THE EVOLUTION OF COLLECTIVE STRATEGIES IN COMMUNITIES

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“Under what conditions will cooperation emerge in a world of egoists without central authority?”

–Robert Axelrod, “The evolution of cooperation” (1984)

EXTREME ALTRUISM IN *DICTYOSTELIUM DISCOIDEUM*

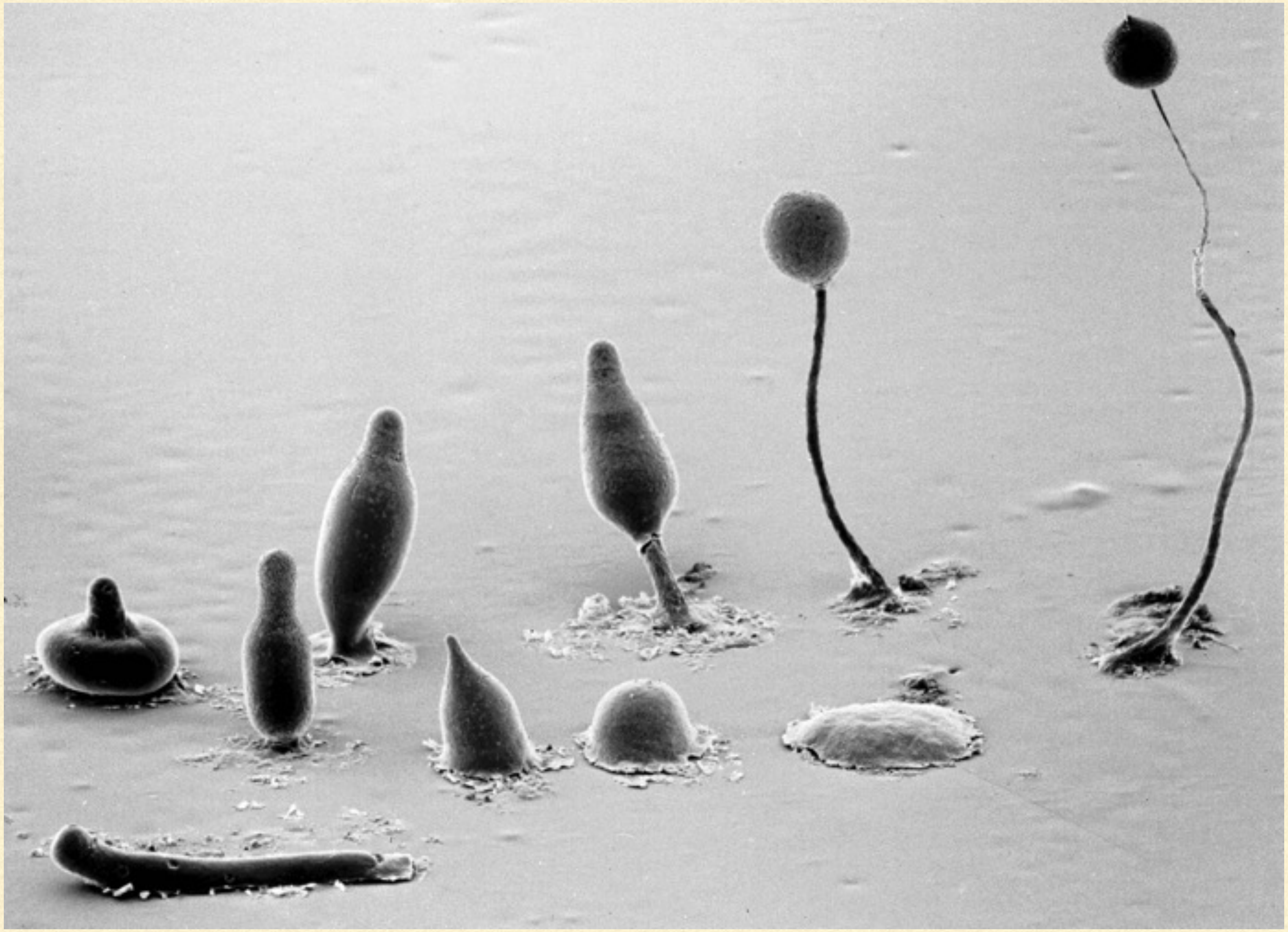


Photo: *M.J. Grimson & R.L. Blanton. Biological Sciences Electron Microscopy Laboratory, Texas Tech University*

WHY COOPERATE?

Photo: *Alamsyah Rauf*

- Cooperation is an organizational mechanism that is observed over a range of scales in the natural world.
- But… why would any individual agent decide to cooperate in a situation when it would be more personally beneficial to act otherwise?
- **Game theory** provides a theoretical framework for the understanding of the evolution of cooperative strategies.
- Perhaps the best known, and most used, paradigm for studying such phenomena is the *Prisoner's Dilemma* (PD) game.

CANONICAL PAYOFF MATRIX FOR THE TWO PLAYER PD GAME

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

T: Temptation [to defect while the other cooperates]
R: Reward [for mutual cooperation]
P: Punishment [for mutual defection]
S: Sucker's payoff [for cooperating while the other defects]

(Typically one sets R=1, P=S=0)

For this to be a PD game, we require:

$$T > R > P > S$$

In this case, defection is the *dominant strategy* for both players, and hence the Nash equlibrium* is **mutual defection.**

* Unilateral deviation from this situation will not benefit either player

THE ITERATED PRISONER'S DILEMMA

- While game theory predicts that interactions of this nature will lead to defection, this is <u>not</u> what is observed in experiments or in society.
- One way of understanding this is through the framework of the *Iterated Prisoner's Dilemma* (IPD)*. Here, the agents choose their **action** (either cooperate [C] or defect [D]) at each step, based on their choice of **strategy**.
- The strategy could be deterministic or probabilistic, and may incorporate memory of previous actions. Note that:
  - If the IPD game is played X times, and it is *known* that the game will be played X times, the only rational strategy for a player (arrived at via induction) is to "always defect".
  - However, if it is *not known* that the game will be played X times, other strategies might be lead to better outcomes.

* Axelrod, R., *"The evolution of cooperation"* (Basic Books, 1984).

AXELROD'S TOURNAMENT

- In 1980, Robert Axelrod organised a tournament to find the "best" possible strategy for the IPD.
- Numerous programs, each of which encoded a particular strategy, competed against each other and themselves.
- The winning strategy was "Tit for tat" developed by Anatol Rapoport, where a player simply mimics the opponent's action in the next round.
- It was subsequently shown that a superior strategy is "Win-stay lose-shift", where a player switches action only if unsuccessful.

Photo: http://skytteprize.statsvet.uu.se/PrizeWinners.aspx

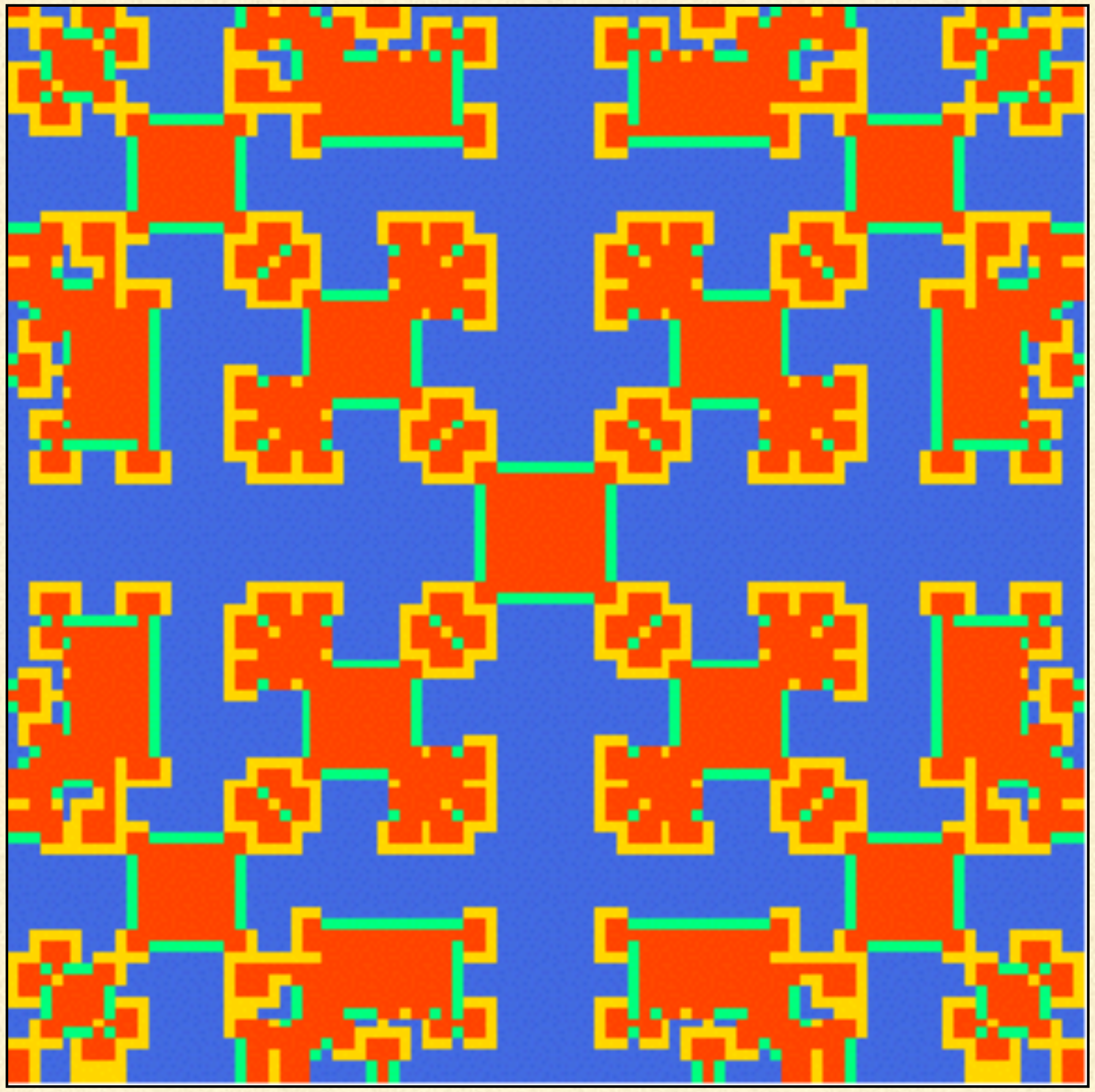
EMERGENCE OF COOPERATION

- These strategies are limited to the context of two-player systems.
- There have been many mechanisms proposed to understand the emergence of cooperation [Nowak, *Nature* **314,** 1560-1563 (2006)].
- In a well-mixed populations, which have an equal likelihood of any two agents interacting, natural selection tends to favour defectors.*
- However, this argument doesn't necessarily hold for interactions over social networks. In this case, cooperation can emerge through **network reciprocity**, i.e. cooperators survive by forming (possibly dynamic) clusters on the network.

* Maynard Smith, J., "*Evolution and the Theory of Games*" (Cambridge Univ. Press, Cambridge, 1982).

NETWORK RECIPROCITY

N=99x99 agents, T=1.9



Blue C->C, Red D->D, Yellow C->D, Green D->C

- Nowak and May* demonstrated that when the IPD is played on a square lattice, chaotically changing spatial patterns arise** if agent employs *unconditional imitation.*
- This is a deterministic strategy in which each agent plays the game with its neighbours and, in the next step, copies the strategy of the most successful neighbour.

* Nowak, M. A & May, R. M., *Nature* **359**, 829-829 (1992).

** Regardless of whether an agent plays with 4 or 8 neighbours, and whether or not self-interactions are considered.

A PROBABILISTIC UPDATING STRATEGY

- Deterministic strategies implicitly assume perfect transfer of information. In a more generalized setting, agents can copy the action of another with some probability.

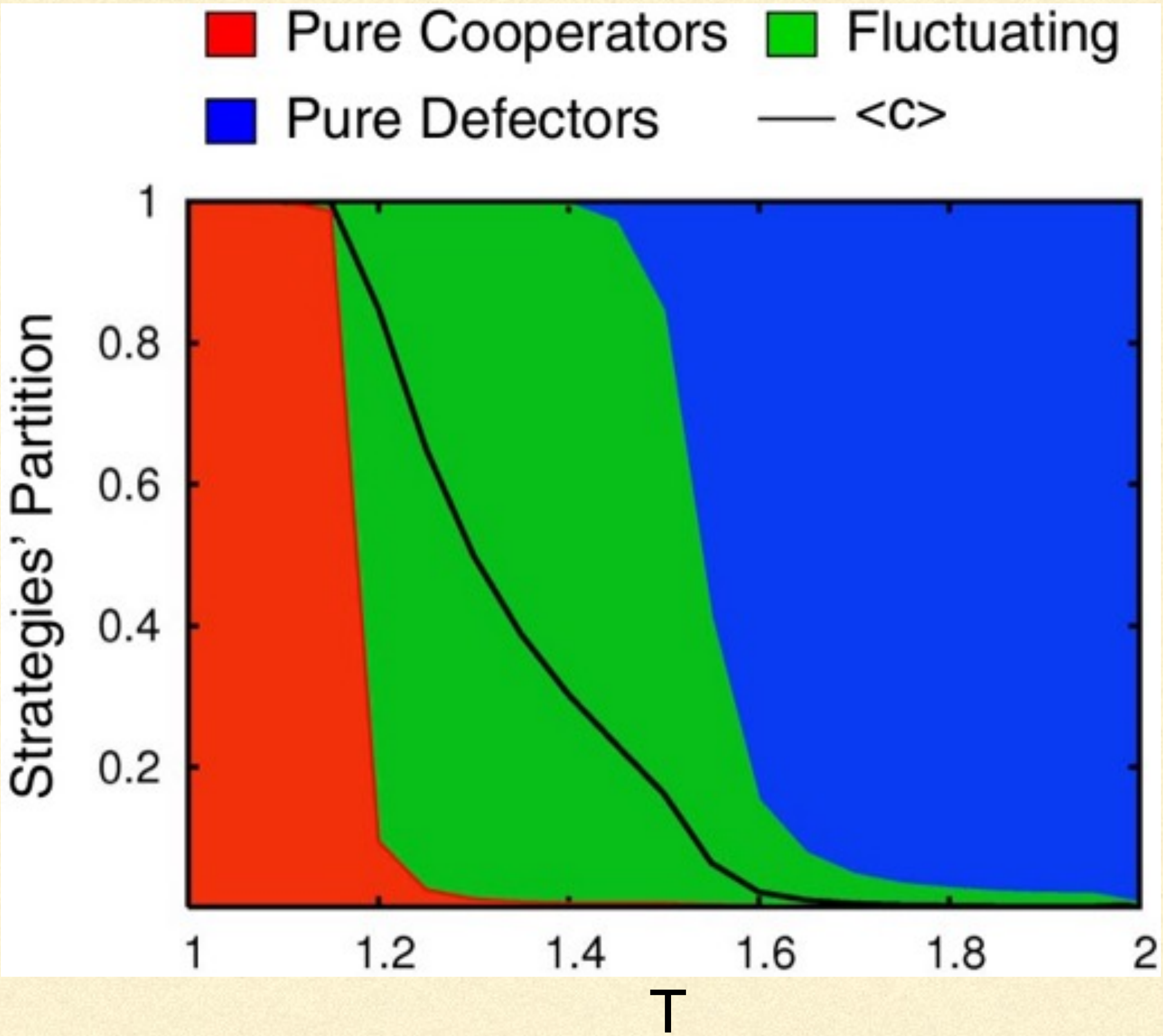
- A commonly used probabilistic strategy* is as follows:

  Each agent i on a network compares its payoff π_i with that of a randomly chosen neighbour j (π_j). If $\pi_i \geq \pi_j$, the agent repeats its action in the next step, otherwise it copies the action of j with a probability proportional to the difference between their respective payoffs, and dependent on the temptation T and their degrees ($k_{i,j}$), namely

$$\Pi_{i \to j} = \frac{\pi_j - \pi_i}{T \max(k_i, k_j)}$$

* Santos, F. C. & Pacheco, J. M., *Phys. Rev. Lett.* **95**, 098104 (2005).

IPD ON ERDÖS-RÉNYI NETWORKS

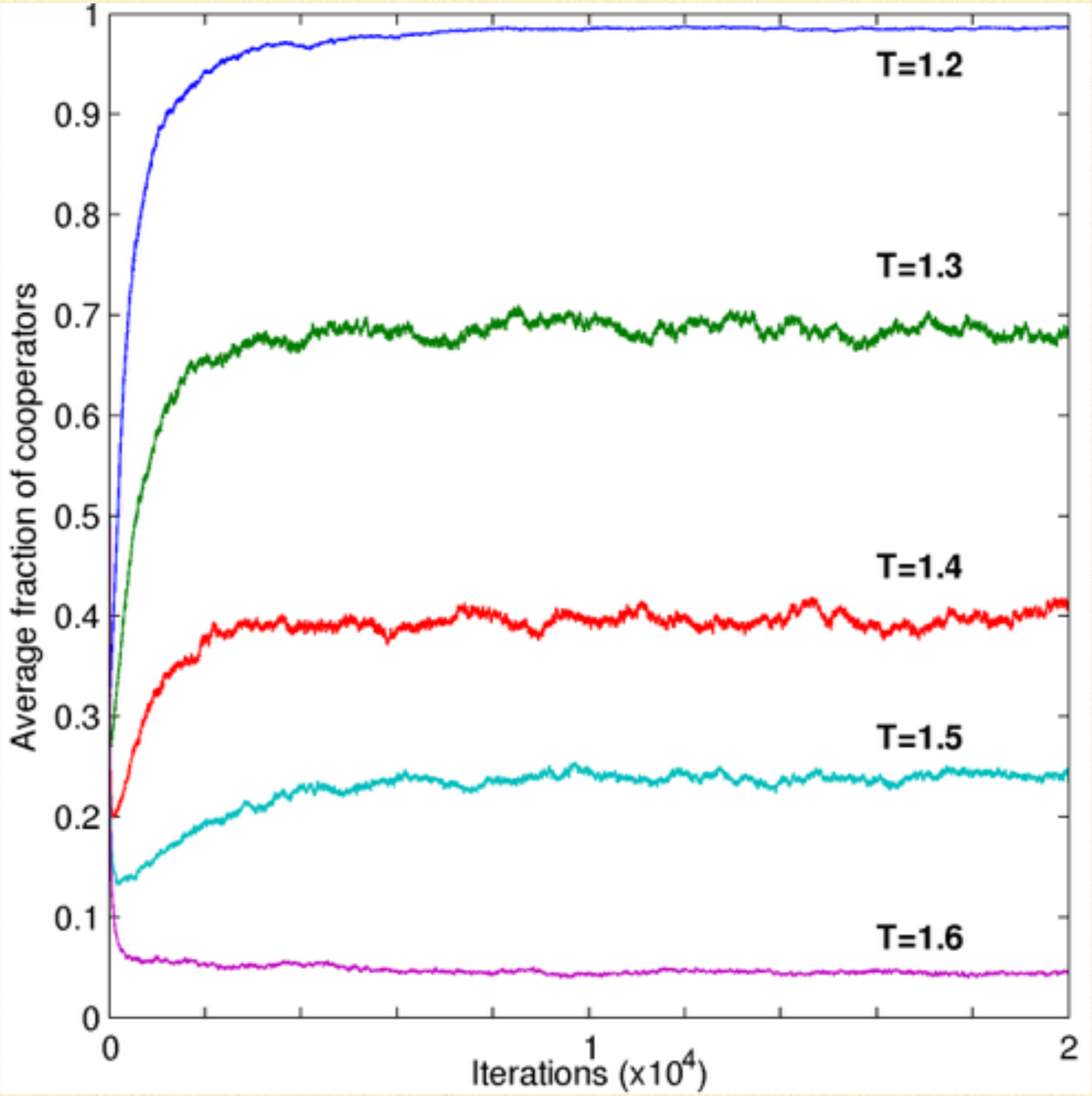


It was found that when agents playing an IPD on an Erdös-Rényi (ER) random network use this probabilistic strategy to update their choice of action, three different strategies emerge*.

The adjacent figure [*taken from Gómez-Gardeñes, J. et al. (2007)*] corresponds to the case N=4000 nodes, and average degree $\langle k \rangle = 4$.

* Gómez-Gardeñes, J. et al., *Phys. Rev. Lett.* **98**,108103 (2007).

IPD ON ERDÖS-RÉNYI NETWORKS



Average fraction of cooperating nodes in ER networks of agents playing the IPD game using replicator dynamics. The networks have N = 1024 agents with average degree $\langle k \rangle = 4.$

The simulations begin with 50% cooperators, and the final result is averaged over 20 trials.

AN UPDATING STRATEGY IN A "NOISY" ENVIRONMENT

In certain situations, stochasticity in the update rule may arise due to some external source (rather than uncertainty in the dynamical state of the system). A simple updating rule that incorporates (tunable external) noise is the **Fermi rule***.

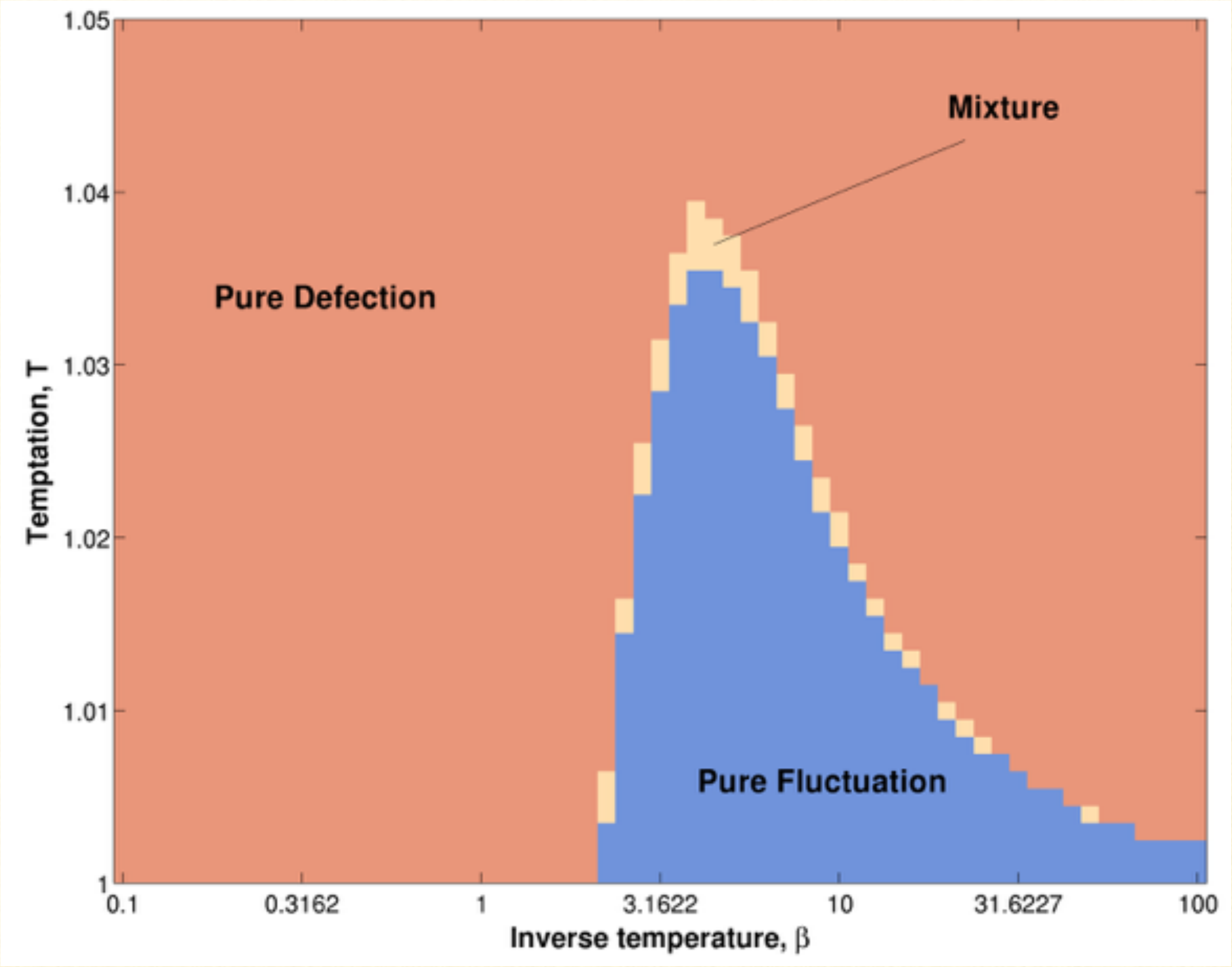
Here, each agent i randomly picks a neighbour j and copies its action with a probability proportional to the Fermi distribution function

$$\Pi_{i \to j} = \frac{1}{1 + \exp(-\beta(\pi_j - \pi_i))}$$

where β can be thought of as the inverse of temperature, or "noise", in the decision-making process.

* Szabó G. & Toke C., *Phys. Rev. E* **58**, 69-73 (1998).

FERMI RULE ON LATTICES

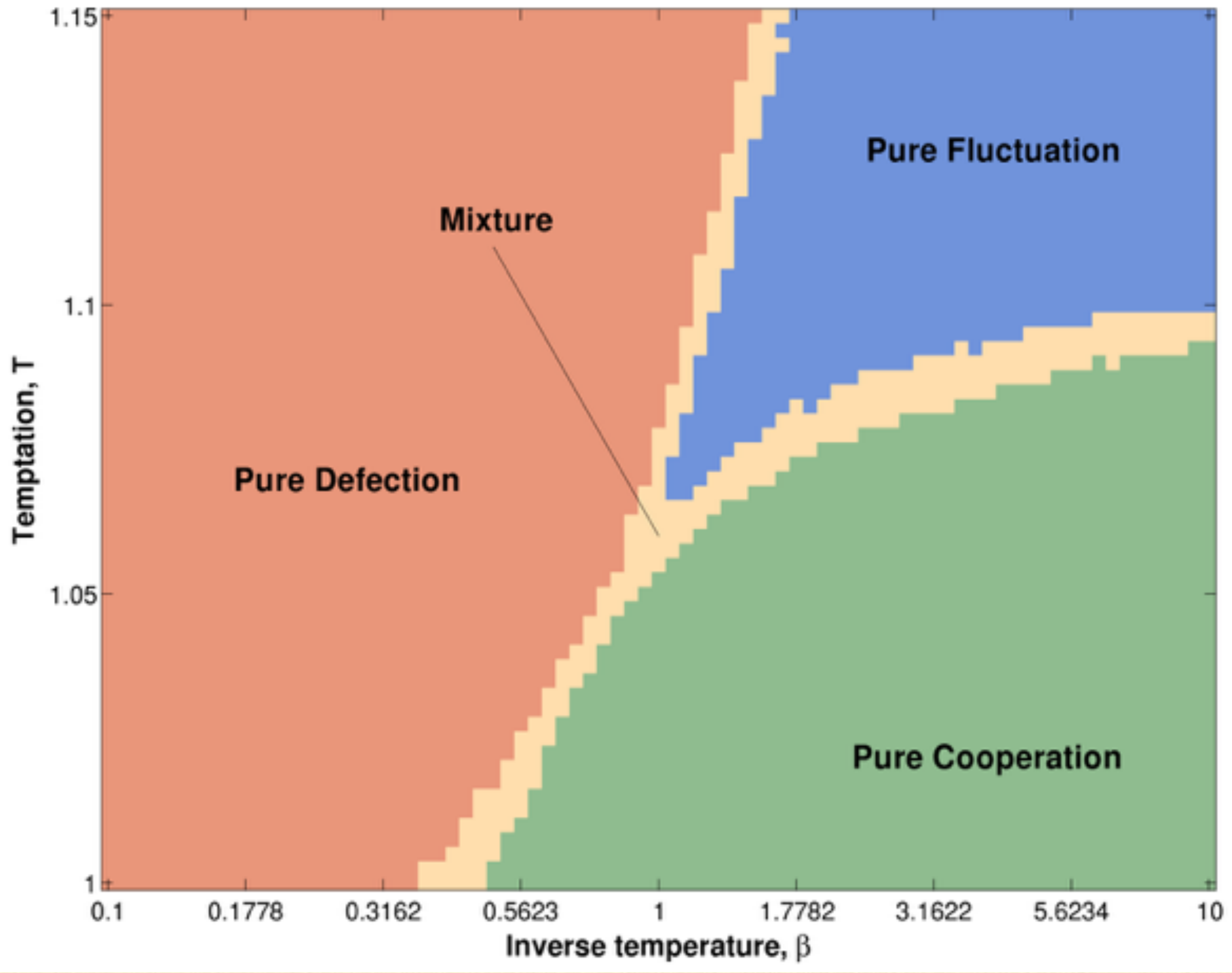


On simulating the IPD on a lattice, using the fermi rule, different collective decisions can arise.

For each realization, 50% of the nodes of an 100x100 square lattice [with 4 neighbours and no self interactions] are randomly chosen to be cooperators.

The regimes are demarcated by identifying those areas where > 50 of 100 trials result in a specific collective outcome.

FERMI RULE ON RANDOM NETWORKS



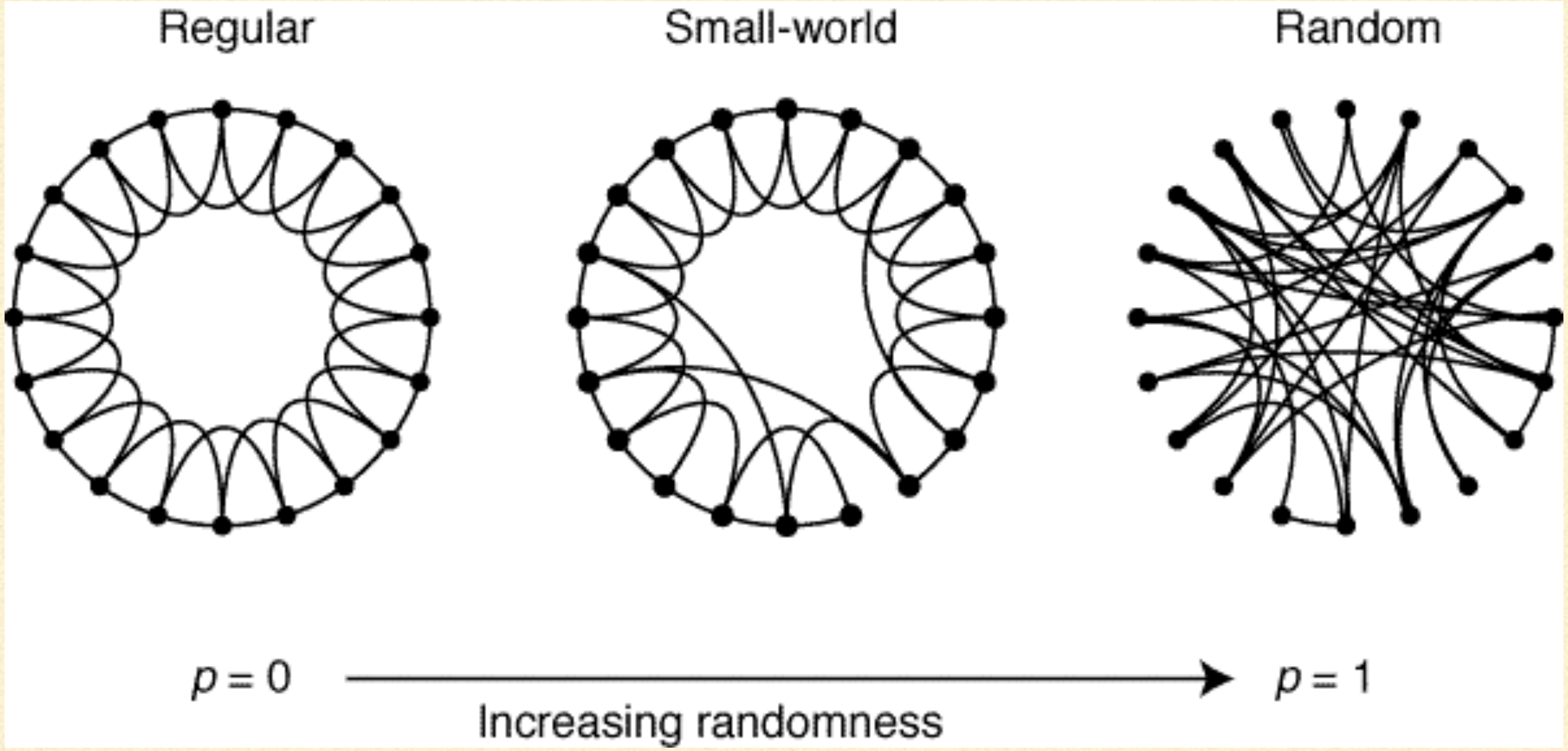
On simulating the IPD on an ER network, using the fermi rule, "pure cooperation" can also be obtained.

For each realization, 50% of the nodes of an ER network [with N=2000 and average degree $\langle k \rangle = 10$] are randomly chosen to be cooperators.

The regimes are demarcated by identifying those areas where > 50 of 100 trials result in a specific collective outcome.

REWIRING REGULAR NETWORKS

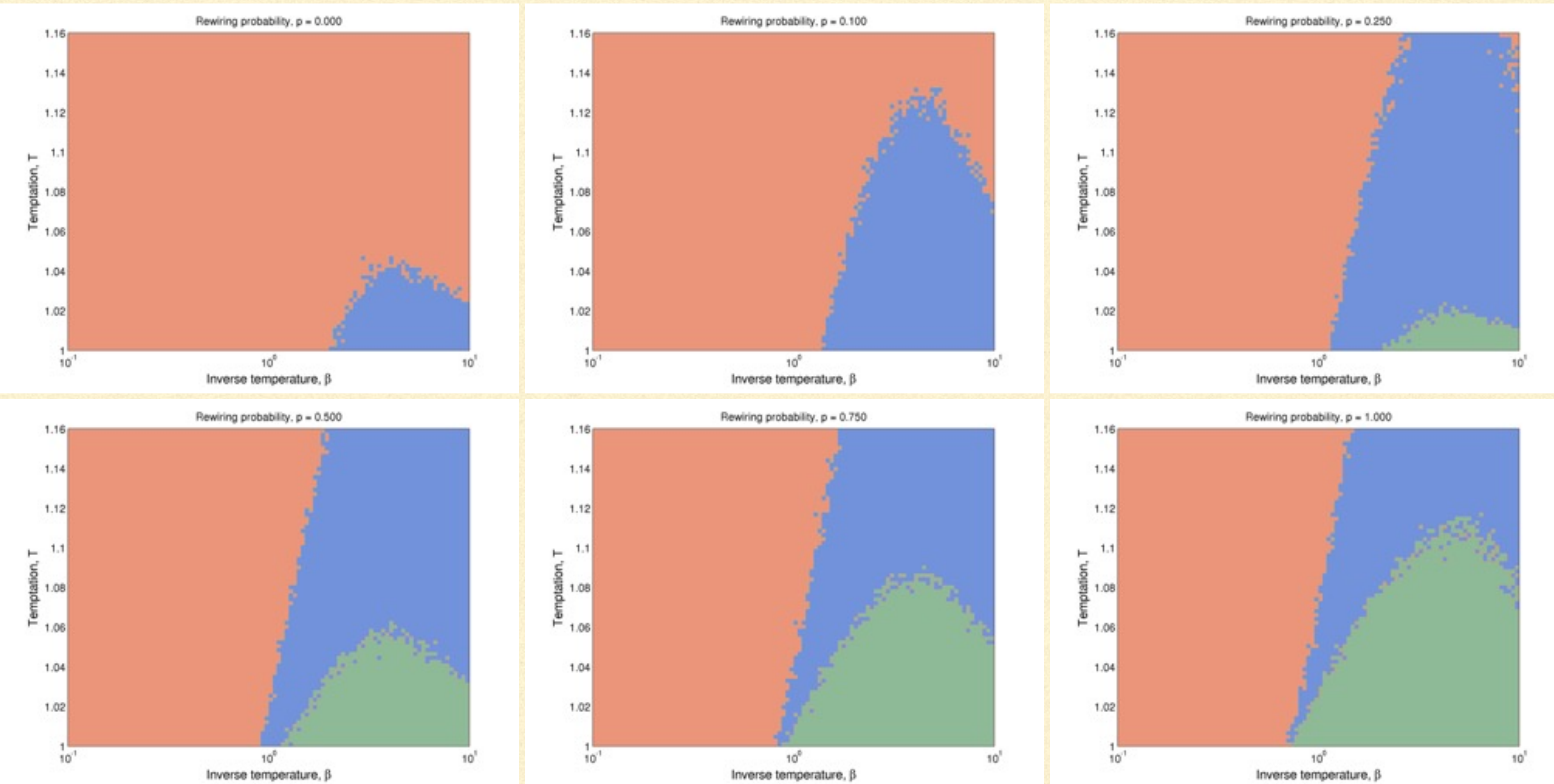
We interpolate between a lattice and a random network using the rewiring procedure similar to that of Watts & Strogatz (1998)* [*from which the figure below is taken*].



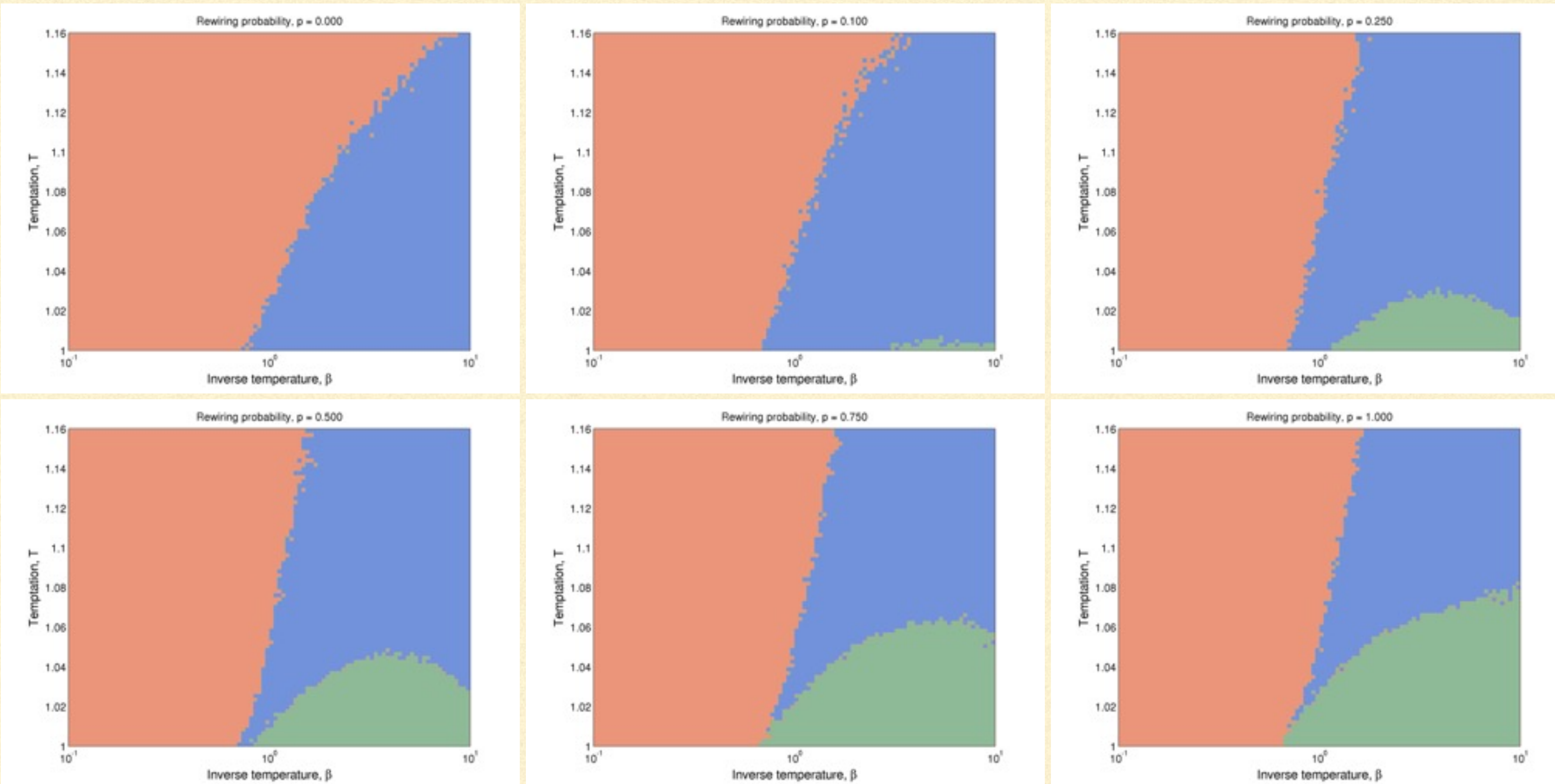
We cycle through all the links randomly and break and rewire each of them (from one of the two originally connected nodes to a randomly chosen node) with a probability p.

* Watts, D. J. & Strogatz, S. H., *Nature* **393**, 440-442 (1998).

We simulate the IPD over a range of T and β, on a 50x50 square lattice [4 neighbours, no self interactions] and on rewired random networks (p=0.1, 0.25, 0.5, 0.75, 1.0).



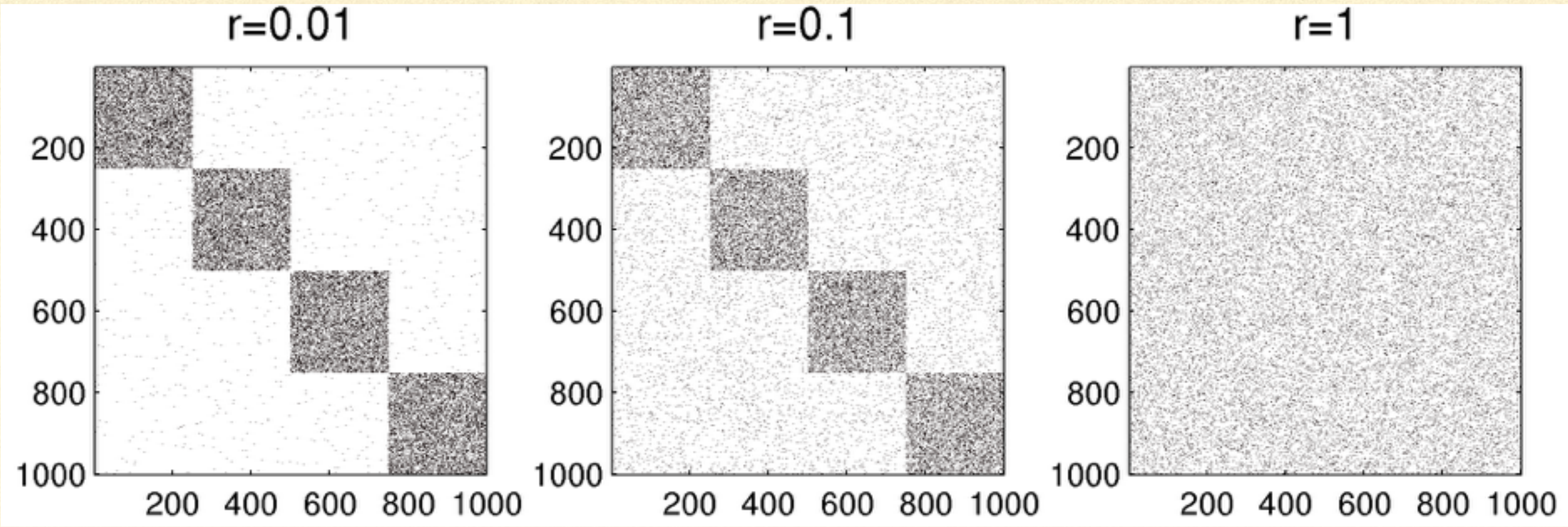
Similar results can be obtained by starting with a 50x50 lattice in which each node has a higher degree [8 neighbours, no self interactions].



OBSERVATIONS

- In the high temperature/noise limit ($\beta \to 0$), pure defection is the only possible outcome.
- The range of temperatures over which pure cooperation exists increases as we increase the rewiring probability p.
- Cooperation emerges at a non-zero value of p, and this critical value is smaller if the average degree increases.
- On increasing the average degree of the network, the tricritical point at the interface of the three phases can be observed at increasingly higher values of T.

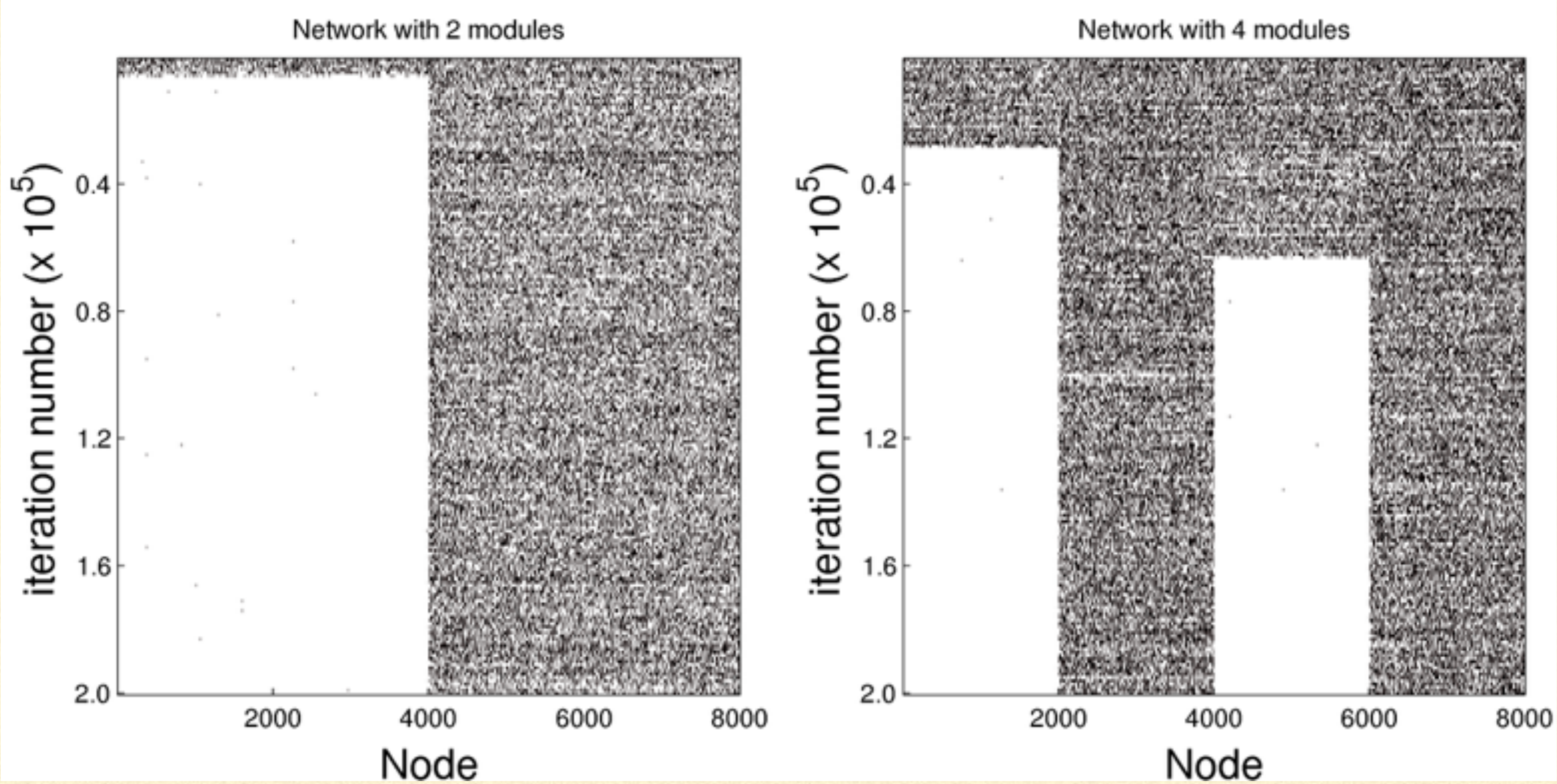
MODULAR NETWORKS



- We next consider the case of modular networks, i.e. networks with community organization.
- We interpolate between ER and modular networks using the modularity parameter $r = \rho_{\rm out}/\rho_{\rm in}$, the ratio of extra to intra-modular connectivity.

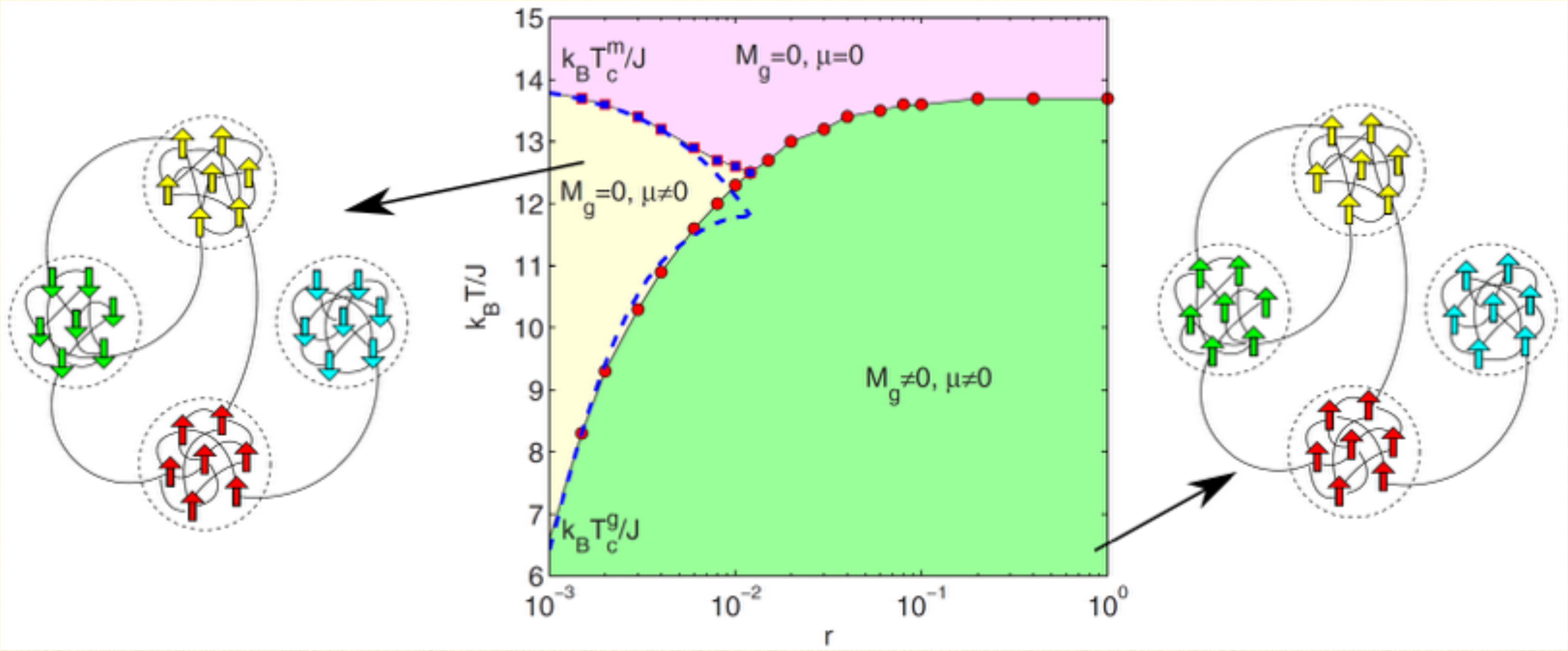
FERMI RULE ON MODULAR NETWORKS

For a range of values of T and β, a highly modular network (r ~ 0.001) can exhibit a pattern characterized by the coexistence of multiple collective strategies.



ORDER-DISORDER TRANSITIONS

This result appears to have an intriguing connection with a recent investigation into Ising spins on modular networks*.



When r << 1 there exists a range of temperatures for which individual modules can evolve to different ordered states.

* S. Dasgupta, R. K. Pan and S. Sinha, *Phys. Rev. E* 80, 025101(R) (2009).

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- S. Dasgupta, R. K. Pan and S. Sinha, *Phys. Rev. E* **80**, 025101(R) (2009).

“The only thing that will redeem mankind is cooperation”

–Bertrand Russell (1954)