Consistent Quantum Histories and the Probability for Singularity Resolution

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In the last decade, rigorous quantization of homogeneous cosmological spacetimes has been performed using techniques of loop quantum gravity. Physical Hilbert space, inner product and observables are known for various loop quantized spacetimes. Singularity resolution has been understood in terms of expectation values of observables such as volume and energy density. How do we extract the probability of singularity resolution? What are the implications for Wheeler-DeWitt quantum universes?

Outline:

- Introduction and Consistent Histories approach
- Wheeler-DeWitt theory and singularities
- Loop quantum cosmology and bounce
- Exactly solvable loop quantum cosmology
- Probabilities in Wheeler-DeWitt and loop quantum cosmologies.
- Towards a covariant generalization
- Summary

Introduction

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How do we assign probabilities in a closed quantum system corresponding to a quantum gravitational spacetime?

Consistent histories formulation: **(Gell-Mann, Griffiths, Hartle, Halliwell, Omnes, ...)**
If predictions are to be extracted from quantum system, then it must satisfy a consistency condition.

Branch wavefunctions of alternative quantum histories must have no interference. When the interference between the histories vanishes, consistent probabilities can be assigned.

Consistent histories approach

Fine grained histories: Most refined descriptions of the Universe. Examples: Individual paths in a path integral formulation; detailed time history of a particle in a two-slit experiment.

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Many physical questions tied to such histories.

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Decoherence functional: Measures the quantum mechanical interference between alternative coarse grained histories and assigns probabilities if interference vanishes.

Sets of histories with negligible interference between all the members of the set decohere. Probabilities can be assigned to these consistent histories.

Histories, class operators and branch wavefunctions

Consider a system with a Hilbert space $\mathcal{H}$, self-adjoint Hamiltonian H and a family of observables A^α with eigenvalues a_k^α.

Propagator: $U(t_i, t_j) = e^{-iH(t_i - t_j)/\hbar}$

Heisenberg projections: $P_{a_k}^\alpha(t) = U^\dagger(t) P_{a_k}^\alpha U(t)$

Sets of alternative histories defined by giving a time sequence of sets of events, such as $\{P_{\alpha_1}^1(t_1)\}...\{P_{\alpha_1}^n(t_n)\}$

Example of a coarse grained history: Observer decides to measure which slit electron went through at t_1, electron passed through lower slit at t_2, and strikes the screen at point y_n at t_3

$$P_{y_n}^3(t_3) P_L^2(t_2) P_{\text{obs}}^1(t_1)$$

An exclusive, exhaustive set of fine-grained histories for the system may be regarded as the set of sequences of eigenvalues (taken at all times t_i)

$$\{h\} = \{(a_{k_1}^{\alpha_1}, a_{k_2}^{\alpha_2}, \ldots, a_{k_n}^{\alpha_n})\}$$

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Each fine-grained history h can be represented by a class operator

$$C_h = P_{a_{k_1}}^{\alpha_1}(t_1) P_{a_{k_2}}^{\alpha_2}(t_2) \cdots P_{a_{k_n}}^{\alpha_n}(t_n) \quad \text{with} \quad \sum_h C_h = 1$$

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Coarse-grained histories correspond to coarse-grainings of the projections at some or all of the times t_i:

$$C_{\overline{h}} = \sum_{a_{k_1} \in \Delta a_{\bar{k}_1}} \sum_{a_{k_2} \in \Delta a_{\bar{k}_2}} \cdots \sum_{a_{k_n} \in \Delta a_{\bar{k}_n}} P^{\alpha_1}_{a_{k_1}}(t_1) P^{\alpha_2}_{a_{k_2}}(t_2) \cdots P^{\alpha_n}_{a_{k_n}}(t_n)$$

(involves a union of alternatives at any time t_i)

Quantum state of the system corresponding to a history h:

$$|\psi_h\rangle \equiv C_h^\dagger|\psi\rangle = P_{a_{k_n}}^{\alpha_n}(t_n)\cdots P_{a_{k_1}}^{\alpha_1}(t_1)|\psi\rangle$$

(Branch wave function)

A pure initial state can be resolved in to branch wave functions:

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Decoherence functional: $d(h,h') = \langle\psi'_h|\psi_h\rangle$

When histories do not interfere, $d(h,h') = 0$ for $h \neq h'$.

Probability for a particular history:

$$p(h) = d(h,h) = \|C_h|\psi\rangle\|^2$$

with $\sum_h p(h) = 1$.

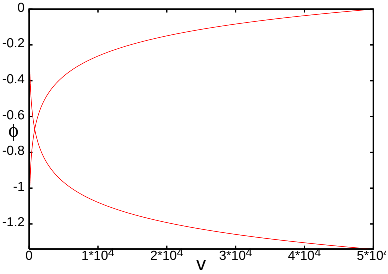
Homoegenous and isotropic universe with a massless scalar

Metric:

$$ds^2 = -N^2 dt^2 + a(t)^2 d\mathbf{x}^2, \qquad a(t) \text{ denotes scale factor}$$

Matter energy density: $\rho = \mathcal{H}_\phi / v$, $\mathcal{H}_\phi = p_\phi^2 / 2v$
(P_ϕ is the field momentum, v is the volume of the universe (a^3))

Einstein's field equations/Hamilton's equations yield:
$p_\phi = \text{constant}, \ \phi \sim \log v, \ \rho \propto a^{-6}$



ϕ is a monotonic function. Acts as a "clock." Measures the way geometry changes with matter. There exist two disjoint solutions: an expanding and a contracting universe (both solutions are singular).

Quantum Cosmological Models

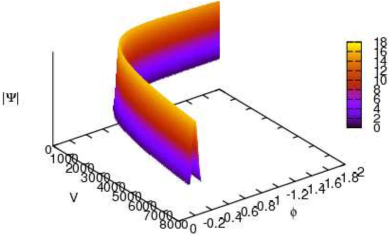
Wheeler-DeWitt framework **(Wheeler, DeWitt 1970's)**:

- Basic variables: $v, p_v \propto \dot{v}$ (geometry), ϕ, p_ϕ (matter).
- Operators: $\hat{v}\,\Psi(v,\phi) = v\,\Psi(v,\phi)$, $\hat{p}_v\,\Psi(v,\phi) = -i\hbar\frac{\partial}{\partial v}\Psi(v,\phi)$
- Hamiltonian constraint $\hat{v}\hat{p}_v\hat{v}\,\hat{p}_v\,\Psi(v,\phi) = \hat{\mathcal{H}}_\phi\,\Psi(v,\phi)$ results in Klein-Gordon type equation:

$$\frac{\partial^2}{\partial\alpha^2}\,\Psi(\alpha,\phi) = \frac{\partial^2}{\partial\phi^2}\,\Psi(\alpha,\phi), \qquad \alpha = \log v$$

- Physical Hilbert space, inner product available.
- Self-adjoint observables: $\hat{v}_{|\phi}, \hat{p}_\phi, \hat{\rho}_{|\phi}$
- To extract departures from general relativity, consider a semi-classical state at late times (present epoch) and evolve backwards towards the big bang. Compute expectation values of observables, and compare with classical theory.

Wheeler-DeWitt states just follow the classical trajectory, all the way to the big bang.



Expectation value of energy density becomes infinite and that of volume goes to zero.

Singularity not resolved.

What is the probability for singularity to occur? Can arbitrary superpositions of contracting/expanding universes avoid big bang?

(Also answered in Struyve's talk using Bohmian formulation)

Loop Quantum Gravity/Cosmology

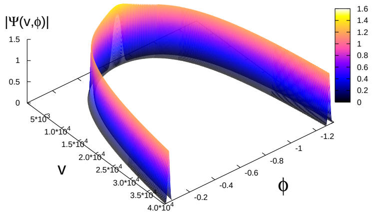
- A non-perturbative quantization of spacetime based on new variables similar to as in electromagnetism (Ashtekar 1986): connection A_a^i (analogous to the vector potential) and the triad E_i^a (analogous to the electric field).
- High mathematical precision (Ashtekar, Baez, Barbero, Bombelli, Corichi, Isham, Gambini, Jacobson, Lewandowski, Marolf, Morau, Pullin, Rovelli, Smolin, Thiemann, Varadarajan ...)
- At a kinematical level classical differential geometry replaced by quantum discrete geometry.

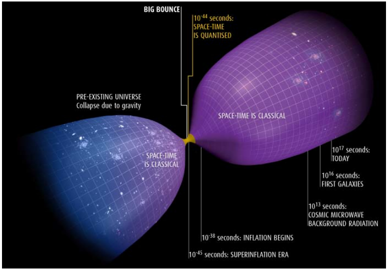
In the last decade, quantization program successfully completed for various cosmological, anisotropic and black hole spacetimes.

Unlike WdW theory, evolution determined by a quantum difference equation. Steps in the difference equation governed by minimum eigenvalue of area λ^2 on quantum geometry (Ashtekar, Pawlowski, PS (2006))

$$\mathcal{C}^+(v)\Psi(v+4,\phi) + \mathcal{C}^0(v)\Psi(v,\phi) + \mathcal{C}^-(v)\Psi(v-4,\phi) = \hat{\mathcal{H}}_\phi \Psi(v,\phi)$$

Quantum Bounce





Feature Story in New Scientist Dec 2008 by A. Ananthaswamy

For states which are sharply peaked at late times, big bang is replaced by a quantum bounce (independent of any fine tuning)

Sharply peaked states bounce at a maximum of energy density $\rho_{\max} = 3/8\pi G\lambda^2 \approx 0.41\rho_{\text{Planck}}$

$\lambda^2 \approx 0.29(G\hbar)^{1/2}$ is the minimum area in quantum geometry
Classical singularity recovered when $\lambda \to 0$.

Robustness of singularity resolution in quantum theory

- Exactly solvable model (flat, isotropic with a massless scalar) **(Ashtekar, Corichi, PS (07))**
- In presence of spatial curvature $k = \pm 1$ **(Ashtekar, Corichi, Kaminski, Karami, Lewandowski, Szulc, Pawlowski, PS, Vandersloot (07-15))**
- Cosmological constant **(Ashtekar, Bentivegna, Kaminski, Pawlowski (07-12))**
- Radiation **(Pawlowski, Pierini, Wilson-Ewing (15))**
- Potentials **(Ashtekar, Pawlowski, PS; Diener, Gupt, Megevand, PS (To appear))**
- Bianchi-I, II and IX spacetimes **(Ashtekar, Corichi, Diener, Joe, Karami, Martin-Benito, Megevand, Mena-Marugan, Montoya, Pawlowski, PS, Wilson-Ewing (09-16))**
- Gowdy models **(de Blas, Garay, Martin-Benito, Mena-Marugan, Olmedo, Pawlowski (09-15))**
- Black holes **(Ashtekar, Boehmer, Bojowald, Corichi, Gambini, Modesto, Pullin, Olmedo, PS, Vandersloot (07-15))**

Exactly solvable LQC

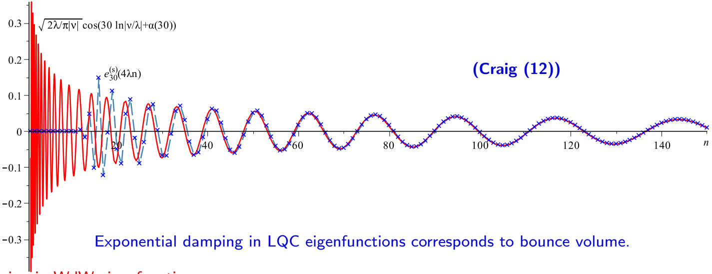
For lapse equal to volume, Hamiltonian constraint in volume rep:

$$\begin{aligned}\partial_\phi^2 \Psi(\nu,\phi) &= \frac{3\pi G}{4\lambda^2}\left[\sqrt{|\nu(\nu+4\lambda)|}|\nu+2\lambda|\Psi(\nu+4\lambda,\phi) - 2\nu^2\Psi(\nu,\phi) + \sqrt{|\nu(\nu-4\lambda)|}|\nu-2\lambda|\Psi(\nu-4\lambda,\phi)\right] \\ &:= -\Theta\Psi(\nu,\phi)\end{aligned}$$

can be written in its conjugate representation as

$$\partial_\phi^2 \Psi(y,\phi) = \partial_y^2 \Psi(y,\phi), \qquad y = (\sqrt{12\pi G})^{-1}\log(\tan(\lambda b/2))$$

Quantum bounce for all the states in the physical Hilbert space. Energy density has a supremum in the Hilbert space which equals $\rho_{\max}$ found in numerical simulations **(Ashtekar, Corichi, PS (07))**



Exponential damping in LQC eigenfunctions corresponds to bounce volume.

No damping in WdW eigenfunctions

Propagator: $U(\phi) = \exp(i\sqrt{\Theta}\phi)$

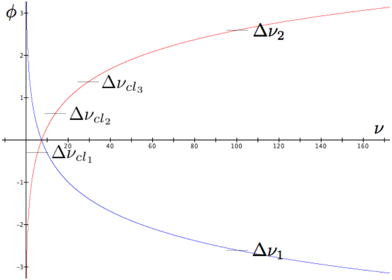
Projection onto range of volumes $\Delta\nu$

$P^{\nu}_{\Delta\nu} = \int_{\nu\in\Delta\nu} d\nu\, |\nu\rangle\langle\nu|$ in WdW

$P^{\nu}_{\Delta\nu} = \sum_{\nu\in\Delta\nu} |\nu\rangle\langle\nu|$ in sLQC

Class operator for history when volume is in range $\Delta\nu$ at ϕ^*

$$C_{\Delta\nu_{|\phi^*}} = U(\phi^* - \phi_0)^\dagger P^{\nu}_{\Delta\nu} U(\phi^* - \phi_0)$$



Class operator describing a coarse grained trajectory for which volume is in $\Delta\nu_1$ at ϕ_1, $\Delta\nu_2$ at ϕ_2, $\Delta\nu_3$ at ϕ_3, and so on is:

$$C_{\Delta\nu_{1|\phi_1};\Delta\nu_{2|\phi_2};\ldots;\Delta\nu_{n|\phi_n}} = P^{\nu}_{\Delta\nu_1}(\phi_1) P^{\nu}_{\Delta\nu_2}(\phi_2)\cdots P^{\nu}_{\Delta\nu_n}(\phi_n)$$

Branch wave function:

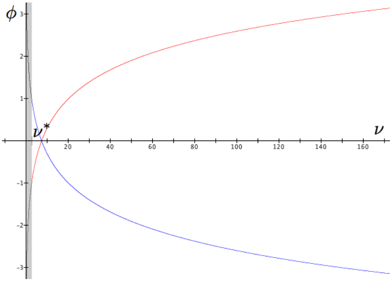
$$|\psi_h(\phi)\rangle = U(\phi - \phi_0) C_h^\dagger |\psi\rangle$$

Decoherence functional:

$$d(\Delta\nu; \Delta\nu') = p_{\Delta\nu}(\phi^*) \delta_{\Delta\nu', \Delta\nu}$$

with probabilities:

$$p_{\Delta\nu}^{\mathrm{WdW}} = \int_{\Delta\nu} d\nu |\Psi^{\mathrm{WdW}}|^2, \qquad p_{\Delta\nu}^{\mathrm{LQC}} = \sum_{\nu \in \Delta\nu} |\Psi|^2$$



What is the probability that the volume of the universe vanishes?

At any given ϕ, partition ν into the range $\Delta\nu^* = [0, \nu^*]$ and its complement $\overline{\Delta\nu^*} = (\nu^*, \infty)$. Calculate probabilities $p_{\Delta\nu^*}$ and $p_{\overline{\Delta\nu^*}}$

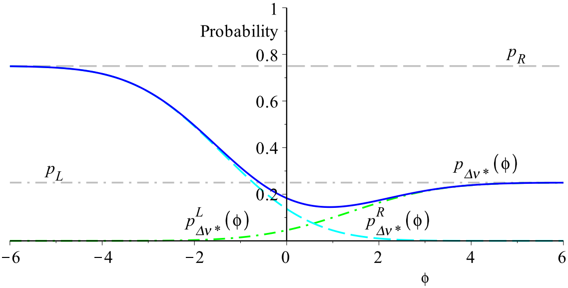
In WdW theory, if wavefunctions are left (contracting) or right (expanding) moving, probability for singularity is unity.

$$\lim_{\phi \to -\infty} p^L_{\Delta\nu^*}(\phi) = 0 \qquad \lim_{\phi \to +\infty} p^L_{\Delta\nu^*}(\phi) = 1$$

$$\lim_{\phi \to -\infty} p^R_{\Delta\nu^*}(\phi) = 1 \qquad \lim_{\phi \to +\infty} p^R_{\Delta\nu^*}(\phi) = 0$$

For an arbitrary superposition of expanding and contracting wavefunctions: $\lim_{\phi \to -\infty} p_{\Delta\nu^*}(\phi) = p_R \quad \lim_{\phi \to +\infty} p_{\Delta\nu^*}(\phi) = p_L$

An example superposition:



Is the probability for bounce non-zero for arbitrary superpositions in WdW theory at a fixed value of ϕ?

Bounce can only be found using at least two time slices.

What is the probability that for an arbitrary superposition, the WdW universe was large in far past and in far future?

Class operator for bounce: $C_{\text{bounce}}(\phi_1, \phi_2) = C_{\overline{\Delta\nu_1^*};\overline{\Delta\nu_2^*}}$

$|\Psi_{\text{bounce}}\rangle = C^\dagger_{\text{bounce}}|\Psi\rangle = 0, \qquad |\Psi_{\text{sing}}\rangle = C^\dagger_{\text{sing}}|\Psi\rangle = |\Psi\rangle$

$p_{\text{sing}} = \langle\Psi_{\text{sing}}\,|\,\Psi_{\text{sing}}\rangle = 1$, $p_{\text{bounce}} = \langle\Psi_{\text{bounce}}\,|\,\Psi_{\text{bounce}}\rangle = 0$

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$$p_{\text{sing}} = \langle\Psi_{\text{sing}}\,|\,\Psi_{\text{sing}}\rangle = 1,\ p_{\text{bounce}} = \langle\Psi_{\text{bounce}}\,|\,\Psi_{\text{bounce}}\rangle = 0$$

In contrast, in sLQC for generic states

$$\lim_{\phi\to-\infty} p_{\Delta\nu^*}(\phi) = 0 \qquad \lim_{\phi\to+\infty} p_{\Delta\nu^*}(\phi) = 0$$
$$\lim_{\phi\to-\infty} p_{\overline{\Delta\nu^*}}(\phi) = 1 \qquad \lim_{\phi\to+\infty} p_{\overline{\Delta\nu^*}}(\phi) = 0$$

$$|\Psi_{\text{bounce}}\rangle = C^\dagger_{\text{bounce}}|\Psi\rangle = |\Psi\rangle, \qquad |\Psi_{\text{sing}}\rangle = C^\dagger_{\text{sing}}|\Psi\rangle = 0$$
$$p_{\text{sing}} = \langle\Psi_{\text{sing}}\,|\,\Psi_{\text{sing}}\rangle = 0,\ p_{\text{bounce}} = \langle\Psi_{\text{bounce}}\,|\,\Psi_{\text{bounce}}\rangle = 1$$

Covariant formulation of sLQC

(Ashtekar, Campiglia, Henderson, Nelson (2009-11); Calcagni, Gielen, Oriti (2011); Craig, PS (To appear))

Path integral in LQC can be understood as a vertex expansion of spin foam models in loop quantum gravity.

The vertex expansion is composed of a discrete sum over M volume transitions. For cosmological dynamics, large M required.

Transition amplitude between $|\nu_i, \phi_i\rangle$ and $|\nu_f, \phi_f\rangle$:

$$G_{\rm H}(\nu_f, \phi_f; \nu_i, \phi_i) = \int_{-\infty}^{\infty} d\alpha\, A_{\rm H}(\Delta\phi; \alpha) A_\Theta(\nu_f, \nu_i; \alpha)$$

where

$$A_{\rm H}(\Delta\phi; \alpha) = \langle \phi_f | e^{i\alpha p_\phi^2} | \phi_i \rangle, \quad A_\Theta(\nu_f, \nu_i; \alpha) = \langle \nu_f | e^{-i\alpha\Theta} | \nu_i \rangle$$

Regroup paths in terms of number of volume transitions and write sum in A_Θ as a vertex expansion. Obtain path amplitudes for $(\nu_f, \nu_{M-1}, \ldots, \nu_1, \nu_i)$.

Class Operators and Consistent Probabilities

For a given family of histories $\{h\}$ associated with M volume transitions, obtain class operators for which $\nu_f \in \Delta\nu_1$ and $\nu_i \in \Delta\nu_2$:

$$C_{\Delta\nu_1;\Delta\nu_2}(\nu_f, \phi_f; \nu_i, \phi_i) = G_{\mathrm{H}}(\nu_f, \phi_f; \nu_i, \phi_i)\delta_{\nu_f,\Delta\nu_1}\delta_{\nu_i,\Delta\nu_2},$$

Class operator corresponding to bounce:

$$C_{\mathrm{bounce}}(\nu_f, \phi_f; \nu_i, \phi_i) \;=\; C_{\overline{\Delta\nu^*};\overline{\Delta\nu^*}}$$

- $\Psi_{\mathrm{bounce}} = C_{\mathrm{bounce}}(\nu_f, \phi_f; \nu_i, \phi_i)\Psi(\nu_i, \phi_i)$
- $p_{\mathrm{bounce}} = d(\mathrm{bounce}, \mathrm{bounce}) = \langle \Psi_{\mathrm{bounce}} | \Psi_{\mathrm{bounce}} \rangle = 1$
- Similarly, $p_{\mathrm{sing}} = d(\mathrm{sing}, \mathrm{sing}) = \langle \Psi_{\mathrm{sing}} | \Psi_{\mathrm{sing}} \rangle = 0$

Summary

- Though many simple but non-trivial models in LQC have led to singularity resolution in terms of expectation values of observables, little was known till recently on the issue of probabilities for singularities and bounces.
- Exactly solvable LQC provides a stage to successfully implement consistent histories framework and compute consistent probabilities for singularities and bounces. (Also for Bohmian formulation: Struyve's talk)
- In the WdW theory, for quantization of FRW spacetime with a massless scalar, probability that bounce ever occured turns out to be zero for arbitrary superpositions.
- In sLQC, the probability for bounce to occur is unity for arbitrary states.
- These results can be generalized to a covariant description using spinfoam techniques.