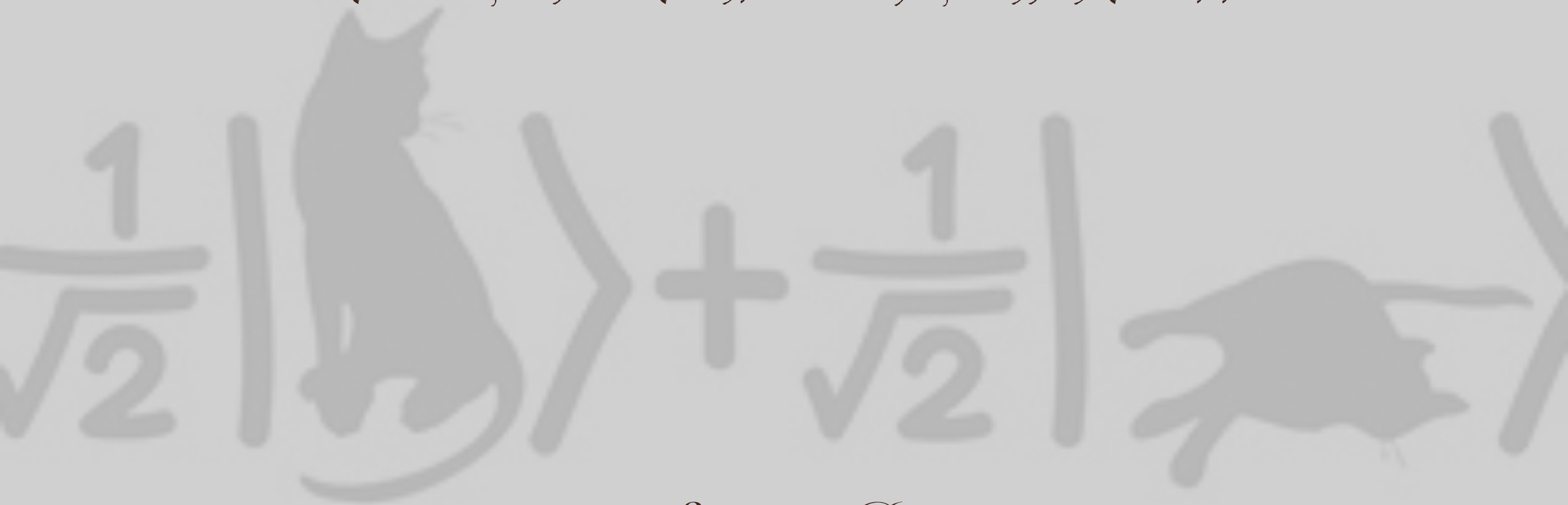


Cosmic Inflation and the measurement problem

(PRD 88, 085020 (2013) & PRD 90, 043503 (2014))

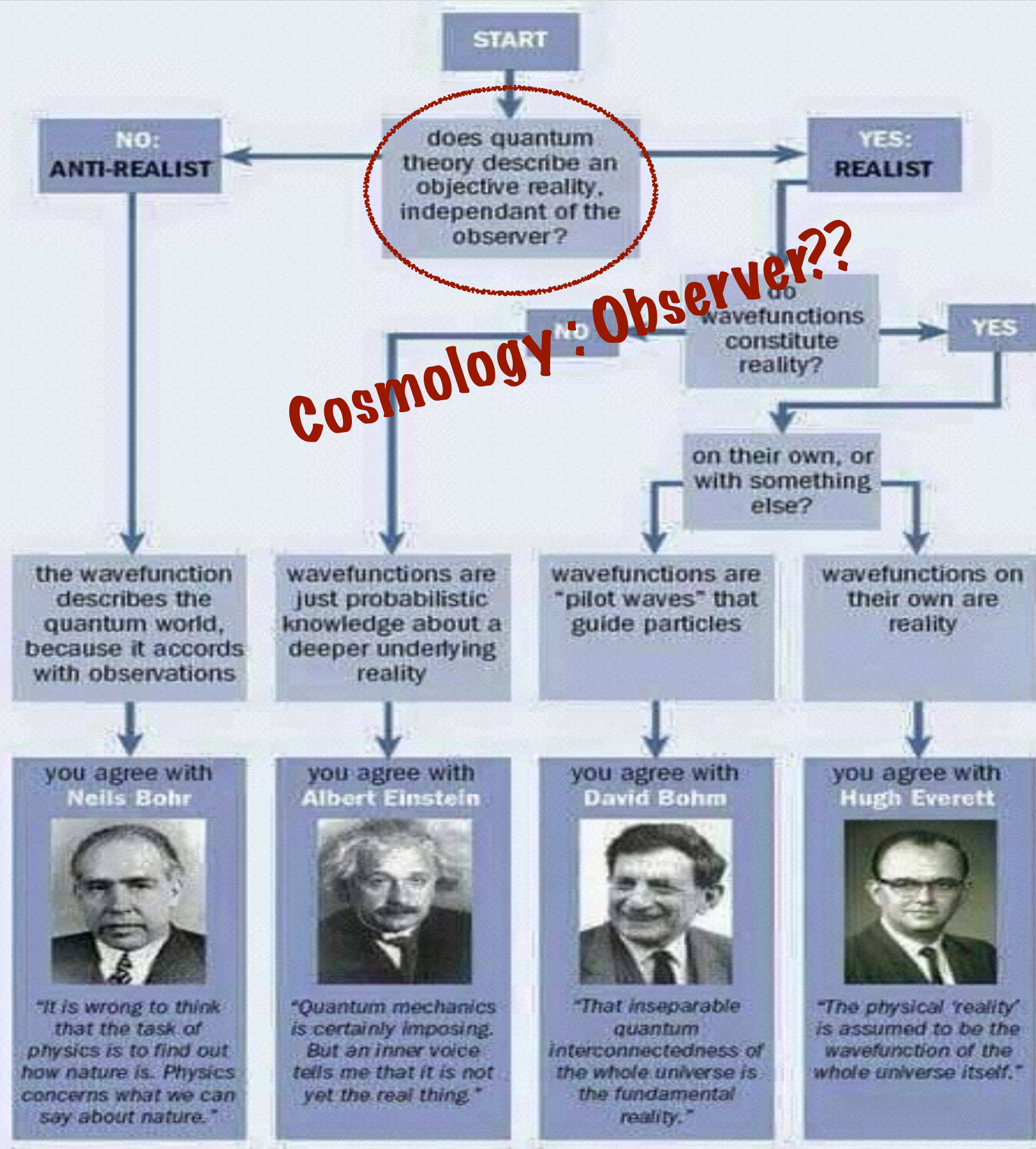


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Inflationary Paradigm: Success

- ◆ Solves Big Bang pathologies
- ◆ Generates (quantum) primordial perturbations → seeds for large scale structures, CMB anisotropy
- ◆ Predicts (Single field models):
 1. Almost scale invariant scalar power spectrum: $n_s = 0.9635 \pm 0.0094$
 2. Almost Gaussian distribution of primordial perturbations : $f_{NL} < 2.7 \pm 5.8$
 3. Consistency relation : $r = -8n_T$

Quantum to Classical transition of primordial perturbations

Origin of perturbations are quantum but observed structures are classical

- Solutions :
 1. Does not modify the basic mechanism of QM : Decoherence
 2. Modifies basic mechanism of QM : Collapse models [Continuous Spontaneous Localization (CSL) Model]

Inherent mechanism evades requirement of observer

Continuous Spontaneous Localization

- ◆ Modifies Schrödinger equation by adding non-linear stochastic terms :

$$d\psi_t = \left[-\frac{i}{\hbar} H dt + \frac{\sqrt{\gamma}}{m_0} \int d\mathbf{x} (M(\mathbf{x}) - \langle M(\mathbf{x}) \rangle_t) dW_t(\mathbf{x}) - \frac{\gamma}{2m_0^2} \int d\mathbf{x} (M(\mathbf{x}) - \langle M(\mathbf{x}) \rangle_t)^2 dt \right] \psi_t$$

- ◆ Non-linear terms break the superposition of wave functions
- ◆ Amplification Mechanism

$$\gamma(m) = \gamma_0 \left(\frac{m}{m_N} \right)^\beta, \quad \gamma(m) = n^2 \gamma_0 \left(\frac{m}{m_N} \right)^\beta$$

- ◆ Hamiltonian not conserved due to non-Hermitian evolution \longrightarrow
Non-conservation of energy

$$\langle E \rangle = \frac{3\gamma\alpha\hbar^2}{4m} t$$

**Aim : Apply collapse mechanism
to evolution equation of primordial
perturbations**

Inflationary perturbations

- ◆ Einstein Field Equations :

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- ◆ Inflation :

- ◆ $T_{\mu\nu}$ is determined by a slowly-rolling scalar field \longrightarrow Inflaton
- ◆ Inflaton \longrightarrow Quantum Field $\longrightarrow \phi_0(t) + \delta\phi(t, \mathbf{x})$
- ◆ Perturbations in $T_{\mu\nu}$ \longrightarrow Induces quantum perturbations in background metric $\longrightarrow g_{\mu\nu}^{(0)}(t) + \delta g_{\mu\nu}(t, \mathbf{x})$
- ◆ Background metric is a tensor and thus the perturbed metric will have scalar, vector and tensor type of perturbations

Scalar perturbations

Schrödinger picture of scalar perturbations

- ♦ Scalar perturbations in terms of Mukhanov-Sasaki variable

$$\zeta(\tau, \mathbf{x}) = a \left[\delta\varphi^{\text{gi}} + \varphi'_0 \frac{\Phi_B}{\mathcal{H}} \right]$$

- ♦ Quantum state wavefunctional satisfy functional Schrödinger equation

$$i \frac{\partial \Psi_{\mathbf{k}}^{\text{R,I}}}{\partial \tau} = \hat{\mathcal{H}}_{\mathbf{k}}^{\text{R,I}} \Psi_{\mathbf{k}}^{\text{R,I}}$$

- ♦ Hamiltonian that of harmonic oscillator

$$\hat{\mathcal{H}}_{\mathbf{k}}^{\text{R,I}} = -\frac{1}{2} \frac{\partial^2}{\partial (\zeta_{\mathbf{k}}^{\text{R,I}})^2} + \frac{1}{2} \omega^2 \left(\zeta_{\mathbf{k}}^{\text{R,I}} \right)^2, \quad \omega^2 \equiv k^2 - \frac{a''}{a}$$

- ♦ Solution of functional Schrödinger equation is a functional Gaussian State

$$\Psi_{\mathbf{k}}^{\text{R,I}} \left[\tau, \zeta_{\mathbf{k}}^{\text{R,I}} \right] = \sqrt{N_k(\tau)} \exp \left(-\frac{\Omega_k(\tau)}{2} \left(\zeta_{\mathbf{k}}^{\text{R,I}} \right)^2 \right)$$

Wigner Function & Squeezing

- ◆ Wigner function recognises the correlation between position (field) and its momentum (conjugate to field)

$$\begin{aligned}\mathcal{W}(\zeta_{\mathbf{k}}^{\text{R}}, \zeta_{\mathbf{k}}^{\text{I}}, p_{\mathbf{k}}^{\text{R}}, p_{\mathbf{k}}^{\text{I}}) &= \frac{1}{(2\pi)^2} \int dx dy \Psi^* \left(\zeta_{\mathbf{k}}^{\text{R}} - \frac{x}{2}, \zeta_{\mathbf{k}}^{\text{I}} - \frac{y}{2} \right) e^{-ip_{\mathbf{k}}^{\text{R}}x - ip_{\mathbf{k}}^{\text{I}}y} \Psi \left(\zeta_{\mathbf{k}}^{\text{R}} + \frac{x}{2}, \zeta_{\mathbf{k}}^{\text{I}} + \frac{y}{2} \right) \\ &= \frac{1}{\pi^2} e^{-\text{Re } \Omega_k (\zeta_{\mathbf{k}}^{\text{R}2} + \zeta_{\mathbf{k}}^{\text{I}2})} e^{-\frac{(p_{\mathbf{k}}^{\text{R}} + \text{Im } \Omega_k \zeta_{\mathbf{k}}^{\text{R}})^2}{\text{Re } \Omega_k}} e^{-\frac{(p_{\mathbf{k}}^{\text{I}} + \text{Im } \Omega_k \zeta_{\mathbf{k}}^{\text{I}})^2}{\text{Re } \Omega_k}}\end{aligned}$$

- ◆ During inflation \longrightarrow on superhorizon scales $\text{Re } \Omega_k \rightarrow 0$

$$\mathcal{W}(\zeta_{\mathbf{k}}^{\text{R}}, \zeta_{\mathbf{k}}^{\text{I}}, p_{\mathbf{k}}^{\text{R}}, p_{\mathbf{k}}^{\text{I}}) \rightarrow \frac{\text{Re } \Omega_k}{\pi} e^{-\text{Re } \Omega_k (\zeta_{\mathbf{k}}^{\text{R}2} + \zeta_{\mathbf{k}}^{\text{I}2})} \delta(p_{\mathbf{k}}^{\text{R}}) \delta(p_{\mathbf{k}}^{\text{I}})$$

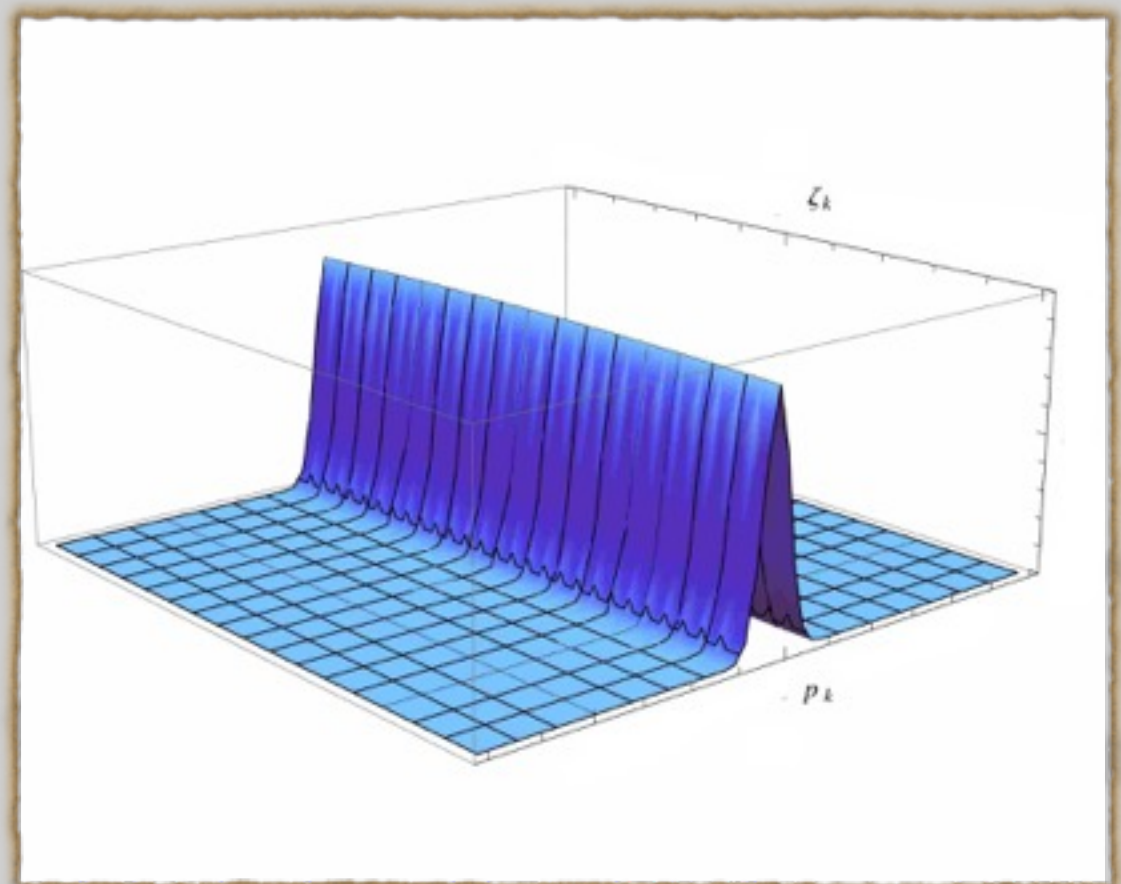
- ◆ Highly squeezed in momentum direction and spread in field
- ◆ Observation shows classicality in field direction



Expect 'collapse models' to
squeeze the modes in field direction



$$\text{Re } \Omega_k \rightarrow \infty$$



CSL-like modification with constant γ

- ◆ Modify functional Schrödinger equation with 'CSL-like' terms

$$d\Psi_{\mathbf{k}}^{\text{R,I}} = \left[-i\hat{\mathcal{H}}_{\mathbf{k}}^{\text{R,I}} d\tau + \sqrt{\gamma} \left(\hat{\zeta}_{\mathbf{k}}^{\text{R,I}} - \langle \hat{\zeta}_{\mathbf{k}}^{\text{R,I}} \rangle \right) dW_{\tau} - \frac{\gamma}{2} \left(\hat{\zeta}_{\mathbf{k}}^{\text{R,I}} - \langle \hat{\zeta}_{\mathbf{k}}^{\text{R,I}} \rangle \right)^2 d\tau \right]$$

- ◆ Frequency of the Harmonic Oscillator Hamiltonian becomes time dependent and complex

$$\omega^2 = k^2 - 2i\gamma - \frac{a''}{a}$$

- ◆ Smaller modes ($2\gamma \ll k^2$)

$$\text{Re } \Omega_k \approx 2k(-k\tau)^2 \rightarrow 0, \quad \mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{16\pi^2 \epsilon M_{\text{Pl}}^2}$$

- ◆ Wigner function not affected by γ
- ◆ Squeezing in momentum direction (can't explain classicality)
- ◆ Power spectrum scale-independent (good for observation)
- ◆ Larger modes ($2\gamma \gg k^2$)

$$\text{Re } \Omega_k \approx \frac{2\gamma}{k}(-k\tau) \rightarrow 0, \quad \mathcal{P}_{\mathcal{R}}(k) = \frac{H^2 k^3}{16\pi^2 \epsilon M_{\text{Pl}}^2 \gamma k_0} e^{-\Delta N}$$

- ◆ Wigner function affected by γ (which we wanted !!)
- ◆ Squeezing in momentum direction (can't explain classicality)
- ◆ Power spectrum scale-dependent (bad for observation)

Modification by scale-dependent γ

- ◆ Modes behave more classically as they start crossing the horizon
- ◆ γ should discriminate between different modes according to their physical length scales \longrightarrow grow stronger as a mode starts crossing the horizon during inflation
- ◆ γ should be a function of time

$$\gamma = \frac{\gamma_0(k)}{(-k\tau)^\alpha}, \quad 0 < \alpha < 2$$

on superhorizon scales

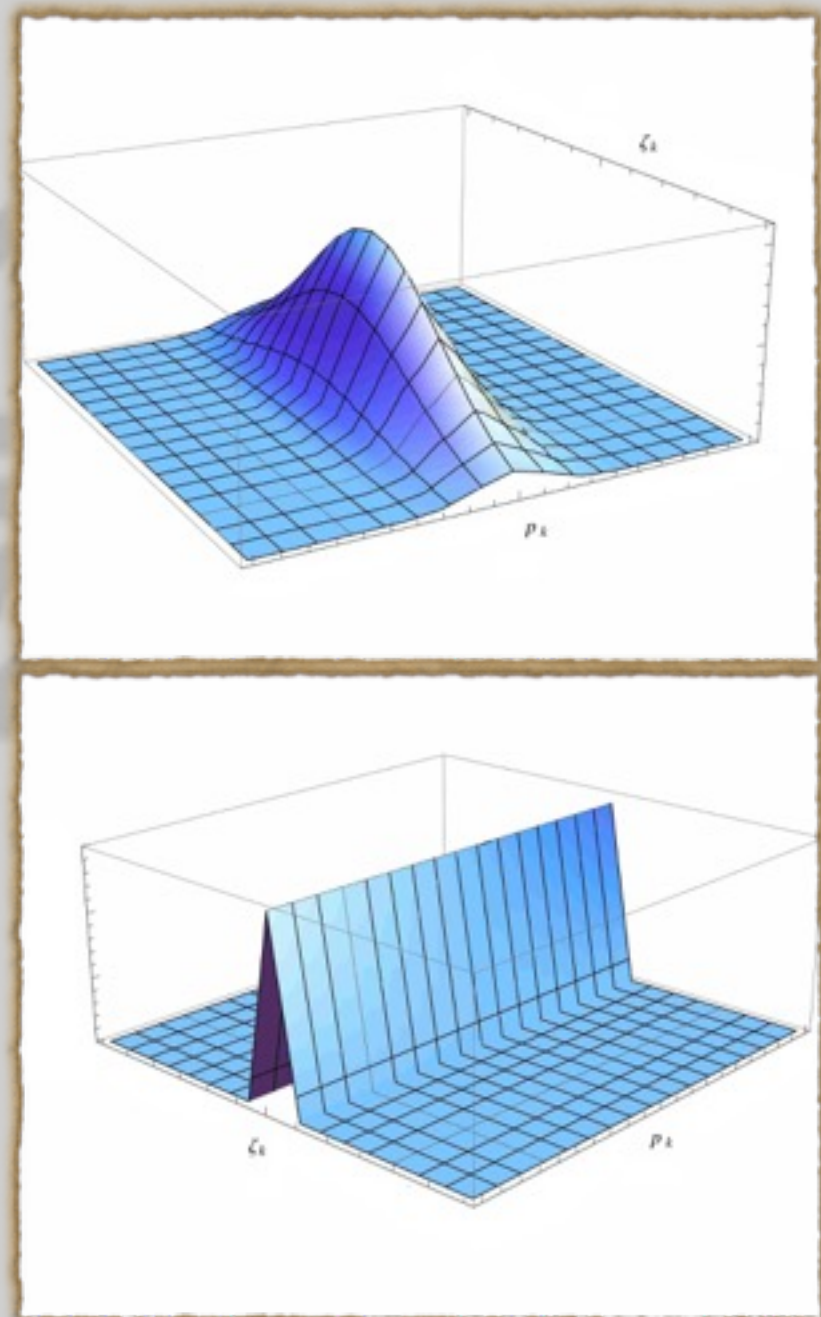
$$\text{Re } \Omega_k \approx \frac{k}{2} (-k\tau)^{1-\alpha} \left(\frac{2\gamma_0(k)}{k^2} \right)$$

$$0 < \alpha < 1 \quad \longrightarrow \quad \text{Re } \Omega_k \rightarrow 0$$

No macro-objectification

$$1 < \alpha < 2 \quad \longrightarrow \quad \text{Re } \Omega_k \rightarrow \infty$$

Macro-objectification occurs



Scale-invariance of power spectrum

- ◆ To obtain a scale-invariant power spectrum make γ mode dependent

$$\gamma_0(k) = \tilde{\gamma}_0 \left(\frac{k}{k_0} \right)^\beta$$

- ◆ The power spectrum becomes

$$\mathcal{P}_{\mathcal{R}}(k) \propto k^{3+\alpha-\beta}$$

- ◆ $\beta = 3 + \alpha$ yields scale-invariant power spectrum

Vector perturbations → Decay
during inflation → Not Important

Tensor perturbations

Macro-objectification of tensor modes

- ◆ Tensor modes \longrightarrow Traceless and transverse part of metric fluctuations \longrightarrow associated with two helicity states $+$ and \times
- ◆ Each helicity states identical to a massless scalar in de Sitter space
- ◆ Previous analysis of scalar perturbations applicable to each helicity states to obtain macro-objectification of tensor modes
- ◆ Assumption : CSL-modified dynamics is essentially same for gravitons and inflatons

Observables

- ◆ Scalar power spectrum

$$\begin{aligned}\mathcal{P}_{\mathcal{R}} &= \frac{1}{8\pi\epsilon M_{\text{Pl}}^2} \frac{k_0^2 H^2}{\tilde{\gamma}_0} e^{-(1+\alpha)\Delta N} \left(\frac{k_*}{k_0}\right)^{3+\alpha-\beta} \left(\frac{k}{k_*}\right)^{3+\alpha-\beta+2\eta-3\epsilon} \\ &\equiv A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s-1}\end{aligned}$$

- ◆ Tensor power spectrum

$$\begin{aligned}\mathcal{P}_h &= \frac{2}{\pi^2 M_{\text{Pl}}^2} \frac{k_0^2 H^2}{\tilde{\gamma}_0} e^{-(1+\alpha)\Delta N} \left(\frac{k_*}{k_0}\right)^{3+\alpha-\beta} \left(\frac{k}{k_*}\right)^{3+\alpha-\beta-2\epsilon} \\ &\equiv A_T(k_*) \left(\frac{k}{k_*}\right)^{n_T}\end{aligned}$$

- ◆ Scalar spectral index

$$n_s - 1 = \delta + 2\eta - 4\epsilon \quad (\delta = 3 + \beta - \alpha)$$

- ◆ δ can be of the order of slow-roll parameters
- ◆ Tensor spectral index

$$n_T = \delta - 2\epsilon$$

- ◆ Tensor-to-Scalar ratio

$$r = -8n_T + 8\delta$$

- ◆ Accurate measurements of r and n_T would be able to distinguish this scenario with the generic one

Summary

- ◆ Scale dependent collapse parameter can yield micro-objectification of modes, both scalar and tensor
- ◆ Wave-number dependence of collapse parameter yield nearly scale-invariance of power spectrum
- ◆ Collapse dynamics changes the consistency relation of single-field model
- ◆ Accurate measurement of tensor-to-scalar ratio and tensor spectral index can distinguish this dynamics from the generic scenario

Thank You