Routing in Cost-shared Networks: Equilibria and Dynamics (Part 2)

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we saw earlier

exponential gap between **best** and **worst** equilibria

which of these equilibria is achievable?

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exponential gap between **best** and **worst** equilibria

which of these equilibria is achievable?

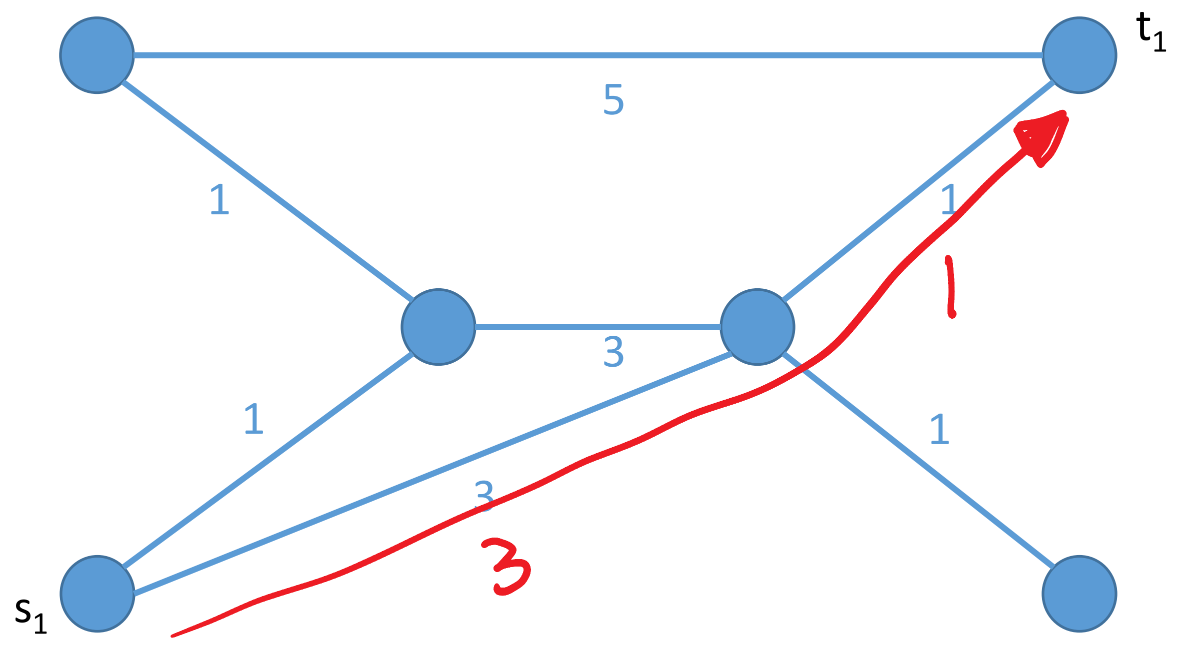
OPEN: Find **any** equilibrium in polynomial time.

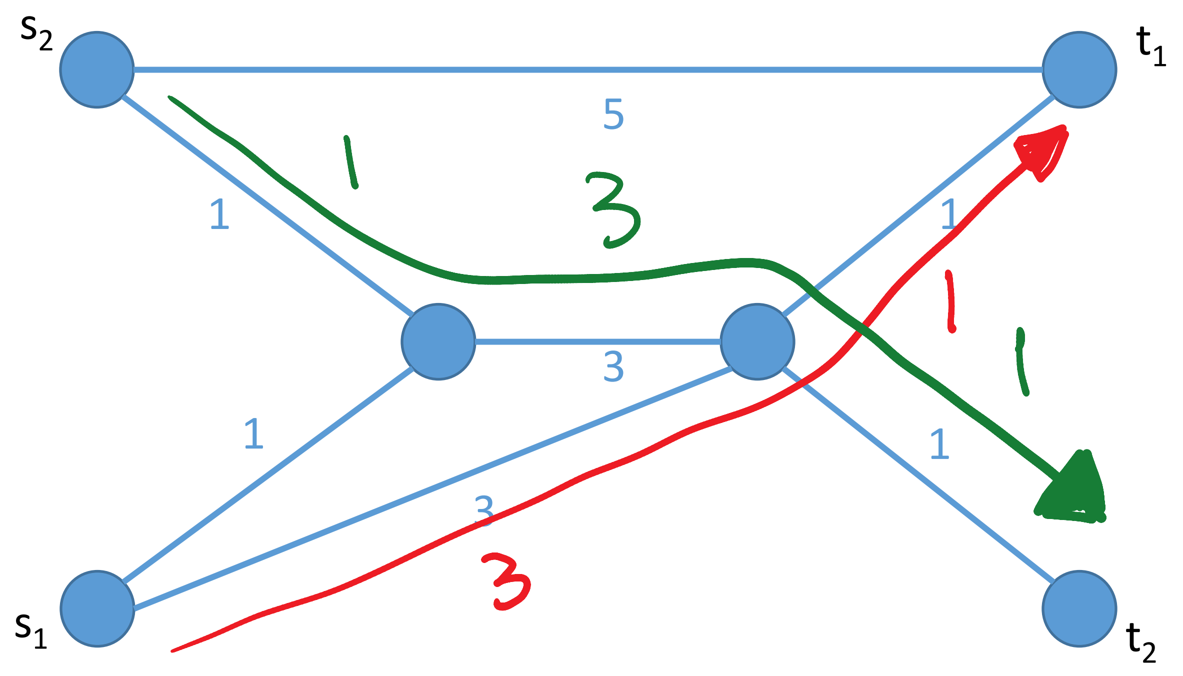
changes in potential can be exponentially small what if agents can join and leave the network?

simplest case

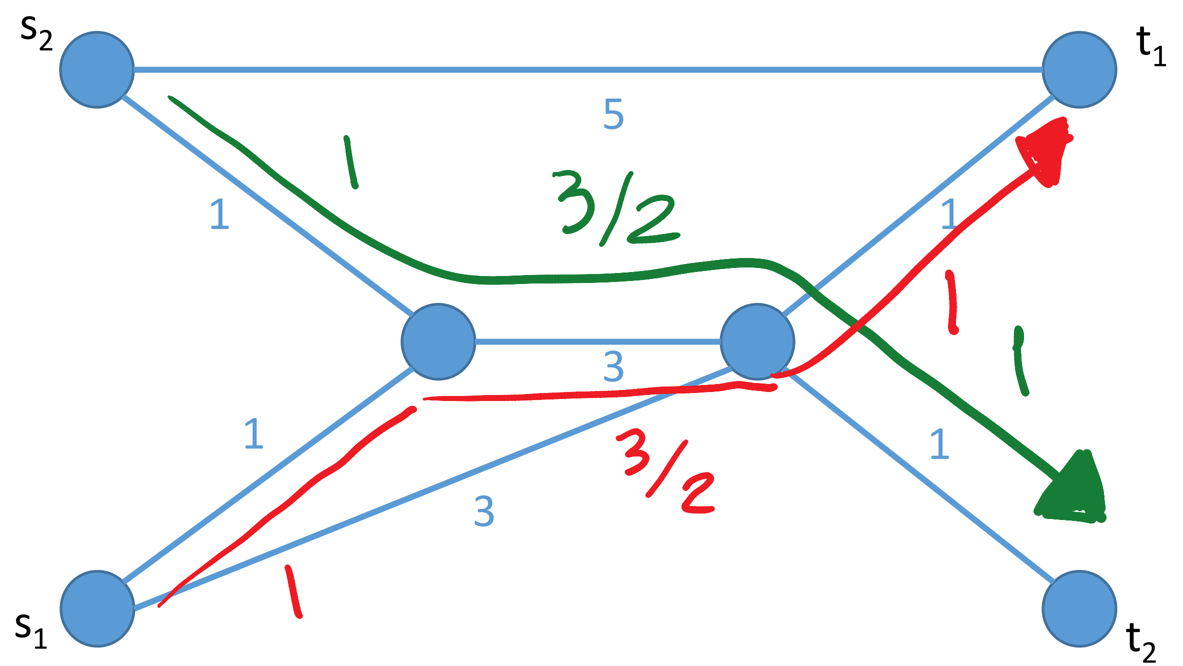
phase 1: agents join the network in sequence, choosing their minimum cost path on arrival

phase 2: agents move to cheaper path from their existing path in arbitrary order until equilibrium is reached


t1
5
1
1
3
1
1
3
1
3
s1



s2
t1
5
1
3
1
1
3
1
3
1
s1
t2



s2
t1
5
1
3/2
1
1
3
1
1
1
3
3/2
1
1
s1
t2

the equilibrium produced at the end costs at most poly-log times optimal cost
[Charikar, Karloff, Matheiu, Naor, Saks ’08]

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only need to show this for phase 1

potential argument works for phase 2

the equilibrium produced at the end costs at most poly-log times optimal cost
[Charikar, Karloff, Matheiu, Naor, Saks ’08]

- A dual fitting argument
- For any vertex **u**, let
  - $\mathbf{b_u}$ = exclusive cost of **u** on arrival
  - $\mathbf{s_u}$ = shared cost of **u** on arrival
- A vertex **u** will have a ball centered at it if
  - $\mathbf{s_u \leq 2\, b_u \log n}$

why is this sufficient?

clearly, $\mathbf{\Sigma_u\ b_u}$ is the overall cost

also,

$$\mathbf{b_u \leq s_u}$$

and

$$\mathbf{\Sigma_u\ b_u \geq \Sigma_u\ s_u / \log n}$$

the equilibrium produced at the end costs at most poly-log times optimal cost
[Charikar, Karloff, Matheiu, Naor, Saks ’08]

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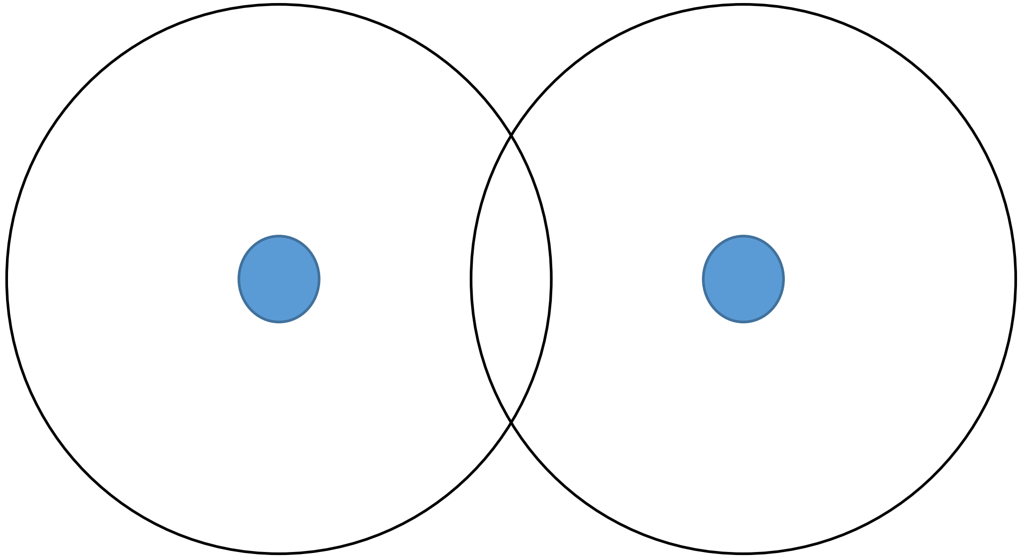
- If $\mathbf{b_u}$ in $\mathbf{(\delta^k, \delta^{k+1}]}$ and $\mathbf{s_u}$ in $\mathbf{(\gamma^j, \gamma^{j+1}]}$, then add a ball of radius $\mathbf{\delta^k/8}$ centered at **u** in dual **(j, k)**

when are the balls non-intersecting?



Lemma: If $\delta = 2$ and $\gamma = 1 + 1/8 \log n$, then the balls in a group are non-intersecting.

OPEN: What is the quality of the equilibrium reached if arrivals and improving moves are interleaved?

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theorem: if **agent departures** is allowed, then **poly(n)**

[Chawla, Naor, P., Singh, Umboh]

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[Chawla, Naor, P., Singh, Umboh]

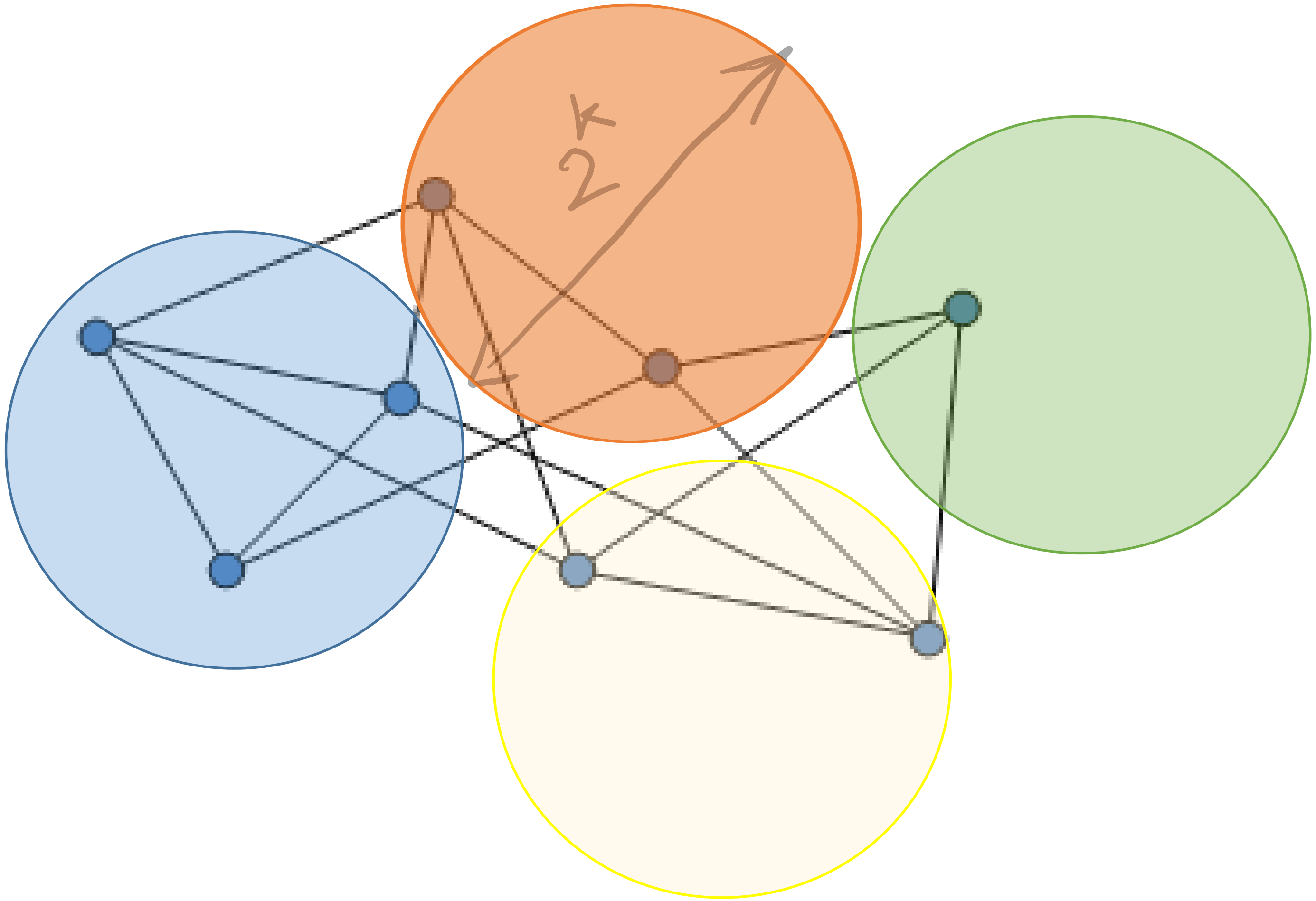
what can a central controller do?

if the controller suggests (improving) moves to attain equilibrium between arrival/departure phases

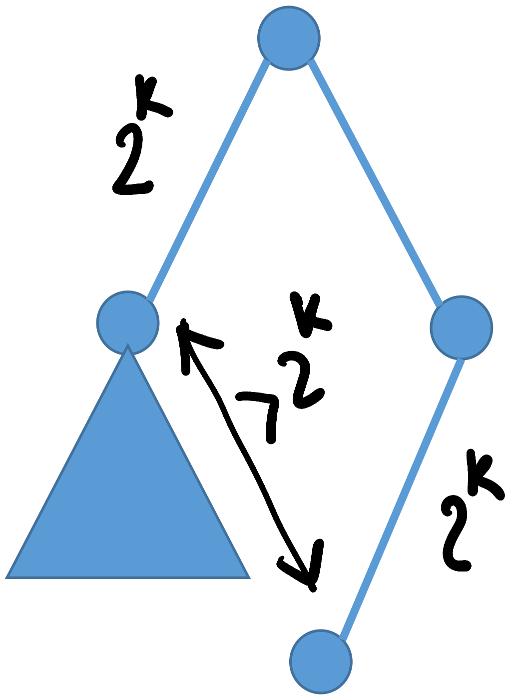
theorem: equilibrium within **log n** of optimal

[Chawla, Naor, P., Singh, Umboh]

partition graph into subgraphs of diameter $\mathbf{2^k}$, for $\mathbf{1 \leq k \leq \log n}$ (embed into a distribution of **HST**s)



hope: vertices with edges of same length are **well-separated**



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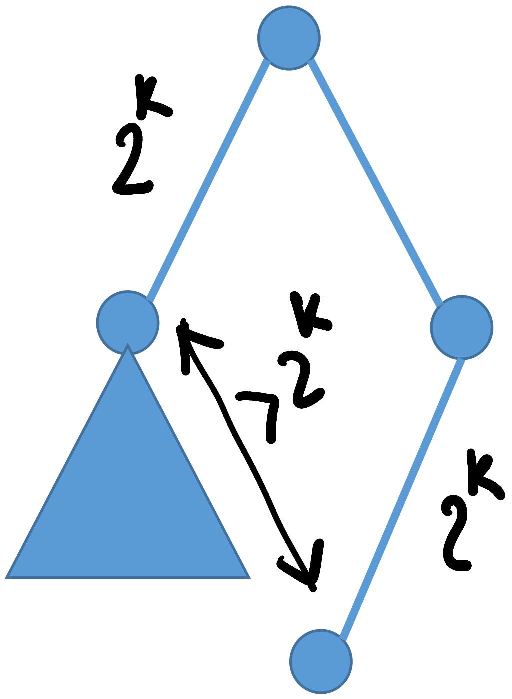
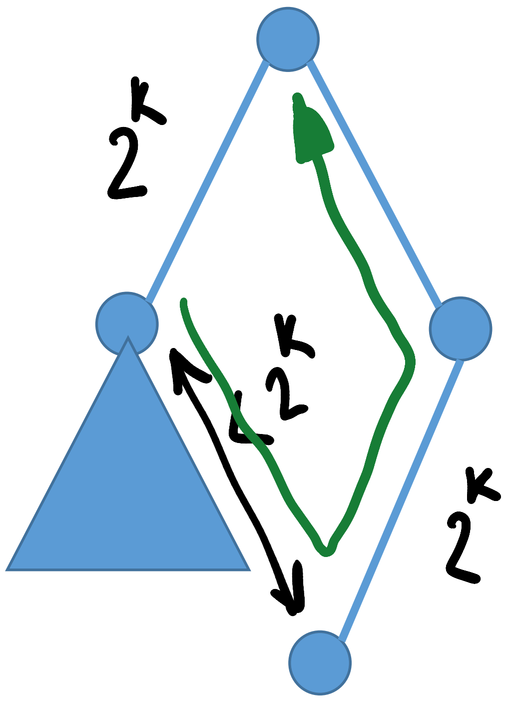
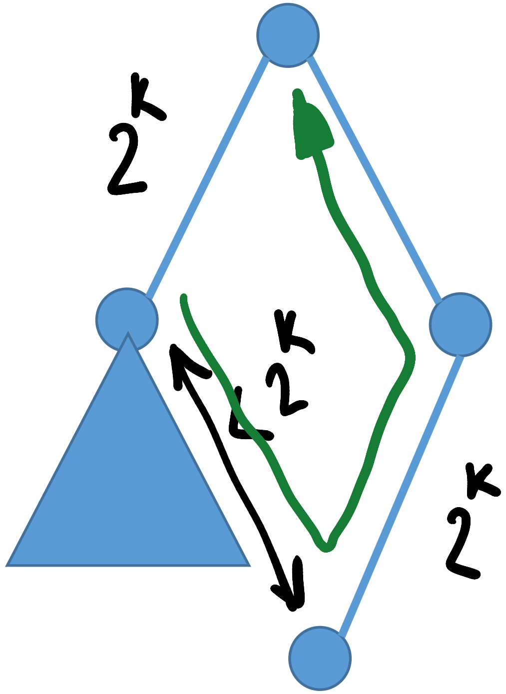
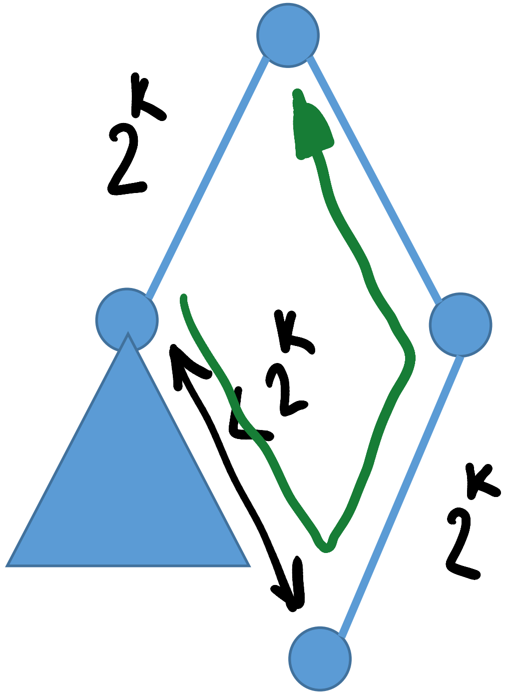




improving move removes an overcharge

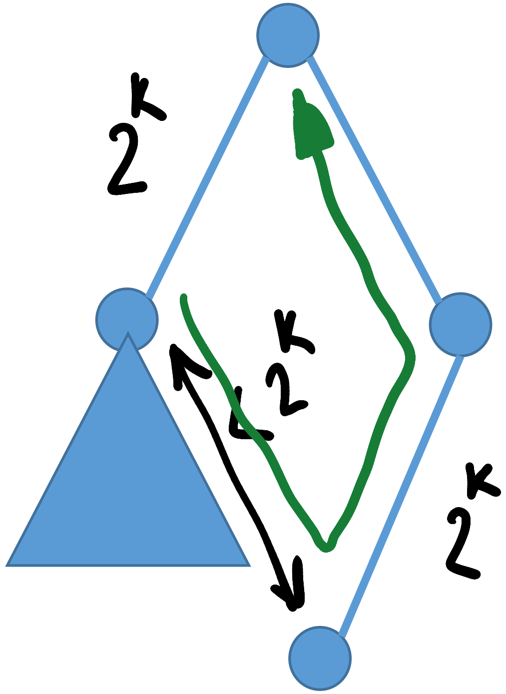


improving move removes an overcharge
but can create a different one



improving move removes an overcharge
but can create a different one

repeat



improving move removes an overcharge
but can create a different one

repeat

potential argument shows sequence is finite
eventually, there is no overcharging

how do we extend to multiple arrivals/departures?

now, overcharging on multiple subgraphs

(1) overcharging only done by leaves of the routing tree
except possibly one subgraph charged by 2 non-leaves

(2) if there is overcharging, then there is an improving move that maintains invariant (1)

(3) potential decreases over time

(4) eventually, there is no overcharging

summary

open: can we find **any** equilibrium in polynomial time?

if agents join/leave/move **arbitrarily**, inefficiency can be **linear**

but controlling the moves yields **log** inefficiency thank you

questions?