QUANTUM MECHANICS FOR NON-INERTIAL OBSERVERS

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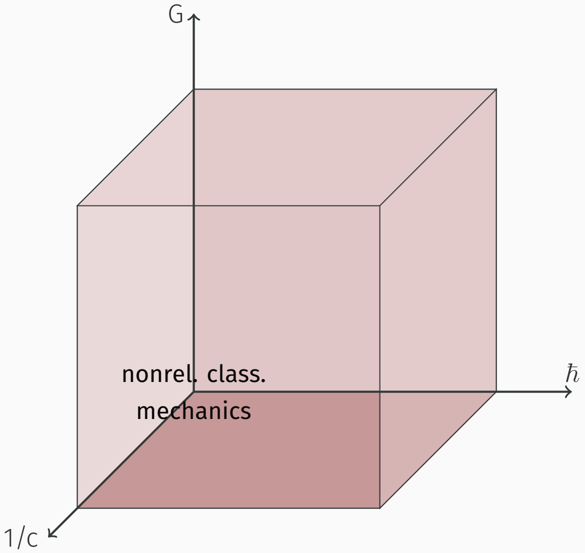
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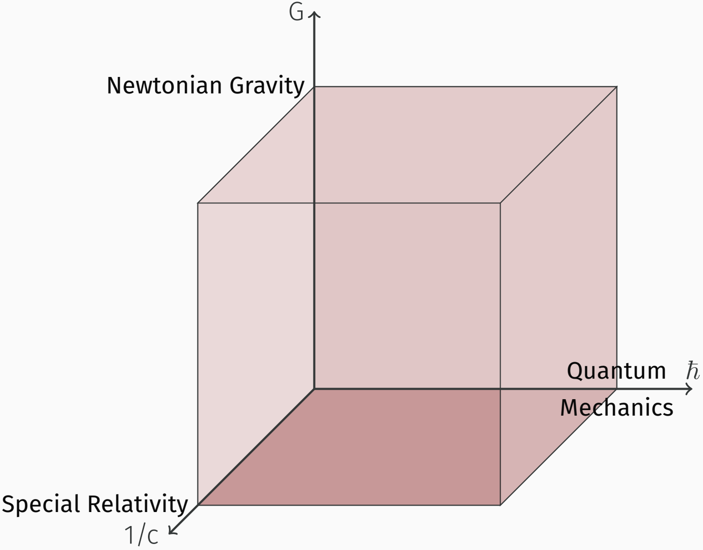
1/c
nonrel. class.
mechanics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Newtonian Gravity
Quantum
Mechanics
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Special Relativity
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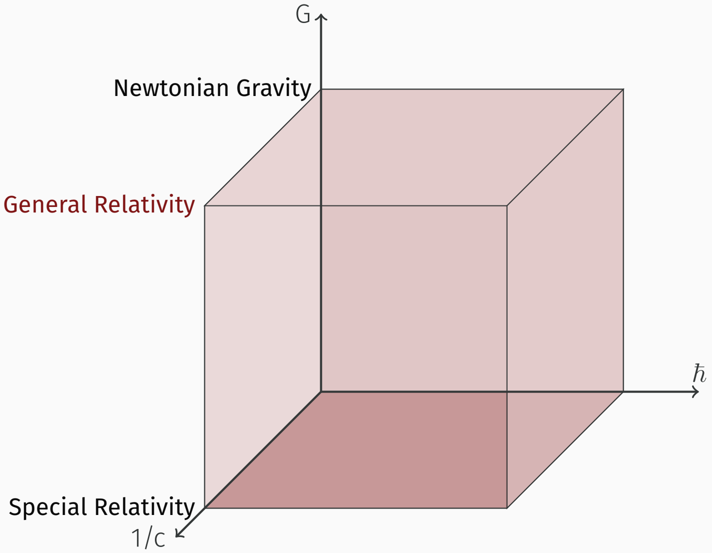
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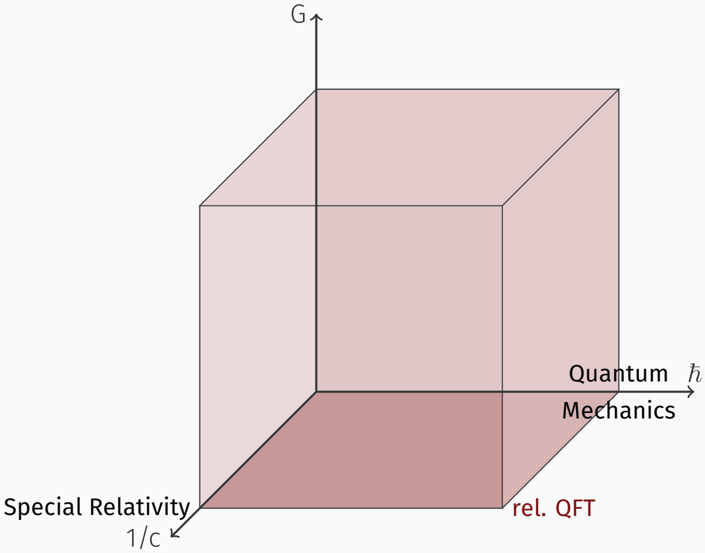
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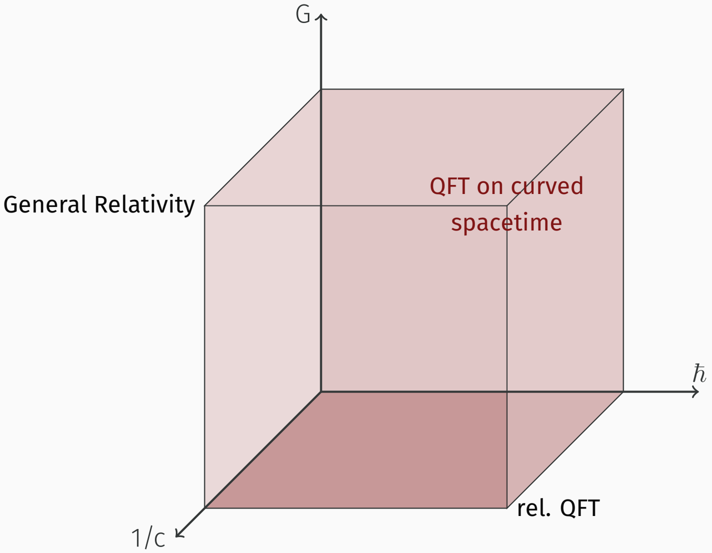
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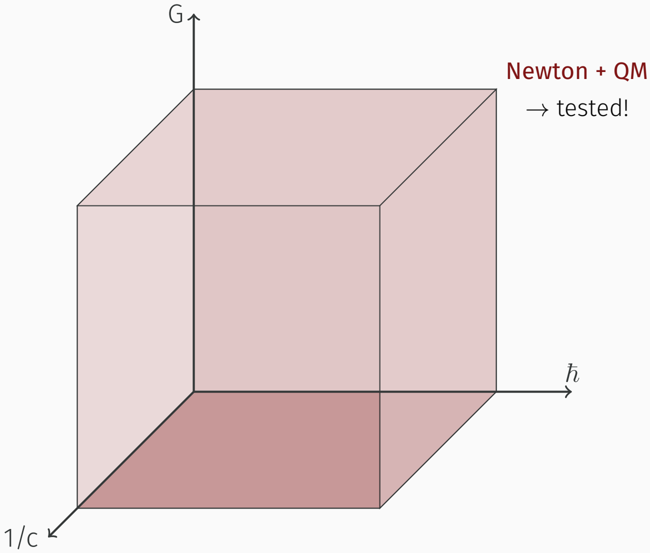
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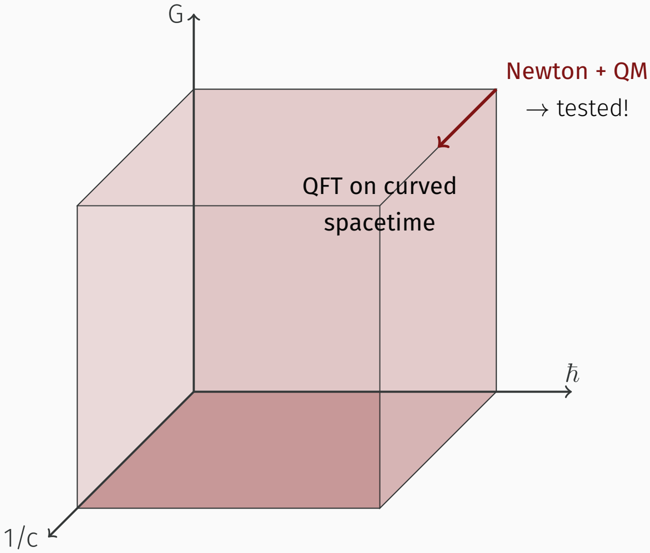
$\hbar$
1/c
Newton + QM
→ tested!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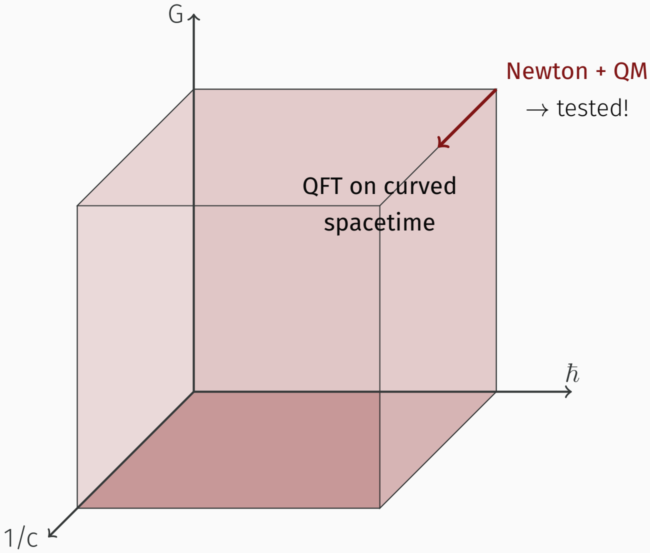


Only **external** gravitational fields!

TIME DILATION INDUCES DECOHERENCE ... OR DOES IT?

Effects of gravitational time dilation on quantum systems:

- Interferometry with clocks
  Zych et al. 2011, Nature Communications 2, 505
- Loss of visibility in photon interference due to Shapiro time delay
  Zych et al. 2012, Classical and Quantum Gravity 29, 224010
- Decoherence due to gravitational time dilation
  Pikovski et al. 2015, Nature Physics 11, 668

Discussion:

Agrees with known principles?

- Diósi 2015
- Bonder et al. 2015/16
- Pang et al. 2016

Is it decoherence? Is it relevant?

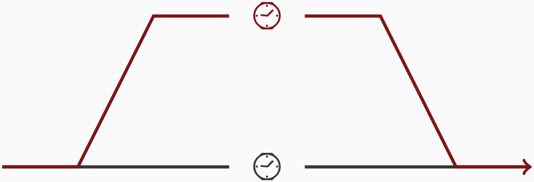
- Zeh 2015
- Adler & Bassi 2016
- Carlesso & Bassi 2016

Here: Relativistic centre of mass? What about external potentials?

TIME DILATION & QUANTUM MATTER

INTERFERENCE OF CLOCKS

- Time evolution of clock: $|\,\psi(\tau)\rangle = \mathrm{e}^{-\mathrm{i}H_{\mathrm{clock}}\tau/\hbar}\,|\,\psi(0)\rangle$



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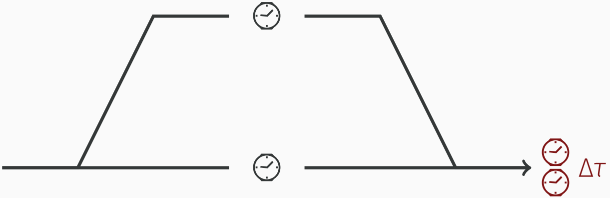
- Clocks on different paths evolve with proper time τ.
  τ is the time coordinate in the co-moving Lorentz frame.



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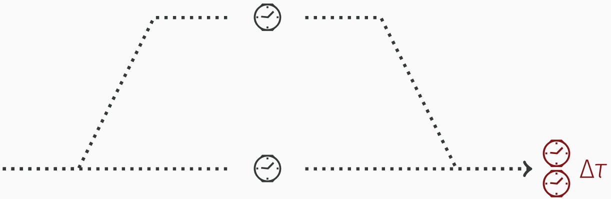

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- **Superposition yields phase difference $\Rightarrow \Delta\varphi = e^{-iH_{\text{clock}}\Delta\tau/\hbar}$?**

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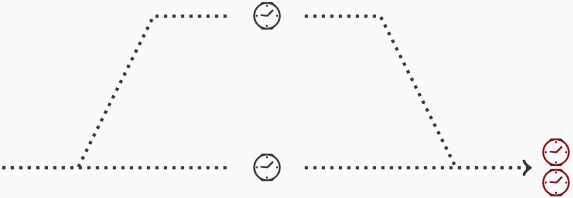

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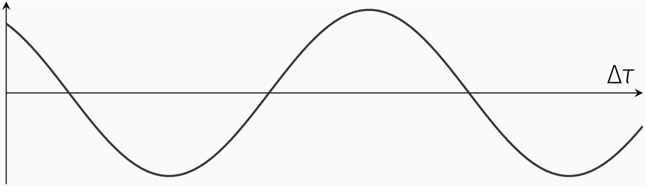
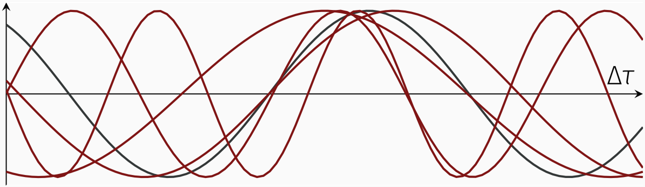
- Superposition yields phase difference $\Rightarrow \Delta\varphi = e^{-iH_{\text{clock}}\Delta\tau/\hbar}$?
- **Different paths in superposition evolve with proper time, i. e. the time coordinate of *different* inertial frames.**
  $\Rightarrow$ *Hypothesis!*



- For a single degree of freedom (“clock”): oscillating visibility of interference pattern



- For a single degree of freedom (“clock”): oscillating visibility of interference pattern
- For many degrees of freedom: recurrence of coherence becomes rare

- Center of mass dynamics described by one particle Hamiltonian

$$H_{\mathrm{cm}} = \frac{p^2}{2m} + mgx + \text{terms of order } 1/c^2$$

SIMPLE "DERIVATION"

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$$H_{\text{cm}} = \frac{p^2}{2m} + mgx + \text{terms of order } 1/c^2$$

- **Relativistic energy:** $mc^2 \to mc^2 + H_{\text{internal}}$

$$H_{\text{total}} = H_{\text{cm}} - \frac{1}{mc^2} H_{\text{internal}} \left(\frac{p^2}{2m} - mgx \right)$$

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- **Coupling of internal Hamiltonian to**
  - kinetic energy ("special relativistic time dilation")
  - gravitational energy ("gravitational time dilation")

CENTRE OF MASS COORDINATES

RELATIVISTIC CENTRE OF MASS

Galilean c.m.	coordinates weighted with rest mass
	not at rest in zero total momentum frame
	no uniform motion for interacting particles
Relativistic:	
centre of...	
...energy	coordinates weighted with energy ("relativistic mass")
	non-covariant, non-canonical
...inertia	previous def. in frame with zero total momentum, translated to other frames by Lorentz transformations
	covariant, non-canonical
...spin	mean of previous definitions
	non-covariant, canonical

- need canonical commutation relations for Quantum Mechanics

Krajcik & Foldy 1974, PRD 10, 1777

Galilei group generators

$$
\begin{aligned}
&[\mathcal{P}_i, \mathcal{P}_j] = [\mathcal{P}_i, \mathcal{H}] = [\mathcal{J}_i, \mathcal{H}] = 0\,, \\
&[\mathcal{J}_i, \mathcal{J}_j] = \mathrm{i}\,\varepsilon_{ijk}\mathcal{J}_k\,, \qquad [\mathcal{J}_i, \mathcal{P}_j] = \mathrm{i}\,\varepsilon_{ijk}\mathcal{P}_k\,, \\
&[\mathcal{J}_i, \mathcal{K}_j] = \mathrm{i}\,\varepsilon_{ijk}\mathcal{K}_k\,, \qquad [\mathcal{K}_i, \mathcal{H}] = \mathrm{i}\,\mathcal{P}_i\,, \\
&[\mathcal{K}_i, \mathcal{P}_j] = \mathrm{i}\,\delta_{ij}M\,, \qquad [\mathcal{K}_i, \mathcal{K}_j] = 0\,.
\end{aligned}
$$

With the centre of mass coordinates $\mathsf{R} = \sum \frac{m_\mu}{M}\mathsf{r}_\mu, \quad \mathsf{P} = \sum \mathsf{p}_\mu$ these assume the **single particle form**:

$$
\begin{aligned}
&\boldsymbol{\mathcal{P}} = \mathsf{P}\,, \qquad \boldsymbol{\mathcal{J}} = \mathsf{R} \times \mathsf{P} \\
&\mathcal{H} = \frac{\mathsf{P}^2}{2M} + H_{\text{internal}}\,, \qquad \boldsymbol{\mathcal{K}} = M\mathsf{R} - t\mathsf{P}\,.
\end{aligned}
$$

Krajcik & Foldy 1974, PRD 10, 1777

Poincaré group generators

$$[\mathcal{P}_i, \mathcal{P}_j] = [\mathcal{P}_i, \mathcal{H}] = [\mathcal{J}_i, \mathcal{H}] = 0\,,$$
$$[\mathcal{J}_i, \mathcal{J}_j] = \mathrm{i}\,\varepsilon_{ijk}\mathcal{J}_k\,, \qquad [\mathcal{J}_i, \mathcal{P}_j] = \mathrm{i}\,\varepsilon_{ijk}\mathcal{P}_k\,,$$
$$[\mathcal{J}_i, \mathcal{K}_j] = \mathrm{i}\,\varepsilon_{ijk}\mathcal{K}_k\,, \qquad [\mathcal{K}_i, \mathcal{H}] = \mathrm{i}\,\mathcal{P}_i\,,$$
$$[\mathcal{K}_i, \mathcal{P}_j] = \mathrm{i}\,\delta_{ij}\mathcal{H}/c^2\,, \qquad [\mathcal{K}_i, \mathcal{K}_j] = -\mathrm{i}\,\varepsilon_{ijk}\mathcal{J}_k/c^2\,.$$

With the centre of mass coordinates $\mathsf{R} =$ **???**, $\quad \mathsf{P} =$ **???**
these assume the **single particle form**:

$$\boldsymbol{\mathcal{P}} = \mathsf{P}\,, \qquad \boldsymbol{\mathcal{J}} = \mathsf{R} \times \mathsf{P}$$
$$\mathcal{H} = \sqrt{\mathsf{P}^2 c^2 + H_{\mathrm{internal}}^2}\,, \quad \boldsymbol{\mathcal{K}} = \frac{1}{2c^2}\{\mathsf{R}, \mathcal{H}\} - t\mathsf{P}\,.$$

- Search coordinates such that the generators have this form

PERTURBATIVE DEFINITION OF CENTRE OF MASS COORDINATES (II)

- Introduce a unitary transformation $\exp(i\varphi)$ that maps Galilean to relativistic variables.

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- **To lowest order in $1/c^2$: $\varphi^{(1)}[\mathbf{R}, \mathbf{P}, \{\boldsymbol{\rho}_\mu\}_N, \{\boldsymbol{\pi}_\mu\}_N]$**

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- **Commutation relations $\Rightarrow$ internal Hamiltonian**

$$H_{\mathrm{internal}} = Mc^2 + H_{\mathrm{i}}^{(0)} + \frac{1}{c^2} H_{\mathrm{i}}^{(1)}[\mathbf{P}, \{\boldsymbol{\pi}_\mu\}_N, \varphi^{(1)}]\,, \quad H_{\mathrm{i}}^{(0)} = \sum_{\mu=1}^{N} \frac{\pi_\mu^2}{2m_\mu} + U_{\mathrm{int}}^{(0)}$$

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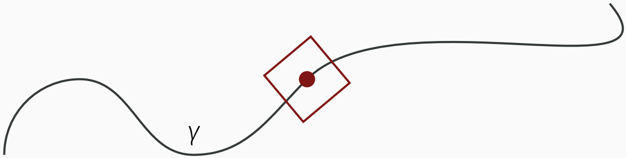
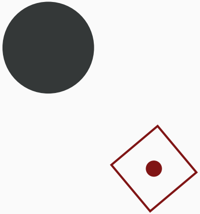
- **Expanding Poincaré generator $\mathcal{H} \Rightarrow$ centre-of-mass Hamiltonian**

$$H_{cm}^{\text{Minkowski}} = Mc^2 + \frac{\mathbf{P}^2}{2M} + H_{\mathrm{i}}^{(0)} + \frac{1}{c^2}\left(-\frac{\mathbf{P}^4}{8M^3} + H_{\mathrm{i}}^{(1)} - \frac{\mathbf{P}^2 H_{\mathrm{i}}^{(0)}}{2M^2}\right)$$

NON-INERTIAL OBSERVERS



- Co-moving observer frame: Fermi normal coordinates



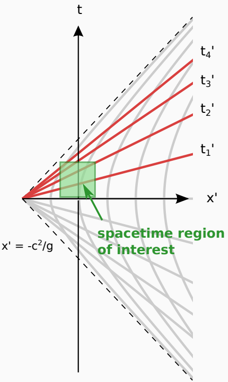
- Co-moving observer frame: Fermi normal coordinates
- Shell observer hovering at constant altitude:
  ≈ Rindler coordinates (homogeneously accelerated observer)

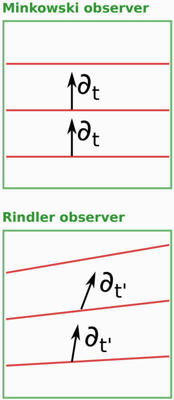
$$\mathrm{d}s^2 = -\left(1 + \frac{gx}{c^2}\right)^2 c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

On Earth: $g \approx$ -10 m/s²

TIME EVOLUTION IN THE RINDLER FRAME

Time-like Killing vectors:





Rindler: (t',x')

Local Lorentz: (t,x)

Killing vector on
$t = 0$ hypersurface:

$$\partial_{t'} = \left(1 + \frac{g\,x}{c^2}\right)\,\partial_t$$

TIME EVOLUTION IN THE RINDLER FRAME (II)

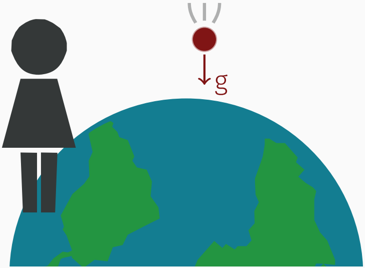
Hamiltonian for the Rindler observer:

$$
\begin{array}{ccc}
\mathcal{H}_{\text{Rindler}} & \overset{?}{\longleftrightarrow} & \mathcal{H}_{\text{Mink.}} + \frac{g}{2c^2}\{x, \mathcal{H}_{\text{Mink.}}\} \\
\updownarrow & & \updownarrow \\
\partial_{t'} & \longleftrightarrow & \left(1 + \frac{g\,x}{c^2}\right)\,\partial_t
\end{array}
$$

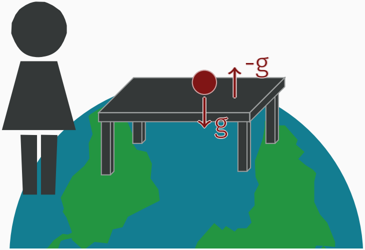
$$
H_{cm}^{\text{Rindler}} = H_{cm}^{\text{Minkowski}} + M\,g\,X + \frac{1}{c^2}\left(\frac{g}{4\,M}\{X, \mathbf{P}^2\} + H_{\text{i}}^{(0)}\,g\,X\right)
$$

SUPPORTING POTENTIAL

- So far: free falling particle seen by accelerated observer



- So far: free falling particle seen by accelerated observer
- What about a particle at rest in the lab frame?



Classically:

$$
\begin{aligned}
0 &= -\dot{\mathrm{P}} \\
&= \frac{\partial H}{\partial X} \\
&= \left(M + \frac{H_{\mathrm{i}}^{0}}{c^2}\right) g\,X + \frac{\partial U_{\mathrm{ext}}}{\partial X}
\end{aligned}
$$

$$\Rightarrow U_{\mathrm{ext}} = -M\,g\,X - \frac{1}{c^2}\,H_{\mathrm{i}}^{(0)}\,g\,X$$

cancels gravity **to all orders**

CLASSICAL STATIONARITY CONDITION

Ehrenfest theorem: $$\frac{\mathrm{d}}{\mathrm{d}t}\langle A\rangle = -\frac{\mathrm{i}}{\hbar}\langle [A, H]\rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$$

Require no class. acceleration: $$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\langle X\rangle = 0$$

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If we want to satisfy this by a potential $U_{\mathrm{ext}}(X,t)$:

$$U_{\mathrm{ext}} = -M\,g\,X - \frac{1}{c^2}\left(\langle H_{\mathrm{i}}^{(0)}\rangle + \frac{\langle \mathbf{P}^2\rangle - 2\langle P_x^2\rangle}{2M}\right) g\,X$$

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- in general **time dependent**, except for stationary states

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- in general **time dependent**, except for stationary states
- must be tuned to internal and external energy
- resulting coupling term: $\frac{g}{c^2}\,X\left(H_{\mathrm{i}}^{(0)} - \langle H_{\mathrm{i}}^{(0)}\rangle\right)$

Consider a Klein-Gordon field, minimally coupled to an electromagnetic field, i. e. $\mathrm{i}\hbar\partial_t \to \mathrm{i}\hbar\partial_t - e\,\Phi$.

To order $1/c^2$: Schrödinger equation

$$\mathrm{i}\hbar\dot{\psi} = \left(\frac{\mathbf{p}^2}{2m} - \frac{\mathbf{p}^4}{8m^3c^2} + V\right)\psi$$

with

$$V = -e\,\Phi + \frac{e^2\Phi^2}{2mc^2} + \frac{e\Phi\,\mathbf{p}^2}{4m^2c^2}$$

- relativistic correction of the interaction **depends on momentum**

If we want to satisfy the condition $\frac{d^2}{dt^2}\langle X \rangle = 0$ by a quantum interaction:

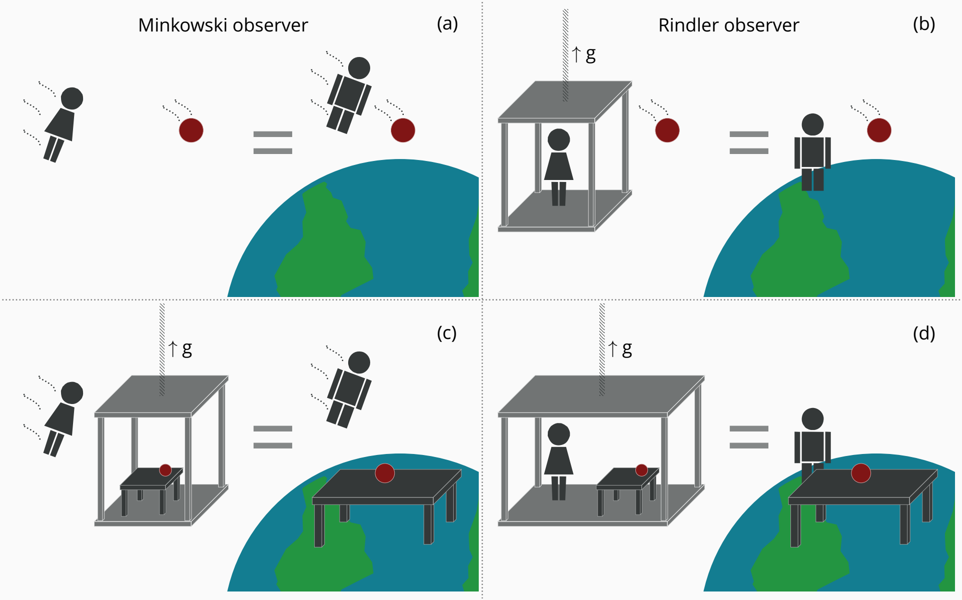
$$U_{\mathrm{ext}}(X, \mathsf{P}) = -M\, g\, X - \frac{1}{c^2}\, H_{\mathrm{i}}^{(0)}\, g\, X - \frac{g}{4Mc^2}\{X, \mathsf{P}^2\} + f(\mathsf{P})$$

This exactly cancels all gravitational terms in the Hamiltonian:

$$H_{cm}^{\text{Rindler, ext. pot.}} = H_{cm}^{\text{Minkowski, free}} + f(\mathsf{P})$$

CONCLUSION

SUMMARY



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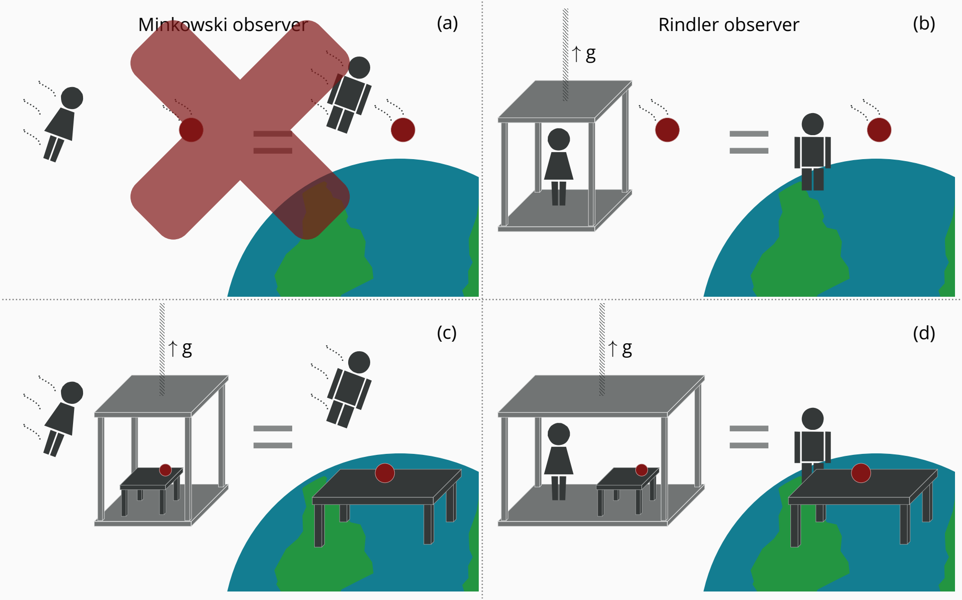

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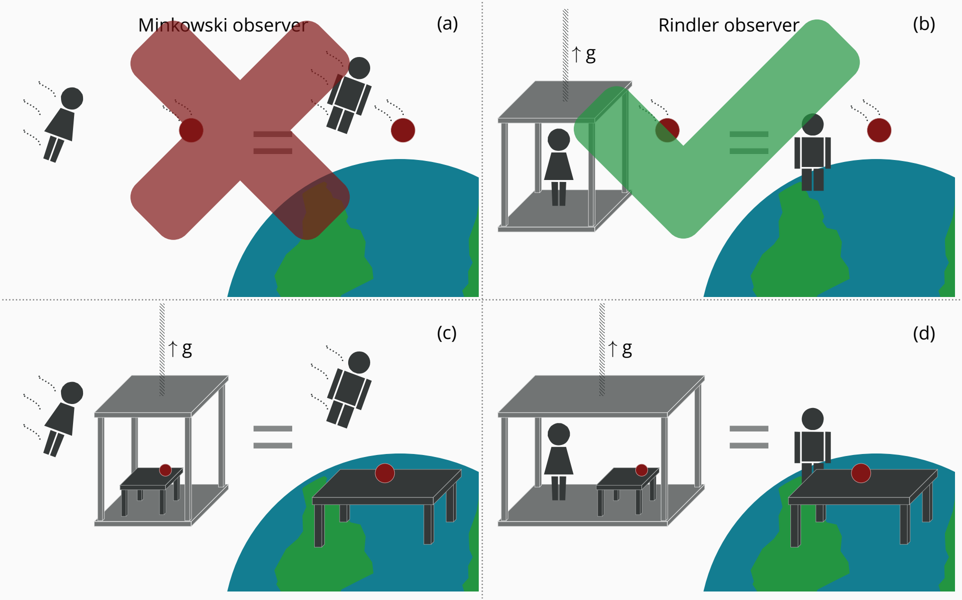

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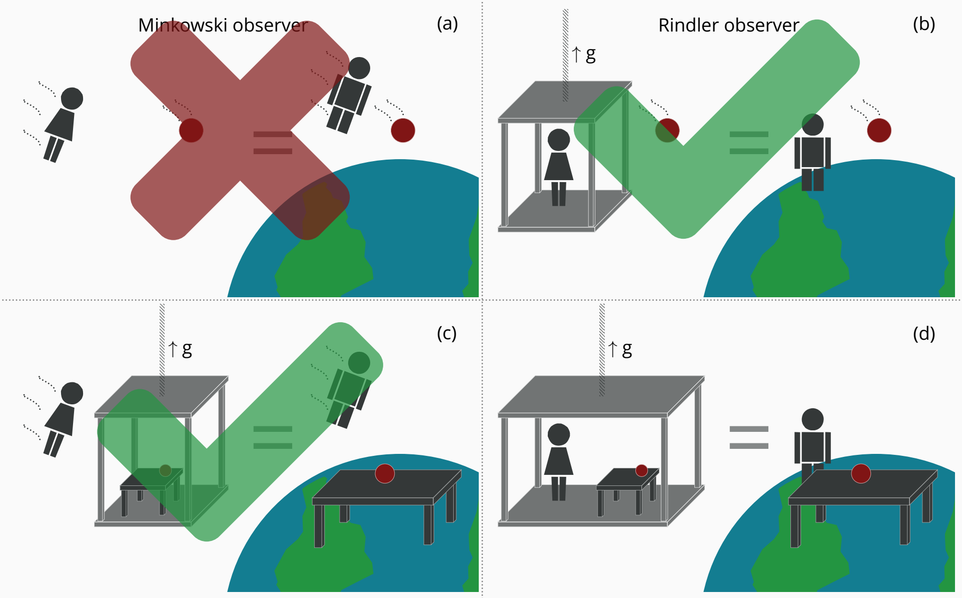

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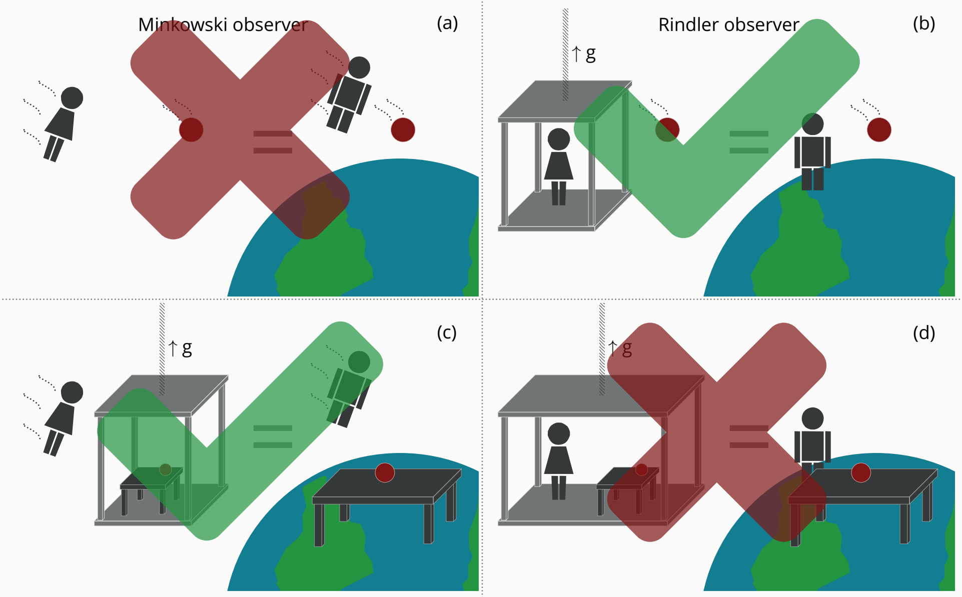

This is joint work with

Marko Toroš & Angelo Bassi

Thank You!

Questions?