SHARING OF NONLOCALITY BY MORE THAN TWO OBSERVERS

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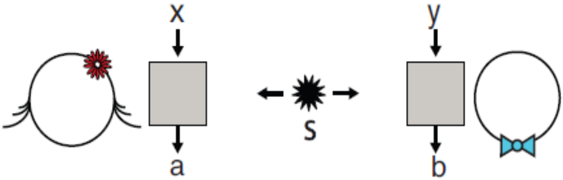
NONLOCALITY AND BELL TEST

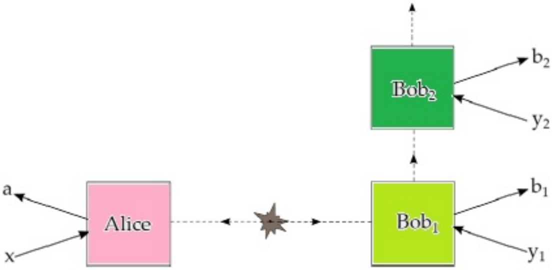


- $x, y \in \{0, 1\}$ and $a, b \in \{-1, 1\}$, $\langle a_x b_y \rangle = \sum abp(ab|xy)$
- CHSH inequality, $S = \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2$.
- According to QT, $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle), x_0 = \hat{x}.\vec{\sigma}, y_0 = \hat{x}.\vec{\sigma}$, $x_1 = -\frac{\hat{e}_1 + \hat{e}_2}{\sqrt{2}}, y_1 = \frac{-\hat{e}_1 + \hat{e}_2}{\sqrt{2}}$, $S = 2\sqrt{2}$.

MONOGAMY OF NONLOCAL CORRELATION

- Let Alice, Bob, and Charlie have two possible dichotomic measurements, represented by observables $\{a_x\}, \{b_y\} and \{c_z\}$ with $x, y, z \in \{0, 1\}$.
- For three qubit state if $S_{ab} > 1$ then $S_{ac} \leq 2$.
- More generally arbitrary tripartite state $S_{ab}^2 + S_{ac}^2 \leq 8$.
- This restriction no longer holds if the non-signaling hypothesis is dropped.

MORE THAN TWO OBSERVERS



- x, y_1, y_2 with $x, y_1, y_2 \in \{0, 1\}$.
- Can the single observer with the first particle observe nonlocal correlations with all of the members in the second group?

MORE THAN TWO OBSERVERS

PRL **114**, 250401 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

Multiple Observers Can Share the Nonlocality of Half of an Entangled Pair by Using Optimal Weak Measurements

Ralph Silva,[1,*] Nicolas Gisin,[2] Yelena Guryanova,[1] and Sandu Popescu[1]

- By weak measurement with optimal pointer state, for spin half system found numerical evidence, no violation for more than two Bobs.
- It was left open how to proof this analytically.

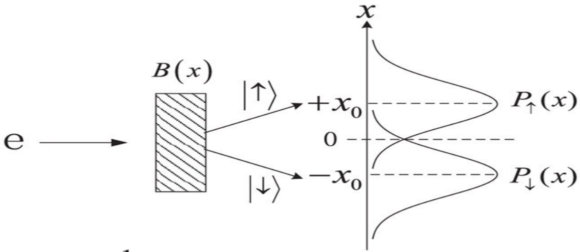
WEAK MEASUREMENT WITHOUT POST SELECTION

- In a von Neumanntype measurement, the pointer is shifted proportional to the eigenvalues of the measured observable.
- $|\psi\rangle|\phi(q)\rangle = \sum\langle a|\psi\rangle|a\rangle \otimes |\phi(q - g_0 a)\rangle$.
- Interaction Hamiltonian $H(t) = g(t)A \otimes p$, where A is the measured observable, p is the momentum operator of the pointer conjugate to q, and $g(t)$ is nonzero only during a short time interval and normalized so that $\int g(t)dt = g_0$.
- In a strong measurement the pointerfs initial state is narrower than the distance between the eigenvalues, i.e., $\langle\phi(q - a)|\phi(q - a'\rangle) = \delta_{a,a'}$; hence, reading the pointerfs position provides full information of the measured physical quantity and collapses the system into the corresponding eigenstate of the observable.

WEAK MEASUREMENT WITHOUT POST SELECTION

- Conversely, if the pointer spread is very large, covering the entire spectrum of eigenvalues, reading the pointer position provides essentially no information since $\langle\phi(q-a)|\phi(q-a'\rangle) = \delta_{a,a'} \approx 1$ and the system is not perturbed.
- $|\psi'\rangle_{q_0} = \sum\langle a|\psi\rangle\langle q_0|\phi(q-a)\rangle|a\rangle \approx \langle q_0|\phi(q)\rangle|\psi\rangle$.

STERN-GERLACH POVM

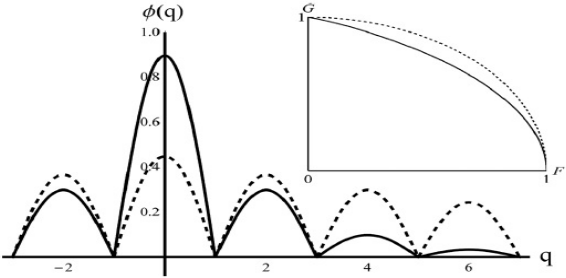


$$P_\sigma(x) = \frac{1}{\sqrt{2\pi D}} \exp[-(x-\bar{x}_\sigma)^2/2D], \sigma = (\uparrow, \downarrow)$$

WEAK MEASUREMENT WITHOUT POST SELECTION

- Now consider measurements in between the two extremes. a spin 1/2 particle with initial state $|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle)$.
- joint state of system and apparatus goes to $\alpha|0\rangle \otimes \phi(q-1) + \beta|1\rangle \otimes \phi(q+1)$.
- On tracing out the pointer state the reduced state of the system is given by $\rho' = F\rho + (1-F)(\pi^+\rho\pi^+ + \pi^-\rho\pi^-)$.
- $F(\phi) = \int_{-\infty}^{\infty} \langle\phi(q+1)|\phi(q-1)\rangle dq$, is called the quality factor of the measurement.
- The probability of getting outcomes '$\pm$' is given by $p(\pm) = G\langle\psi|\pi^{\pm}|\psi\rangle + (1-G)\frac{1}{2}$. Here, $G = \int_{-1}^{1} \phi^2(q) dq$, which quantifies the precision of the measurement.

WEAK MEASUREMENT WITHOUT POST SELECTION



- $F = 0$ and $G = 1$ corresponds to a strong measurement.
- An optimal pointer is defined as, which gives the best trade-off, i.e., for a F, it provides the largest precision G.
- optimal information-disturbance trade-off condition given by $F^2 + G^2 = 1$.

ANALYTIC PROOF

mathematics

MDPI

Article

Sharing of Nonlocality of a Single Member of an Entangled Pair of Qubits Is Not Possible by More than Two Unbiased Observers on the Other Wing

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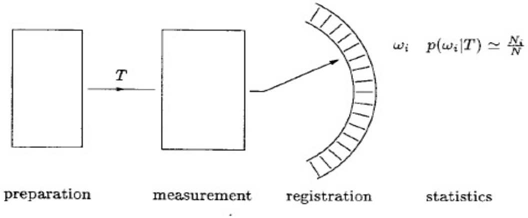
- Casting weak measurement in terms of POVM

POSITIVE OPERATOR VALUED MEASURE

- In quantum theory any measurement is described by POVM $\{E_i\}$, whose special case is projective von Neumann measurement.
- Positivity condition, $E_i \geq 0$ and normalisation, $\sum_i E_i = \mathbb{I}$.
- For state T, the probability functional $T \rightarrow p_T(\omega_i) = tr[TE_i]$.
- Ludder transformation rule upto unitary freedom, $\rho \rightarrow \sqrt{E_i}\rho\sqrt{E_i}/tr[\sqrt{E_i}\rho\sqrt{E_i}]$.
  For projective measurement $\rho \rightarrow P_i\rho P_i/tr[P_i\rho P_i]$.

SCHEMATICS OF MEASUREMENT



- Measurement scheme is $\mathcal{M} := \langle \mathcal{H}_A, \rho_A, \mathcal{U}, Z \rangle$.
- $\mathcal{U}(\rho_S \otimes \rho_A) \mapsto \mathcal{U}\rho_S \otimes \rho_A \mathcal{U}^*$
- State transformation rule is given by $\mathcal{I}_M(X)(\rho) := tr_A[\mathbb{I} \otimes Z(\mathcal{U}\rho \otimes \rho_A \mathcal{U}^*)\mathbb{I} \otimes Z]$.

SCHEMATICS OF MEASUREMENT

- Now corresponding POVM element $E(x)$, recovers from the probability reproducibility condition i.e., $tr[E(X)\rho] = tr[\mathcal{I}_M(X)\rho] \forall X \in \mathcal{F}, \rho \in T(H_S)$
- What is the appropriate POVM for the weak measurement scheme?

UNSHARP EFFECT OPERATOR

- Two outcome POVM correspond to weak measurement $E^{\lambda} \equiv \{E_{+}^{\lambda}, E_{-}^{\lambda} | E_{+}^{\lambda} + E_{-}^{\lambda} = \mathbb{I}\}$.
- $E_{\pm}^{\lambda} = \frac{1+\lambda}{2} P_{\pm} + \frac{1-\lambda}{2} P_{\mp} = \lambda P_{\pm} + \frac{1-\lambda}{2}\mathbb{I}$.
- Non-selective un-normalised state after measurement is
  $\rho' = \sqrt{E_{+}^{\lambda}}\rho\sqrt{E_{+}^{\lambda}} + \sqrt{E_{-}^{\lambda}}\rho\sqrt{E_{-}^{\lambda}}$
  $\rho' = \sqrt{1-\lambda^2}\rho + (1-\sqrt{1-\lambda^2})(P_{+}\rho P_{+} + P_{-}\rho P_{-})$.
- $p(\pm) = tr[E_{\pm}^{\lambda}\rho] = \lambda tr[P_{\pm}\rho] + \frac{1-\lambda}{2}$.
- Comparing two formalism $G = \lambda$ and $F = \sqrt{1-\lambda^2}$.
- For $G = \lambda = 1$, F becomes zero, this being the limit of sharp measurement. Unsharp measurement yields the maximum information about the system while disturbing the original state minimally.

SHARING OF NONLOCALITY

- Alice measures in the direction $\hat{X}$ and $\hat{Z}$ and Bob measures in directions $\frac{-(\hat{Z}+\hat{X})}{\sqrt{2}}$, $\frac{-\hat{Z}+\hat{X}}{\sqrt{2}}$. For nonorthogonal measurements this result also holds.
- Each Bob measures independently of the previous Bobs. Hence, any Bob has to consider the average effect of possible choices of measurements done by previous Bobs.
- For two Bob measuring in succession, the joint probability is given by
  $p(a, b_2) = \frac{\sqrt{1-\lambda_1^2}}{2}\frac{1-ab_2\lambda_2\hat{y_2}.\hat{x}}{2} + \frac{1-\sqrt{1-\lambda_1^2}}{2}\frac{1-ab_2\lambda_2\hat{x}.\hat{y_1}\hat{y_1}.\hat{y_2}}{2}$.
- CHSH values are given by $CHSH_{AB_1} = 2\sqrt{2}\lambda_1$, and $CHSH_{AB_2} = \sqrt{2}(1+\sqrt{1-\lambda_1^2})$. Both Bob obtained violation when $1/\sqrt{2} < \lambda_1 < \sqrt{2(\sqrt{2}-1)}$.

SHARING OF NONLOCALITY

- Now consider 1st and 2nd Bobs both measure weakly, while the last Bob measures sharply.
- $p(a, b_3) = \frac{1}{2}[\sqrt{1-\lambda_1^2}\sqrt{1-\lambda_2^2}\frac{1-ab_3\lambda_3\hat{y}_3.\hat{x}}{2} + (1-\sqrt{1-\lambda_1^2})\sqrt{1-\lambda_2^2}\frac{1-ab_3\lambda_3\hat{x}.\hat{y}_1\hat{y}_1.\hat{y}_3}{2} + \sqrt{1-\lambda_1^2}(1-\sqrt{1-\lambda_2^2})\frac{1-ab_3\lambda_3\hat{x}.\hat{y}_2\hat{y}_2.\hat{y}_3}{2} + (1-\sqrt{1-\lambda_1^2})(1-\sqrt{1-\lambda_2^2})\frac{1-ab_3\lambda_3\hat{x}.\hat{y}_1\hat{y}_1.\hat{y}_2\hat{y}_2.\hat{y}_3}{2}]$
- $C_3 = \lambda_3[\sqrt{1-\lambda_1^2}\sqrt{1-\lambda_2^2}\hat{y}_3.\hat{x} + (1-\sqrt{1-\lambda_1^2})\sqrt{1-\lambda_2^2}\hat{x}.\hat{y}_1\hat{y}_1.\hat{y}_3 + \sqrt{1-\lambda_1^2}(1-\sqrt{1-\lambda_2^2})\hat{x}.\hat{y}_2\hat{y}_2.\hat{y}_3 + (1-\sqrt{1-\lambda_1^2})(1-\sqrt{1-\lambda_2^2})\hat{x}.\hat{y}_1\hat{y}_1.\hat{y}_2\hat{y}_2.\hat{y}_3]$

SHARING OF NONLOCALITY

- As any Bob is ignorant about inputs of previous Bobs, this correlation has to be averaged over all possible earlier inputs $\bar{C}_3 = \sum_{y_1 y_2} C_3 P(y_1) P(y_2)$.
- $\mathcal{I}^3 = \frac{(1+\sqrt{1-\lambda_1^2})(1+\sqrt{1-\lambda_2^2})}{\sqrt{2}}$.
- $CHSH_{AB_1} = 2\sqrt{2}\lambda_1$ and $CHSH_{AB_2} = \lambda_2\sqrt{2}(1+\sqrt{1-\lambda_1^2})$.
- For 1st Bob to obtain violation $\lambda_1 > 1/\sqrt{2}$ and for the 2nd Bob this condition is $\lambda_2 > \frac{2}{\sqrt{2}+1}$.
- With the choice of λ_is $\mathcal{I}^3$ never get violated. In the worst case scenario of violation of CHSH by Bob^1 and Bob^2, i.e., when both of them obtain minimal violation, $\mathcal{I}^3$ can not becomes greater than 1.88

EXPERIMENTAL VERIFICATION

Experimental Sharing of Nonlocality among Multiple Observers with One Entangled Pair via Optimal Weak Measurements

Meng-Jun Hu,[1,2,*] Zhi-Yuan Zhou,[1,2,*] Xiao-Min Hu,[1,2]
Chuan-Feng Li,[1,2] Guang-Can Guo,[1,2] and Yong-Sheng Zhang[1,2,†]

- arXiv:1609.01863v1 [quant-ph] 7 Sep 2016
- It reports an observation of double CHSH-Bell inequality violations for a single pair of entangled photons with strength continuous tunable optimal weak measurements in photonic system.

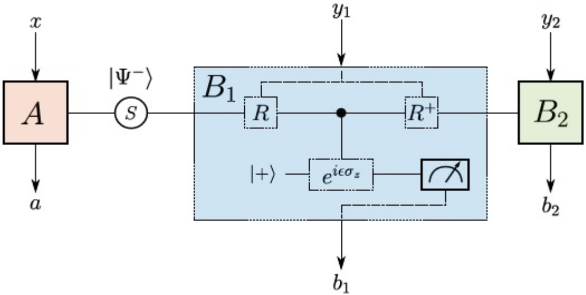
EXPERIMENTAL VERIFICATION

Three-observer Bell inequality violation on a two-qubit entangled state

Matteo Schiavon,[1] Luca Calderaro,[1] Mirko Pittaluga,[1] Giuseppe Vallone,[1,2] and Paolo Villoresi[1,2]
[1]*Department of Information Engineering, University of Padova, I-35131 Padova, Italy*
[2]*Istituto di Fotonica e Nanotecnologie, CNR, Padova, Italy*

- arXiv:1611.02430v1 [quant-ph] 8 Nov 2016
- Experimentally demonstrate the violation of two simultaneous CHSH inequalities by exploiting a two-photon polarization maximally entan- gled state.

EXPERIMENTAL VERIFICATION



- Bob1 apply $CP_\epsilon = |H\rangle\langle H| \otimes \mathbb{I} + |V\rangle\langle V| \otimes e^{i\epsilon\sigma_z}$. Controllable strength measurement $\epsilon = \pi/2 \Rightarrow \{|H\rangle, |V\rangle\}$.
- $x \in \{\frac{-(\hat{Z}+\hat{X})}{\sqrt{2}}, \frac{-\hat{Z}+\hat{X}}{\sqrt{2}}\}$ and Bob Z or X corresponds to $R_0 = \mathbb{I}, R_1 = (\sigma_z + \sigma_x/2)$.

EXPERIMENTAL VERIFICATION

- Bob2 measures strongly on 1st qubit.
- $\mathcal{I}^1_{CHSH} = 2\sqrt{2}\sin^2\epsilon$.
- $\mathcal{I}^2_{CHSH} = \sqrt{2}(1 + \cos\epsilon)$.
- With $\epsilon = 1.049 \pm 0.002$ they obtain $\mathcal{I}^1_{CHSH} = 2.125 \pm 0.003$ and $\mathcal{I}^2_{CHSH} = 2.096 \pm 0.003$.

Thank You