13.6.12

Talk @ ICTS

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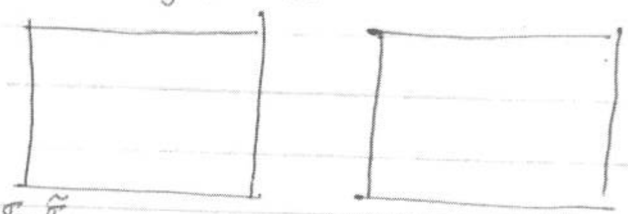
§1. Holography:

AdS/CFT

AdS/CMT



Geometry / FT : Holography $\longrightarrow$ RG $\quad \sigma_0, \tilde\sigma \searrow$ Pictures

~~Can we~~ $\int \mathcal{D}\Phi_0 \exp\left(S_{ST}[\Phi_0] + \int \sigma_{0m} \operatorname{Tr}\Phi_0^m \right) = \int \mathcal{D}\tilde\Phi \exp\left(S_{ST}[\tilde\Phi] + \tilde\sigma_m \operatorname{Tr}\tilde\Phi^m + S^{(d+1)}_{bulk}[\tilde\sigma]\right)$

How general is this correspondence? What sort of RG flows can be described by holography?

~~Can we ho~~ Could we have invented a holographic dual to any RG flow?

RG $\longrightarrow$ holography

§2. Large N ~~requixed~~ field theory (vector / matrix)

Model: Matrix

$$S_{ST} = \int d^dx \; N \operatorname{Tr}\left[+\tfrac{1}{2} \partial_\mu \Phi_0 \partial_\mu \Phi_0 + \sum_n \lambda_{0n} \Phi^n \right] \qquad N\operatorname{Tr} \sim N^2$$

Deform by ~~single trace~~ ~~f~~ double trace operators

$$S[\Phi_0] = S_{ST} + \int d^dx \; \frac{f_{mn}}{2} \operatorname{Tr}\Phi_0^m \operatorname{Tr}\Phi_0^n \qquad \text{`Tr' `Tr'} \sim N^2$$

$$\Rightarrow \; S_{ST} + \int d^dx \left(\sigma_{0m} \operatorname{Tr}\Phi_0^m - \tfrac{1}{2} f^{mn} \sigma_{0m}\sigma_{0n} \right) \ne \cancel{S[\Phi_0, \sigma_0]} \qquad \sigma \sim \text{`Tr'} \sim$$

$$Z = \int \mathcal{D}\Phi_0 \, e^{S[\Phi_0]} = \int \mathcal{D}\sigma_0 \, e^{-\frac{1}{2}\int f^{mn}\sigma_{0m}\sigma_{0n}} \underbrace{\int \mathcal{D}\Phi_0 \, e^{S[\Phi_0, \sigma_0]}}_{Z[\sigma_0]}, \qquad S[\Phi_0, \sigma_0] = S_{ST}[\Phi_0] + \int \sum_m \sigma_{0m} \operatorname{Tr}\Phi_0^m$$

All physics is d dimensional so far.

(a) Note $N\,\text{`Tr'} \sim N^2$

(b) Will only discuss $U(N)$ singlets [could enforce by $\partial_\mu \to D_\mu$]

(2)

Vectors:

$$S_{ST} = \int d^d x \left(\frac{1}{2} \partial_\mu \vec{\Phi}_0 \cdot \partial_\mu \vec{\Phi}_0 + \cancel{\sum_n \lambda_{0n} (\vec{\Phi}^2)^n +} \; m_0^2 \vec{\Phi}^2 \right)$$

~~For $d=2$, $n=2$ and $\vec{\Phi}$ = fermions, this is~~

Deform by 'double trace'

$$S[\vec{\Phi}_0] = S_{ST} + \int d^d x \, f(\vec{\Phi}^2)^2$$

For $\vec{\Phi}$ a fermion, examples include Gross-Neveu model, NJL model et

[See Luca Vecchia 1005.492

We will discuss matrices first.

§3. RG:

Integrating out 1 step $\Lambda_0 \to \Lambda_0 - \delta\Lambda = \Lambda_1$

- ~~We will work with the hybrid model $S[\Phi_0, \sigma_0]$~~
- ~~We will first ignore derivative term in σ_0 which appear at first~~

Consider $Z[\sigma_0] \equiv \int \mathcal{D}\Phi_0 \exp S[\Phi_0, \sigma_0]$

$$\Phi_0 = \{ \Phi_1, \hat{\Phi}_1 \} \qquad = \int \underbrace{\mathcal{D}\Phi_1}_{\Lambda_1} \int \underbrace{\mathcal{D}\hat{\Phi}_1}_{[\Lambda_1, \Lambda_0]} \exp S[\Phi_1 + \hat{\Phi}_1, \sigma_0]$$

More exactly, [Polchinski ERG]

$$\int \mathcal{D}\Phi_0 \exp\left[-\frac{1}{2} \int_p \frac{N p^2}{K(p/\Lambda_0)} \mathrm{Tr}\, \Phi_0(p) \Phi_0(-p) - S_I(\Phi_0) \right]$$

$$= \int \mathcal{D}\Phi_1 \exp\left[-\frac{1}{2} \int_p \frac{N p^2}{K(p/\Lambda_1)} \mathrm{Tr}\, \Phi_1(p) \Phi_1(-p) \cancel{- S_I} \right] \times$$

$$\times \int \mathcal{D}\hat{\Phi}_1 \exp\left[-\frac{1}{2} \int_p \frac{N p^2}{K(p/\Lambda_0) - K(p/\Lambda_1)} \mathrm{Tr}\, \hat{\Phi}_1(p) \hat{\Phi}_1(-p) - S_I(\Phi_1 +\right.$$

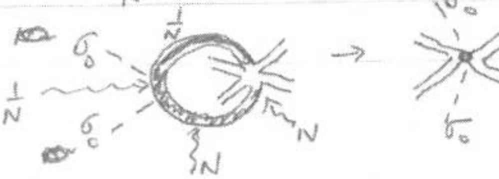
ⓐ RG of ST sector $_{Tr}$  $\lambda_{02} + \delta\Lambda\, \lambda_{04}\, \Lambda^{\#} = \lambda_{1,2}$

$\Rightarrow \int d^dx \Lambda_1^d \underbrace{\sum \lambda_{1m}\, \Phi_1^m\, \Lambda_1^{-m[\Phi]}}$

ⓑ ~~RG of~~ Generation of double trace terms

$\frac{1}{N}$ Λ $N\lambda_6$ $O(1)$

$$\lambda_{22}^{(1)} = \frac{\delta\Lambda}{\Lambda_1}\, \beta_{22} \qquad \beta_{22} = b_{22}^{(6)}\lambda_{06}$$



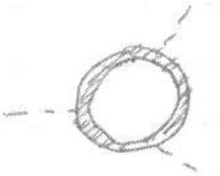
σ_0 $\frac{1}{N}$ σ_0 $\frac{1}{N}$ N N σ_0 $\frac{1}{N^2}$ $\boxed{\sigma_0 \sim N\,\underset{\sim}{\tilde{\sigma}_0}}$

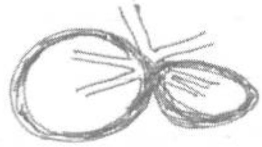
$$\Rightarrow \frac{\delta\Lambda}{\Lambda_1}\int d^dx\, \Lambda_1^d \left[\frac{1}{2}\, \Lambda_1^{-(m+n)[\Phi]} A_{mn}(\lambda_0, \Lambda_1^{-[\sigma_m]}\sigma_{0m})\, \mathrm{Tr}\,\Phi_1^m\, \mathrm{Tr}\,\Phi_1^n \right.$$

(c) 

$$+ \Lambda^{-m[\Phi]} B_m\, \mathrm{Tr}\,\Phi_1^m + C(\lambda_0, \Lambda_1^{-[\sigma]}\ldots$$

$\cdot\ [\sigma_m] = d - m[$

(d) 

(e)  $\propto \delta\Lambda^2 \Rightarrow$ multi trace (e.g. triple trace) not generated.

$$A_{mn} = \beta_{mn}(\lambda) + \beta_{mn}^{l}(\lambda)\,\Lambda_1^{-[\sigma_l]}\sigma_{0l} + \cdots$$
$$B_m = \left(-\frac{1}{2}\gamma(\lambda)\,\delta_m^n + \beta_m^n(\lambda)\right)\Lambda_1^{-[\sigma_n]}\sigma_{0n} + O(\sigma^2)$$

[no σ-indep term since that is included in S'_{ST}]

$$C = \beta(\lambda) + \beta^m(\lambda)\,\Lambda_1^{-[\sigma_m]}\sigma_{0m} + \cdots$$

(4)

§4. <u>Reduction to single trace</u> : <u>getting rid of double trace by introducing auxiliary fields</u>

$$\int \frac{A_{mn}}{2}\,\mathrm{Tr}\,\Phi_1^{\,m}\,\mathrm{Tr}\,\Phi_1^{\,n}\;\overbrace{\Lambda_1^{\,d-(m+n)[\Phi]}}^{\Lambda_1^{-d+[\sigma_m]+[\sigma_n]}}\;\frac{\delta\Lambda_1}{\Lambda_1}\;d^dx$$

$$\longrightarrow \int -\underbrace{\frac{A^{mn}}{2}}_{\frac{\Lambda_1}{\delta\Lambda}\Lambda_1^{d-[\sigma_m]-[\sigma_n]}}\tilde{\sigma}_{1m}\tilde{\sigma}_{1n} + \underbrace{\tilde{\sigma}_{1m}}_{\substack{d-m[\Phi]\\ =[\sigma_m]}}\underbrace{\mathrm{Tr}\,\Phi_1^{\,m}}_{m[\Phi_1]}\quad , \text{ with } \int \mathcal{D}\tilde{\sigma}_{1m}$$

$$= \Lambda_1^{\,1+d-[\sigma_m]-[\sigma_n]}\;\frac{1}{\delta\Lambda_1}$$

$$\sigma_0 + \tilde{\sigma}_1 = \sigma_1 \qquad \tilde{\sigma}_1 = \sigma_1 - \sigma_0 \qquad \int\mathcal{D}\tilde{\sigma}_{1m} \to \int\mathcal{D}\sigma_{1m}$$

$$\frac{\tilde{\sigma}_1^{\,2}}{\delta\Lambda} = \left(\frac{\sigma_1-\sigma_0}{\delta\Lambda}\right)^2\cdot\delta\Lambda$$

$$\sigma_{0m}\,\mathrm{Tr}\,\Phi_1^{\,m} + \tilde{\sigma}_{1m}\,\mathrm{Tr}\,\Phi_1^{\,m} \longrightarrow \sigma_{1m}\,\mathrm{Tr}\,\Phi_1^{\,m}$$

$$\frac{1}{2}A_{mn}\vartheta_m\vartheta_n + B_m\vartheta_m + C = \cancel{A_{mn}\vartheta_m +} = \frac{1}{2}A(\vartheta + A^{-1}B)(\vartheta+A^{-1}B) + C - \frac{1}{2}BA^{-1}B$$

Get

$$Z[\sigma_0] = \int\mathcal{D}\sigma_1\,\mathcal{D}\Phi_1\,\exp\left[S_{ST}(\lambda_1) + \int \sigma_{1m}\,\mathrm{Tr}\,\Phi_1^{\,m}\,d^dx\right]\times$$

$$\times\exp\left[-\int\frac{1}{2}A^{mn}\Lambda_1^{\,1+d-[\sigma_m]-[\sigma_n]}\frac{\delta\sigma_m}{\delta\Lambda}\frac{\delta\sigma_n}{\delta\Lambda} - \Lambda_1^{\,d-[\sigma_m]}D^m + V\right]$$

$$= \int\mathcal{D}\Phi_0\,\exp\left[S_{ST}(\lambda_0) + \sigma_0\,\mathrm{Tr}\,\Phi_0^{\,m}\right]$$

Repeat this on $S_{ST}(\lambda_1) + \int\sigma_{1m}\,\mathrm{Tr}\,\Phi_1^{\,m}\,d^dx$ etc.

§5. Emergence of bulk: ~~Redefine~~

$$= \int \mathcal{D}\Phi_0 \exp\left(S_{ST}(\lambda_0 \Phi) + \int \sigma_{0m} \operatorname{Tr} \Phi_0^m\right)$$

$$Z[\sigma_0] = \int \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 \cdots \mathcal{D}\sigma_k \int \mathcal{D}\Phi_k \exp\left[S_{ST}(\Lambda_k) + \int \sigma_{km} \operatorname{Tr} \Phi_k^m \, d^d x \right] \quad \Lambda_k \equiv \Lambda'$$

$$\times \exp\Big[-\tfrac{1}{2}\delta\Lambda \int \Big(A^{mn}_{(1)} \Lambda_1^{1+d} \frac{\delta^{(1)}\sigma_m}{\delta\Lambda} \frac{\delta^{(1)}\sigma_m}{\delta\Lambda} - \Lambda_1 D^m_{(1)} \frac{\delta^{(1)}\sigma_m}{\delta\Lambda} + V_{(1)} \Big) d^d x$$

$$- \tfrac{1}{2}\delta\Lambda \int A^{mn}_{(2)} \Lambda_2^{1+d} \frac{\delta^{(2)}\sigma_m}{\delta\Lambda} \frac{\delta^{(2)}\sigma_n}{\delta\Lambda} - \Lambda_2 D^m_{(2)} \frac{\delta^{(2)}\sigma_m}{\delta\Lambda} + V_{(2)} \, d^d x$$

$$- \cdots \Big]$$

[We have used $\Lambda^{-[\sigma_m]}\sigma_m \to \sigma_m$]

$$= \int \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 \cdots \mathcal{D}\sigma_k \exp\left[S_{ST}(\Lambda') + \int \sigma'_m \operatorname{Tr} \Phi'^m d^d x \right] \times \exp\left[-S_{bulk}[\sigma]\right]$$

$$e^{-S_{bulk}} \equiv \exp\left[- \int_{\Lambda'}^{\Lambda_0} d\Lambda \left(\Lambda^{d+1} \partial_\Lambda \sigma_m \partial_\Lambda \sigma_n \frac{A^{mn}(\Lambda)}{2} + \Lambda^d D^m(\Lambda) \partial_\Lambda \sigma_m + \cdots \right.\right.$$

$$= \exp\left[- \int_{r_{IR}}^{r_{UV}} dr \left\{ r^{d+1} (\partial_r \sigma_m \partial_r \sigma_n) \frac{A^{mn}(r)}{2} + \underbrace{r^d D^m(r) \cdot \partial_r \sigma_m}_{-\sigma_m \partial_r r^d D^m(r)} + V(r) \right\} \right]$$

$\boxed{\text{N.B}}$ Using $\boxed{\Lambda = r}$ $\boxed{\text{WITTEN-SUSSKIND}}$

If $A^{mn}(r) =$ constant, ~~then~~ then, after diagonalizing A^{mn} we will get

$$S_{bulk} = - \int_{r_{IR}}^{r_{UV}} dr \Big[\underbrace{r^{d-1} \frac{r^2}{2}}_{\text{AdS - metric}} (\partial_r \sigma_m)^2 \frac{1}{a_m} + \tilde{V}(\sigma(r)) \Big]$$

$$\cancel{A^{mn} = \beta^m} \qquad A_{mn}(\Lambda) = \beta_{mn}(\lambda) + \cdots \equiv \beta_{mn}(\lambda(\Lambda)) \longrightarrow \beta_{mn}(r)$$

If $\lambda = \lambda_*$ (ST fixed point) then $\beta_{mn}(r) \Rightarrow \beta_{mn}(\lambda_*)$

$$A^{mn}_k \simeq \beta^{mn}(\lambda_*) = \frac{1}{a_m} \quad \text{(constant)}$$

~~§6 Role of large N :~~

Comments :

$\underline{\underline{a.}}$ momentum ranges : In Polchinski RG $p \in [0, \infty]$

$$S_{ST} = \int d^d p \, \mathrm{Tr} \left[\frac{\Phi_1(p) \, \Phi_1(-p) \, p^2}{K(p/\Lambda_0)} \cdot \frac{1}{2} + (\lambda_{1m} + \sigma_{1m}(-p)) \, \mathrm{Tr} \, \Phi_1^{m}(p) \right]$$

After the n-th iteration $K(p/\Lambda_n)$

$$S_{bulk} = d\Lambda \int \left(\frac{\delta\sigma_m}{\delta\Lambda} \frac{\delta\sigma_n}{\delta\Lambda} \right) \frac{\Delta^{mn}}{2} \Lambda^{d+1}$$

no K here ! $p_r \in [0, \infty]$

$\underline{\underline{b.}}$ $p_\mu p^\mu$ term for σ

Does this have a $K(p/\Lambda_0)$ or not ? If yes, is it $K(p/\Lambda_0)$ or $K(p/\Lambda_1)$ or ...

According to Bervilliers Eq. (10) $p_\mu p^\mu$ for σ does not come with a K (in a specific RG scheme)

We get $\int \sqrt{g} \, r^{-2} p_\mu p^\mu \, \sigma(p) \, \sigma(-p)$ at fixed pt

[also follows if scale $\Rightarrow$ CFT $\Rightarrow$ AdS]

$$ds^2 = \frac{dz^2 + f\left(\frac{x^2}{z^2}\right) dx_\mu dx^\mu}{z^2}$$

has conformal sym iff $f\left(\frac{x^2}{z^2}\right) = 1$

$\underline{\underline{c.}}$ de Sitter

We get AdS (bosons vs fermions can have different signs ....

$\delta S = \int \frac{\delta\lambda}{2} (\mathrm{Tr}\,\Phi^2)^2 \Rightarrow \int -\frac{1}{\delta\lambda}\frac{\sigma^2}{2} + \sigma(\mathrm{Tr}\,\Phi^2)$

note − sign

$\delta S = \int \frac{\delta\lambda}{2} (\mathrm{Tr}(\bar\psi\psi))^2 = \int + \frac{1}{\delta\lambda}\frac{\sigma^2}{2} - \sigma\,\mathrm{Tr}\,\bar\psi\psi$

Correct sign for a +ve potential.

+ sign

However, the sign here depends actually on the sign of β_{mn}

If β_{mn} has signs different for bosons & fermions, then the $(\partial_r\sigma)^2$ can have the same sign again.

d. Vasiliev

For matrix theories

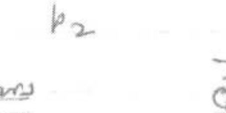
σ_m is actually $\sigma_{\{m\}}(p_1 \ldots p_m)$ $\sigma_m(p_1 \ldots p_m)$

e.g. the GKP-W relation is

~~$\sigma_{\{m\}}(p_1, \ldots p_m)$ $\mathrm{Tr}\,\Phi^{m_1}(p_1)$ $\mathrm{Tr}\,\Phi^{m_2}(p_2) \ldots \mathrm{Tr}\,\Phi$~~

$\sigma_m(p_1, \ldots, -p_m)\ \mathrm{Tr}\,\underbrace{(\Phi(p_1)\,\Phi(p_2) \ldots \Phi(p_m))}$

many words possible



$\sigma_m(p_1, p_2)$

Vectors $\quad \vec\Phi(p_1)\cdot\vec\Phi(p_2) \qquad \sigma(p_1, p_2) \sim \underbrace{\sigma_n(p)}$

single Regge trajectory ?

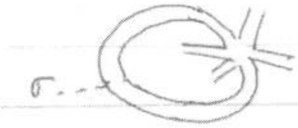
Rephrase / [illegible] / [illegible]

§6. Role of Large N

$$A_{mn} = \underbrace{\beta_{mn}}_{1} + \underbrace{\beta_{mn}^{\ l}}_{\frac{1}{N}} \sigma_l + \qquad \sigma_l \to N\tilde{\sigma}_l$$

$\frac{1}{N}$ N

$O(1)$ $\underline{\mathrm{Tr}\,\mathrm{Tr} \sim N^2}$,



$$S_{bulk}[\sigma] = N^2 S_{bulk}[\tilde{\sigma}] \qquad \cancel{N \to \infty \text{ classical limit}}$$

~~This~~ ~~All diagrams~~

After $\sigma = N\tilde{\sigma}$ N^2 comes out of the entire action

- Large N limit exists.
- Large N limit is classical.

§7. Wilson - Fisher [Preliminary]

$$S_{ST}[\Phi_0, \lambda_0; \Lambda_0] = \int d^dx \, \mathrm{Tr}\left[-\tfrac{1}{2}(1+\lambda'_{02})(\partial\Phi_0)^2 + \Lambda^2\lambda_{02}\Phi^2 + \Lambda^{1+2\epsilon}\lambda_{04}\Phi_0^4 + \Lambda^{4\epsilon}\lambda_{06}\Phi_0^6\right]$$

$$\beta_6 = -4\epsilon\lambda_6 + b_6\lambda_4^2 + \cdots$$

$$\beta_4 = -\lambda_4 + b_4\lambda_6 + \cdots$$

$$\beta_2 = -2\lambda_2 + b_2\lambda_4$$

Fixed pt @

$$\lambda_4^* = b_4\lambda_6^*, \quad \lambda_6^* = \frac{4\epsilon}{b_6 b_4^2}, \quad \lambda_2^* = \frac{b_2}{2}\lambda_4^* \qquad ??$$

[Check:]

- See that higher order terms really do not matter in $\epsilon \to 0$
- See if

$$\beta_{2,2} = b_{2,2}\lambda_6 + \cdots = b_{2,2}\lambda_6^* \text{ at fixed pt}$$

hence $A_{2,2} \xrightarrow{r \to 0}$ constant ~~AdS~~ Emergent $\boxed{AdS}$

$A_{2,2}^{-1} \propto \frac{1}{\lambda_6^*}$ exists if $\lambda_6^* \neq 0$ [need a nontrivial IR fixed

§8. Grosx-Neveu model

$$`S_{ST}' \xrightarrow{\text{replace}} \quad S = \int d^2x \left(\bar\psi_{0i} \partial\!\!\!/ \psi_{0i} + \frac{\lambda_0}{2N} (\bar\psi_{0i}\psi_{0i})^2 + \sigma_0 \bar\psi_{i0}\psi_{i0} \right)$$

~~Expect:~~

$$Z[\sigma_0] = \int \mathcal{D}\tilde\sigma_0 \, \exp\left[N \int d^2x \left(\frac{\tilde\sigma_0^2}{2\lambda} - \mathrm{Tr} \ln (\partial\!\!\!/ + \tilde\sigma_0 + \sigma_0) \right) \right]$$

For constant σ_0, saddle point

$$\frac{\tilde\sigma_0}{\lambda} = \mathrm{Tr} \int^{\Lambda} \frac{d^2p}{p\!\!\!/ + \tilde\sigma_0 + \sigma_0} = 2(\tilde\sigma_0 + \sigma_0) \underbrace{\int_0^{\Lambda} \frac{d^2p}{p^2 + (\tilde\sigma_0+\sigma_0)^2}}$$

$$\left(\frac{\tilde\sigma_0}{\tilde\sigma_0 + \sigma_0} \right) \frac{1}{2\lambda} = \ln \frac{\Lambda^2}{(\tilde\sigma_0+\sigma_0)^2}$$

future directions

§9. $d+1$ dimensional covariance

- The p^2 term
- p^4, p^6 etc. $\Rightarrow$ interpret
- vector $\rightarrow$ Vasiliev
- Matrix models $c=1$, ..
- General covariance in $d+1$

$\lambda \rightarrow \underline{\underline{\lambda(x^\mu)}}$ lapse vector, ...

[Sung. ~~Shik~~ Sik Lee]