Contagion, Coordination & Communities

Diffusion of innovations on social networks with modular organization

CELEBRATING 50 years of CREATIVE SCIENCE 1962 – 2012

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What and Why ?

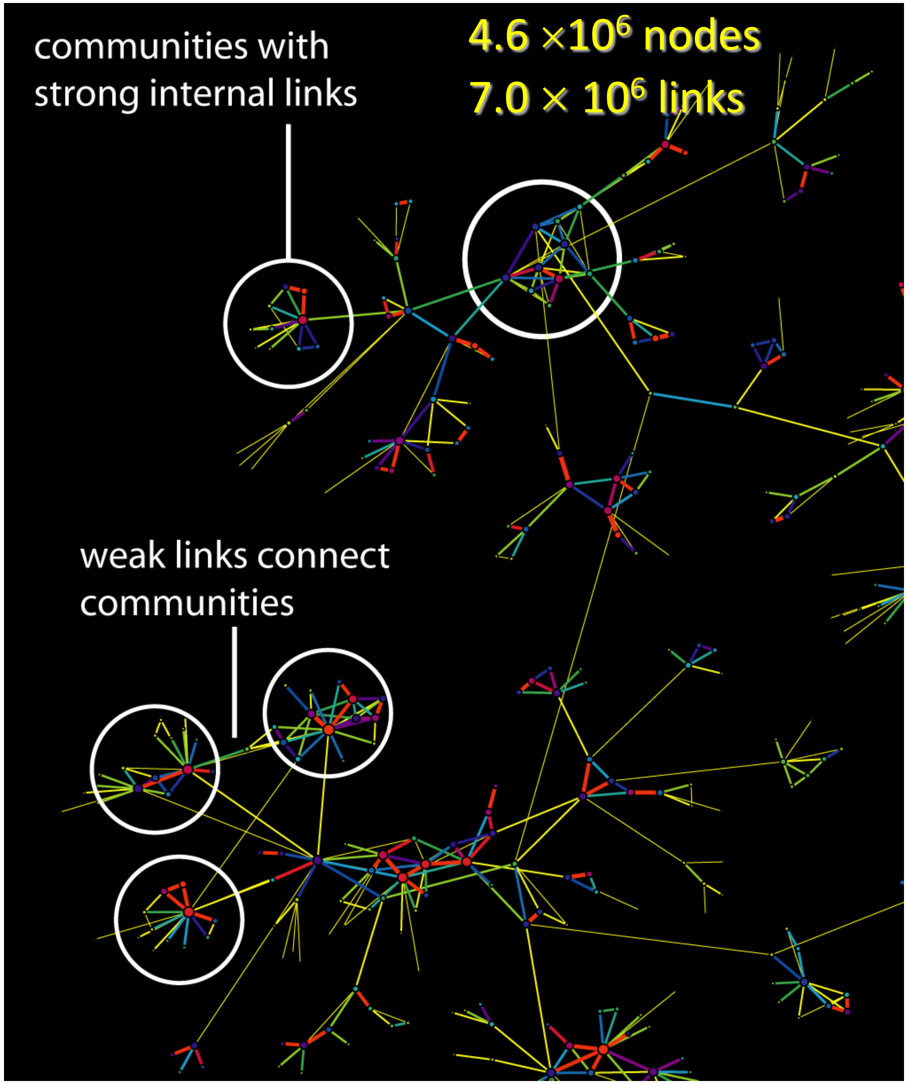
- Success of attempts to change public perception on key issues depend critically on whether the attitude of a significant fraction of the population can be changed

- Opinion of individuals are affected by that of their peers (neighbors in social network) $\Rightarrow$ change in attitude has to spread through a major part of the network connecting the entire population

- Mass media can influence people but by itself may not be sufficient to induce individuals to change their habits, especially if it involves costs (e.g., increased efforts, or just going against the inertia of ingrained habits) – the group to which an individual belongs can reinforce the opinion she presently holds and thereby resist the efforts of external agencies to change it.

- Important to understand how structural organization of social network – in particular the occurrence of communities or modules - affects the process of changing collective opinion, i.e., diffusion of innovation

Modularity of social networks



J P Onnela et al. *PNAS*,**104**,7332 (2007)

Modules: Cohesive groups

communities with dense internal & sparse external connections

Examples of modular social networks

- Cell phone communication
- Scientific collaborators
- e-mail communication
- PGP encryption ”web-of-trust”
- non-human animals

Diffusion of Innovation: The Bass Model

F. M. Bass, "A new product growth for model consumer durables," *Management Science*, **15** (1969) 215-227

A mean-field dynamical description of how innovation spreads in a population

F: fraction of population that has adopted a particular innovation

$$dF/dt = (p + q F)(1 - F)$$

p: coefficient of innovation   q: coefficient of imitation

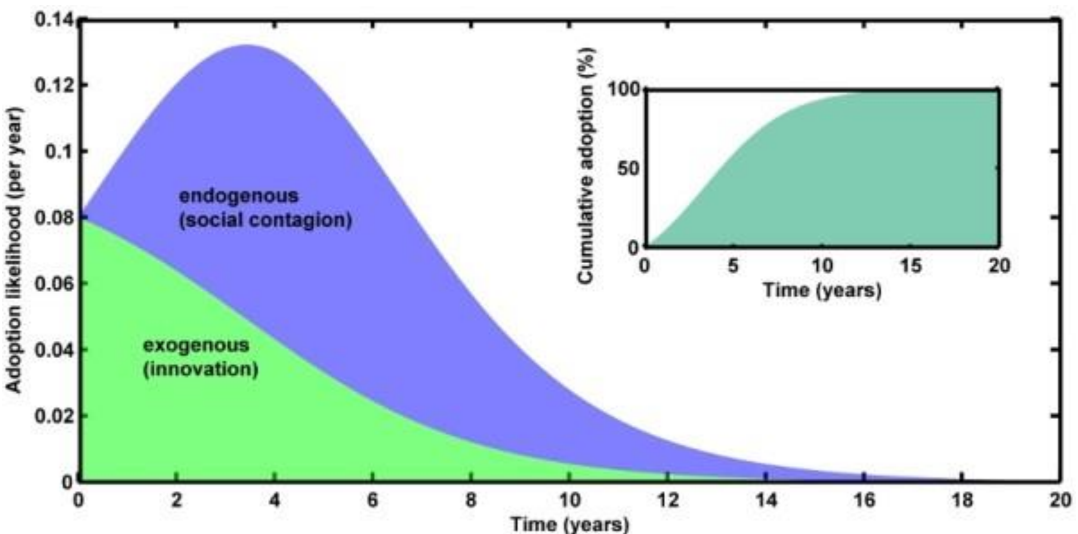
Data-fitting allows estimation of p, q for different consumer products in almost all cases p is at least 1 to 2 orders of magnitude smaller than q

Essentially identical to SI compartmental model of epidemic spreading

A Dunn *et al* *BMC Health Services Research* **12**(1):248 (2012)

q: probability that a susceptible agent gets contagion from infected neighbor

p:probability of spontaneous infection even when none in the network are infected (i.e., the infection arrives from outside population being considered).



Bass Model on a Network

- A population of N agents: represented by nodes of a network
- Composed of two subpopulations:
  - S(t): number of susceptibles at time t
  - I(t) : number of infected individuals at time t
- Each link between a pair of nodes is a contact along which infection can propagate.
- Susceptible nodes with k_{inf} infected neighbors become infected with probability

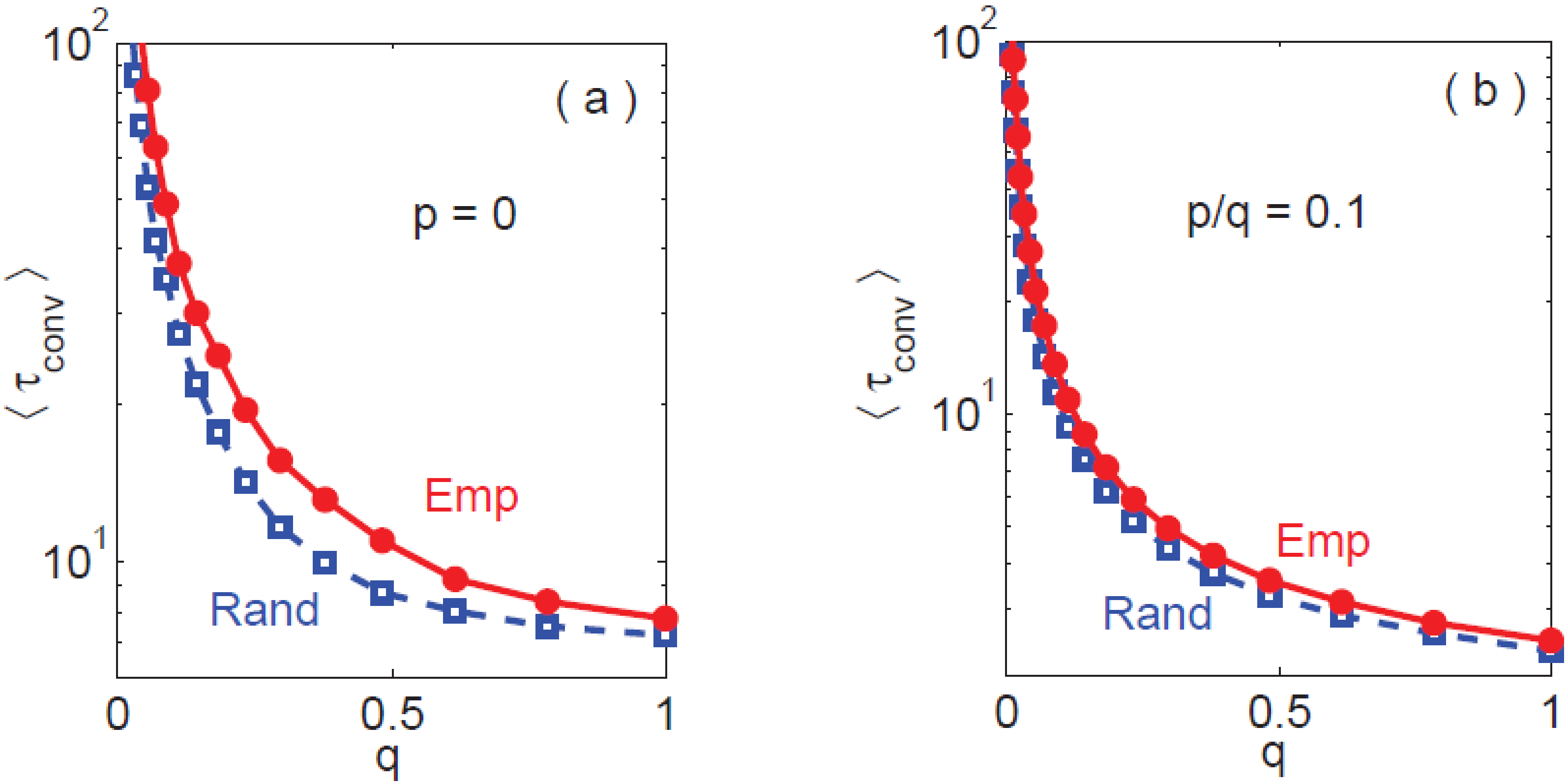
$$\alpha = [1 - (1-p)(1-q)^{k_{inf}}]$$

q : rate of contagion transmission from an infected to a susceptible individual

If p = 0, the innovation can only spread through contagion spreading $\Rightarrow$ require to have a few nodes initially in the infected (adopted) state to begin the process of diffusion
[if p has a finite value, the process can start even in a population where no one has initially adopted the innovation]

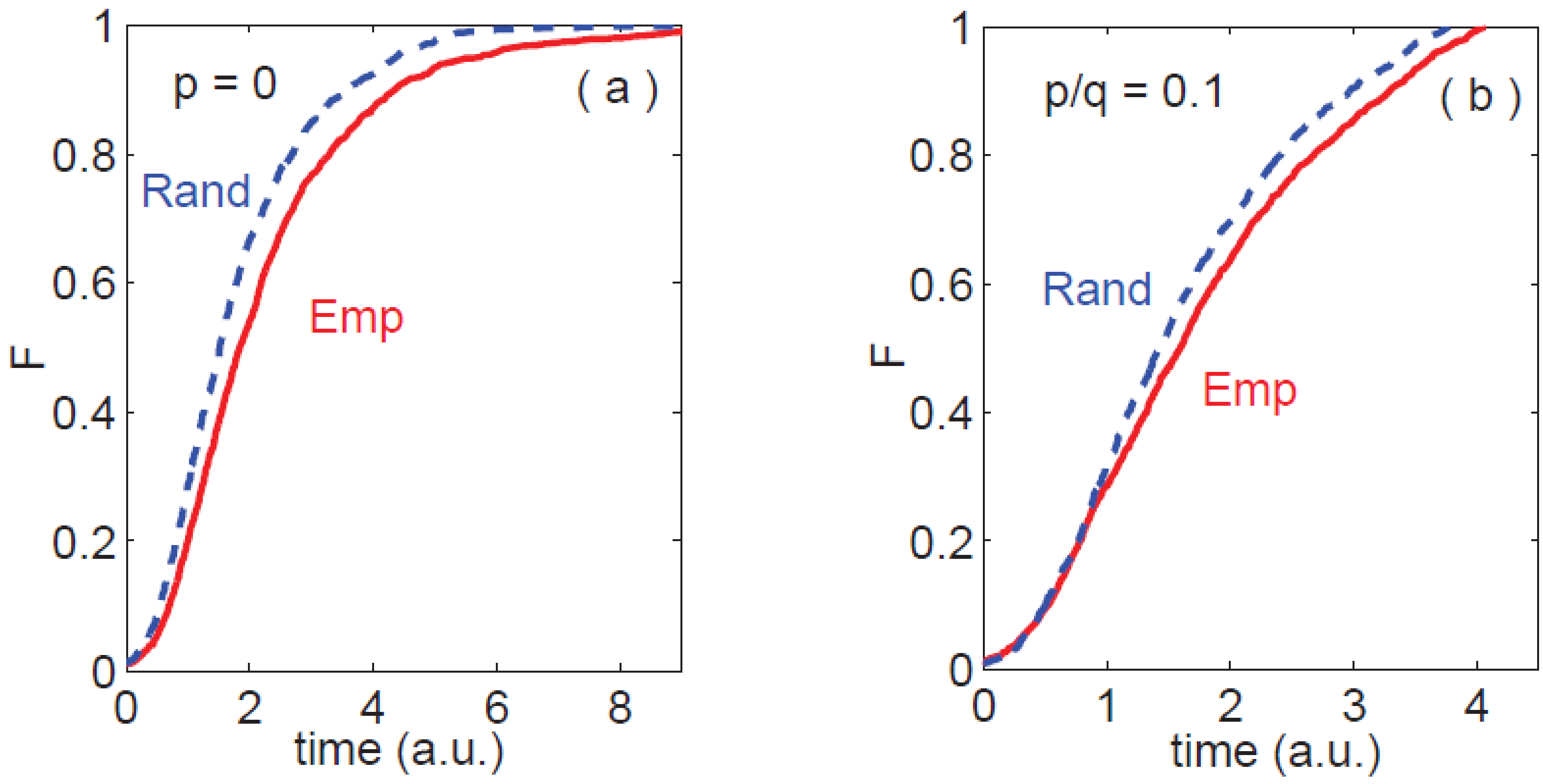
The dynamical equations for the network are solved using the Gillespie algorithm and the system behavior is analyzed for 100 realizations, with only 1% of the population chosen to be initial adopters of the innovation.

Bass Model on Real Social Networks



Average time required for 98% of a population of N = 1497 individuals connected through the empirical social network (solid curve) and its degree-preserved randomized surrogate (broken curve), to adopt an innovation as a function of q (rate of contagion transmission)

Bass Model on Real Social Networks



Time-evolution of the fraction of population of N = 1497 individuals connected through the empirical social network (solid curve) and its degree-preserved randomized surrogate (broken curve), who have adopted an innovation for the rate of contagion transmission q = 0.379

Limitations of Bass model:

- Any innovation is destined to eventually be adopted by the entire population
- Cannot explain why certain innovations get adopted rapidly while others with very similar perceived advantages do not ?
- Nor can it explain why the speed of adoption of fairly similar innovations can vary over a wide range ?
- Does not capture the cooperative nature of interactions within a social group, whereby the effect of multiple neighbors adopting a particular innovation can affect the other members of the group in a nonlinear manner

A Spin Network Model for Diffusion of Innovation

(Chandrashekar & Sinha, Proc. IEEE COMSNETS 2015 Social Networks Workshop)

Binary (Ising) spin at each node with states $\sigma_i = \pm 1$, i = 1, 2, …, N

+1 : Adoption   – 1: Non-adoption

The behavior of the entire network is governed by the energy function

$$\mathcal{H} = -\sum_{i,j} J_{ij}\sigma_i\sigma_j - \sum_i \theta_i\sigma_i - h\sum_i \sigma_i$$

Interaction strength J_{ij} = J if agents i and j are connected in a social network, else 0

J > 0 $\Rightarrow$ Each link tries to align the two spins connected by it

The system is subject to noise (random perturbations arising from the environment) characterized by a “temperature” T

Local field θ_i : individual thresholds of agents

Uniform external field h: perceived advantage of innovation being adopted

Order parameter M = $(1/N)\Sigma_i\sigma_i$: population fraction adopting

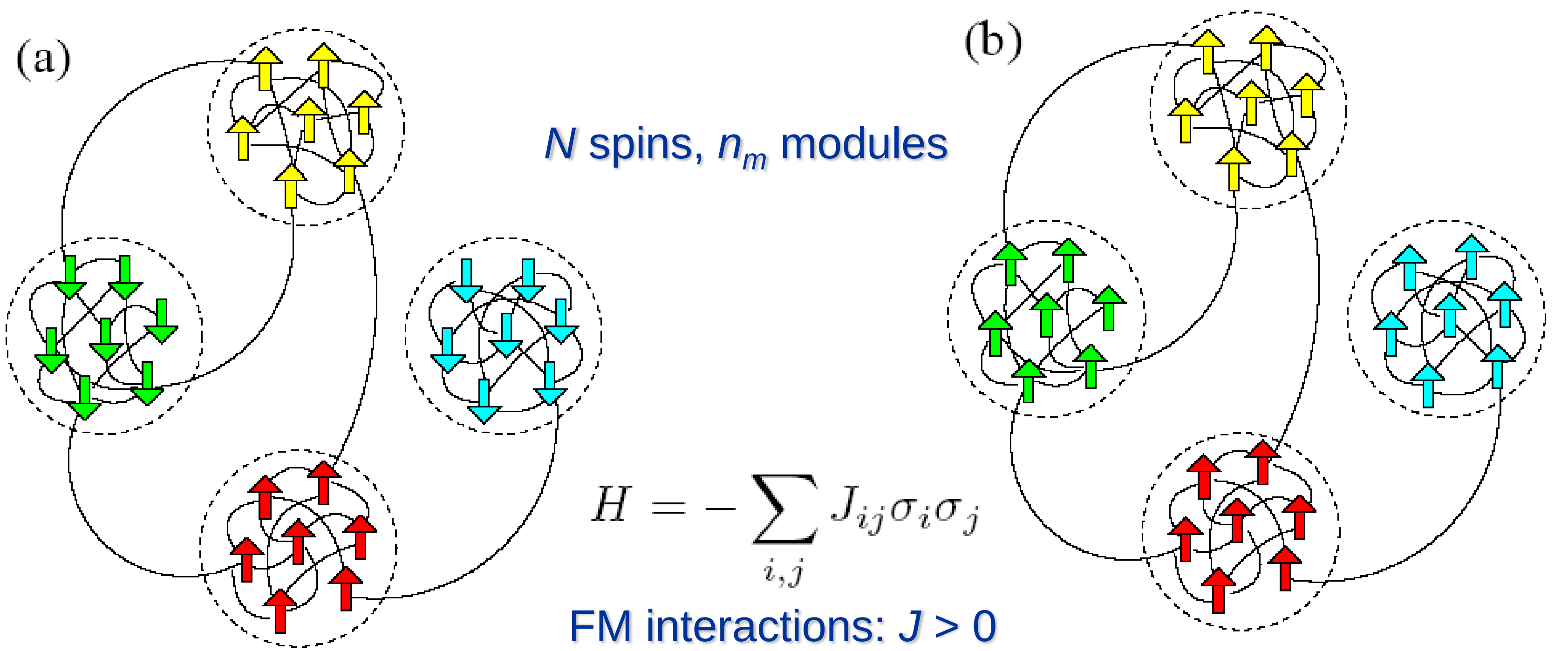
In the absence of fields, a modular network of Ising spins shows

S Dasgupta, R K Pan & S Sinha, *Phys Rev E* **80** 025101 (2009)

Two types of order

Modular order $M_g = 0, \mu \neq 0$

Global order $M_g \neq 0, \mu \neq 0$



N spins, n_m modules

$$H = -\sum_{i,j} J_{ij}\sigma_i\sigma_j$$

FM interactions: *J* > 0

Avg magnetic moment / module $\mu = \langle | \sum_{i \in s} \sigma_i | \rangle_{s = 1, \ldots, n_m}$

Total or global magnetic moment $M_g = \sum_i \sigma_i$

Two types of order

fraction of "up" spins in module

Magnetic moment of a single module $\mu = n(2f_+ - 1)$

$= N/n_m$

fraction of modules with + μ

At eqlbm, for strong modularity (r << 1)

Total magnetic moment $M_g = n_m \mu (2f_+^m - 1)$

Minimizing free energy w.r.t. f_+

$$\frac{1}{4f_+ - 2} \log \frac{f_+}{1 - f_+} = \frac{T_c^m}{T} + \frac{J\rho_o n(n_m - 1)(2f_+^m - 1)^2}{k_B}$$

Continuous transition to "modular order" phase below

"Modular" critical temp $T_c^m = 2JL_i/(nk_B)$

links within module

As T is lowered,

Another continuous transition to "global order" phase below

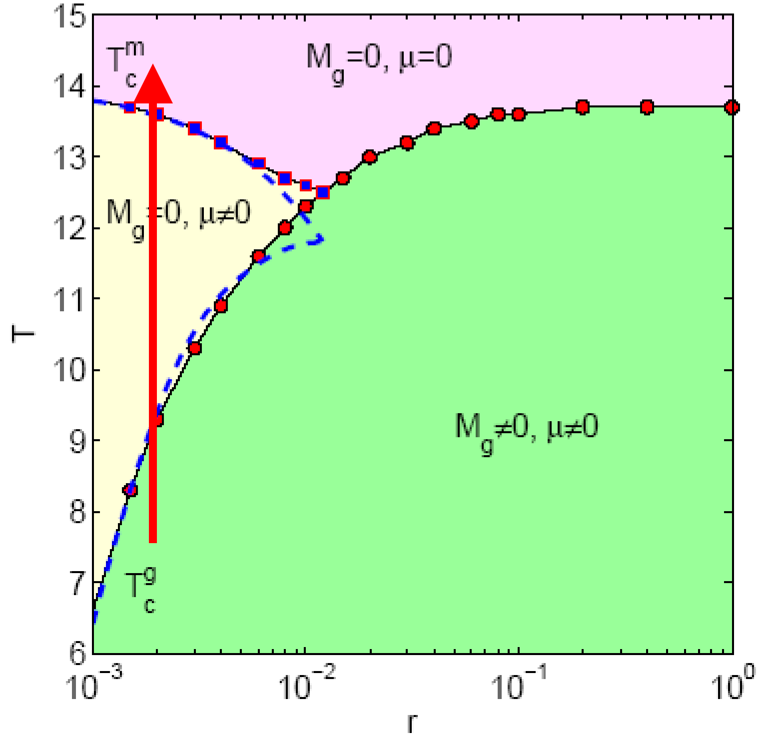
conn prob betn modules

"Global" critical temp $T_c^g = \frac{J\rho_0}{k_B}(n_m - 1)n^2(2f_+ - 1)^2$

Phase diagram: two types of order

S Dasgupta, R K Pan & S Sinha, *Phys Rev E* **80** 025101 (2009)

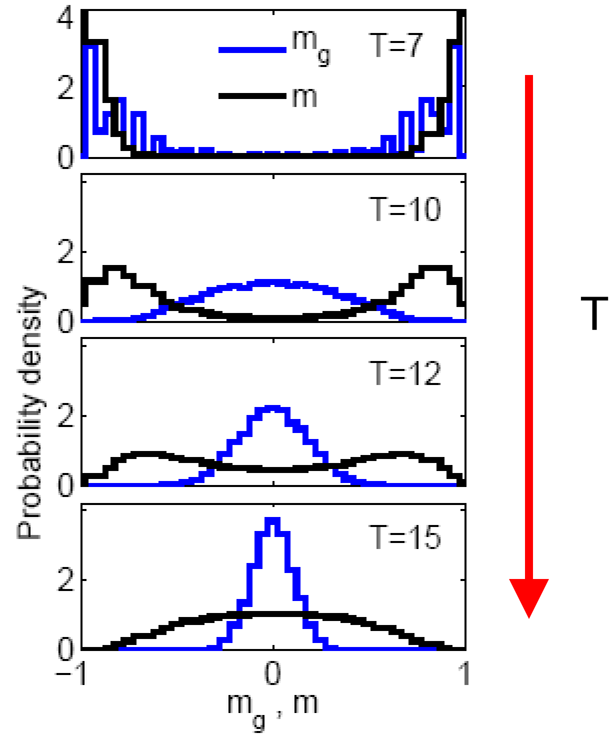
$$T_c^m = 2JL_i/(nk_B)$$



$$T_c^g = \frac{J\rho_0}{k_B}(n_m - 1)n^2(2f_+ - 1)^2$$

r = 0.002

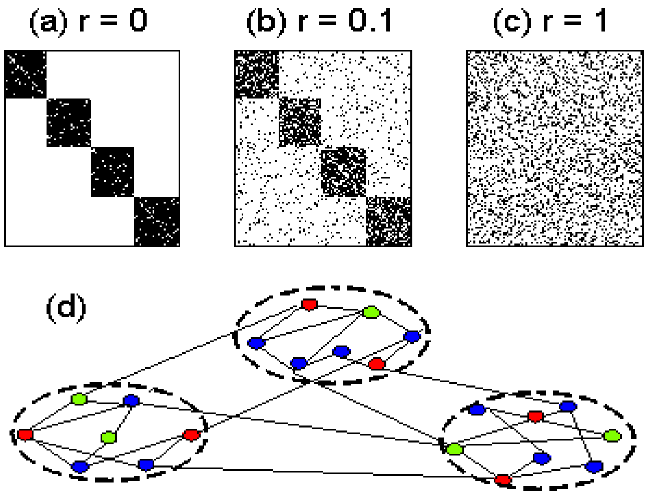
$N = 512$, $n_m = 16$ and $\langle k \rangle = 14$.

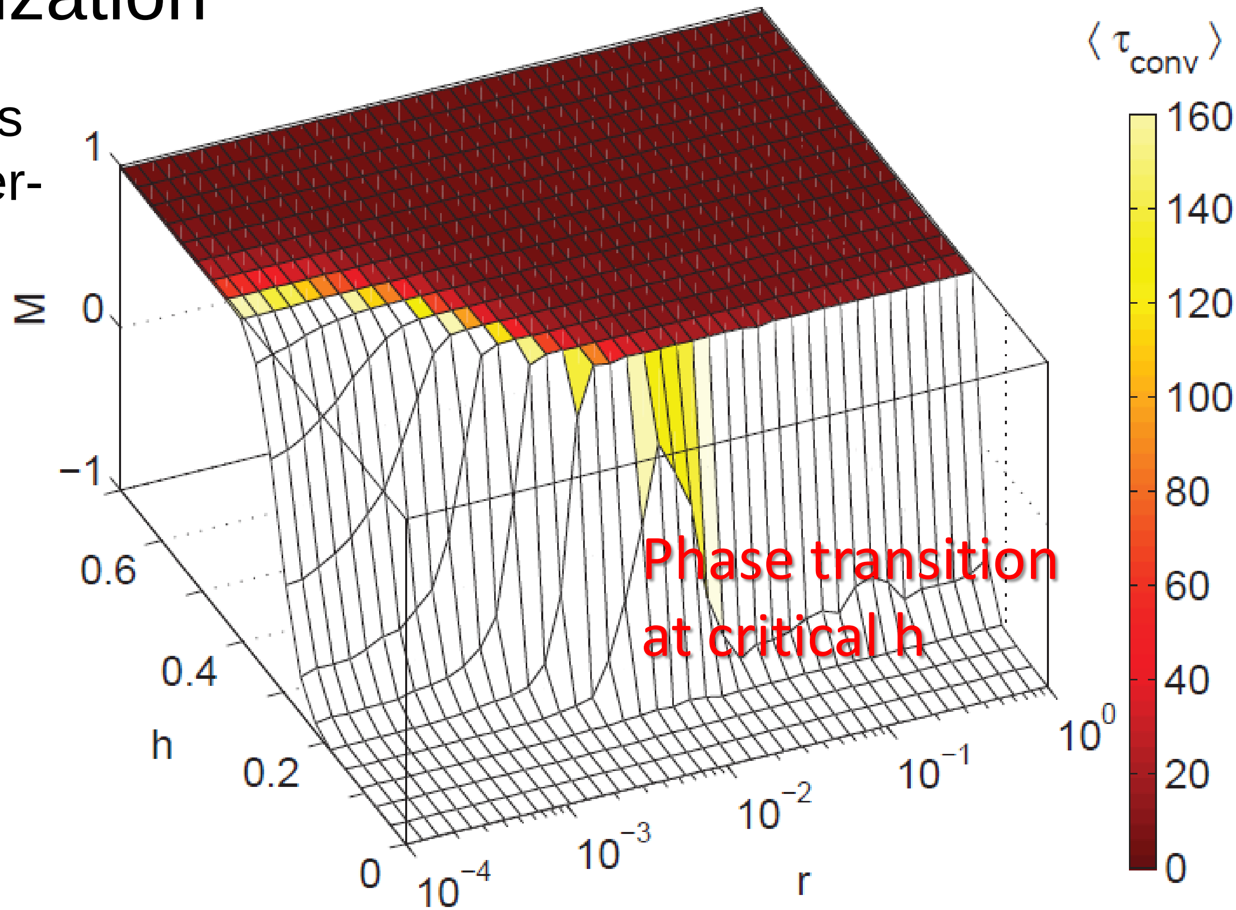


Existence of phase corresponding to modular but not global order (*coexistence of contrary opinions*) even when all mutual interactions are FM (*favor consensus*) !

Innovation Spreading in Model Networks with Community Organization

Modular nature of contacts varied using r (ratio of inter- to intra-modular links)



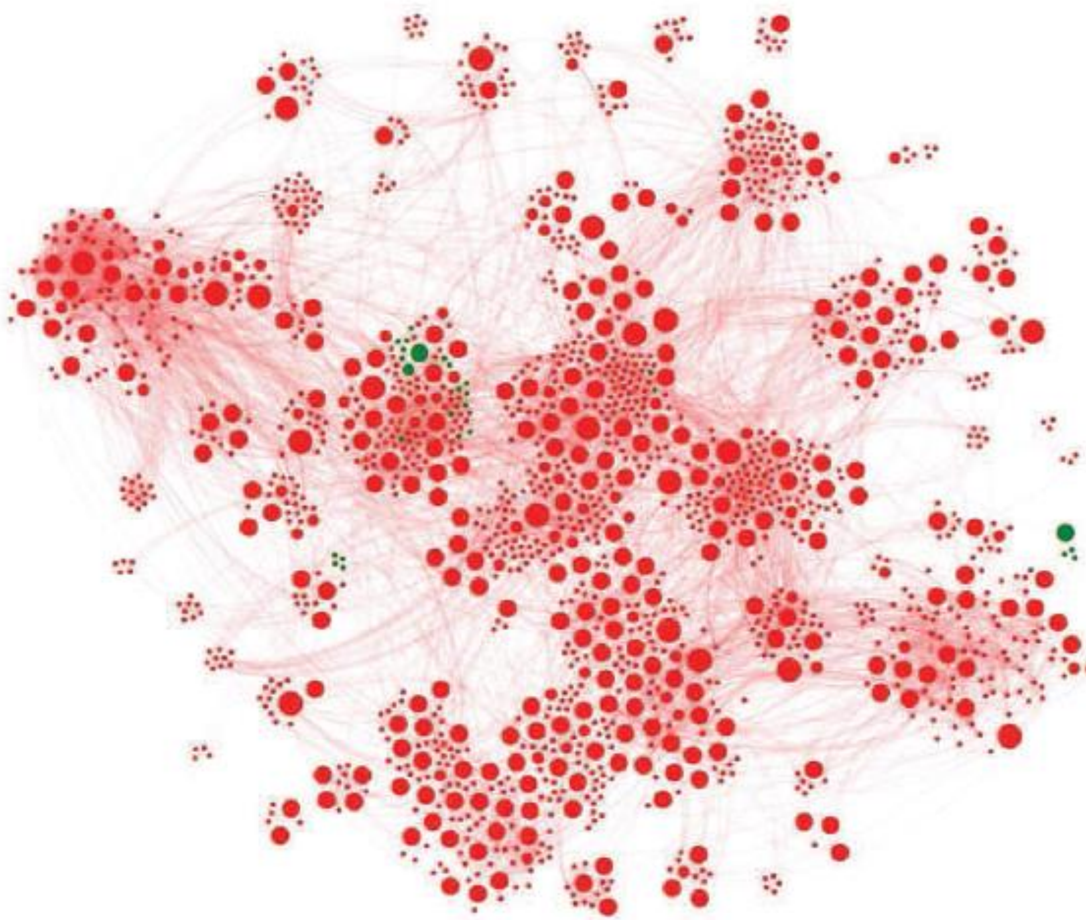


Fraction of population adopting an innovation within t= 200 MC steps for different h and r

Color: avg time required for at least 98% of the population to adopt
No color: less than 98% of the population has adopted the innovation by end of simulation
N = 1024, avg degree k = 12, M = 64 modules
Nodes evolve under noise $k_B T/J = 0.1$
Local thresholds from a Gaussian (0, 0.2) distribution

Innovation Spreading in Real Social Networks

Data: Bharatha Swamukti Samsthe microfinance institution
For 75 villages from Southern Karnataka
Nodes: Individuals
Links: Social relations



Classification of nodes in terms of functional role

- Nodes can be classified in terms of functional roles according to their pattern of intra- and inter-module connections.

$$z_i = \frac{k_i - \bar{k}_{si}}{\sigma_{si}}$$

- Intra-modular connectivity defined in terms of within-module degree z-score:

k_i: number of links of node i to other nodes in its module s_i,
$\bar{k}_{si}$: average of k over all the nodes in s_i
σ_{si} : the standard deviation of k in s_i.

$$P_i = 1 - \sum_{s=1}^{N_M} \left(\frac{k_{is}}{k_i} \right)^2$$

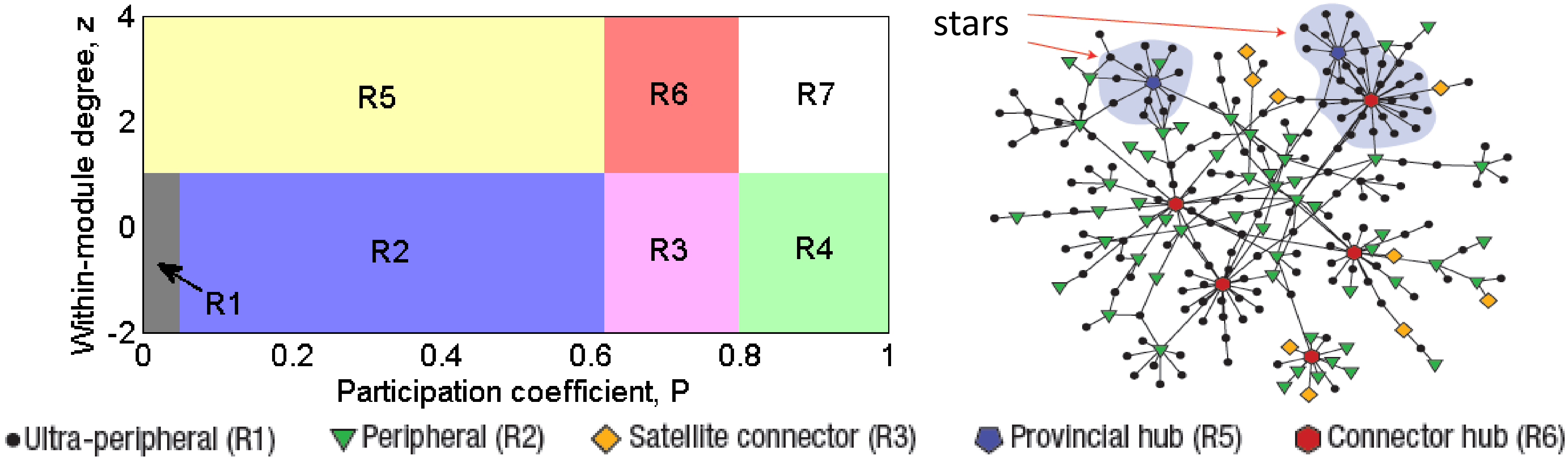
- Extra-modular connecitivity defined in-terms of the participation coefficient P_i of node i:

k_{is}: number of links of node i to nodes in module s
k_i : total number of links of node i.

P $\rightarrow$ 1 for a node if links are uniformly distributed among all modules
P $\rightarrow$ 0 if all its links are within its own module.

R Guimera and L A N Amaral *Nature* **433** 895 (2005)

What do different regions in P-z space mean ?



Seven different universal roles, each defined by a different region in the P-z parameter space.
R Guimera and L A N Amaral *Nature* **433** 895 (2005)

- ❑ The within-module degree z defines **hubs** (nodes with z ≥ 1) and **non-hubs** (z< 1).
- ❑ **Non-hub** nodes are divided into four different roles:
- **(R1) ultra-peripheral nodes**: *all* their links within their own module (P ≤ 0.05)
- **(R2) peripheral nodes**: *most* links within their module (0.05 <P ≤ 0.62)
- **(R3) non-hub connector nodes**: *many* links to other modules (0.62 < P ≤ 0.80)
- **(R4) non-hub kinless nodes**: links *homogeneously distributed* among all modules (P>0.80)
- ❑ **Hub** nodes are divided into three different roles:
- **(R5) provincial hubs**: *most* links within their own module (P ≤ 0.62)
- **(R6) connector hubs:** *many* links to most of the other modules (0.62< P ≤ 0.8)
- **(R7) global hubs**: links *homogeneously distributed* among all modules (P> 0.8)

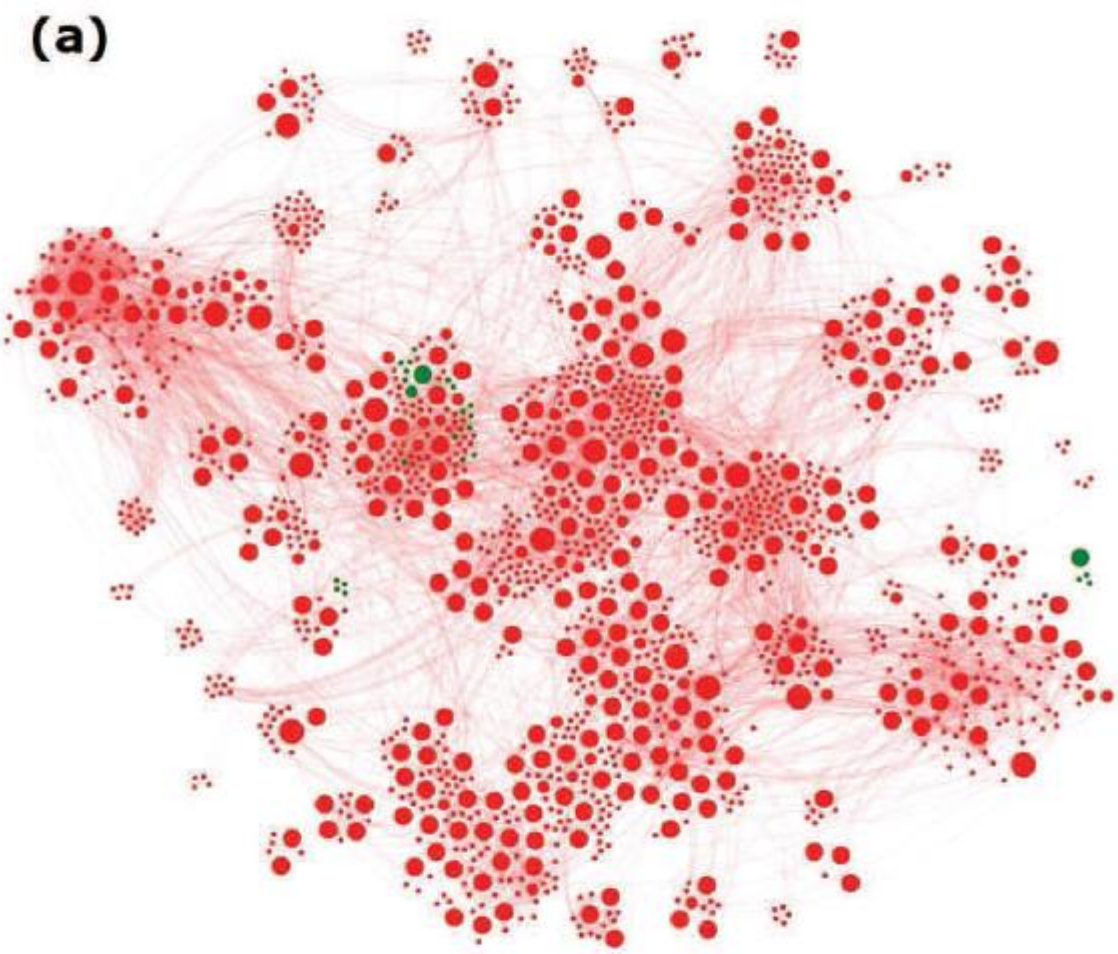
Successful adoption of an innovation

In the real social network of a Karanataka village comprising N = 1497 individuals

Size of each node represents its functional role

Largest size: connector hubs
Next largest size: local hubs
Next largest nodes: satellite connectors and kinless nodes
Smallest size nodes: peripheral and ultra-peripheral nodes.

Successful adoption of an innovation

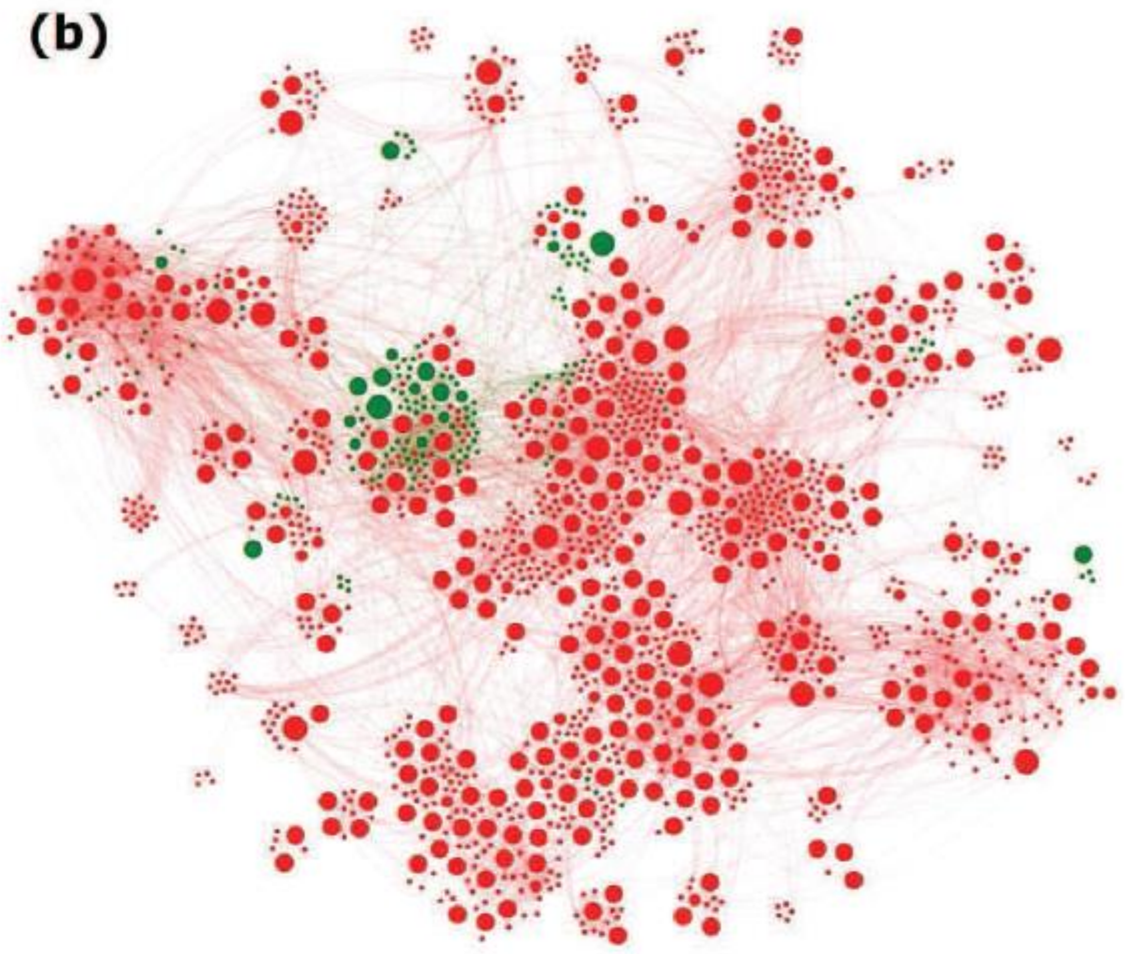


Adopted

Not adopted

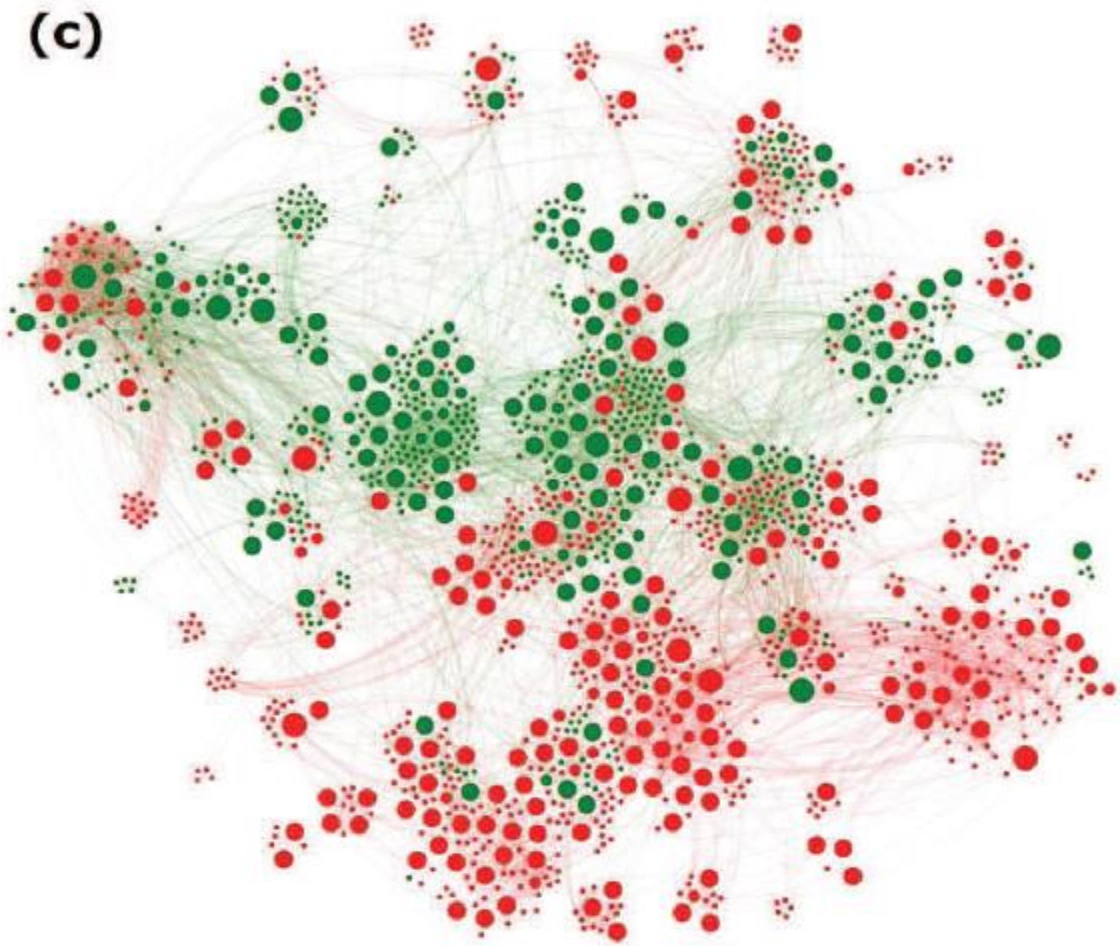
t = 110 MC steps

Successful adoption of an innovation



Adopted   Not adopted   t = 120 MC steps

Successful adoption of an innovation

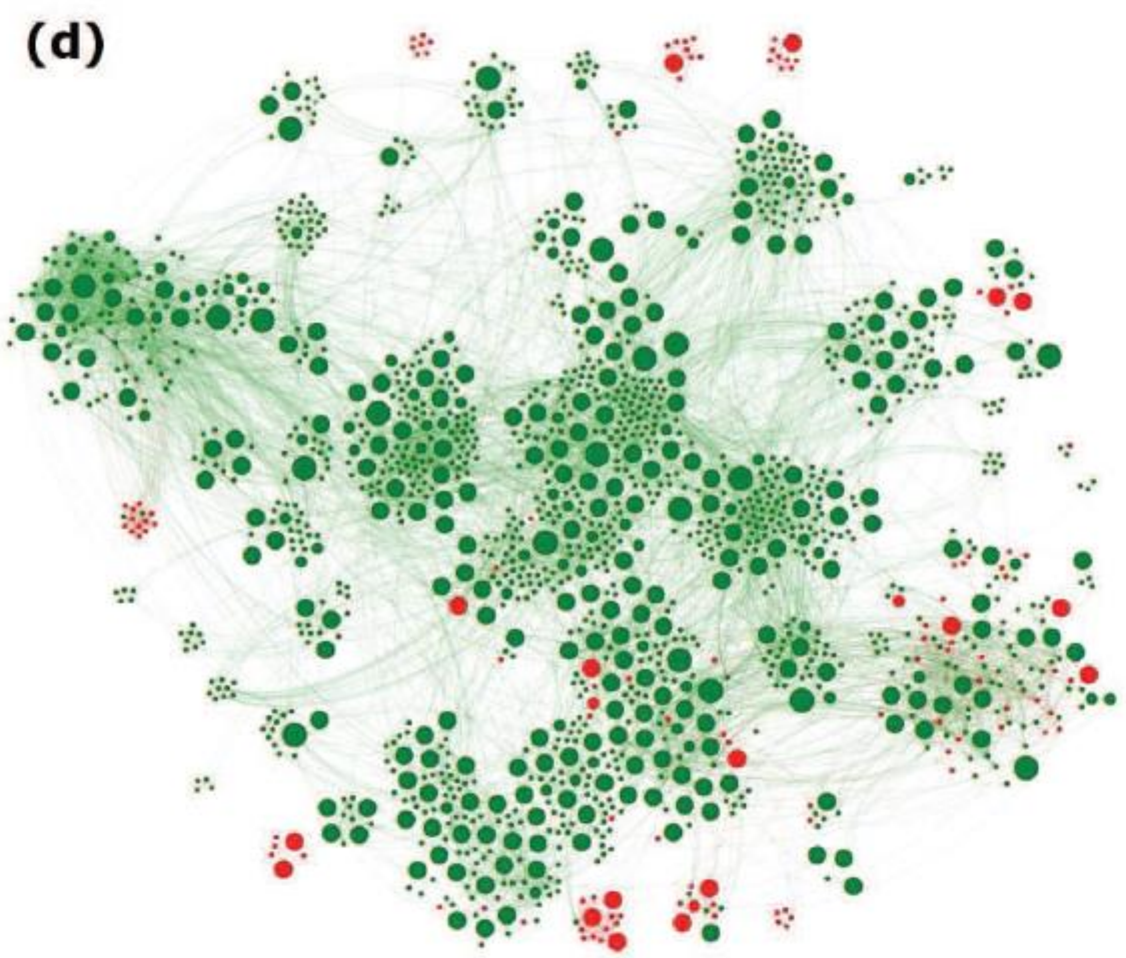


Adopted

Not adopted

t = 125 MC steps

Successful adoption of an innovation



Adopted   Not adopted   t = 130 MC steps

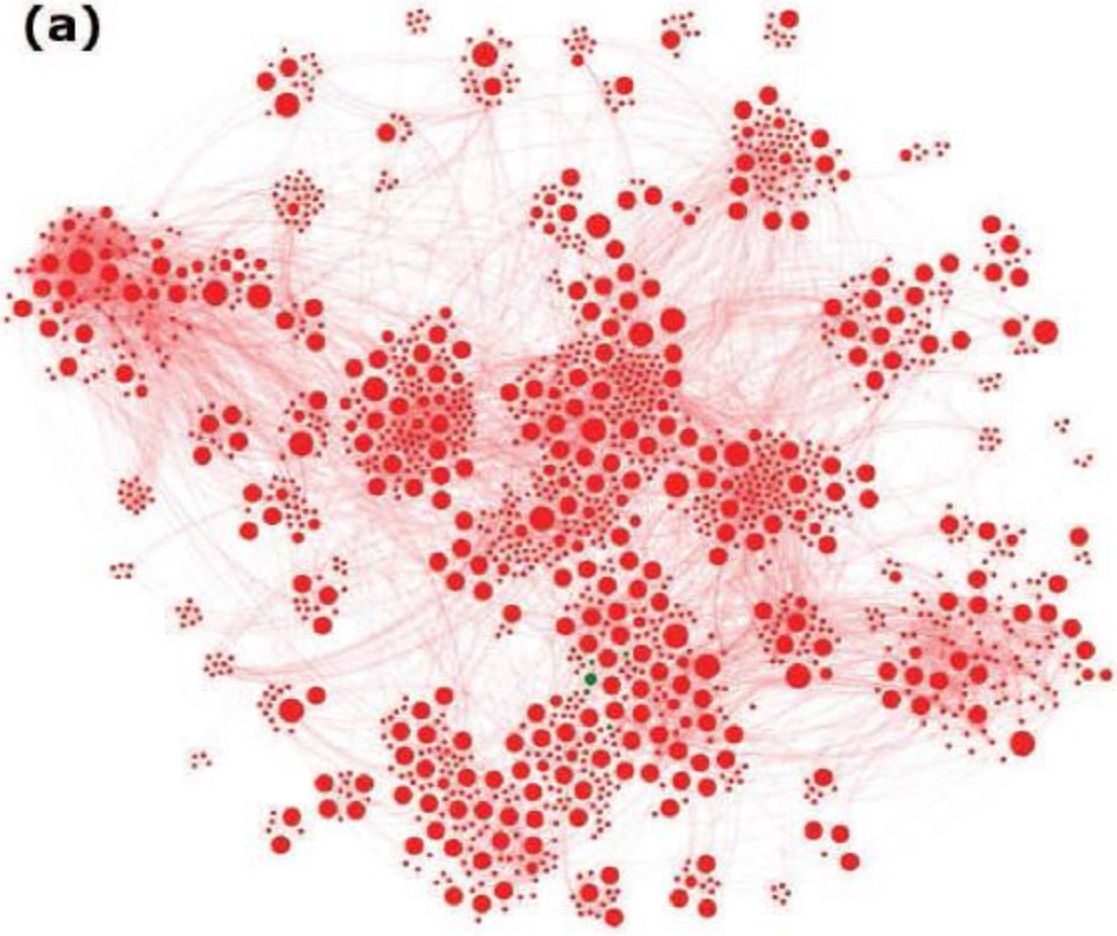
Failed adoption of an innovation

In the real social network of a Karanataka village comprising N = 1497 individuals

Size of each node represents its functional role

Largest size: connector hubs
Next largest size: local hubs
Next largest nodes: satellite connectors and kinless nodes
Smallest size nodes: peripheral and ultra-peripheral nodes.

Failed adoption of an innovation

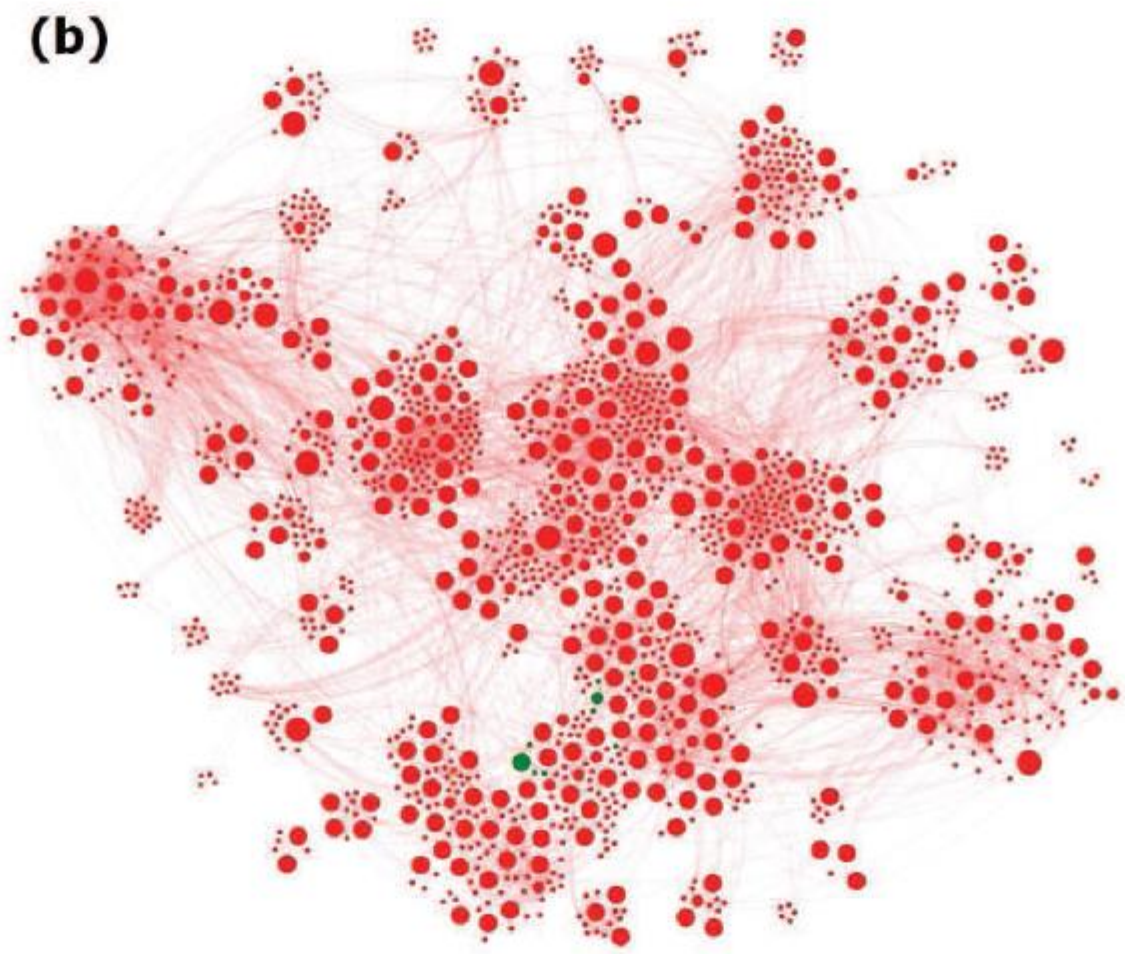


Adopted

Not adopted

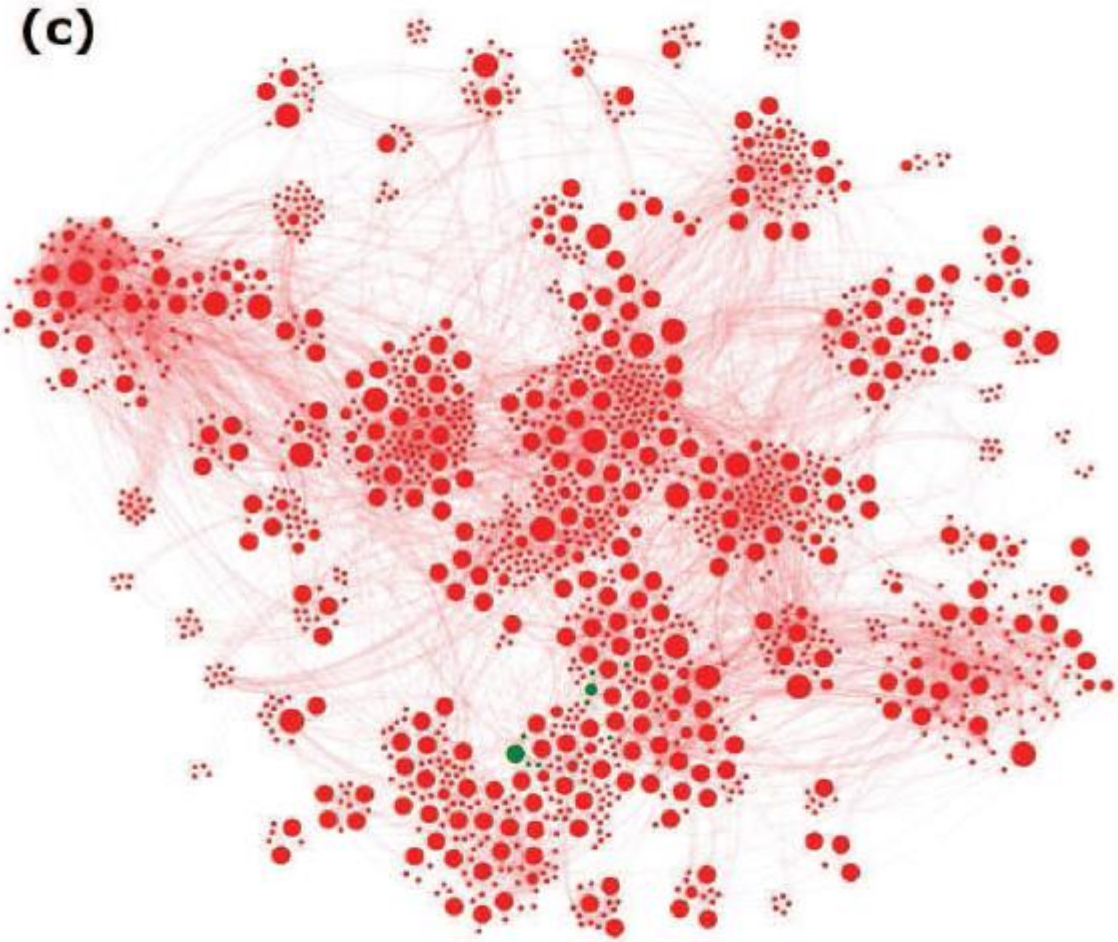
t = 40 MC steps

Failed adoption of an innovation



Adopted   Not adopted   t = 90 MC steps

Failed adoption of an innovation

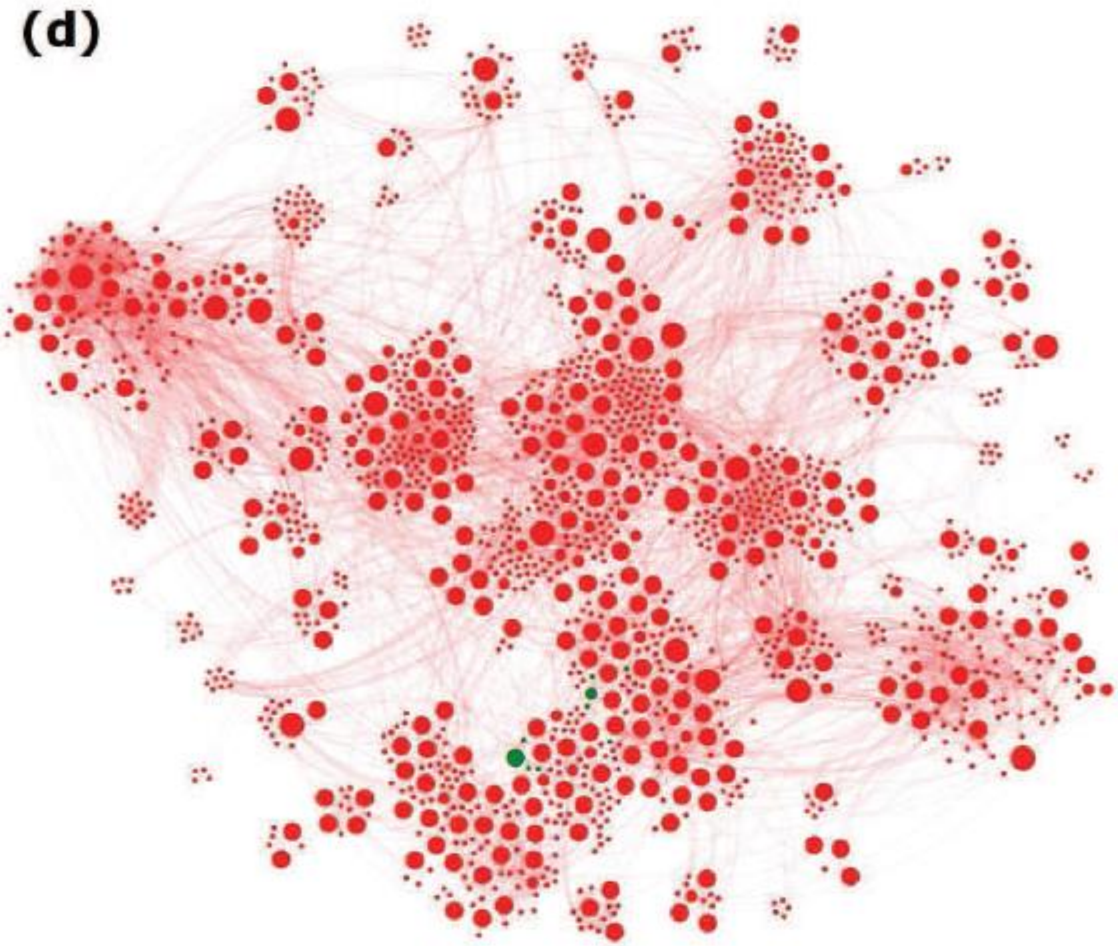


Adopted

Not adopted

t = 140 MC steps

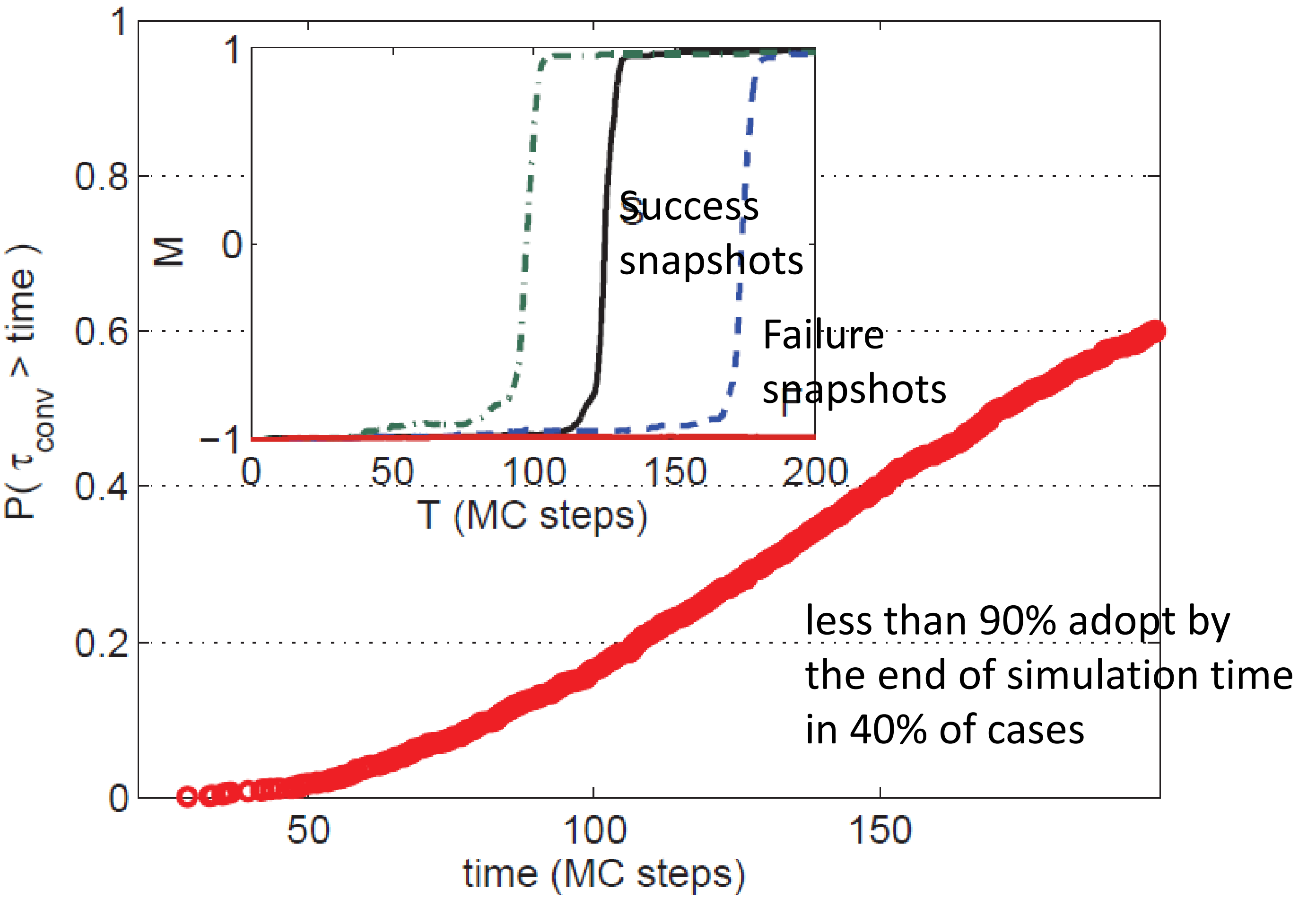
Failed adoption of an innovation



Adopted Not adopted t = 190 MC steps

Spin Model of Innovation on Real Social Networks

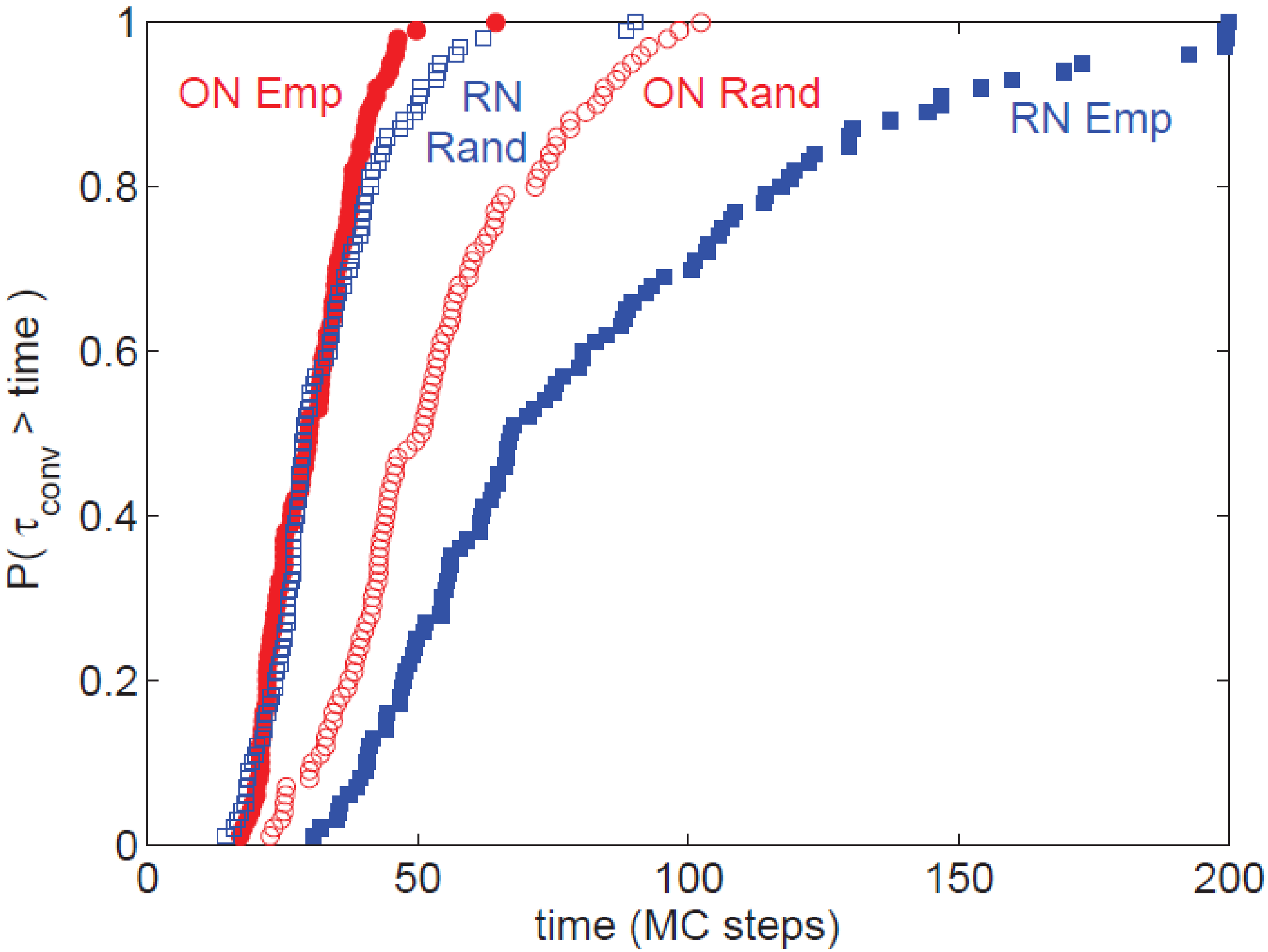
Eventual outcome can be very different depending upon the random thresholds assigned to the nodes & exact sequence of switching events



Cumulative probability distribution of time required for 90% of population to adopt an innovation with a perceived advantage h = 0.33

Spin Model of Innovation on Real Social Networks

Innovation adoption can occur *faster* in the empirical network with community organization than in the corresponding randomized surrogate



ON Emp: original empirical social network

ON Rand: its degree-preserved randomized surrogate

RN Emp: a reduced version of the empirical social network constructed by removing 80% of the inter-modular connections

RN Rand: its degree-preserved randomized surrogate

Cumulative probability distribution of time required for 98% of population to adopt an innovation with a perceived advantage h = 0.41

Implications

Spreading occurs in the original empirical network with community organization faster than in a randomized homogeneous network …

… but is slower than the corresponding randomized system when most inter-modular connections have been removed

$\Rightarrow$ role of nodes with specific functional roles in terms of their position in the modular organization, e.g., connector hubs and local hubs

Thanks

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