Routing in Cost-shared Networks: Equilibria and Dynamics (Part 1)

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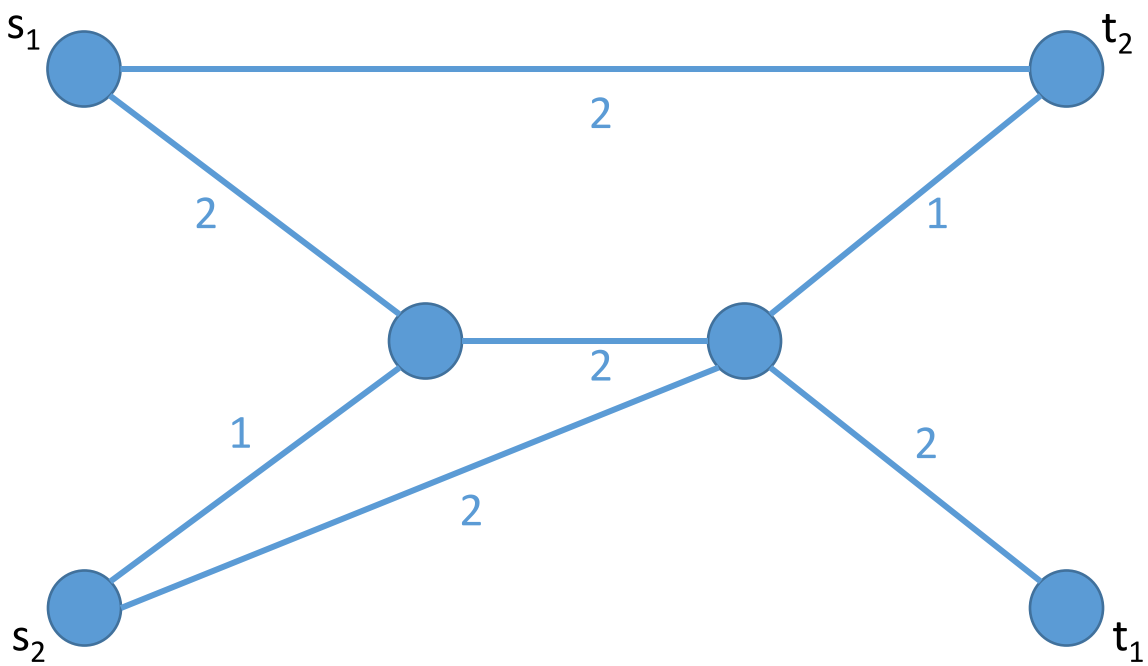
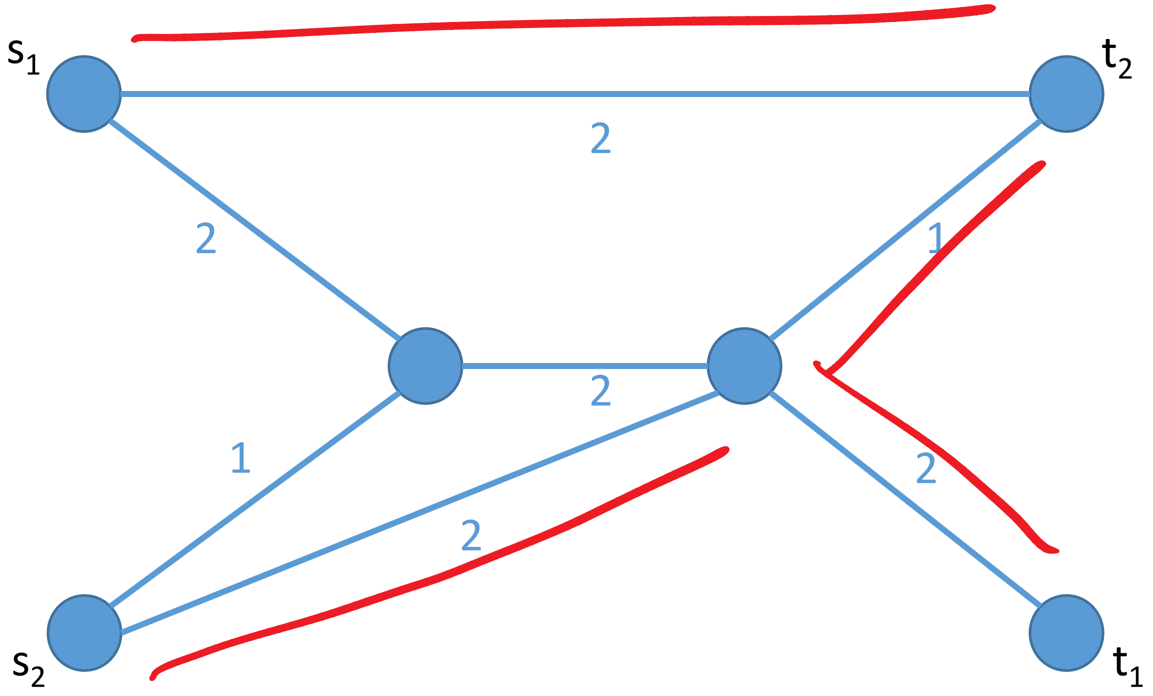
on an undirected network

a set of **agents** want to route traffic from their respective source to sink vertices

each edge used in routing has a **fixed cost** that is **shared equally** by agents using the edge

minimize **sum of cost of edges** used in routing


s_1
t_2
2
2
1
2
1
2
2
s_2
t_1



s1
t2
2
2
1
2
1
2
2
s2
t1

Steiner forest problem

2-approx [Agarwal-Klein-Ravi ’91, Goemans-Williamson ’92]

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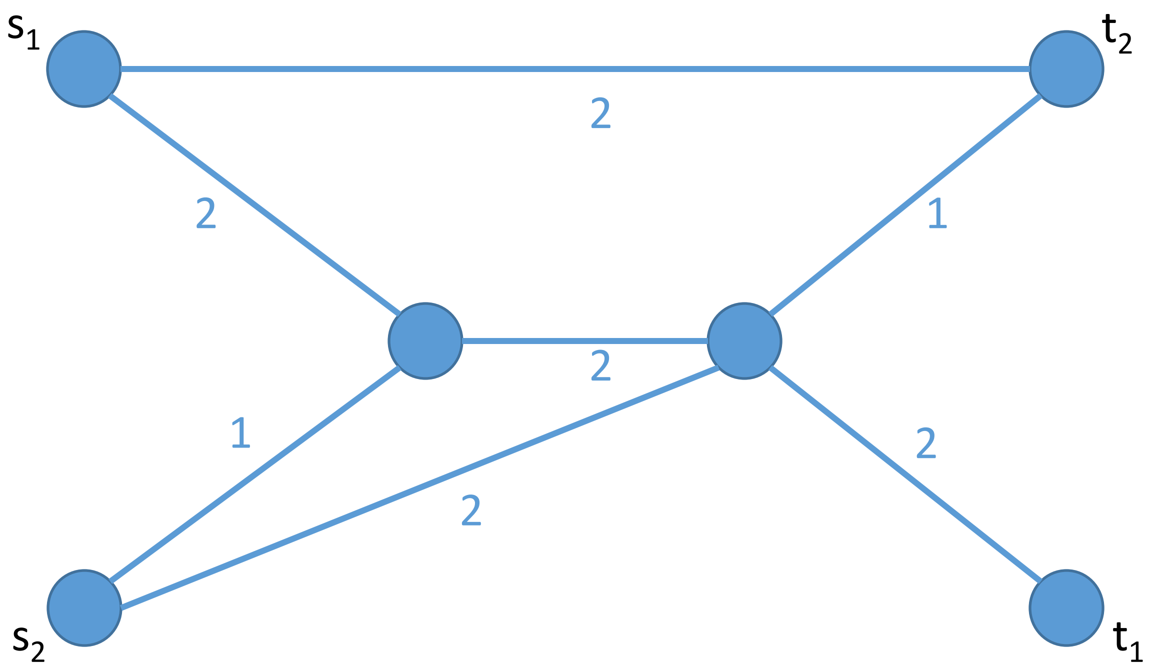
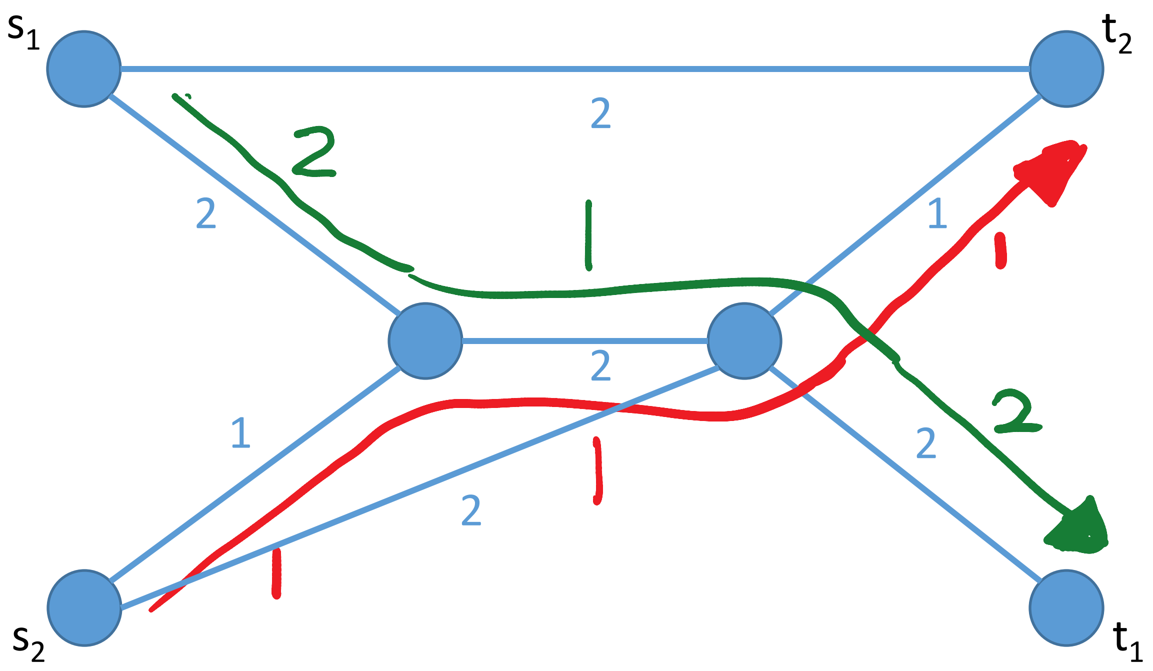
However…

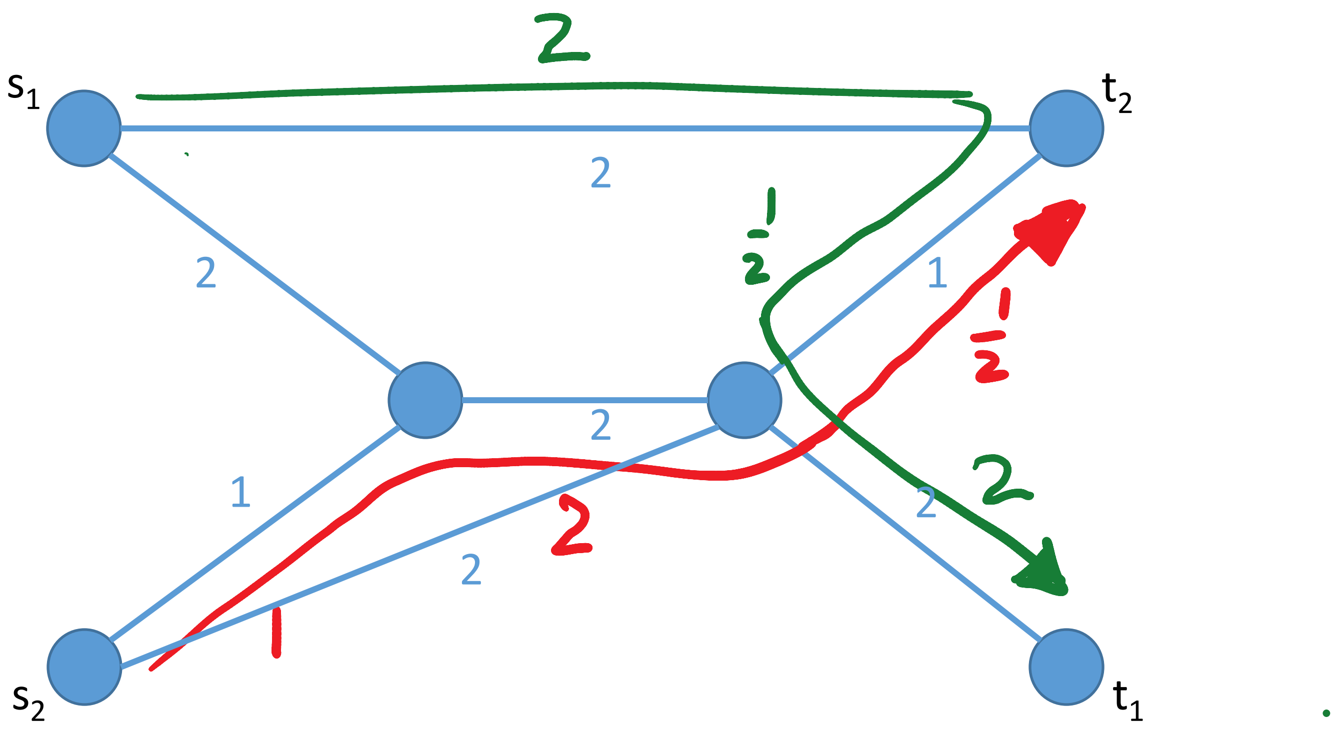
agents are strategic!

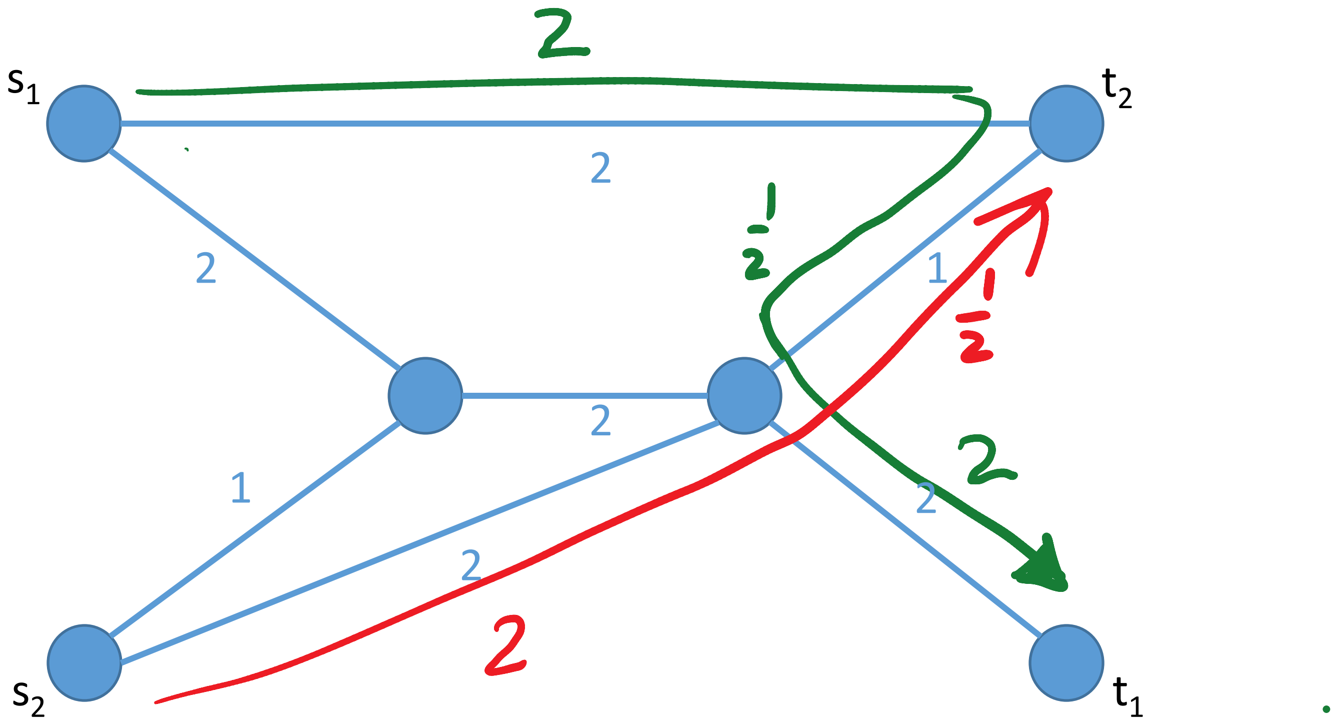
(want to minimize their own cost)


s_1
t_2
2
2
1
2
1
2
2
s_2
t_1



s1
t2
2
2
1
2
1
2
s2
t1


s1
t2
2
2
1
2
2
1
2
2
s2
t1
2
1/2
1/2
2
2
1

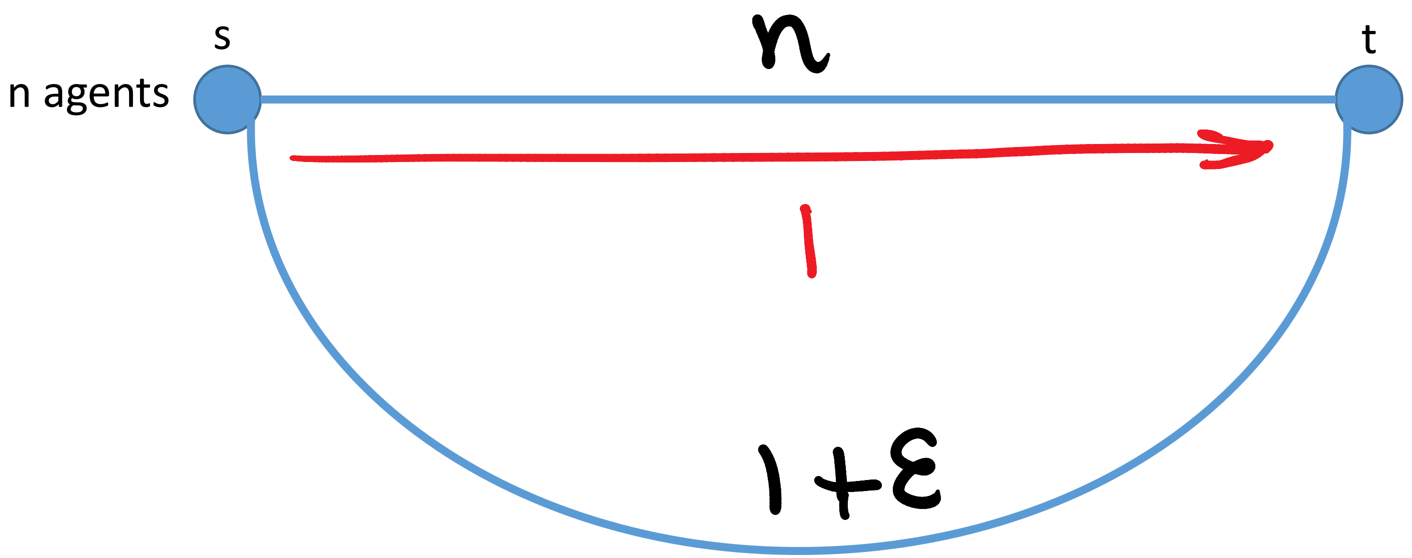


s1
t2
2
2
2
1
1/2
1/2
2
2
1
2
2
2
s2
t1

this is optimal!

this is optimal!

But, the situation can be much worse …


s
t
n agents
n
1
1+ε

This is (just) a game!

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agent’s strategy: routing path

agent’s payoff: negative of the shared cost

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agent's strategy: routing path

agent's payoff: negative of the shared cost

each agent aims to maximize payoff, i.e., minimize cost

This is (just) a game!

equilibrium: no agent has a less expensive routing path

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do equilibriums always exist?

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yes, reason coming up soon …

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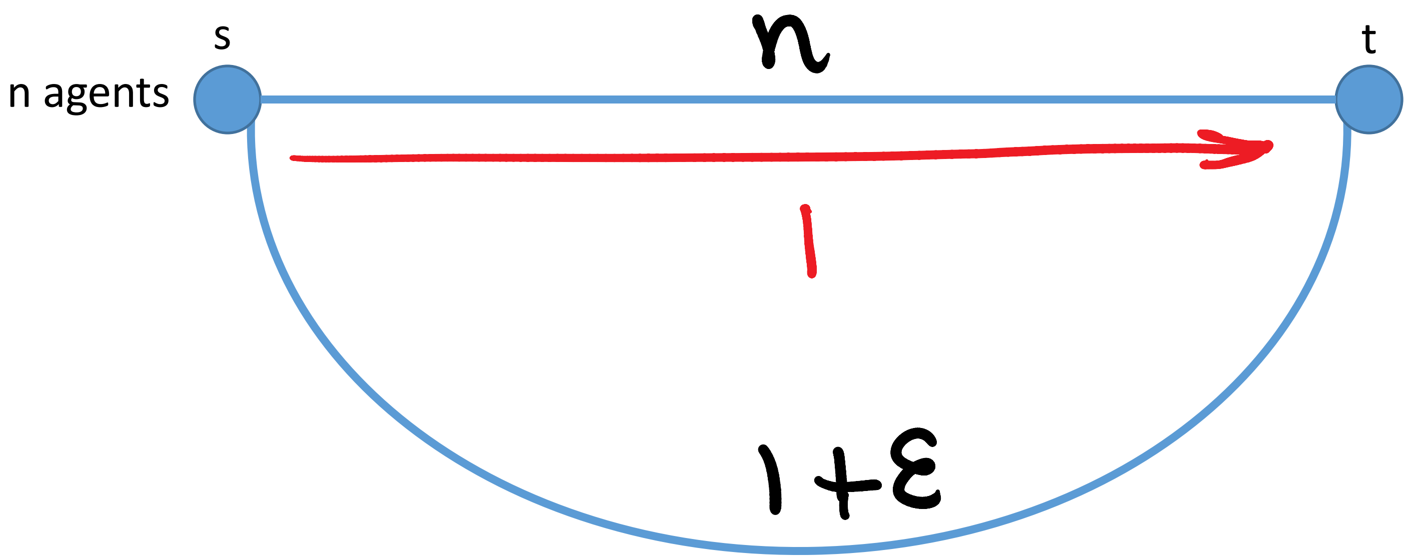
how suboptimal can an equilibrium be?

unfortunately, **very** suboptimal



price of anarchy: max (over all equilibria) ratio of total cost at an equilibrium state to optimal cost (inefficiency of **worst** equilibrium)

how inefficient is the **best** equilibrium?
i.e., controller chooses routing paths
but they need to be **in equilibrium**

price of stability: min (over all equilibria) of total cost at an equilibrium state to optimal cost

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this is a **potential game:** there exists a global function of the strategies played by all agents which strictly decreases (or strictly increases) for every valid move (change in strategy by an agent that decreases her cost)

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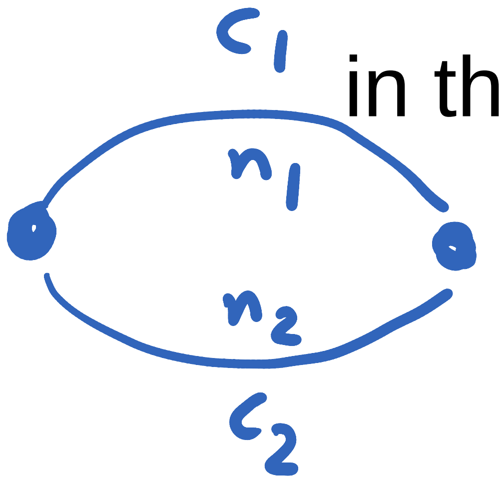
this is a **potential game:** there exists a global function of the strategies played by all agents which strictly decreases (or strictly increases) for every valid move (change in strategy by an agent that decreases her cost)

corollary: there always exists an equilibrium edge **e** used by $\mathbf{n_e}$ agents

potential of edge **e** is $\varphi_e = c_e\,(1 + 1/2 + 1/3 + \ldots + 1/n_e)$



in the example, if agent moves from 1 to 2

$\Delta\,\varphi = c_2/(n_2+1) - c_1/n_1$

= difference in shared cost

Initialize with optimal solution and run to equilibrium

$$\sum_{e \in EQ} c_e \le \phi_{EQ} \le \phi_{OPT} \le \Big(\sum_{e \in OPT} c_e\Big) H_n$$

[Anshelevich, Dasgupta, Kleinberg, Tardos, Wexler, Roughgarden ’04]

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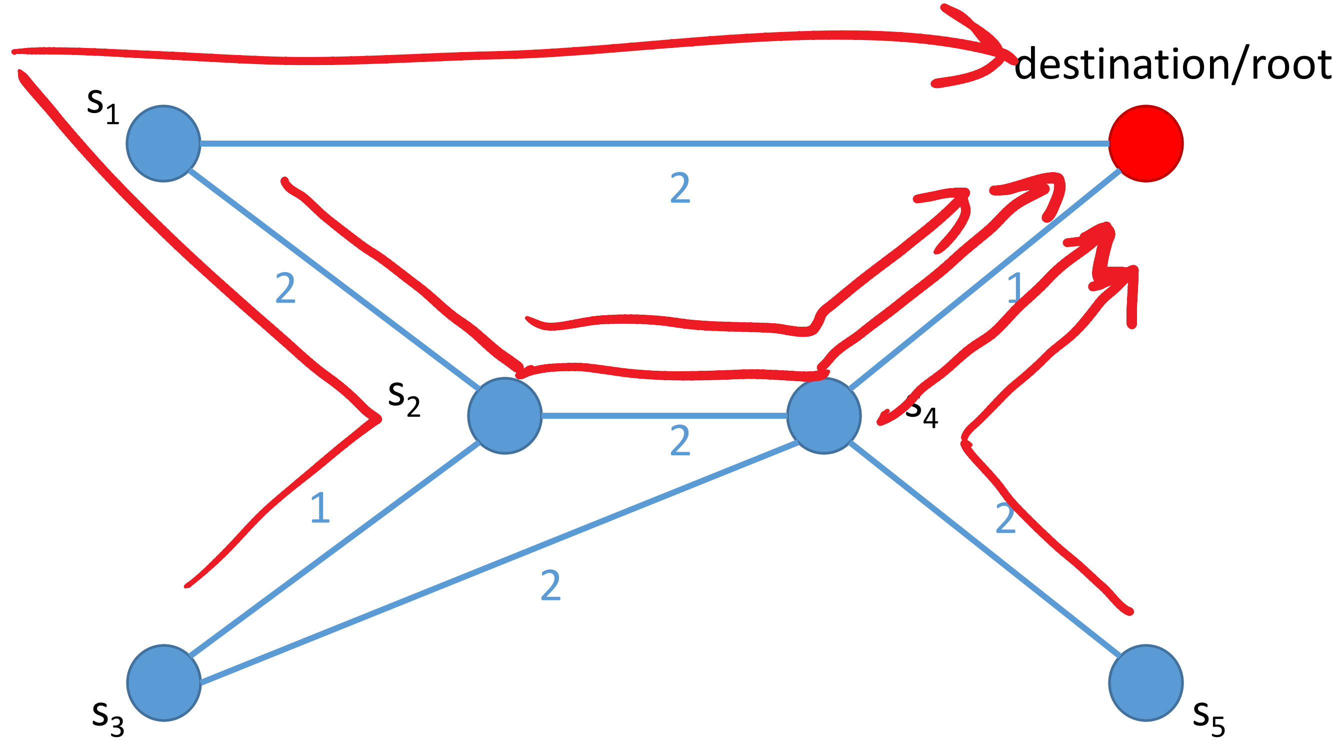
<u>special case: **broadcast games**</u>
- each vertex has an agent
- all agents route to a common gateway destination

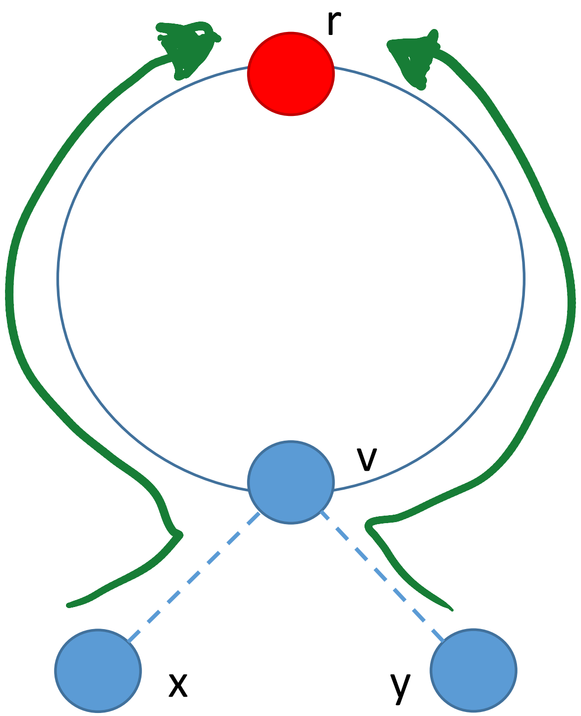
Fiat-Kaplan-Levy-Olonetsky-Shabo ’06: **O(log log n)**
Liggett-Lee ’13: **O(log log log n)**
Bilo-Flammini-Moscardelli ’13: **O(1)**



destination/root
s_1
s_2
s_3
s_4
s_5
2
2
1
2
2
1
2

broadcast games

- Property: at equilibrium, the routing paths form a tree



- if there is a cycle, then there exists a vertex **v** through which two different routing paths are used to go to the root (say by agents **x** and **y**)

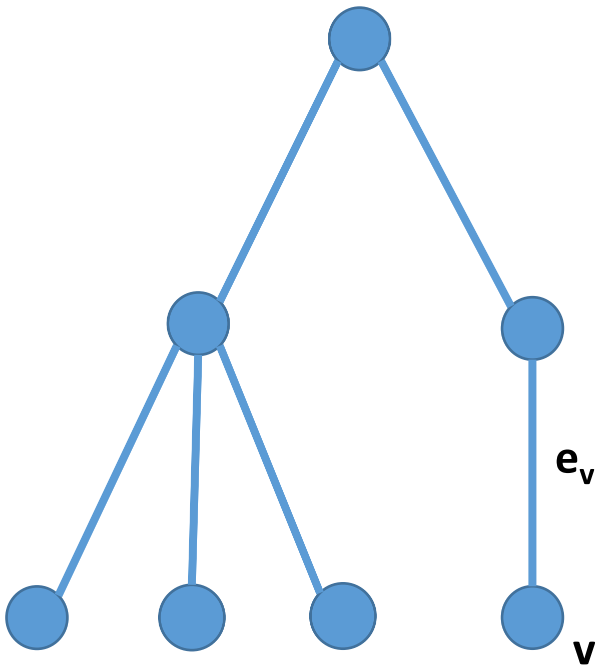
- if shared cost of agent **x**

≤ shared cost of agent **y**, then

x can move to **y**'s path

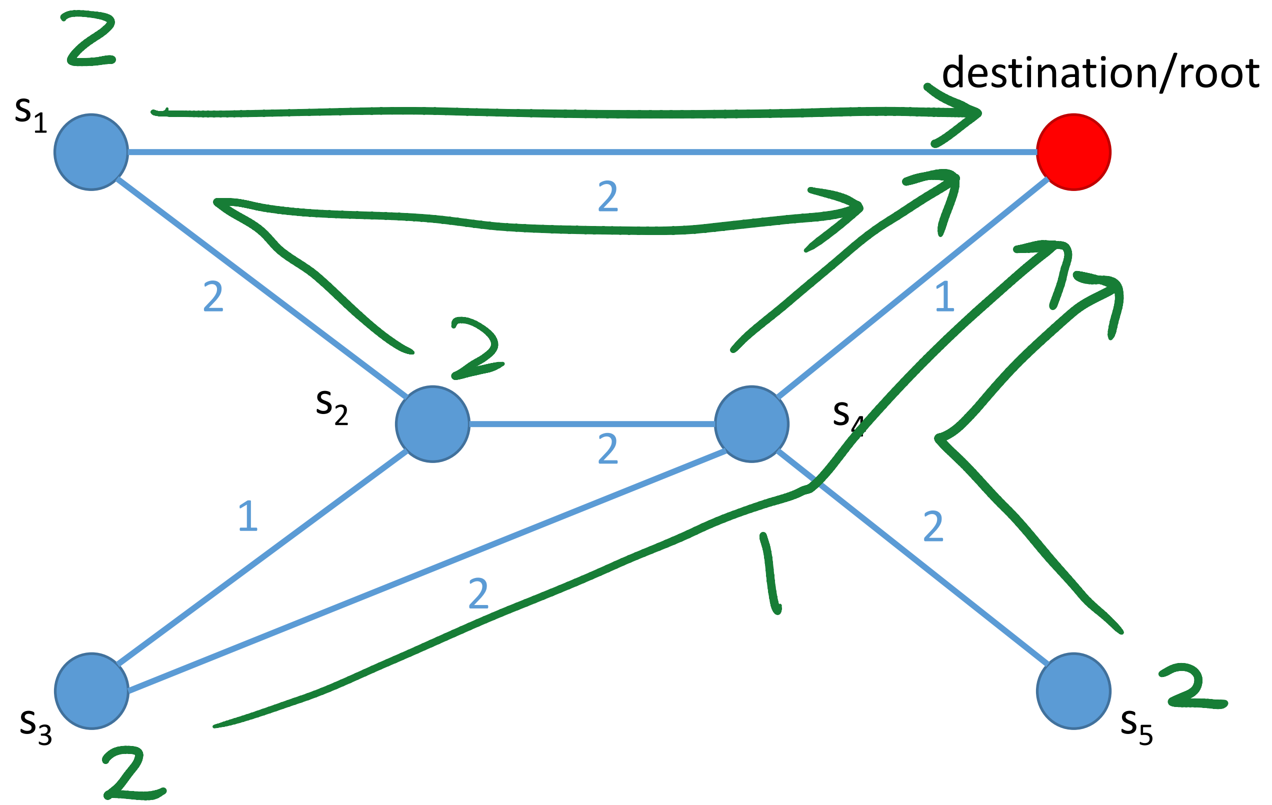
contradicting equilibrium condition

broadcast games



v is responsible for edge $\mathbf{e_v}$

the dual fitting technique



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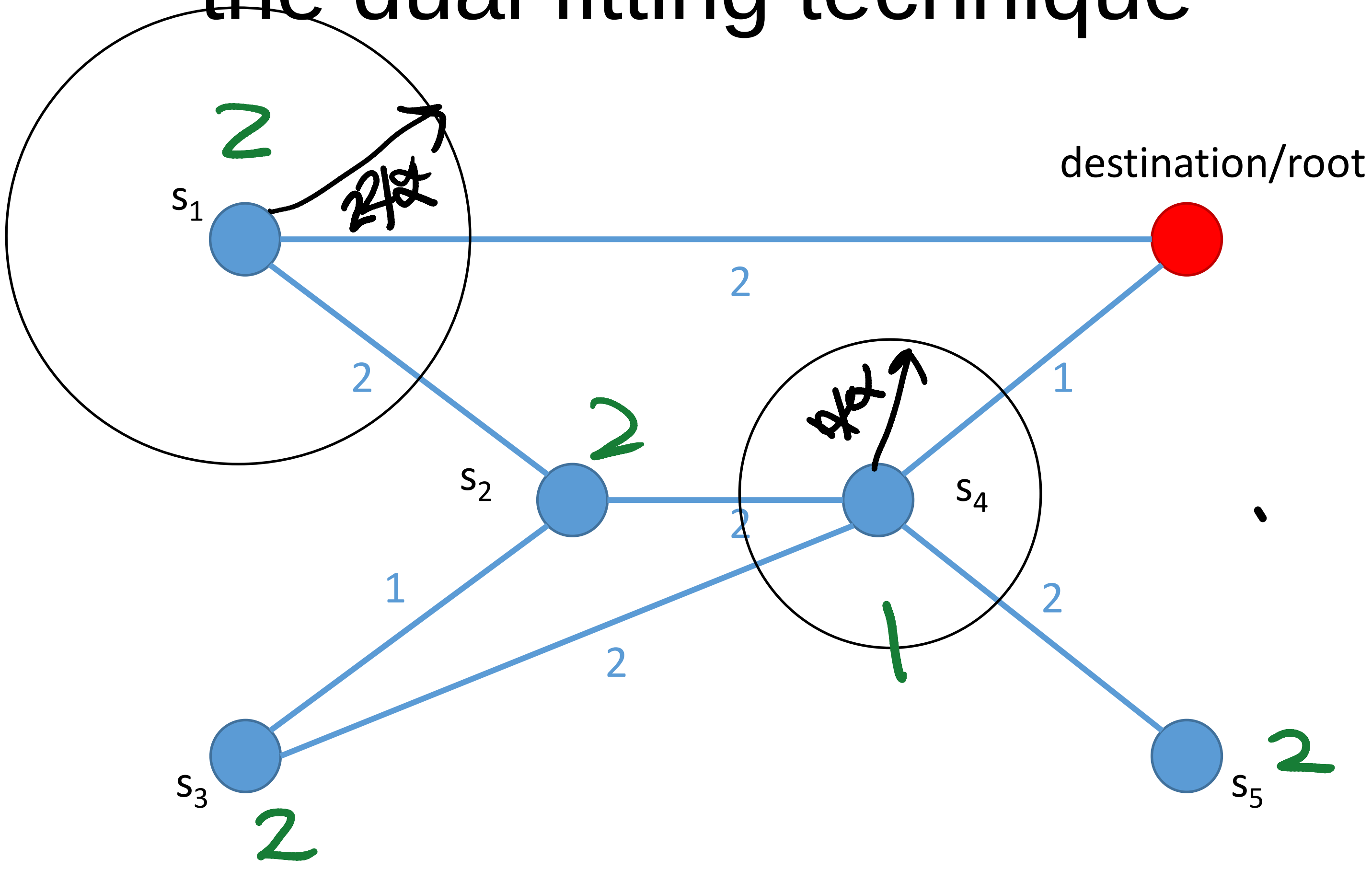
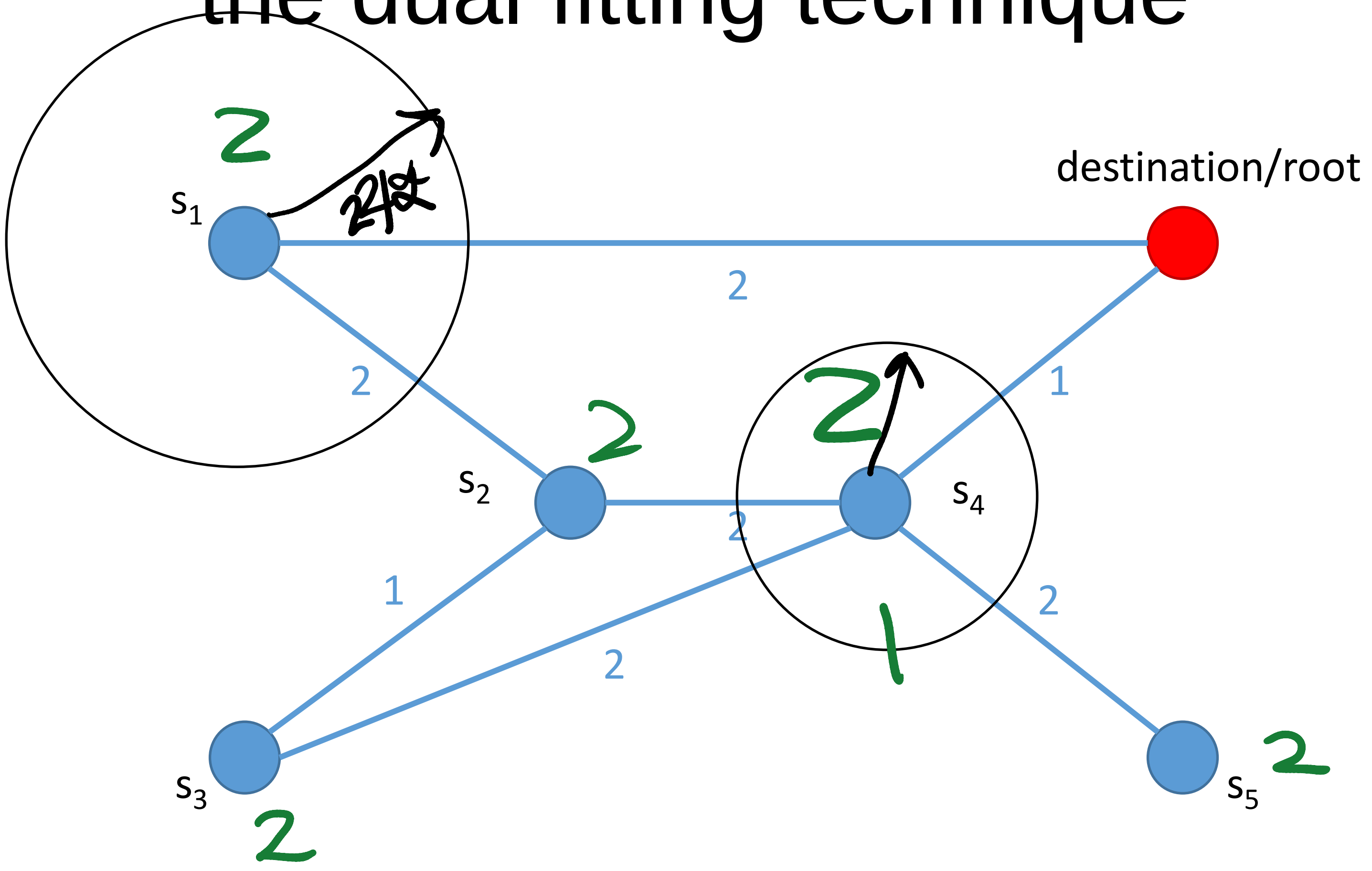
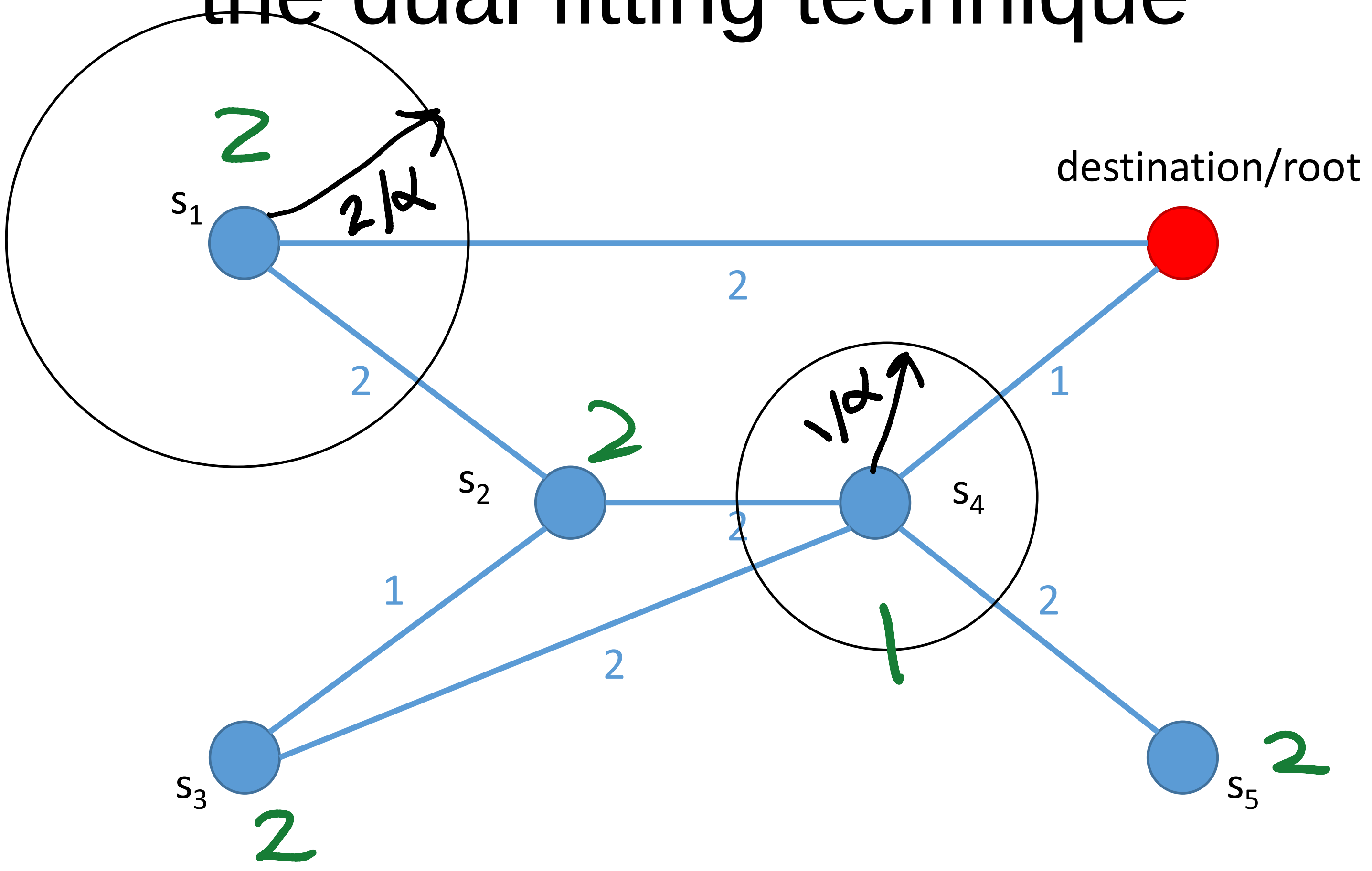


the dual fitting technique

if the balls do not intersect, then ratio is α



the dual fitting technique



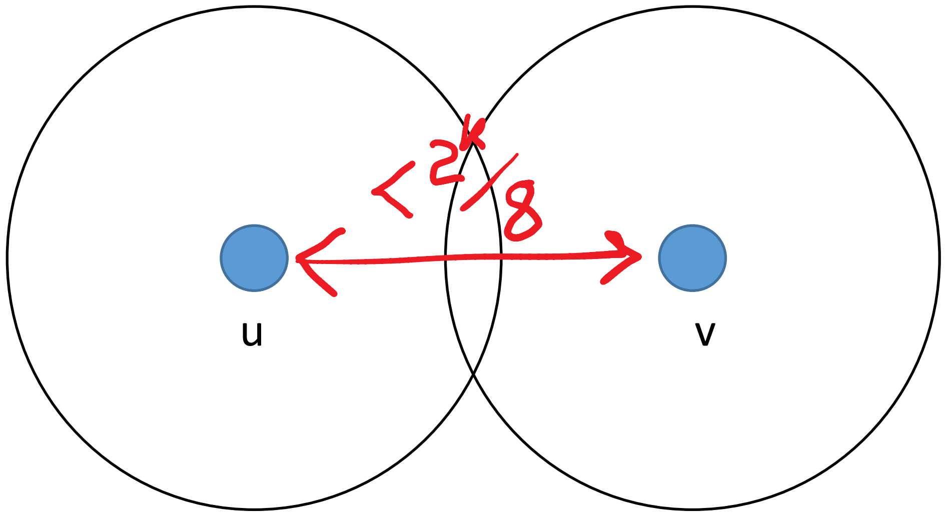
if the balls do not intersect, then ratio is α

if the balls are grouped into β groups, and the balls in any group do not intersect, then the approximation factor is α β broadcast games: an **O(log n) pos** bound

log n groups of balls

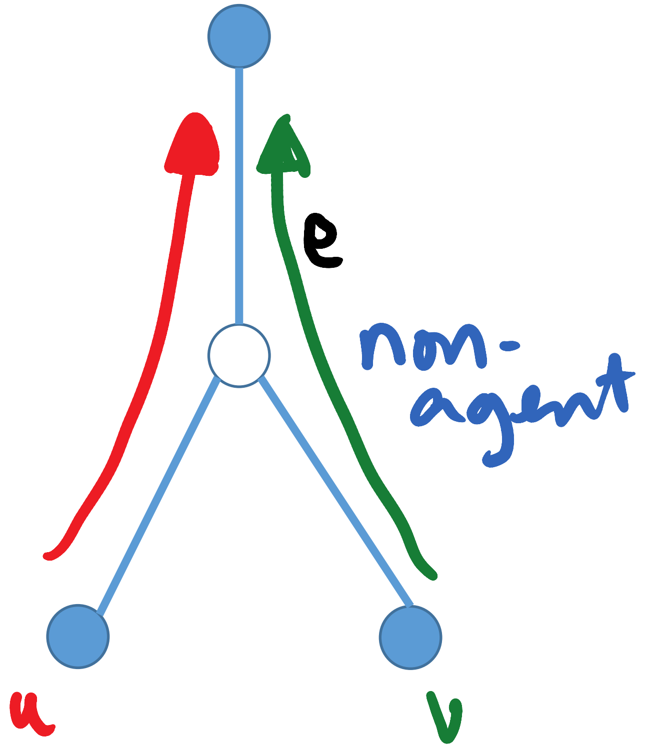
if **cost(e_v) in [2^{k-1}, 2^k]** place ball of radius **2^k/16** in group **k**

claim: no two balls in the same group intersect



what about multicast games?

<u>Main challenge</u>
Mechanism for transferring responsibility

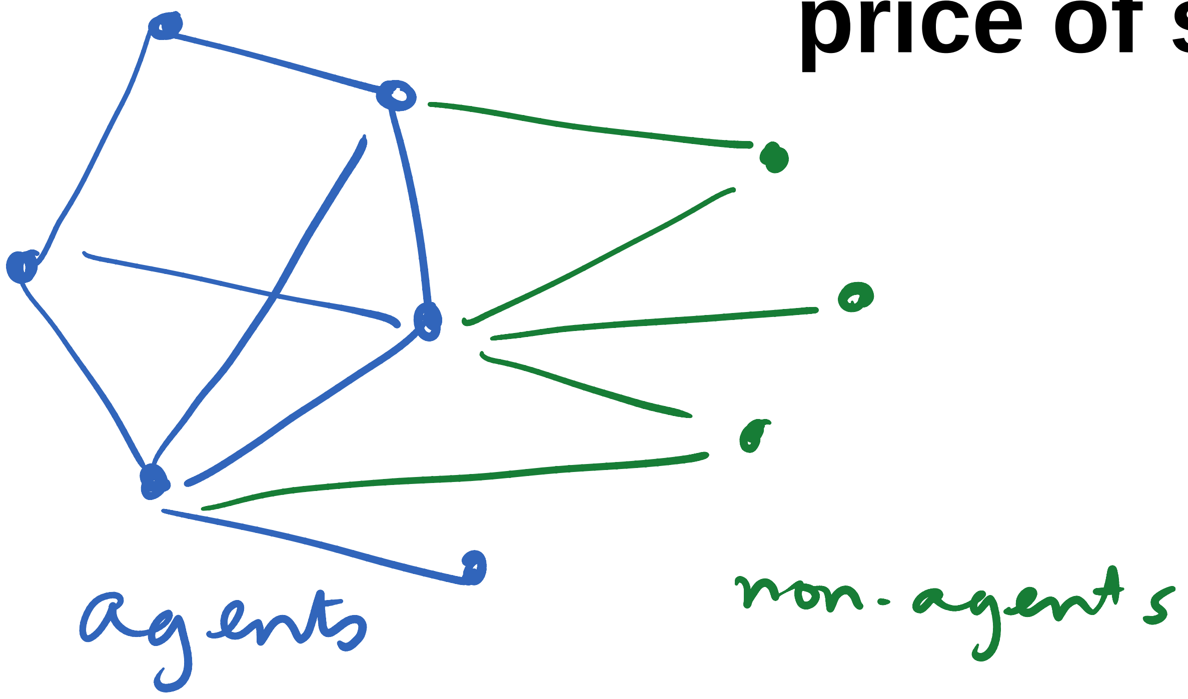


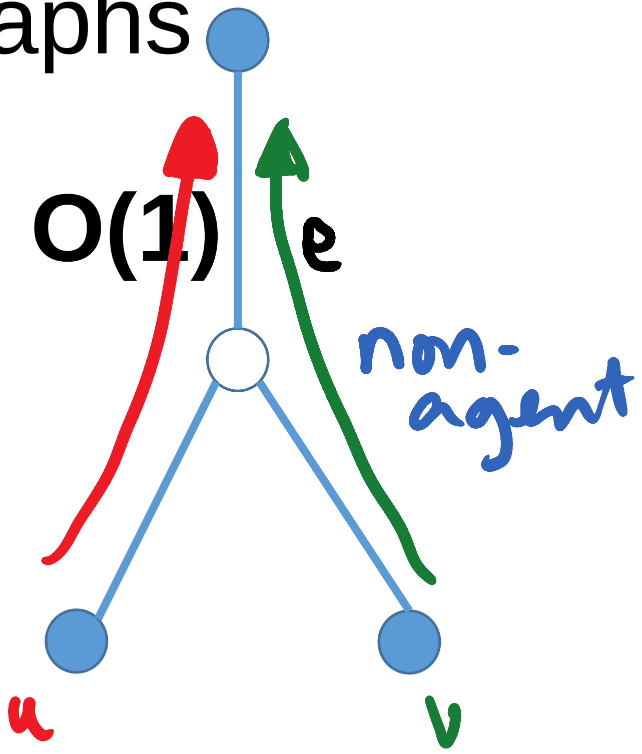
who is responsible for edge e?

recent progress [Freeman, Haney, P.]

multicast games on quasi-bipartite graphs

price of stability is O(1)





agent-agent path is of length ≤ 2

summary

equilibria in network games can have linear inefficiency

but the best equilibrium has **log** inefficiency

open: does it only have **constant** inefficiency?

yes, for broadcast and multicast on quasi-bipartite thank you

questions?