The Information Paradox and State-Dependence

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Fundamentals of Quantum Physics
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References

1. *"The Breakdown of String Perturbation Theory for Many External Particles"*, Sudip Ghosh and Suvrat Raju, arXiv:1611.08003
2. *"Smooth Causal Patches for AdS Black Holes"*, Suvrat Raju, arXiv:1604.03095.
3. *"A toy model of black hole complementarity"*, Souvik Banerjee, Jan-Willem Bryan, Kyriakos Papadodimas and Suvrat Raju, JHEP (2016)
4. *"Remarks on the necessity and implications of state-dependence in the black hole interior"*, Kyriakos Papadodimas and Suvrat Raju, Phys. Rev. D (2016)
5. *"Local Operators in the Eternal Black Hole"*, Kyriakos Papadodimas and Suvrat Raju, Phys. Rev. Lett. (2015).

References and Collaborators

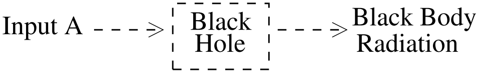
6. *"State-Dependent Bulk-Boundary Maps and Black Hole Complementarity"*, Kyriakos Papadodimas and Suvrat Raju, Phys. Rev. D (2014).
7. *"Black Hole Interior in the Holographic Correspondence and the Information Paradox"*, Kyriakos Papadodimas and Suvrat Raju, Phys. Rev. Lett. (2014)
8. *"An Infalling Observer in AdS/CFT"*, Kyriakos Papadodimas and Suvrat Raju, JHEP (2013)

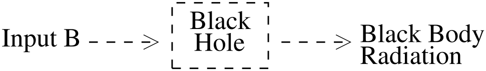
Collaborators: Kyriakos Papadodimas (CERN/Groningen), Souvik Banerjee (Uppsala), Jan-Willem Bryan (Groningen), Sudip Ghosh (ICTS).

Outline

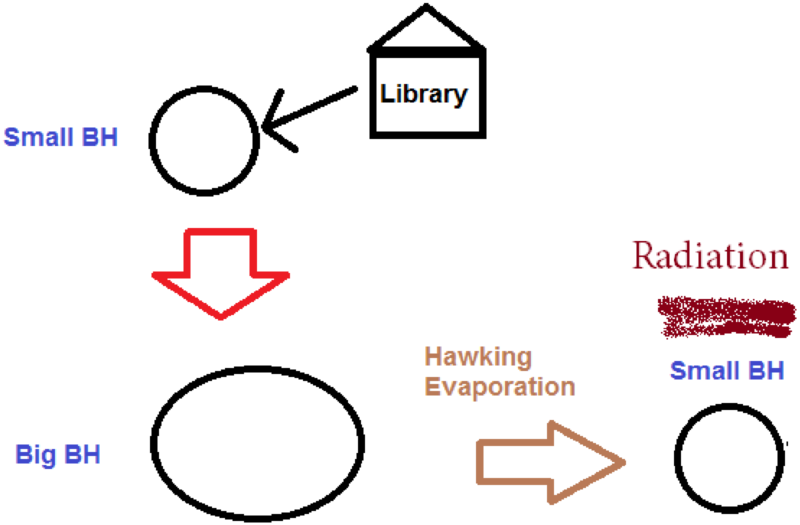
The Naive Information Paradox

- Hawking radiation leads to a seeming paradox.
- The matter that collapses into a black-hole contains information about the initial state.
- But the Hawking radiation is apparently black-body radiation.
- Seemingly, for different inputs, we get the same output.





Where is the Information?



Resolution to the Naive Information Paradox

- This is just thermalization of pure states.
- The difference between expectation values in a generic pure state and the microcanonical ensemble is $e^{\frac{-S}{2}}$

$$\int \left[\langle \Psi_U | A | \Psi_U \rangle - \mathrm{Tr}_{\text{micro}}(A)\right]^2 dU = \frac{1}{e^S} \sigma_A$$

[Lloyd, 1988]

- Hawking radiation is thermal to excellent approximation does not imply that the final-state is mixed.

Resolution to the Naive Information Paradox

- Very small corrections of the order of $e^{-S/2}$ can restore unitarity to Hawking radiation.

  [Maldacena, 2001]

- A pure density matrix in a very large system can mimic a thermal density matrix to extreme accuracy

$$\mathrm{Tr}\left(\rho_{\text{pure}} A_\alpha\right) = \frac{1}{\mathcal{Z}} \mathrm{Tr}\left(e^{-\beta H} A_\alpha\right) + \mathrm{O}\left(e^{-S/2}\right),$$

- Another way to state this is

$$\rho_{\text{pure}} = \frac{1}{\mathcal{Z}} e^{-\beta H} + e^{-S} \rho_{\text{corr}}; \quad \rho_{\text{pure}}^2 = \rho_{\text{pure}}$$

  [Kyriakos Papadodimas, S.R., 2013]

- In words: "information can be encoded in tiny correlations between the Hawking quanta."

Path Integral Perspective

- A semi-classical spacetime is a saddle point of the QG path-integral.

$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}$$

- Perturbative effective field theory (used to derive the Hawking answer) is an asymptotic series expansion of this path-integral, which can only provide limited accuracy.

- Exponentially small corrections to restore unitarity can easily come from other saddle points.

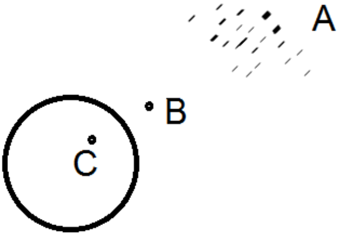
Summary: Naive Information Paradox

Naive Paradox	Resolution
Hawking radiation appears to be thermal for different initial states.	Standard in pure state thermalization
No mistake has been found in Hawking's computation.	Hawking's calculation is not precise enough to lead to a paradox.

Outline

The information paradox can be sharpened by making reference to the black hole interior.

Three Subsystems



They key point is to think of three subsystems

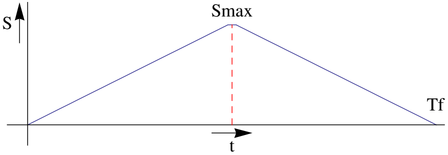
1. The radiation emitted long ago – A
2. The Hawking quanta just being emitted – B
3. Its partner falling into the BH – C

Entropy of A

- Say the Black Hole is formed by the collapse of a pure state.
- Consider the entropy of system A

$$S_A = -\mathrm{Tr} \rho_A \ln \rho_A$$

- Very general arguments due to Page tell us this must eventually start decreasing.



Strong Subadditivity contradiction?

- Now, consider an old black hole, beyond its “Page time” where S_A is decreasing. We must have

$$S_{AB} < S_A$$

since B is purifying A.

- Second, the pair B, C is related to the Bogoliubov transform of the vacuum of the infalling observer, and almost maximally entangled.

$$S_{BC} < S_C$$

- However, strong subadditivity tells us

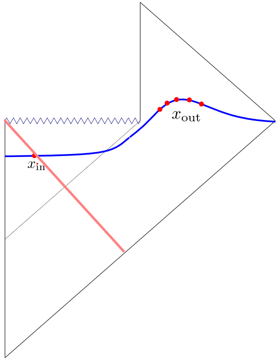
$$S_A + S_C \leq S_{AB} + S_{BC}$$

[Mathur, AMPS, 2009–12]

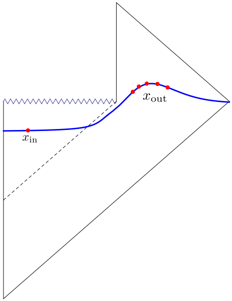
- We seem to have a violation at $O(1)$.

Cloning Paradox

- The Strong subadditivity paradox is closely related to the cloning paradox.
- Quantization on nice slices seems to lead to a "cloning" paradox.



Black Hole Complementarity



- To avoid quantum cloning in Hawking evaporation, we require a version of black hole complementarity

$$\phi(x_{in}) \cong P(\phi(x_2), \phi(x_3), \ldots)$$

Complementarity and Locality

- Our world is approximately local. So,

$$\phi(x_{in}) \cong P(\phi(x_2), \phi(x_3), \ldots)$$

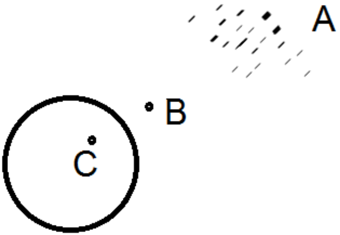
must be consistent with

$$\langle [\phi(x_1), \phi(x_2)] \phi(x_3) \ldots \phi(x_n) \rangle = 0$$

unless n is too large.

- Complementarity ensures no violation of QM; subtle loss of locality instead.

Complementarity and Strong Subadditivity



In the presence of

$$\phi(x_C) \cong P(\phi(x_{A_1}), \phi(x_{A_2}), \ldots)$$

A, B, C are not independent subsystems. So, strong subadditivity inapplicable.

[K.P, S.R., 2013–15]

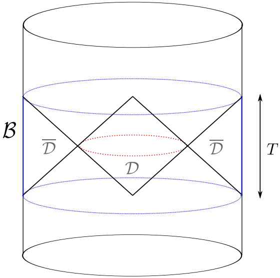
Outline

The idea of complementarity can be realized precisely in empty anti-de Sitter space.

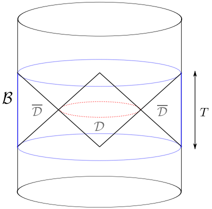
[S. Banerjee, J.W. Bryan, K. Papadodimas, S. R. ,2016]

Complementarity in empty AdS

Operator at center of AdS can be written as complicated polynomial of operators that are uniformly spatially separated!



Empty AdS Complementarity



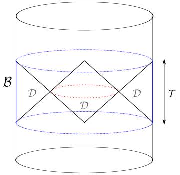
$$\phi(\mathcal{D}) = \sum_{n,m} c_{nm} |n\rangle \langle m|$$

Use version of the Reeh-Schlieder theorem to write

$$|n\rangle = X_n[\phi(\bar{\mathcal{D}})]|0\rangle$$

where X is a simple polynomial.

Empty AdS complementarity



- Use complicated polynomials in $\mathcal{B}$ to approximate $P_0 = |0\rangle\langle 0| \approx \mathcal{P}[\phi(\bar{\mathcal{D}})]$.

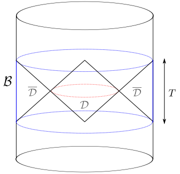
$$\mathcal{P} = \lim_{\alpha\to\infty} e^{-\alpha H} \approx \sum_{n=0}^{n_c} \frac{(\alpha H)^n}{n!}$$

- But H is an operator in $\bar{\mathcal{D}}$. So with

$$\alpha = \log\left(\frac{l_{\text{ads}}}{l_{\text{pl}}}\right); \quad n_c = \frac{l_{\text{ads}}}{l_{\text{pl}}} \log\left(\frac{l_{\text{ads}}}{l_{\text{pl}}}\right)$$

$\mathcal{P}$ is a very complicated polynomial in $\bar{\mathcal{D}}$.

Empty AdS complementarity



Combining the previous formulas we get

$$\boxed{\phi(\mathcal{D}) = \sum_{nm} c_{nm} X_n[\phi(\bar{\mathcal{D}})]\mathcal{P}[\phi(\bar{\mathcal{D}})]X_m^\dagger[\phi(\bar{\mathcal{D}})]}$$

where X_n are explicit simple polynomials, and $\mathcal{P}$ is a complicated polynomial. Explicitly realizes complementarity and also consistent with approximate locality.

Flat-space analogue

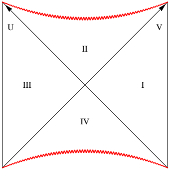
- We do not have an explicit realization of complementarity in flat-space.
- But a strong hint is provided by the breakdown of string perturbation theory for many external particles, even when they have low energy.

  [Sudip Ghosh, S.R., 2016]
- Suggests that locality breaks down if we probe spacetime at too many points.

Outline

A consideration of the black-hole interior in AdS leads to sharper paradoxes also dependent on conflict between unitarity and effective field theory.

Thermofield Doubled State



The eternal black hole is believed to be dual to an entangled state of two CFTs

$$|\Psi_{\rm tfd}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_E e^{-\frac{\beta E}{2}} |E, E\rangle$$

Time Shifted States

- We now consider time-shifted states

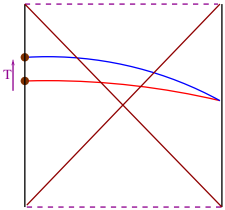
$$|\Psi_{\mathrm{T}}\rangle = e^{iH_L T}|\Psi_{\mathrm{tfd}}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum e^{i\phi_E - \frac{\beta E}{2}} |E, E\rangle$$

- Geometrically, time-translations by the boundary Hamiltonian correspond to large diffeomorphisms.

- $e^{iH_L T} \leftrightarrow$ large diffeomorphisms that die off at the right boundary, but not at the left boundary.

Large Diffeomorphisms

- A naive picture of such a diff may be



- There is no unique diffeomorphism that implements this flow.

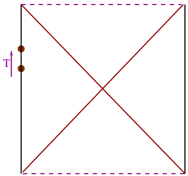
- For example, we can take

$$U \to U\big[\gamma\hat{\theta}(-X) + \hat{\theta}(X)\big];$$
$$V \to \frac{V}{\gamma}\big[\hat{\theta}(-X) + \gamma\hat{\theta}(X)\big],$$

where $\hat{\theta}$ is a smooth version of the theta function and $X = V - U$ and $\gamma = e^{\frac{2\pi T}{\beta}}$.

Smoothness of Time Shifted States

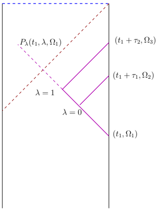
- However, large diffeomorphisms that differ by trivial diffeomorphisms are equivalent. We can even undo the diff everywhere, except infinitesimally close to the boundary.



- This makes it clear that the large diffeomorphism leaves the geometry essentially invariant, but just slides the left boundary.

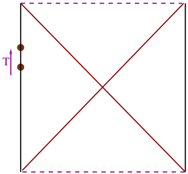
Right Relational Observables

- Consider the following process: jump from the right boundary, wait for a certain proper time, measure the field.



- Right relational observables are invariant under diffeomorphisms that do not reach the right boundary.

Right-relational observables



In any given state

$$|\Psi_{\mathrm{T}}\rangle = e^{iH_L T}|\Psi_{\mathrm{tfd}}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum e^{i\phi_E - \frac{\beta E}{2}} |E, E\rangle$$

we understand that right-relational observables are

$$\widetilde{\mathcal{O}}(t) \sim O_L(-t + T)$$

But this is a state-dependent prescription of an observable.

It turns out that we cannot do better. The correct right-relational observable can only be realized as a “state-dependent” operator.

[K.P, S. R. ,2015]

Semi-Classical Quantization of Time-Shifted States

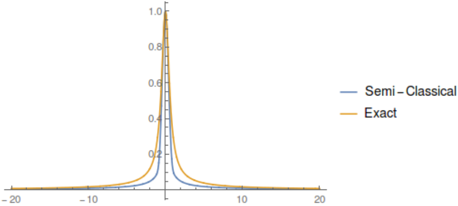
- Each time-shifted state is a solution to the equations of motion — point on the phase space of gravity.
- We understand local degrees of freedom behind the horizon as functions on this part of phase space
- Usually, given a function on phase space $f(z)$ we lift to an operator via

$$\hat{f} = \int f(z) |z\rangle \langle z| dz$$

Fat-tail obstruction to state-independence

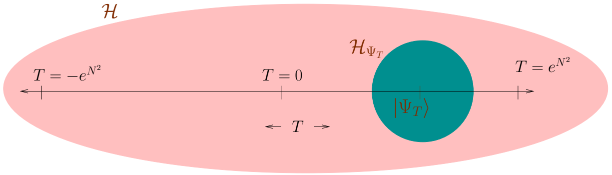
- This relies on
$$|\langle z|z'\rangle|^2 = e^{-|z-z'|^2}$$
allowing the expression above to converge.

- Here
$$|\langle \Psi_{\text{tfd}}|\Psi_{\text{T}}\rangle| = e^{\frac{-CT^2}{2\beta^2}}, \quad T \ll 1$$
$$|\langle \Psi_{\text{tfd}}|\Psi_{\text{T}}\rangle| = \mathrm{O}\left(e^{\frac{-S}{2}}\right), \quad T \gg 1$$



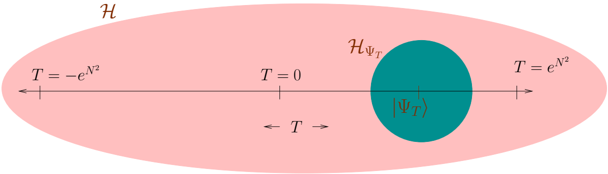
Mirrors defined in patches of Hilbert space

- The states $|\Psi_{\mathrm{T}}\rangle = e^{iH_L T}|\Psi_{\mathrm{tfd}}\rangle$ considered for exponentially long T give a one-parameter geometrization of a large number of microstates.
- But description of interior-operators works patch-wise.



- We can lift functions on this curve in phase-space to operators in a range of T. After exponentially long T, need new operators.

Overcompleteness



- Coherent states are always overcomplete, but state-dependence arises because in the CFT, states dual to well defined geometries are even more overcomplete than one would expect from a semi-classical analysis.

Implications of state-dependence

- Usually, in decoherence theory we ask which observables become classical.
- But in holography, local-observables are emergent.
- Which operators in the full theory of quantum-gravity provide a good classical description may be state-dependent.

Seems to require a fundamental new understanding of measurement.

Outline

Summary

Paradox	Description	Resolution
Naive Information paradox	Hawking radiation is universal	simply reflective of thermalization.
Cloning and strong subadditivity	Information appears outside and inside the black hole simultaneously	Complementarity — well understood in AdS; not so well in flat-space
Black Hole interior in AdS	Interior operators must be state dependent	??

Appendix

Non-perturbative observables

- Consider

$$S(k_1, \ldots k_S)$$

if S is "too large", then even for "small" $|k_n|$, string perturbation theory breaks down.

- Breakdown of gravitational perturbation theory $\Rightarrow$ breakdown of locality.

- So if we probe spacetime at too many points, locality breaks down.

String Perturbation theory limits

- Consider a limit where string-scale and Planck scale are widely separated, number of particles is large, and energy-per-particle is small.

$$g_s^2 = \frac{2\pi l_{pl}^{d-2}}{(2\pi\sqrt{\alpha'})^{d-2}} \to 0$$
$$n \to \infty$$
$$\frac{\log(E\sqrt{\alpha'})}{\log(n)} \to -\gamma \leq 0$$

- String perturbation theory breaks down n that satisfies

$$\frac{\log(g_s)}{\log(n)} = \frac{1}{2}((d-2)\gamma - 1)$$
$$\Rightarrow n \propto g_s^{\frac{2}{(d-2)\gamma-1}}$$

Requires $(d-2)\gamma \leq 1$.

Black Hole Evaporation in d-dimensions

- Recall that in d-dimensions

$$T \propto \left(\frac{M_{\text{pl}}^{d-2}}{M}\right)^{\frac{1}{d-3}} \propto \frac{1}{R_h}$$

$$S \propto \left(M_{\text{pl}} R_h\right)^{d-2}$$

- To observe the cloning-contraction, we need to measure at least some connected S-point correlators in the emitted Hawking radiation.

Breakdown of perturbation theory

- Corresponds to S-matrix elements with S-insertions and typical momentum of order T
- But perturbation theory breaks down for

$$\frac{(2-d)\log\frac{T}{M_{\rm pl}}}{\log(n)} = 1 + \mathrm{O}\left(\frac{1}{\log(n)}\right)$$

Reached precisely at n = S

- So S-point correlators may receive non-perturbative corrections. Suggests a version of complementarity.