Non-linear Quantum Mechanics and de Broglie double solution program

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Content

1. **SOME HISTORY**
   - Poincaré pressure
   - de Broglie's double solution
   - semi-classical gravity, Schrödinger-Newton equation: a non-linear modification of Schrödinger equation.
2. **RECENT RESULTS: Factorization ansatz**
   - Factorization ansatz applied to quantum objects/elementary particles.
   - applied to "walkers"[1] phenomenology.

[1] Bouncing oil droplets: Y. Couder and E. Fort. Single-Particle Diffraction and Interference at a Macroscopic Scale. *Phys. Rev. Lett.*, 97:15410, 2006.

Introduction

- Poincaré [2]: droplet model for the electron; introduces a self-focusing interaction, negative pressure, aimed at preventing electronic density from spreading.
- 1909: letter to Lorentz by Einstein (about the photon):
  ... the essential thing seems to me to be not the assumption of singular points but the assumption of field equations of a kind that permit solutions in which finite quantities of energy propagate with velocity c in a specific direction without dispersion...

[2] Henri Poincare. Sur la dynamique de l'electron. 1906. Rendiconti del Circolo matematico di Palermo 21: 129-176

Recurrent problem in field theory

In particular in quantum Physics (due to Wave-Particle duality).

- waves spread but particles are sharply localized
- Born 1926: probabilistic interpretation
- 1927: de Broglie's alternative interpretation: double solution program[3]:

  1. only waves

  2. particles: either singularities or "humps" of which dispersion gets compensated by non-linearity (SOLITONS: subject of today's talk.).

[3] G. Bacciagaluppi and A. Valentini. Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference. Cambridge University Press, Cambridge, 2010.

Remark 1

- de Broglie's double solution is WAVE MONIST (no wave-particle duality)
- BUT: there are two waves
- Ψ_L the pilot-wave
- ϕ_{NL} the (peaked) soliton

Remark 2

- Since 1927, there were a few attempts to consider non-linear generalisations of Schrödinger equation: Bialynicki-Birula, Bohm-Bub, Weinberg, Kibble, Fargue (de Broglie school), Goldin and so on.
- More recently: Schrödinger-Newton equation (Lieb, Penrose, Diosi, Carlip, Tod, Moroz, Giulini, Grossart, Jones, van Meter and many others...)[4]

[4] see S. Colin, T. Durt, and R. Willox. Would a quantum particle succumb to its own gravitational attraction? Class. Quantum Grav., 31:245003, 2014. and references therein.

Schrödinger-Newton equation

- 60's Möller and Rosenfeld: matter is quantum, but space-time is classical[5]
- single particle case:

$$i\hbar\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = -\hbar^2\frac{\Delta\Psi(t,\mathbf{x})}{2m} - Gm^2\int d^3x'\frac{|\Psi(t,\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}\Psi(t,\mathbf{x}), \quad (1)$$

- Lieb[6] shows the existence of a unique (up to translations) ground state.
- numerical studies establish that when the L2 norm of the solution is unity, the size of the ground state is of the order of $\hbar^2/G\cdot M^3$

[5] R. Penrose: "On the Gravitization of Quantum Mechanics 1: Quantum State Reduction", Foundations of Physics, 2014, Vol. 44, Issue 5

[6] E.H. Lieb. Existence and Uniqueness of the Minimizing Solution of Choquard's Nonlinear Equation. Studies in Applied Mathematics, 57:93-105, 1977.

Quantum-Classical transition: Diosi-Penrose (80's)

- the S-N ground state can be considered as a SELF-COLLAPSED GROUND STATE (spontaneously localized).
- $\hbar^2/G \cdot M^3$ goes to zero when M goes to infinity (Classical limit: material point)
- $\hbar^2/G \cdot M^3$ goes to infinity when M goes to zero (Quantum limit: delocalized object)
- Quantum-Classical transition[7]: consider a sphere of homogenous density (normal density) and impose $\hbar^2/G \cdot M^3$=R=radius of the sphere.
- One finds: $(\frac{\hbar^2}{G \cdot (\frac{4\pi}{3}\rho R)^3})^3 = R$
- Solution: R $\approx$ 100 nanometer.

Conclusion: we expect that around and beyond 100 nanometer, effects of <u>self-gravity begin to be important</u>

[7] L. Diosi. Gravitation and quantum-mechanical localization of macro-objects. Phys. Lett. A, 105:199, 1984.

OPEN PROBLEMS WITH S-N EQUATION

1. NO EXPERIMENTAL CONFIRMATION OR FALSIFICATION. WHY?

- self-gravity is very weak, and likely to get dominated by decoherence[8] Let us define the critical parameter $\Lambda_{crit.}$ through $\Lambda_{crit.} = \frac{G^4 M^{11}}{\hbar^7}$ in the "quantum" regime $\sqrt{< r_{BS}^2 >} \gg 2R$, and $\Lambda_{crit.} = \frac{GM^2 R^{-3}}{\hbar}$ in the "classical" regime $\sqrt{< r_{BS}^2 >} \ll 2R$.
- with $\sqrt{< r_{BS}^2 >} = \hbar^2 / GM^3$ in the "quantum" regime and $\sqrt{< r_{BS}^2 >} = (\frac{\hbar^2}{GM^3})^{\frac{1}{4}} R^{\frac{3}{4}}$ in the "classical" regime (Diosi);
- At the mesoscopic transition $\hbar^2 / GM^3$=R and $\frac{G^4 M^{11}}{\hbar^7} = \frac{GM^2 R^{-3}}{\hbar}$.
- **If $\Lambda_{decoherence} > \Lambda_{crit.}$ then decoherence dominates self-gravity and no experiment is conclusive.**

[8] S. Colin, T. Durt, and R. Willox. Crucial tests of macrorealist and semi-classical gravity models with freely falling mesoscopic nanospheres. Phys. Rev. A, 93, 062102, 2016.

OPEN PROBLEMS WITH S-N EQUATION.

- Decoherence also "kills" quantum superpositions (linear Schrödinger equation) when $\Lambda_{decoherence}$ is too large.
- $\Lambda_{decoherence}$=rate of collisons with environmental photons, atoms and so on per unit of time divided by square of their size.
- It is necessary to realize extreme vacuum and temperature conditions-difficult...

Last but not least self-gravity is weaker than van der Waals interactions so that electrical neutrality must be ensured and optical trapping is required. Now, in optical traps, the size of the ground state is of the order of 10^{-11} meter at the mesoscopic transition; but then the size of the wave function is quite smaller than $\hbar^2/GM^3 \approx R \approx 100$ nanometer, so that kinetic energy is quite larger than self-gravitational energy[9] and self-gravity gets masked/dominated by the kinetic energy.

[9] van Meter criterion: for gaussian states, self-collapse occurs if $\sqrt{<r>^2 - <r>^2} \geq 1.48\hbar^2/GM^3$, J. R. van Meter, Schrödinger-Newton "collapse" of the wave-function.Class. and Quant. Grav. 28:215013, 2011.

2. NO-GO THEOREMS.

A. DERRICK NO-GO THEOREM[10].

- Concerns the stability of self-collapsed solutions.

B. GISIN NO-GO THEOREM[11].

- Concerns the link between non-linearity and non-locality (supraluminal communication/signaling).

[10] G.H. Derrick. Comments on Nonlinear Wave Equations as Models for Elementary Particles. J. Math. Phys., 5(9):1252-1254, 1964.

[11] N. Gisin. Weinberg's non-linear quantum mechanics and superluminal communications. Phys. Lett. A, 143(1,2):1-2, 1990.

2. NO-GO THEOREMS.

A. DERRICK NO-GO THEOREM.

- Concerns a very large class of non-linear Schrödinger equations.
- It is possible by rescaling to generate states with energy lower than the energy of the ground state.
- Flaw in the argument: requires to increase the L2 norm of the state; contradicts the norm-preserving nature of Newton-Schrödinger equation.

OPEN PROBLEMS WITH S-N EQUATION

2. NO-GO THEOREMS.

B. GISIN NO-GO THEOREM.

- Concerns the link between non-linearity and non-locality (supraluminal communication/signaling).
- Roughly summarized, Gisin's argument goes as follows: non-linear corrections to the linear Schrödinger equation make it possible, in principle, to distinguish different realizations of the same density matrix. By performing a local measurement on a system A that is entangled with a distant system B, one is able, by collapsing the full wave function, to obtain realizations of the reduced density matrix of the system B which differ according to the choice of the measurement basis made in the region A. Therefore, in principle, non-linearity can be a tool for sending classical information faster than light, contradicting the no-signaling property valid in the framework of linear quantum mechanics.

OPEN PROBLEMS WITH S-N EQUATION

2. NO-GO THEOREMS B. CIRCUMVENTING GISIN NO-GO THEOREM. One way to circumvent Gisin's theorem is to add stochasticity to the non-linearity in order that Born's rule is satisfied at the end.

Examples:

- Bohm-Bub theory-1966 (T. Durt. About the possibility of supraluminal transmission of information in the Bohm-Bub theory. Helv. Phys. Acta, 72:356-376, 1999.)
- GRW-CSL , Gisin-Percival (see A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht. Models of wave-function collapse, underlying theories, and experimental tests. Rev. Mod. Phys., 85:471-527, 2013. and references therein).
- de Broglie-Bohm dynamics at equilibrium (see A. Valentini. Signal locality, uncertainty and the subquantum H-theorem. I. Phys. Lett. A, 156:5-11, 1992.).
- Copenhagen with collapse (see N. Gisin. Stochastic Quantum Dynamics and Relativity. Helv. Phys. Acta, 62(4):363-371, 1989.).

2. NO-GO THEOREMS B. CIRCUMVENTING GISIN NO-GO THEOREM.

PROBLEM: N-S equation is deterministic. THIS JUSTIFIES THE QUESTION:
CAN ONE DERIVE BORN RULE FROM S-N EQUATION?
ANSWER: WE DO NOT KNOW BUT IT IS NOT IMPOSSIBLE.

- We know from chaos theory that deterministic systems behave stochastically.
- As we shall show, there exist connections between de Broglie-Bohm trajectories and S-N equation.
  Now, de Broglie-Bohm trajectories seemingly admit the Born distribution as equilibrium distribution (Colin, Efthymiopoulos, Struyve, Westman, Valentini and others).
- Other possibility: add stochastic noise to the evolution in order to ensure Q equilibrium. [12]

[12] D. Bohm and J.P. Vigier. Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations. Phys. Rev., 96(1):208-216, 1954.

A. Factorization ansatz applied to quantum objects/elementary particles.

- We try to solve the Schrödinger-Newton (S-N) equation in presence of external forces
$i\hbar\frac{\partial\Psi(t,\mathbf{x})}{\partial t} =$
$-\hbar^2\frac{\Delta\Psi(t,\mathbf{x})}{2m} + V^L(t,\mathbf{x})\Psi(t,\mathbf{x}) - Gm^2\int d^3x'(\frac{|\Psi(t,\mathbf{x}')|^2}{|\mathbf{x}-x'|})\Psi(t,\mathbf{x})$,
with an ansatz solution such that
Ψ factorizes (2) into the product of two functions Ψ_L and ϕ_{NL}:

$$\Psi(t,\mathbf{x}) = \Psi_L(t,\mathbf{x})\cdot\phi_{NL}(t,\mathbf{x}), \tag{2}$$

where Ψ_L, the pilot wave, is in good approximation a solution of the linear Schrödinger equation:

$$i\hbar\cdot\frac{\partial\Psi_L(t,\mathbf{x})}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi_L(t,\mathbf{x}) + V^L(t,\mathbf{x})\Psi_L(t,\mathbf{x}), \tag{3}$$

- We also do NOT require that the L2 norm of Ψ is equal to 1.

PART 2A. Factorization ansatz: Applied to a pair of quantum objects/elementary particles.

- We consider two quantum objects/particles A and B and represent them by self-collapsed solitons of small size.
- We force them to follow de Broglie-Bohm dynamics[13], and consider the feedback of the de Broglie solitons on the pilot wave.
- We find

$$i\hbar \cdot \frac{\partial \Psi_L(t,\mathbf{x})}{\partial t} + \frac{\hbar^2}{2m}\Delta\Psi_L(t,\mathbf{x}) - V^L(t,\mathbf{x})\Psi_L(t,\mathbf{x})$$
$$= -4\pi\frac{\hbar^2}{2m}\Psi_L(t,\mathbf{x})\cdot(L_A\delta^{Dirac}(\mathbf{x}-\mathbf{x_0^A}) + L_B\delta^{Dirac}(\mathbf{x}-\mathbf{x_0^B})), \qquad (4)$$

with L_A (L_B) the sizes of the A(B) particles.

- We look for a factorisable solution in the form $\Psi = \Psi_L^{hom}(t,\mathbf{x}) + \Psi_L^{inhom}(t,\mathbf{x})$ with $\Psi_L^{inhom}(t,\mathbf{x}) = \Psi_L^{hom}(t,\mathbf{x})(-\phi^G(t,\mathbf{x})/c^2)$

[13] T. Durt Generalized guidance equation for peaked quantum solitons and effective gravity. Euro. Phys. Lett., 114, nr 1, 2016.

PART 2A. Factorization ansatz:Applied to a pair of quantum objects/elementary particles

- Then ϕ^G obeys the effective Poisson equation (of quantum origin)

$$\Delta(\phi^G(t,\mathbf{x})/c^2) = 4\pi \cdot (L_A\delta^{Dirac}(\mathbf{x}-\mathbf{x_0^A}) + L_B\delta^{Dirac}(\mathbf{x}-\mathbf{x_0^B})) \tag{5}$$

Making use of the well-known properties of the Green functions associated to the Laplace equation, it is easy to check that the solution of (5) is

$$\phi^G(t,\mathbf{x})/c^2 = -\left(\frac{L_A}{|\mathbf{x}-\mathbf{x_0^A}|} + \frac{L_B}{|\mathbf{x}-\mathbf{x_0^B}|}\right). \tag{6}$$

- At this level, an effective gravitational interaction between A and B emerges which satisfies $\phi^G_{A(B)}(t,\mathbf{x})/c^2 = -(\frac{L_{B(A)}}{|\mathbf{x_0^{B(A)}}-\mathbf{x_0^{A(B)}}|})$.

PART 2A. Factorization ansatz:Applied to a pair of quantum objects/elementary particles

- Conclusion, if we consider two quantum objects/particles A and B.
- An effective gravitational interaction between A and B emerges which satisfies $\phi^G_{A(B)}(t,\mathbf{x})/c^2 = -(\frac{L_{B(A)}}{|\mathbf{x_0^{B(A)}}-\mathbf{x_0^{A(B)}}|})$.
- Requiring the equivalence of this effective potential with the Newtonian gravitational potential $\phi^{Newton} = -\frac{G^{Newton} m_A m_B}{|\mathbf{x_0^{B(A)}}-\mathbf{x_0^{A(B)}}|}$ imposes the constraint:

$$L_{A(B)} = \frac{G^{Newton} \cdot m_{A(B)}}{2c^2}, \qquad (7)$$

which means that particles are associated to solitons of size of the order of their Schwarzschild radius ("mini black holes").

PART 2A. Factorization ansatz:Applied to a pair of quantum objects/elementary particles

REMARK 1:

- $$L = \frac{G^{Newton} \cdot m}{2c^2}, \tag{8}$$

  is quite smaller than the Lieb radius $\hbar^2/Gm^3$
- Example, for an electron, $\frac{G^{Newton} \cdot m}{2c^2} \approx 10^{-57}$ meter, while $\hbar^2/Gm^3 \approx 10^{+32}$ meter.
- Reason: we do not impose that the L2 norm of Ψ is 1, and rescale the ground state accordingly.

PART 2A. Factorization ansatz:Applied to a pair of quantum objects/elementary particles

REMARK 2:

- A virial-like relation can be derived in the case of S-N equation: Kinetic energy=-2 times Potential energy
- therefore the full energy is of the order of $-mc^2$.
- To destabilize the soliton requires[14] an energy of the order of $+mc^2$.
- In our approach, particles are thus self-collapsed since they were created (like in de Broglie-Bohm interpretation).

[14] Stability criterion of Ruiz-Arriola and Soler: E.Ruiz-Arriola an J.Soler, J. Stat. Phys.103, 1069, 2001.

QUESTION:

Can we derive de Broglie-Bohm dynamics from our factorization ansatz?
Answer: We find a modified dB-B dynamics[15]:

- when the soliton size is small enough, its velocity obeys the generalized guidance equation

$$\mathbf{v}_{drift} = \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x_0}(t), t) + \frac{< \phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL} >}{< \phi_{NL}|\phi_{NL} >} = \mathbf{v}_{dB-B} + \mathbf{v}_{int.}, \quad (9)$$

  which contains the well-known Madelung-de Broglie-Bohm contribution ($\mathbf{v}_{dB-B} = \frac{\hbar}{m}\nabla\varphi_L(\mathbf{x_0}(t), t)$) plus a new contribution due to the internal structure of the soliton ($\mathbf{v}_{int.} = \frac{<\phi_{NL}|\frac{\hbar}{im}\nabla|\phi_{NL}>}{<\phi_{NL}|\phi_{NL}>}$).
- Recent numerical simulations show that $\mathbf{v}_{int.}$ may not be neglected in general.

[15] T. Durt,quant-ph arxiv 2016+ "de Broglie double solution and gravitation", to be published soon in the annales de la fondation de Broglie-special issue 2017.

PART 2A. Factorization ansatz:Applied to a pair of quantum objects/elementary particles

CONCLUSION:

- We have to conjecture here that in the practice some stochasticity is present so that averaging over this stochastic contribution we expect thus (and this is our main conjecture):
  $<< \mathbf{v}_{int.} >>= 0$ and thus $<< \mathbf{v}_{drift} >>=<< \mathbf{v}_{dB-B} >>$,
  where the bracket $<<, >>$ represents an averaging over this extra-stochastic preturbation of the velocities that we conjecture to be present in nature.

Remark:

- In the past Bohm, Vigier and de Broglie suspected the existence of a stochastic noise superposed to the quantum potential, necessary in their eyes in order to explain the irreversible in time convergence to quantum equilibrium.

PART 2A. Factorization ansatz:Applied to a pair of quantum objects/elementary particles

de Broglie wrote for instance[16] the following sentences
...Finally, the particle's motion is the combination of a regular motion defined by the guidance formula, with a random motion of Brownian character... any particle, even isolated, has to be imagined as in continuous "energetic contact" with a hidden medium, which constitutes a concealed thermostat. This hypothesis was brought forward some fifteen years ago by Bohm and Vigier, who named this invisible thermostat the "subquantum medium"....If a hidden sub-quantum medium is assumed, knowledge of its nature would seem desirable...
Our conjecture is a re-expression of the subquantum medium hypothesis invoked by Bohm and Vigier in 1954.

[16] L. de Broglie. Interpretation of quantum mechanics by the double solution theory Annales de la Fondation Louis de Broglie, 12, 4, 1987, (chapter XI: On the necessary introduction of a random element in the double solution theory. The hidden thermostat and the Brownian motion of the particle in its wave).

PART 2B. Factorization ansatz:Applied to walkers (bouncing oil droplets)

- Studies concerning de Broglie-Bohm (dB-B) trajectories regained interest since they were realized in the lab. with artificial macroscopic systems, the bouncing oil droplets or walkers, which were shown experimentally to follow dB-B like quantum trajectories.
- e.g., when the walker passes through one slit of a two-slit device, it undergoes the influence of its "pilot-wave" passing through the other slit, in such a way that, after averaging over many dB-B like trajectories, the interference pattern typical of a double-slit experiment is restored, despite of the fact that each walker passes through only one slit.

PART 2B. Factorization ansatz:Applied to walkers (bouncing oil droplets)

- Pseudo-gravitational interaction has also been reported between two droplets.
- For instance, in the paper Walking droplets, a form of wave-particle duality at macroscopic scale[17], we can read: *...We find that, depending on the value of d*, (d represents here the impact parameter of the collision)*the interaction is either repulsive or attractive. When repulsive, the drops follow two approximately hyperbolic trajectories. When attractive, there is usually a mutual capture of the two walkers into an orbital motion similar to that of twin stars ...*

[17] Y. Couder, A. Boudaoud, S. Protière and E. Fort, Europhysics News, Vol. 41, No. 1, 14-18 2010

PART 2B. Factorization ansatz:Applied to walkers (bouncing oil droplets)

- Let us identify the droplets with the self focused solitons of our model ϕ_{NL} and the liquid on which they are floating with the pilot wave Ψ_L, which implies that the state of the system is represented by $\Psi(\mathbf{x}, t) = \Psi_L(\mathbf{x}, t) \cdot (\frac{\phi_{NL}^{0(A)}(\mathbf{x},t)}{A_L(\mathbf{x_0^A},t)} + \frac{\phi_{NL}^{0(B)}(\mathbf{x},t)}{A_L(\mathbf{x_0^B},t)})$, where A and B refer to the presence of two droplets, and A_L is the amplitude of the linear wave.

PART 2B. Factorization ansatz:Applied to walkers (bouncing oil droplets)

- We now describe wave propagation in the container by the d' Alembert equation

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \Delta)\Psi_L = 0, \tag{10}$$

with $v = \lambda_0 \cdot f_0$ where v is the velocity of sound propagation in the medium, and the label 0 refers to the forcing. f_0 is the forcing frequency imposed to the bath by the shaker.

- When two droplets are present we must solve an inhomogeneous d' Alembert equation with a source term $4\pi(L_A \cdot \delta(\mathbf{x} - \mathbf{x_0^A}) + L_B \cdot \delta(\mathbf{x} - \mathbf{x_0^B}))$.

PART 2B. Factorization ansatz:Applied to walkers (bouncing oil droplets)

- Repeating the same process as for the inhomogeneous Schrödinger equation, we predict the appearance of a pseudo-gravitational interaction ϕ^{PG} which obeys

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \Delta)\phi^{PG} = -4\pi(L_A \cdot \delta(\mathbf{x} - \mathbf{x_0^A}) + L_B \cdot \delta(\mathbf{x} - \mathbf{x_0^B})).$$

PART 2B. Factorization ansatz:Applied to walkers (bouncing oil droplets)

- Due to the forcing, we must replace d' Alembert equation by Helmholtz equation

$$(\frac{(2\pi)^2 f_0^2}{v^2} + \Delta)\phi^{PG} = (k_0^2 + \Delta)\phi^{PG} = 4\pi(L_A \cdot \delta(\mathbf{x} - \mathbf{x_0^A}) + L_B \cdot \delta(\mathbf{x} - \mathbf{x_0^B})), \tag{11}$$

  where $k_0 \equiv 2\pi/\lambda_0$.

- The associated Green function is not $1/r$ but $cos(kr)/r$ which leads to the appearance of attractive and repulsive gravitational zones periodically distributed among the punctual sources, IN AGREEMENT WITH EXPERIMENTAL OBSERVATIONS.

CONCLUSIONS:

- WE STUDIED SELF-FOCUSING NON-LINEAR GENERALISATIONS OF SCHROEDINGER EQUATION IN THE LIGHT OF DE BROGLIE DOUBLE SOLUTION PROGRAM;
- WE APPLIED A FACTORIZATION ANSATZ ACCORDING TO WHICH SOLUTIONS CONSIST OF THE PRODUCT OF A PILOT WAVE AND A SOLITON (PARTICLE)
- IN THE CASE OF QUANTUM SYSTEMS THE FEEDBACK ON THE PILOT WAVE RESULTS IN AN EFFECTIVE GRAVITATION
- IN ORDER TO DERIVE DE BROGLIE TRAJECTORIES FROM THE INTERACTION WITH THE PILOT WAVE, A STOCHASTIC DISTURBANCE (QUANTUM BROWNIAN MOTION) IS NECESSARY
- IN THE CASE OF DROPLETS, WHERE A BROWNIAN MOTION IS ALREADY PRESENT, A GOOD AGREEMENT WITH PHENOMENOLOGY IS REACHED