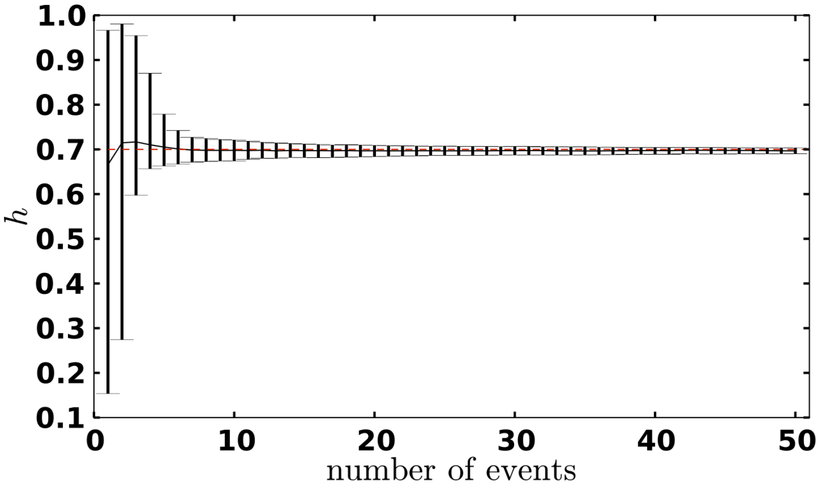
  Standard candles need to be calibrated using closer-by sources

  "Cosmic distance ladder"

Probing the large-scale evolution of the Universe

Binary neutron star and black hole coalescences are *standard sirens*

- Distance can be inferred from gravitational wave signal itself!
- Variety of ways to get redshift
- No need for a cosmic distance ladder!
- Measure cosmological quantities with completely different systematics



With advanced detector network:

- Completely independent measurement of the Hubble constant

Probing the large-scale evolution of the Universe

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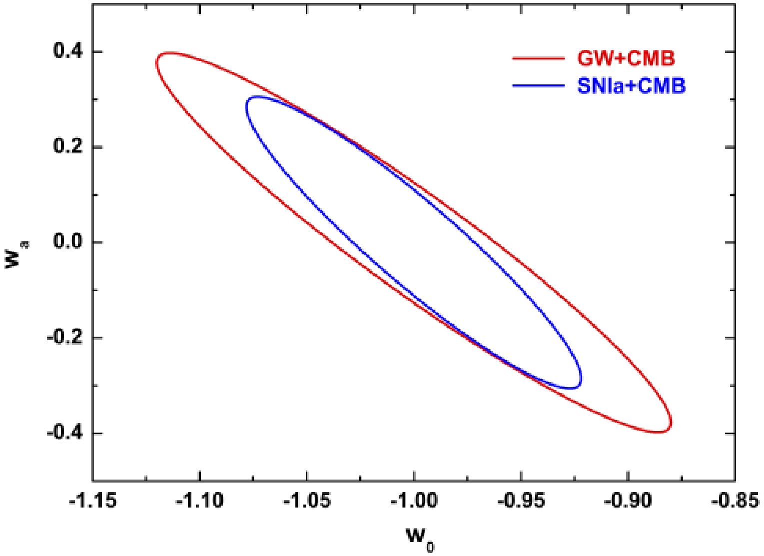
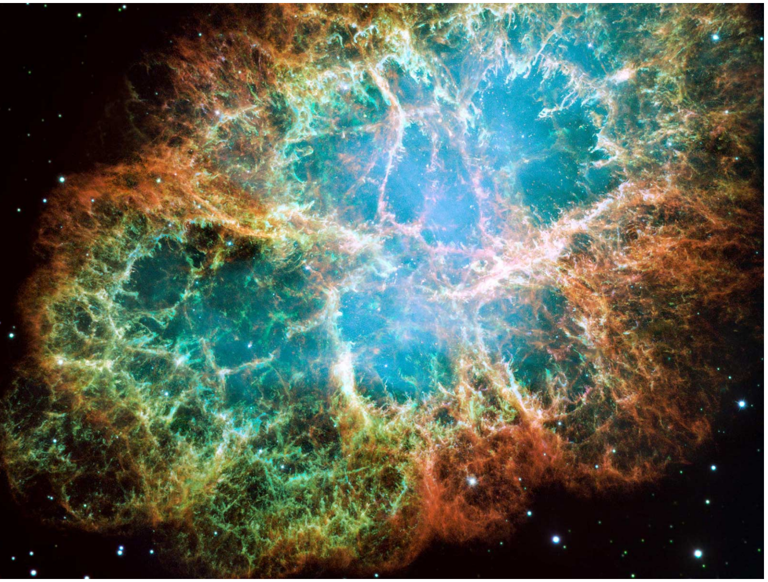
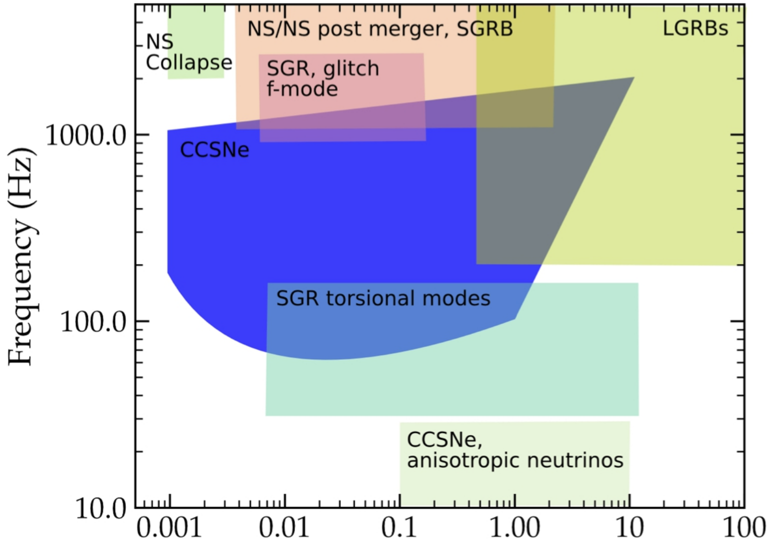


With Einstein Telescope:

- Measure equation of state of dark energy $w = P/\rho < 0$
- Does this quantity evolve?

"Burst" sources





Transient sources

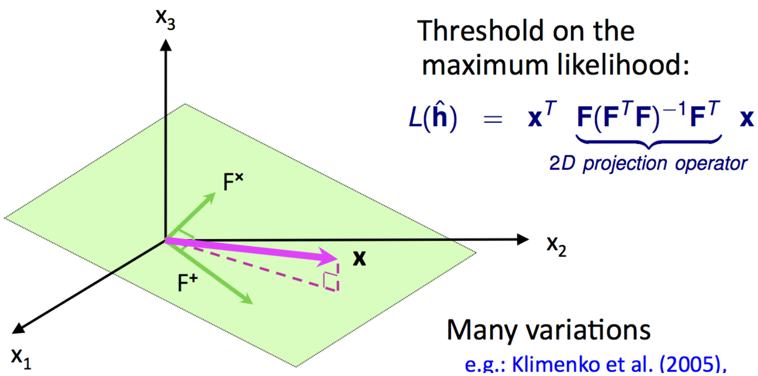
- Supernovae in or near our galaxy
- Long gamma ray bursts
- Neutron star instabilities
- Intermediate mass and/or eccentric binaries
- Cosmic strings
- ...
- The unknown!

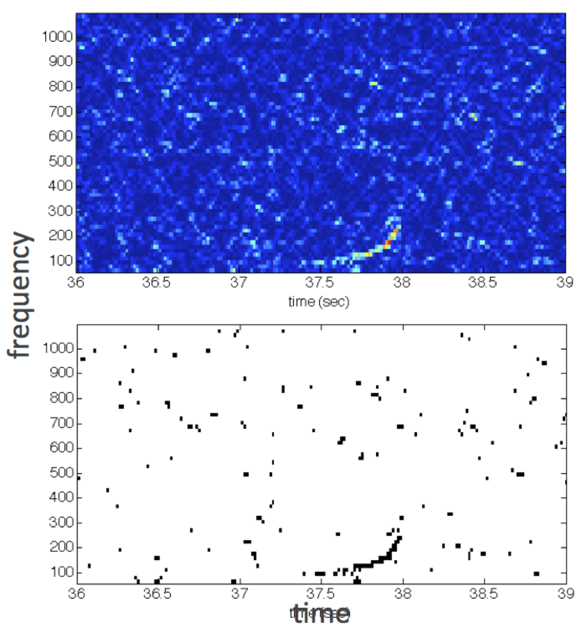
Many of these are poorly modeled

- Can't necessarily use matched filtering

“Burst” sources

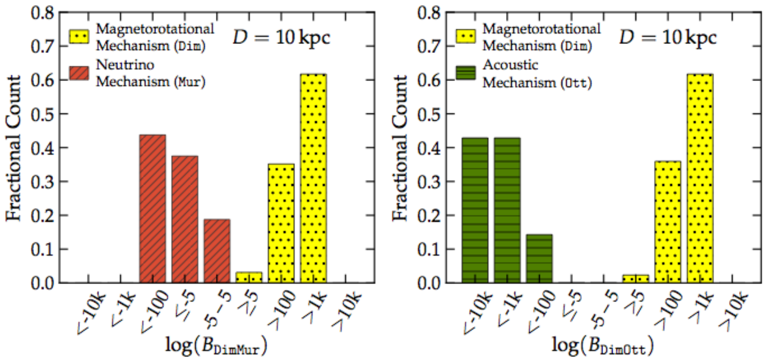
Detection without a (good) signal model

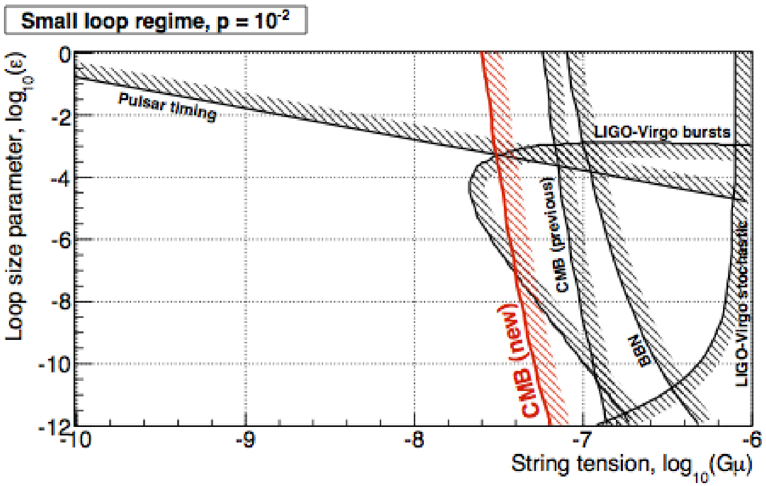




- Look for “coherent” signals
  - Present in multiple detectors
  - Consistent given detector responses
- Look for “excess power”
  - Decompose data
    - Fourier, wavelets, …
  - Make time-frequency maps
  - Identify and cluster brightest pixels
- Here too: timeslides to calculate significance (false alarm probability) of candidate events

“Burst” sources

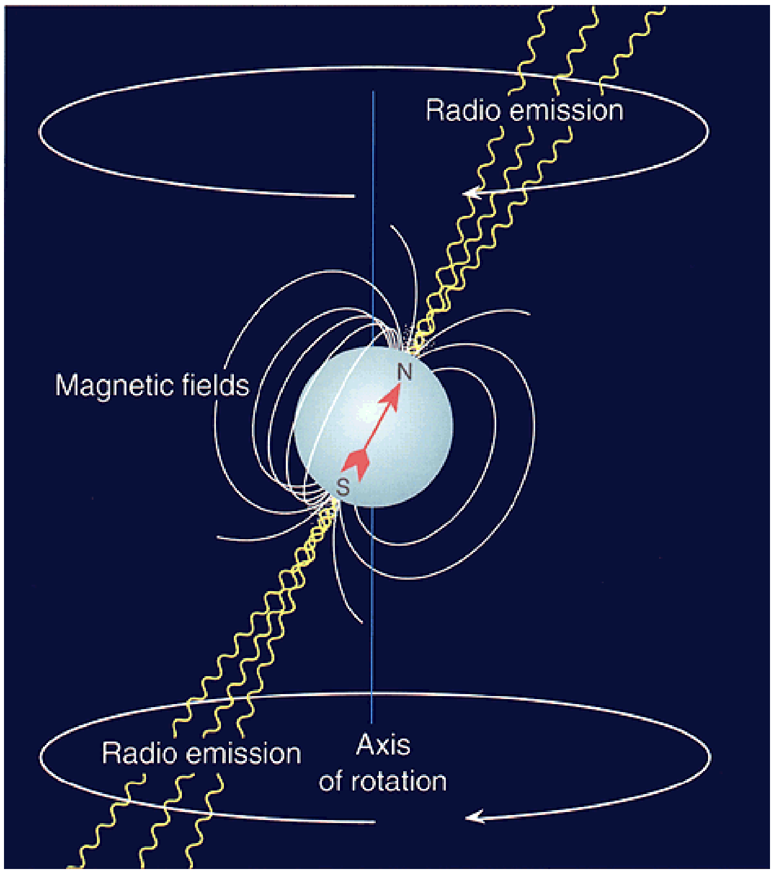




Parameter estimation?

- General case:
  Try to characterize signal rather than source
  - Time-frequency content, polarization
- For e.g. supernovae some amount of theoretical modeling has been done
  - Large-scale simulations with different assumptions give “catalogs” of waveforms
  - Identify main features of waveforms
  - Model selection to see which physical mechanism dominates
- Cosmic strings: matched filtering is possible!
  - Cusps $h \propto f^{-4/3}$, kinks $h \propto f^{-5/3}$

Continuous gravitational waves

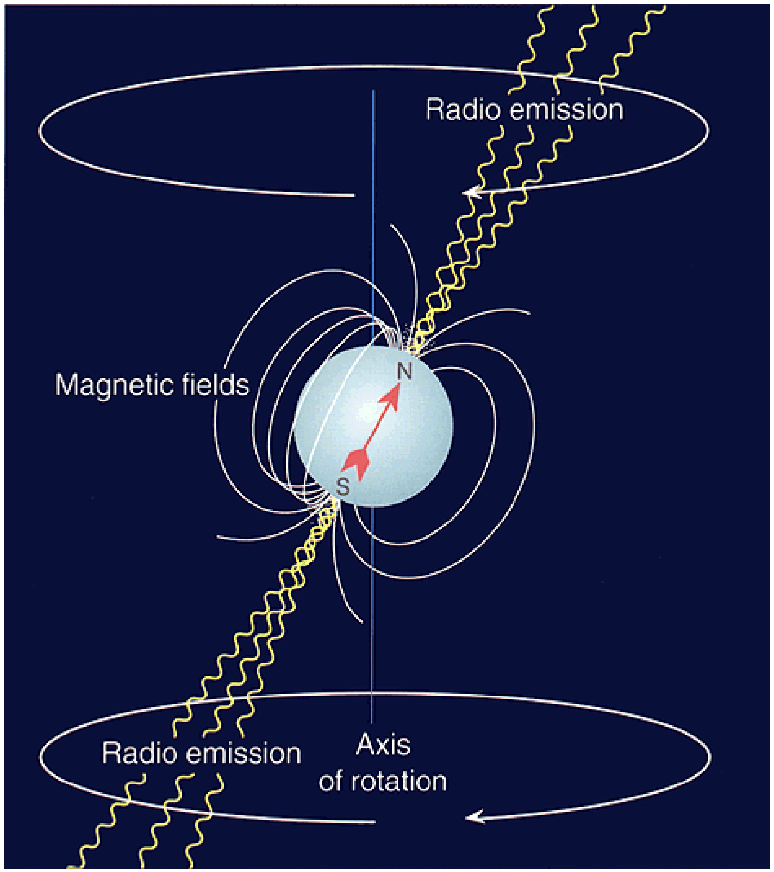


Neutron stars

- Mostly composed of neutrons in superfluid state
- Crust composed of dense but ordinary matter
- Strong magnetic fields:
  Precessing magnetic dipole
  ⇒ Emission of EM radiation
  - Carries away rotational energy
  - Star becomes less oblate
  - Cracking of the crust
- "Mountains" (~0.1 mm)
  ⇒ Asymmetry
  ⇒ Varying quadrupole moment

Continuous gravitational waves

$$h \propto \frac{M}{D}\,\frac{M}{R}\,\left(\frac{v}{c}\right)^{\alpha}\,\epsilon$$



Neutron stars: detectability

- Compactness: M/R ~ 0.1
- Typical speed at surface:
  $v/c \sim R\omega/c \sim (10^4\ \mathrm{m})(2\pi\ 10^3\ \mathrm{Hz})/c$
  ~ 0.2
- Many sources in the Milky way
  D ~ 10 kpc [1 pc = 3.26 lightyear]
- But, asymmetry expected to be small
  $\varepsilon \sim 10^{-6}$ as an optimistic estimate

Continuous gravitational waves

$$h \propto \frac{M}{D}\frac{M}{R}\left(\frac{v}{c}\right)^{\alpha}\epsilon$$

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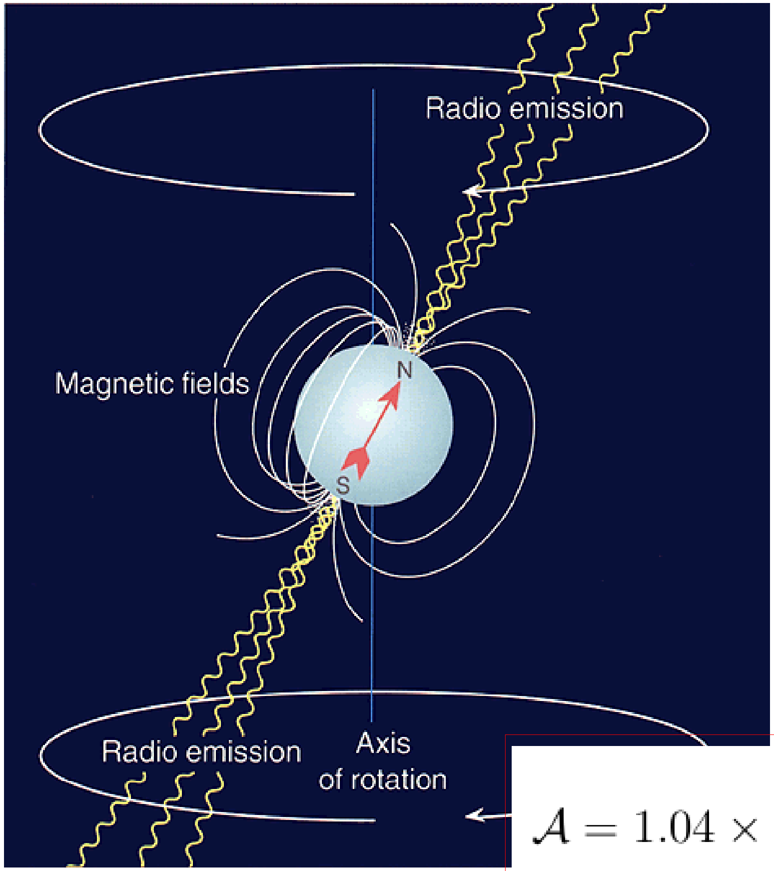
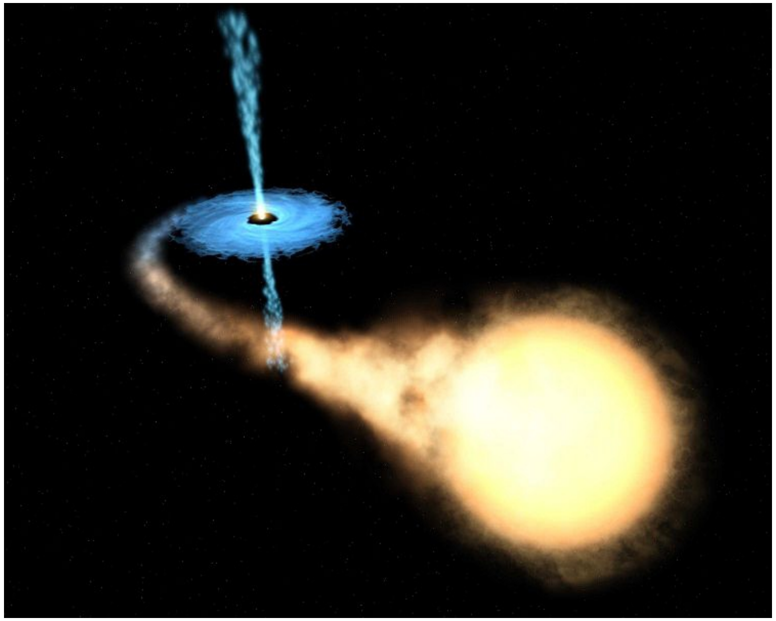


$$\mathcal{A} = 1.04 \times 10^{-25}\left(\frac{10\,\mathrm{kpc}}{r}\right)\left(\frac{I_3}{10^{38}\,\mathrm{kg\,m^2}}\right)\left(\frac{f}{1\,\mathrm{kHz}}\right)^2\left(\frac{\epsilon}{10^{-6}}\right)$$

Small, but can be integrated over entire observation time (~ 1 year)

Continuous gravitational waves

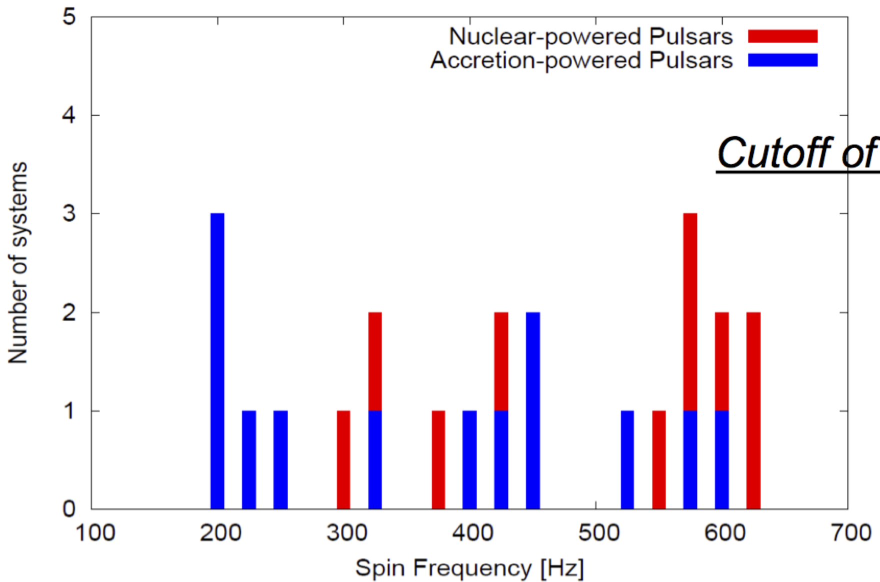


Neutron stars: accretion

- Low-mass X-ray binaries (LMXBs):

  Neutron star (or black hole) in orbit with an ordinary star
- No LMXB with spin > 700 Hz
- Accretion tries to "spin up" the neutron star, but GW try to spin it down?

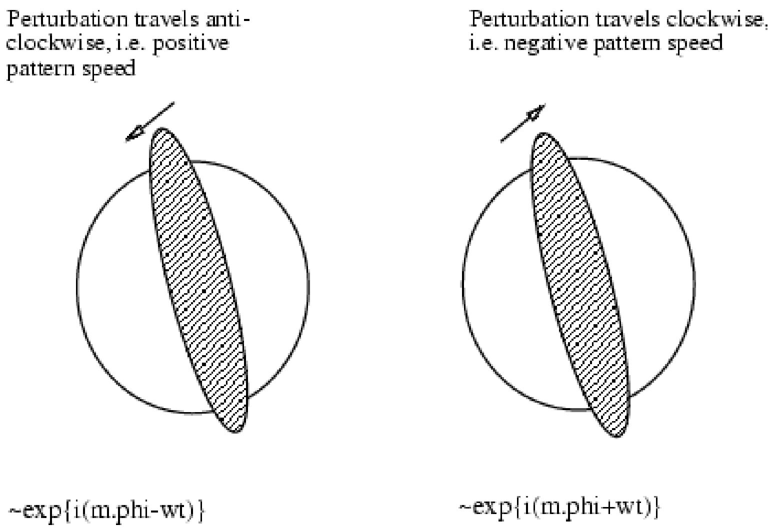
Indirect evidence for gravitational waves



Cutoff of distribution at ~730 Hz

Continuous gravitational waves

Neutron stars: fluid modes



- Fluid excitation causes star to get deformed
- Suppose deformation moves *retrograde* in the co-rotating frame
- If star spins fast enough, deformation may move *prograde* in inertial frame

  → GW carry away positive angular momentum, star spins down
- In co-rotating frame: angular momentum of the deformation increasingly more negative

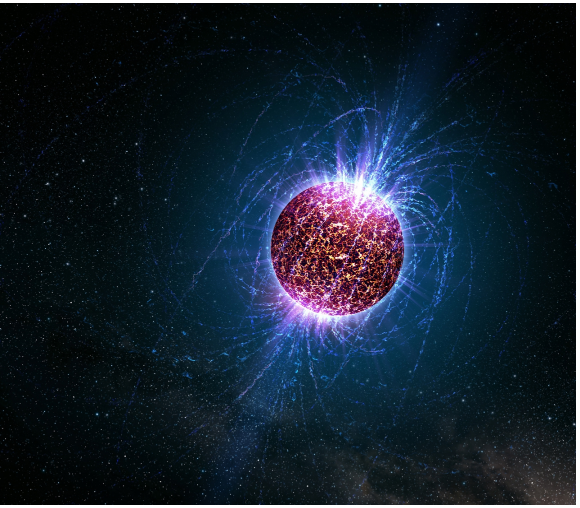
  → Deformation grows!

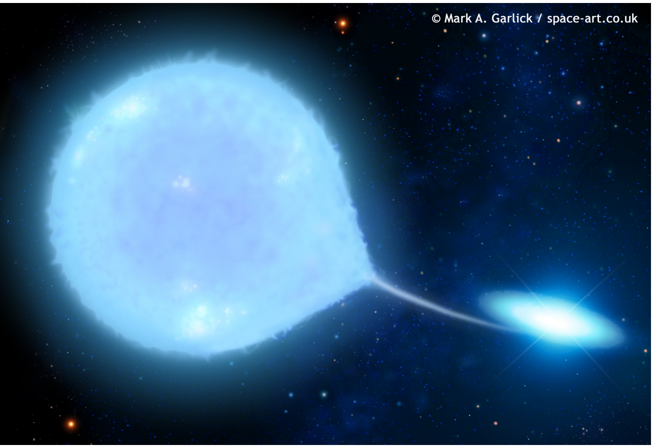
- If star can be observed electromagnetically, measure rotation frequency f_{rot}

  If f_{GW} is the gravitational wave frequency:

  $f_{GW} = (4/3)\, f_{rot}$ instead of $f_{GW} = 2\, f_{rot}$

Continuous gravitational waves





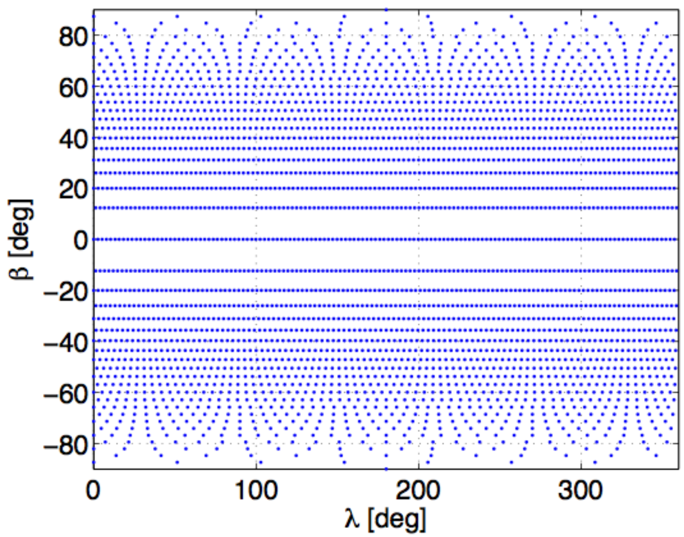
- Long-lived emission by fast-spinning neutron stars
- Signals are weak:

$$\mathcal{A} = 1.04 \times 10^{-25} \left(\frac{10\,\mathrm{kpc}}{r}\right) \left(\frac{I_3}{10^{38}\,\mathrm{kg\,m^2}}\right) \left(\frac{f}{1\,\mathrm{kHz}}\right)^2 \left(\frac{\epsilon}{10^{-6}}\right)$$

  … but can integrate for a long time

- Searches:
  - All-sky
    - Includes stars that have not (yet) been seen electromagnetically
  - "Directed"
    - Sky position known but not frequency
  - "Targeted"
    - Sky position and frequency known

Continuous gravitational waves



All-sky searches for isolated neutron stars

- Account for Doppler modulation due to motion of the Earth:

$$f(t) \simeq f_0 \left(1 + \frac{\vec{v} \cdot \hat{n}}{c}\right) \approx f_0 \left(1 + \frac{\Omega_{orb} R_{orb} \cos\beta \sin(\Omega_{orb} t)}{c}\right)$$

  – Binning in sky position:

    up to O(10^5) points

- Given a sky position, need to account for change in intrinsic frequency due to spin-down:

$$f_0(t) = f_0 + \dot{f}_0 (t - t_0) + \frac{\ddot{f}_0}{2}(t - t_0)^2 + \ldots$$

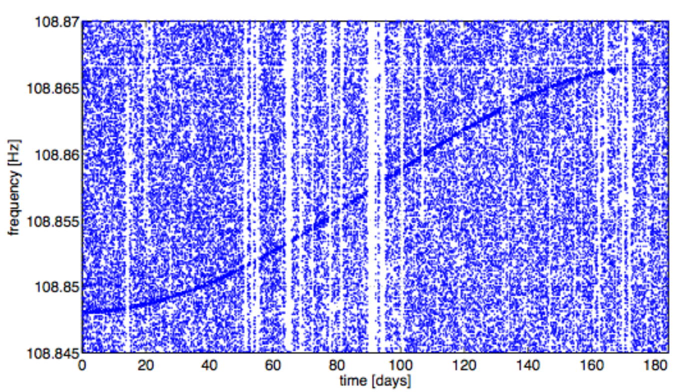
  – Binning in spin-down coefficients:

    typically O(10^6) points

- Hierarchical approach

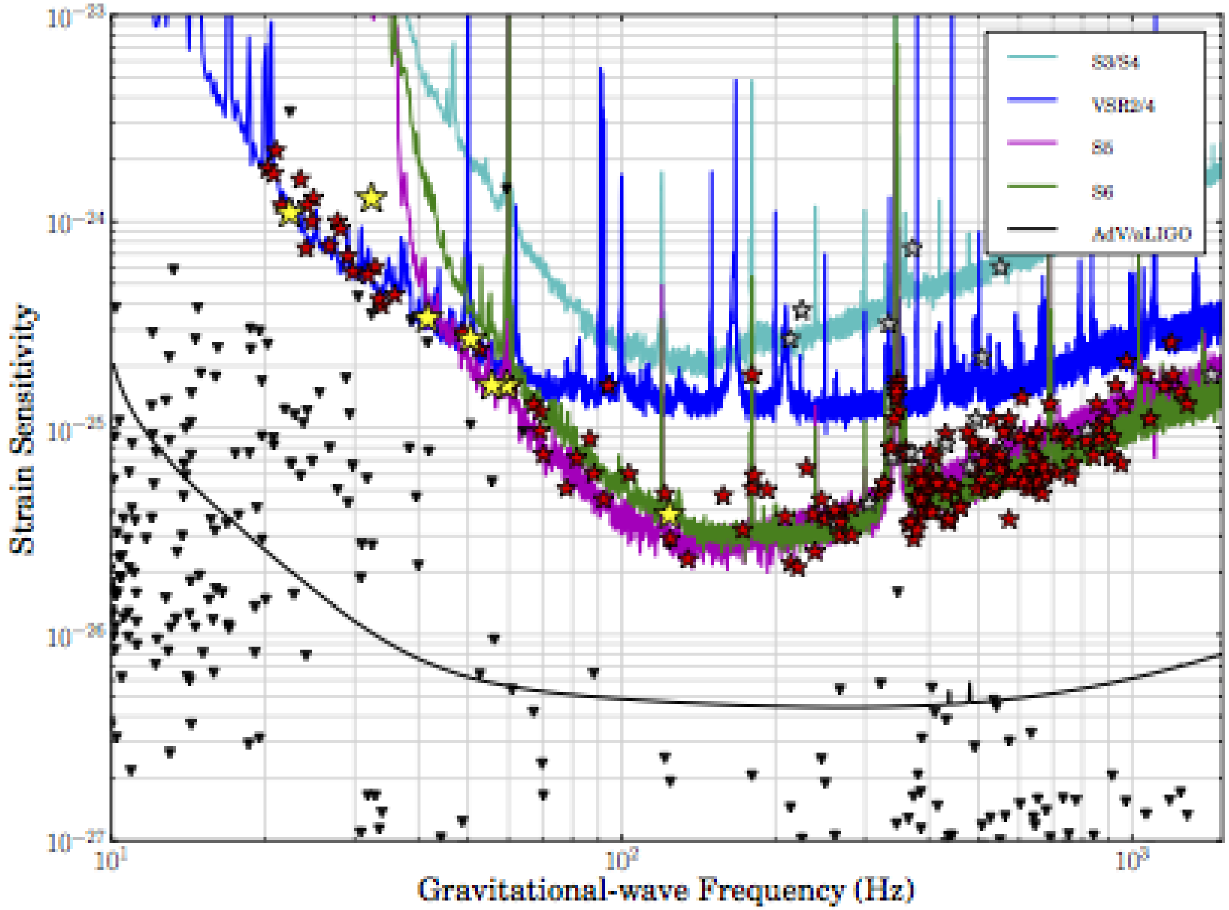
  – Refine grids once candidate events

  – Signal reconstruction



Continuous gravitational waves

“Targeted” searches: sky position and frequency both known



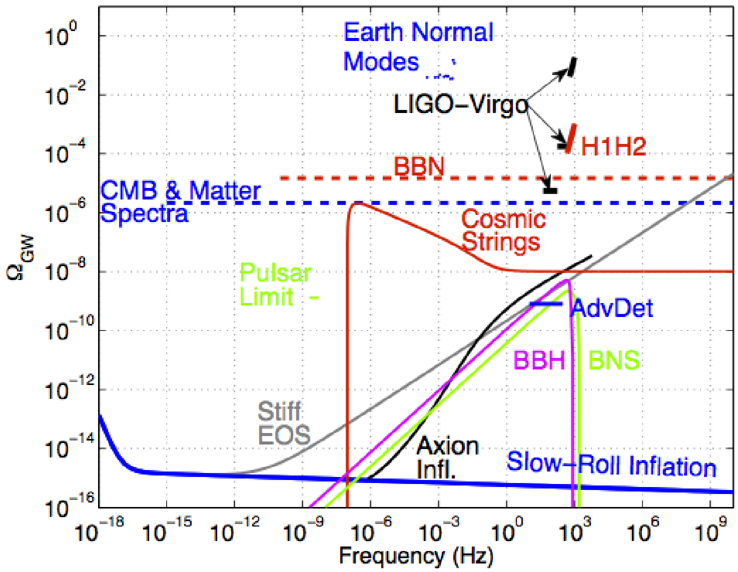
☆ Upper limits on amplitude for 195 pulsars from initial LIGO/Virgo runs

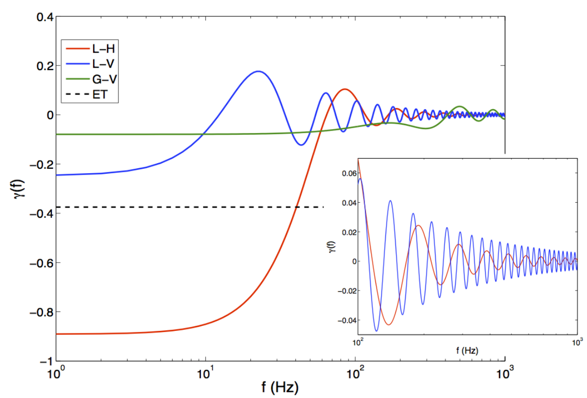
- Crab pulsar: GW emission not more than 1% of spin-down

▼ Upper limits on GW emission from other known pulsars

Stochastic gravitational waves

$$\Omega_{gw}(f) = \frac{d\rho_{gw}(f)}{\rho_c d(\ln f)}$$





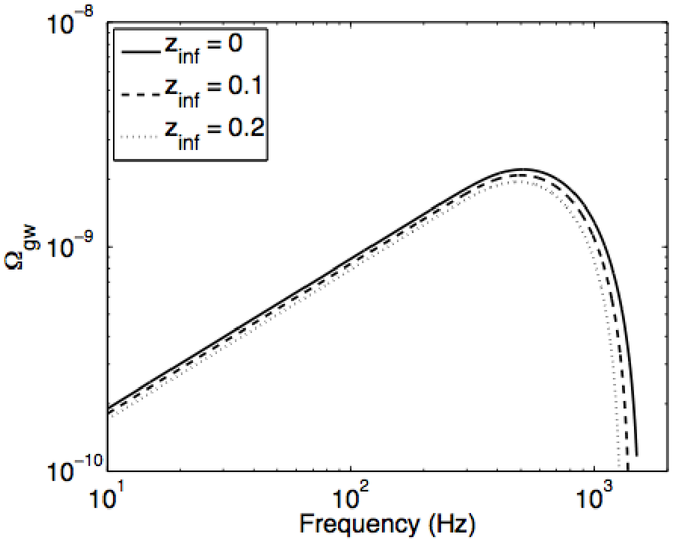
- Gravitational wave backgrounds of a fundamental nature
  - Inflation:
    - Period of exponential growth of the Universe
  - Phase transitions
  - Cosmic strings
  - …
- Searched for by cross-correlating between detectors:

$$Y = \int \tilde{s}_1^*(f)\tilde{Q}(f)\tilde{s}_2(f)\,df$$

  - Optimal filter:

$$\tilde{Q}(f) \propto \frac{\gamma(f)\Omega_{gw}(f)}{f^3 P_1(f) P_2(f)} \quad \text{with } \Omega_{gw}(f) \equiv \Omega_0 f^{\alpha}$$

Stochastic gravitational waves



- Stochastic background of weak signals from far-away sources (e.g. coalescing binaries) that are not individually resolvable
- Might be detectable after ~1 year of operations at design sensitivity!

"High" event rate

"Realistic" event rate

"Low" event rate