## First GAP-exercises

0 . Download and install GAP on your machine. Choose an editor you wish to use. For beginners on Windows machines PSPad is a good and free choice.

1. Let $G=S_{4}$ and $H=\mathrm{GL}(2,5)$. Find the number of subgroups and normal subgroups in $G$ and $H$. How many conjugacy classes of elements of order 3,4 and 6 are there in $G$ and $H$ respectively?
2. Write a program which takes as input a finite group $G$ and a prime $p$ and returns the number $n_{p}(G)$ of Sylow $p$-subgroups in $G$.
3. How many groups of order 64 are there? How many of these groups $G$ have the following properties:

- $G$ is abelian.
- $G / Z(G)$ is abelian.
- $G$ has a non-trivial normal cyclic subgroup $N$ such that $G / N$ is abelian.

4. We call a group $G$ wide if the derived subgroup of $G$ is different from the set of commutators of elements in $G$.

- Write a program which checks if a finite group is wide.
- Find the smallest group which is wide. What is the StructureDescription of this group?

5. The prime graph of a group $G$ is an undirected loop-free graph whose indices are labeled by the primes appearing as orders of elements of $G$ and the vertices $p$ and $q$ are connected by an edge if and only if $G$ contains an element of order $p q$. Find the smallest group not isomorphic to the symmetric group $S_{5}$ which has the same prime graph as $S_{5}$.
6. Give explicitly the structures of the groups you found in the previous two exercises.
