The Quantum Measure – And How To Measure It

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Work with

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References

- http://arxiv.org/abs/1610.02087 (IJTP, 2017)
- Three lectures I delivered at ICTS, Bengaluru
- Matt Leifer, quant-ph/0509193

ABSTRACT

When utilized appropriately, the path-integral offers an alternative to the ordinary quantum formalism of state-vectors, selfadjoint operators, and external observers – an alternative that seems closer to the underlying reality and more in tune with quantum gravity. The basic dynamical relationships are then expressed, not by a propagator, but by the **quantum measure**, a set-function μ that assigns to every (suitably regular) set E of histories its generalized measure $\mu(E)$. (The idea is that μ is to quantum mechanics what Wiener-measure is to Brownian motion.) Except in special cases, $\mu(E)$ cannot be interpreted as a probability, as it is neither additive nor bounded above by unity. Nor, in general, can it be interpreted as the expectation value of a projection operator (or POVM). Nevertheless, I will describe how one can ascertain $\mu(E)$ experimentally for any specified E, by means of an arrangement which, in a well-defined sense, filters out the histories that do not belong to E. This raises the question whether in favorable circumstances we can claim to know that the event E actually did occur.

A histories-based quantum mechanics?

Quantum Foundations $\rightarrow$ Quantum Gravity

QG needs (may need?) QuFo

no external agents

no tests under our control

may lead us to "Quantum Bell Causality"

Quantum Gravity $\rightarrow$ Quantum Foundations

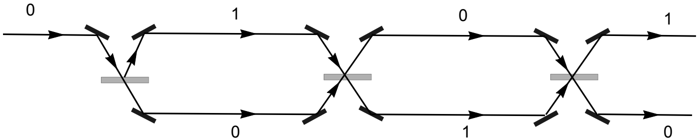
QG needs histories formulation (path $\int$) to define BH horizon, for example

Claim. All probabilities of instrument-events follow from path $\int$ alone

So can we do QM with histories and no more?

(This question will lead us to a new class of measurements)

The two-site hopper — an optical model



Amplitude $=$ $1/\sqrt{2}$ for transmission and $i/\sqrt{2}$ for reflection

Examples of histories are $[0000\cdots]$ $[0100\cdots]$ $[0011\cdots]$

Will write a history as $\gamma = [0\ x_1\ x_2\ x_3 \cdots]$

Amplitudes $A(\gamma)$ and event-measures $\mu(E)$ for our hopper

$A(\gamma)$ “is” product of amplitudes $1/\sqrt{2}$ or $i/\sqrt{2}$

$$\mu(E) = \sum_{\gamma,\gamma' \in E} A(\gamma)\, A(\gamma')^*\, \Theta(\gamma, \gamma')$$

Here Θ controls which histories interfere — those that share same “final outcome”

The rule is familiar in this form:

interfere $\implies$ add first then square

do not interfere $\implies$ square first then add

Single versus double path-sums

QMT is thus defined by a **double sum over paths (DPI)**

It computes a "generalized probability" $\mu(E)$ rather than
a "transition amplitude" $\langle \mathit{final} | \mathit{initial} \rangle$

Mathematically, it is a mild generalization of the single path integral (SPI)

But there's a difference in emphasis

We don't care so much about " getting from A to B "

What we care about is what happened on the way! compare weak measurements

There's no final time really, so everything that happens happens along the way

Such a change of focus is necessary if we hope to free QM
from external agents, wave functions, observable operators, ...

Types of event

singleton

instrumental

“terminal” (cf. records)

temporally extended (“spacetime”, better name?) **(example: double double slit)**

Almost all events are of this last type, but
ordinary quantum mechanics (OQM) doesn’t recognize them!

They lead beyond OQM.

Compare black hole horizons

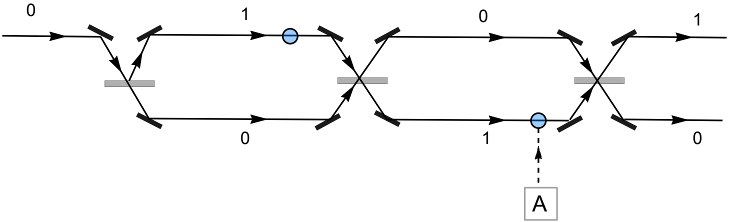
Does $\mu(E)$ have experimental content for such events?

operationalists like to ask this

The (surprising?) answer is YES

It turns out that, as we are about to demonstrate, one can “measure the measure” without going beyond the narrow confines of the “Copenhagen-Lüders” conception of measurement. For lack of time, we will here treat only the general case. But the special cases of singleton events and events of just two histories can help to bring out the basic ideas in a simpler setting. (See our paper or my ICTS lectures, if interested.)

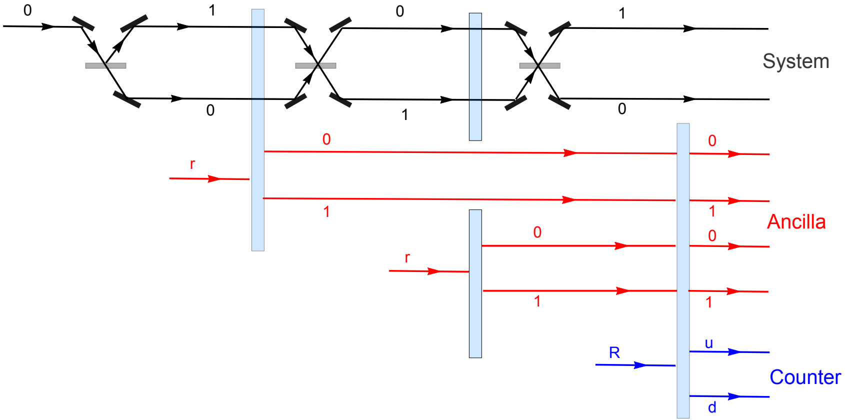
Two-site hopper with ancillas coupled in



The j^{th} ancilla "copies x_j to y_j"

much like a CNOT gate

Schematic of the measurement scheme



Stages of the measurement scheme

Stage 1. Ancillas copy the system history γ (counter is spectator)

What happens in this stage is independent of the event E we are interested in

Stage 2. Ancillas interact with each other and with "counter" (system is spectator)

The couplings in this stage are designed specifically for the event E

Stage 3. We listen for the counter to click

The couplings in stage 2

Since the particle is a spectator in stage 2, we ignore it to simplify our notation:

ancillas: $\lambda = (y_1 y_2 \cdots y_n)$ = momentary configuration of ancillas

counter: R = ready-state $\rightarrow$ either u = click or d = silent

The important transition amplitudes are then

$\lambda \notin E \implies [\lambda\ R] \rightarrow [\lambda\ d]$ Amp = 1

$\lambda, \lambda' \in E \implies [\lambda\ R] \rightarrow [\lambda'\ u]$ Amp = 1 / $|E|$

$\lambda \in E, \lambda' \notin E \implies [\lambda\ R] \rightarrow [\lambda'\ u]$ Amp = 0 (ie forbidden)

other cases $\implies$ choose amps as you please, subject to overall unitarity

In another language, we are "measuring" the projector $|\psi\rangle\langle\psi|$ onto the ancilla-state $\psi = \sum |\lambda\rangle$ for $\lambda \in E$. (cf. "quantum eraser")

The click event

Let Γ represent a joint history

$$\Gamma = \begin{bmatrix} \textit{system} & \\ \textit{ancillas} & \textit{counter} \end{bmatrix} = \begin{bmatrix} \gamma & \\ \lambda & q \end{bmatrix}$$

where $q = u, d$

Let μ_0 = the measure for system alone

Let μ_E = the measure for system + ancillas + counter

We want $\mu_E(\mathcal{C})$ to reveal $\mu_0(E)$, where

$$\mathcal{C} = \textit{click event} = \begin{bmatrix} * & \\ * & u \end{bmatrix}$$

Computing the probability that the counter clicks: $\mu_E(\mathcal{C})$

If you trace through how the joint history develops, you find that only histories $\Gamma = [\gamma\ \lambda\ u]$ such that $\gamma, \lambda \in E$ contribute, and their amplitudes are

$$A_E(\Gamma) = A_0(\gamma) \times 1\ /\ |E|$$

The remaining calculation is then short

$$\mu_E(\mathcal{C}) = \sum_{\Gamma,\Gamma' \in \mathcal{C}} A(\Gamma)\, A(\Gamma')^*\ \Theta(\Gamma, \Gamma')\ = \sum_{\gamma,\gamma',\lambda,\lambda' \in E} \frac{A_0(\gamma)}{|E|} \frac{A_0(\gamma')^*}{|E|}\ \Theta_0(\gamma, \gamma')\ \delta(\lambda, \lambda')$$

But

$$\sum_{\lambda,\lambda' \in E} \delta(\lambda, \lambda') = |E|$$

so

$$\mu_E(\mathcal{C}) = |E|^{-1} \sum_{\gamma,\gamma' \in E} A_0(\gamma)\ A_0(\gamma')^*\ \Theta_0(\gamma, \gamma') = \mu_0(E)\ /\ |E|$$

Therefore $\mu_0(E)$ **= (click frequency)** $\times |E|$ **and thus we "measure"** $\mu_0(E)$

CODA Does a click teach us that E has really happened?

And if so was our intervention "minimally disturbing"?

Suppose that the counter does click . . .

The “Lüders collapse rule” would say to project Ψ onto the “counter-state” $|u\rangle$.

“Stochastic conditioning” would say to omit all $\gamma \notin E$ from our history space Ω .

We don’t really know whether either rule is true.

But they are equivalent if we treat $\Psi(\textit{collapsed})$ as the initial amplitudes for the subseqent path integral,

because $\psi(\textit{collapsed})$ for the particle is precisely what you get by evolving $\psi(\textit{initial})$ via a **restricted** path-integral (SPI) taken over only histories $\gamma \in E$.

Notice also that the measure of the conjunction, $\mathcal{C} \cap \bar{E}$, vanishes for $\bar{E} = \Omega \backslash E$. For classical logic, this would be the implication $\mathcal{C} \implies E$.

In this provisional sense, we can interpret a click as a minimally disturbing verification that E actually happened.

Next up — building one of these event-filters in Urbasi Sinha’s lab