Covariant Observables in Causal Set Quantum Gravity

Sumati Surya

Raman Research Institute

Fundamental Problems of Quantum Physics 2016

Outline

- Covariant Observables in Quantum Cosmology
- Sequential Growth Dynamics in Causal Set Cosmology
- The Quantum Measure Formulation
  - The Histories Hilbert Space
  - Principle of Preclusion
- Quantum Cosmology with Complex Percolation Dynamics:
  - Covariant Observables
  - Examples of Precluded Events

–Work in collaboration with Rafael Sorkin

Twin Horns of Quantum Cosmology

- Covariance and the "Problem of Time"
  - Moment of Time functions: ~~Spatial Distance~~, ~~Spatial Area~~, ~~Spatial Volume~~

    Non Covariant!
  - Spacetime Functions: Spacetime Volume, Proper Time, Causal Order
  - Covariant observables are teleological: Blackholes, Cosmological Horizons..

Histories Formulation

Twin Horns of Quantum Cosmology

- Dilemma of Quantum Cosmology:

  Cosmologists, even more than laboratory physicists, must find the usual interpretive rules of quantum mechanics a bit frustrating.

  When the system is the whole world (universe) where is the measurer to be found?

  What exactly qualifies some subsystems to play this role?

  Was the world wave function waiting to jump for thousands of millions of years .... for some .. highly qualified measurer ... with a PhD?

  Is there ever then a moment when no jumping and the Schrodinger equation applies?

  – J.S Bell, 'Quantum Mechanics for Cosmologists', 1981

Quantum Measure Formulation

Twin Horns of Quantum Cosmology

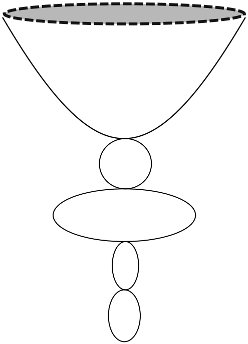
- Problem of Time – Histories Formulation
- Measurement Problem – Quantum Measure Formulation

Candidate Covariant Observables in Quantum Cosmology

- Covariant observables require a *fully-evolved* spacetime.
- But we have access only to limited spacetime regions.
- *"Acorn's intrinsic telos is to become a fully grown oak tree"* — Aristotle
- What are the covariant acorns of quantum cosmology?

Candidate Covariant Observables in Quantum Cosmology

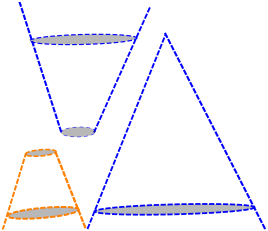
- Cyclic universe:



  - Number of Epochs
  - Spacetime Volume of Each Epoch

Candidate Covariant Observables in Quantum Cosmology

- Observables as Past Sets or Future Sets

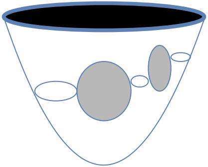


  - $S = J^{-}(S)$, $T = J^{+}(T)$
  - $(N, g) \subset (M, g)$ such that $N = J^{-}(N)$
  - Observable is the set of $(M, g) \supset (N, g)$
  - Examples: TIFS and TIPS

Candidate Covariant Observables in Quantum Cosmology

- Hartle-Hawking Wavefunction: Is there an "Initial Condition" Observable?

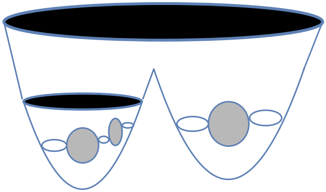
$$\Psi_{HH}(\Sigma, h) = \sum_{M, \partial M = \Sigma} \int_{g|_\Sigma = h} Dg \exp^{-\frac{1}{\hbar} S_E} \quad (1)$$



Candidate Covariant Observables in Quantum Cosmology

- Hartle-Hawking Wavefunction: Is there an "Initial Condition" Observable?

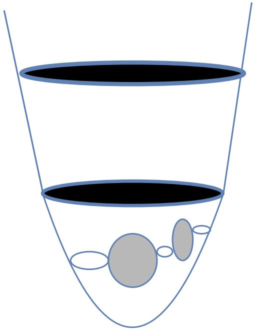
$$\Psi_{HH}(\Sigma, h) = \sum_{M,\partial M=\Sigma} \int_{g|_\Sigma = h} Dg \exp^{-\frac{1}{\hbar}S_E} \tag{1}$$



Candidate Covariant Observables in Quantum Cosmology

- Hartle-Hawking Wavefunction: Is there an "Initial Condition" Observable?

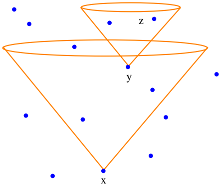
$$\Psi_{HH}(\Sigma, h) = \sum_{M, \partial M = \Sigma} \int_{g|_\Sigma = h} Dg \exp^{-\frac{1}{\hbar} S_E} \tag{1}$$



A teleological initial condition – and hence a covariant observable.

The Causal Set Hypothesis – L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

- The Causal Structure Poset $(M, \prec) \subset (M, g)$

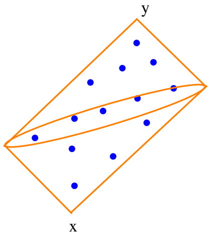


- M: the set of events.
- $\prec$:
  - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
  - Irreflexive: $x \nprec x$
  - Transitive: $x \prec y$ and $y \prec z \Rightarrow x \prec z$

The Causal Set Hypothesis

– L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

- The Causal Structure Poset $(M, \prec) \subset (M, g)$
- Spacetime Discreteness



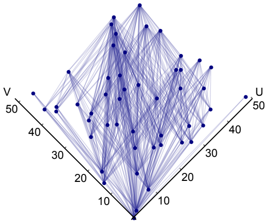
Finite number of "atoms" of spacetime $\sim V/V_p$

The Causal Set Hypothesis

– L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

- The Causal Structure Poset $(M, \prec) \subset (M, g)$
- Spacetime Discreteness

The underlying structure of spacetime is a *causal set* or locally finite poset $(C, \prec)$



The Causal Set Hypothesis

– L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

- The Causal Structure Poset $(M, \prec) \subset (M, g)$

Causal Structure + Volume Element = Spacetime

–Hawking-King-McCarthy-Malament Theorem

Causal Structure $\rightarrow$ Partially Ordered Set

Spacetime Volume $\rightarrow$ Number

Order + Number $\sim$ Spacetime geometry

Causal Set Cosmology

- Classical Sequential Growth Dynamics – D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

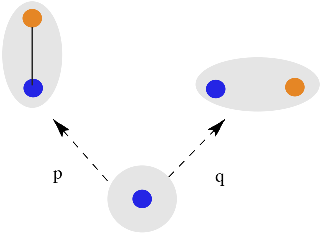
Causal Set Cosmology

- Classical Sequential Growth Dynamics – D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

And then there was one..

Causal Set Cosmology

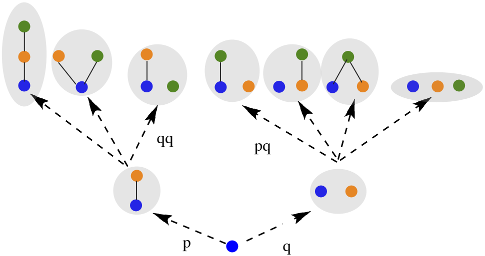
- Classical Sequential Growth Dynamics — D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)



$$p + q = 1$$

Causal Set Cosmology

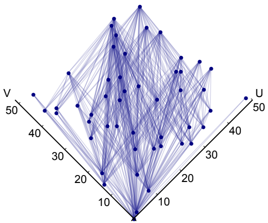
- Classical Sequential Growth Dynamics — D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)



Causal Set Cosmology

- Classical Sequential Growth Dynamics — D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

Until..



Causal Set Cosmology

- Classical Sequential Growth Dynamics – D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

- Example: Transitive percolation
  - p: probability of adding in a link
  - $q = 1 - p$: probability for being unrelated

Causal Set Cosmology

- Classical Sequential Growth Dynamics – D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

- Example: Transitive percolation
  - p: probability of adding in a link
  - $q = 1 - p$: probability for being unrelated

- Principles:
  - General Covariance or Label Independence,
  - Bell-causality condition

Classical Measure Space

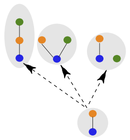
- Sample Space: Ω
  Growth generates *countable (teleological)* labelled causal sets

- Event Algebra: $\mathcal{A}$

  $\alpha \subset \Omega$: collection of histories.

  Events are generated from *cylinder sets* (acorns)

  Example: **cyl**(C_2):

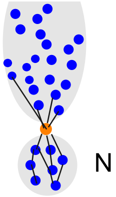


- Measure Space: $(\Omega, \mathcal{A}, \mu)$

- $(\Omega, \mathcal{A}, \mu)$ can be "extended" to infinite time events – Covariant Events

Covariant Observables: Examples

- Post Event:

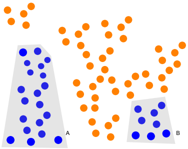


- Stem Events : Covariant Acorns

- Hawking Hartle Event.

Covariant Observables: Examples

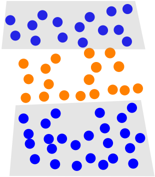
- Post Event:
- Stem Events : Covariant Acorns



- Hawking Hartle Event.

Covariant Observables: Examples

- Post Event:
- Stem Events : Covariant Acorns
- Hawking Hartle Event.



The Quantum Measure

Quantum Dynamics as a Quantum Measure Space: $(\Omega, \mathcal{A}, \mu)$

- $\Omega, \mathcal{A}$: as in classical stochastic theories.
- $\mu : \mathcal{A} \to \mathbb{R}^+$ is non-additive: $\mu(\alpha \sqcup \beta) = \mu(\alpha) + \mu(\beta) + I(\alpha, \beta)$

$$\mu(\alpha_1 \sqcup \alpha_2 \sqcup \alpha_3) = \mu(\alpha_1 \sqcup \alpha_2) + \mu(\alpha_1 \sqcup \alpha_3) + \mu(\alpha_2 \sqcup \alpha_3) - \mu(\alpha_1) - \mu(\alpha_2) - \mu(\alpha_3).$$

Quantum Sum Rule

The Quantum Measure as a Vector Measure

- Histories Hilbert space: $\mathcal{H}$ — H. F. Dowker, S. Johnston and R. Sorkin, J. Phys. A (2010)
  - V: free vector space of functions $f : \mathcal{A} \to \mathbb{C}$.
  - $\langle u, v \rangle_V \equiv \sum_{\alpha \in \mathcal{A}} \sum_{\beta \in \mathcal{A}} u^*(\alpha) v(\beta) D(\alpha, \beta)$
  - $\{u_i\} \sim \{v_i\} \quad \text{if} \quad \lim_{i \to \infty} \| u_i - v_i \|_V = 0,$
  - Hilbert Space $\mathcal{H} = V / \sim, \qquad [\{u_i\}] \in \mathcal{H}$.

- Vector measure: $\mu_V : \mathcal{A} \to \mathcal{H}$. — H. F. Dowker, S. Johnston and S. Surya , J. Phys. A (2010)
  - $\langle \mu_V(\alpha), \mu_V(\beta) \rangle = D(\alpha, \beta)$
  - Finite additivity: $\mu_V(\sqcup_{i=1}^n \alpha_i) = \sum_{i=1}^n \mu_V(\alpha_i)$

Principle of Preclusion

- $|\alpha\rangle = 0$:

  α does not happen

  OR

  Histories in the event $\alpha \subset \Omega$ *do not happen*.

- Eg. $|\mathrm{post}\rangle = 0$:

  There are no causal sets which have posts.

  The universe did not bounce!

- "Prediction" of events that do not happen.

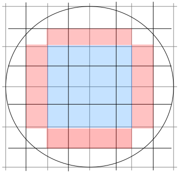
- If you want to do more.. Coevent Interpretation

Complex Percolation

- Complex Percolation:
  - $p, q \in \mathbb{C}$
  - $p + q = 1, \qquad |p| + |q| = 1 + \zeta, \qquad \zeta \geq 0$
  - $\mathcal{H} \sim \mathbb{C}$
- Gives a Quantum Measure Space $(\Omega, \mathcal{A}, \mu)$
- Extension of $(\Omega, \mathcal{A}, \mu)$ to Infinite Time Events (Covariant Events).

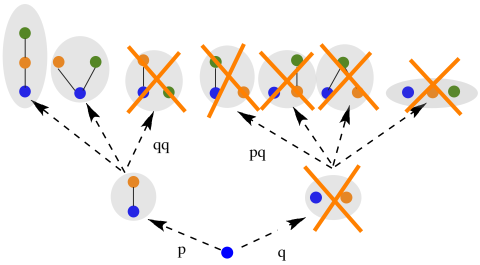
Complex Percolation

- Complex Percolation:
- Gives a Quantum Measure Space $(\Omega, \mathcal{A}, \mu)$
- Extension of $(\Omega, \mathcal{A}, \mu)$ to Infinite Time Events (Covariant Events).
    - Method of Canonical Approximants
    - Approximants: $A_1 \subset A_2 \subset \dots \quad , A_i \subset E$.
    - $A_n \to E$
    - Even convergence: $|A_n\rangle \to |E\rangle$ (ensuring consistency).



An Example: The Originary Event

- ω: there is a single element to the past of all other elements.

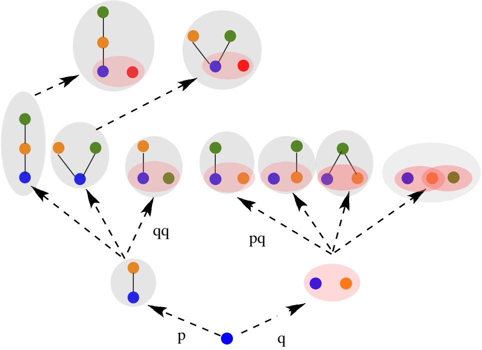


- $|\text{stem}(\bullet \;\; \bullet)\rangle = |\Omega\rangle - |\omega\rangle$

An Example: The Originary Event

- ω: there is a single element to the past of all other elements.

- $|\mathrm{stem}(\bullet\ \ \bullet)\rangle = |\Omega\rangle - |\omega\rangle$



An Example: The Originary Event

- ω: there is a single element to the past of all other elements.

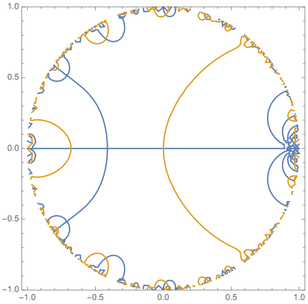
- $|\mathrm{stem}(\bullet\ \ \bullet)\rangle = |\Omega\rangle - |\omega\rangle$

- $|\mathrm{stem}(\bullet\ \ \bullet)\rangle = \lim_n |A_n\rangle = (1 - \varphi(q))|\Omega\rangle, \quad \varphi(q)$ – Euler Function.

- $|\omega\rangle = \varphi(q)|\Omega\rangle$: converges for $|q| < 1$.

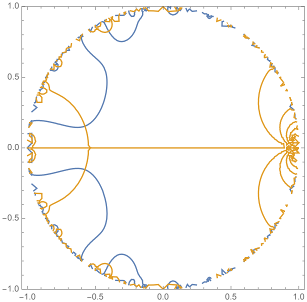
Preclusion of A Stem Event

- $|\omega\rangle = \varphi(q)|\Omega\rangle$
- $\varphi(q) = 0 \Rightarrow q = 1$: Originary event is not precluded for $q \neq 1$.
- $\varphi(q_0) = 1 \;\Rightarrow\; |\omega\rangle = |\Omega\rangle$
- $|\mathrm{stem}(\bullet\;\;\bullet)\rangle = |\Omega\rangle - |\omega\rangle = 0$: $\mathrm{stem}(\bullet\;\;\bullet)$ is a precluded event.



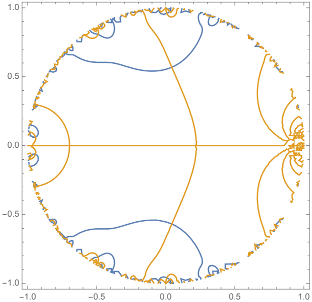
Preclusion of $\overline{\mathrm{post}}_k$

- $|\mathrm{post}_2\rangle = (1-q)\varphi(q)|\Omega\rangle$
- $\exists q_0 \in \mathbb{C}, |\mathrm{post}_2\rangle = |\Omega\rangle$
- $|\overline{\mathrm{post}}_2\rangle = 0$: $\overline{\mathrm{post}}_2$ is precluded.



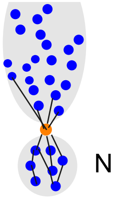
Preclusion of $\overline{\mathrm{post}}_k$

- $|\overline{\mathrm{post}}_3\rangle = q(1-q)^2\varphi(q)|\Omega\rangle$
- $\exists q_0 \in \mathbb{C}, |\mathrm{post}_3\rangle = |\Omega\rangle$:
- $|\overline{\mathrm{post}}_3\rangle = 0$: $\overline{\mathrm{post}}_3$ is precluded.



Predictions without External Measuring Devices

- Can we do more?
- For example, is there a q_0 such that $|\overline{\mathrm{post}}_N\rangle = 0$ for $N = 100$?



- Are Bouncing Universes Precluded for some values of q_0?

Conclusions and Open Questions

- It is possible to construct covariant observables in causal set quantum gravity: $\omega,\ \mathrm{post}_k,\ \mathrm{post},\ \mathrm{fstem},\ \mathrm{hh}$

- $|\omega\rangle,\ |\mathrm{post}\rangle,\ |\mathrm{fstem}\rangle,\ |\mathrm{hh}\rangle$ evenly converge in Complex Percolation .

- Find values of $q \in \mathbb{C}$ for which certain covariant observables are precluded.

- These are "predictions" of the theory, consistent with covariance without recourse to external observers.

Conclusions and Open Questions

- Generalisations to a more generic dynamics.
- Start with covariant events (Stems)
- Bell Causality Condition: No idea what to do in the Quantum Context.