First GAP-exercises

- 0. Download and install GAP on your machine. Choose an editor you wish to use. For beginners on Windows machines PSPad is a good and free choice.
- 1. Let $G = S_4$ and H = GL(2, 5). Find the number of subgroups and normal subgroups in G and H. How many conjugacy classes of elements of order 3, 4 and 6 are there in G and H respectively?
- 2. Write a program which takes as input a finite group G and a prime p and returns the number $n_p(G)$ of Sylow p-subgroups in G.
- 3. How many groups of order 64 are there? How many of these groups G have the following properties:
 - G is abelian.
 - G/Z(G) is abelian.
 - G has a non-trivial normal cyclic subgroup N such that G/N is abelian.
- 4. We call a group G wide if the derived subgroup of G is different from the set of commutators of elements in G.
 - Write a program which checks if a finite group is wide.
 - Find the smallest group which is wide. What is the StructureDescription of this group?
- 5. The prime graph of a group G is an undirected loop-free graph whose indices are labeled by the primes appearing as orders of elements of G and the vertices p and q are connected by an edge if and only if G contains an element of order pq. Find the smallest group not isomorphic to the symmetric group S_5 which has the same prime graph as S_5 .
- 6. Give explicitly the structures of the groups you found in the previous two exercises.