QUANTUM DISCORD-A TOOL FOR COMPARING POSSIBLE SOLUTIONS TO QUANTUM MEASUREMENT PROBLEM

Shreya Banerjee

Tata Institute of Fundamental Research (TIFR), India

Talk presented at FPQP, ICTS Bangalore, 2016

Talk Outline

1. Quantum Discord-Mathematical and Conceptual Overview
2. Collapse/Decoherence models studied
3. Experimental Set-Up-Brief Description
4. Quantum Discord in the presence of CSL, GID and ED
5. Results/Inferences
6. Summary

Quantum Discord-Mathematical and Conceptual Overview- [Concept by H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 8, 2001]

Basic Concept:

• Two **classically identical** expressions for **mutual information**/correlations **differ** when the systems involved are **quantum**.

• This difference defines the quantum discord → measure of the quantumness of correlations.

Mathematical Description: For two subsystems X, Y

• *First Measure*: provided by the von Neumann entropy,

$$\boxed{\mathcal{I}(X:Y) = \underbrace{S(\hat{\rho}_X) + S(\hat{\rho}_Y)}_{\text{von}-\text{Neumann entropies for } X \text{and } Y} - \underbrace{S(\hat{\rho})}_{\text{joint quantum entropy}}} \quad (1)$$

- *Second measure*: information gained about X due to the measurement on Y (vice versa).

$$\mathcal{J}(X:Y) = S(\hat{\rho}_X) - \overbrace{\sum_j p_j S(\hat{\Pi}_j \hat{\rho}_X \hat{\Pi}_j)}^{\text{conditional quantum entropy}} \quad \text{where } \hat{\Pi}_j \hat{\rho}_X \hat{\Pi}_j = \mathrm{Tr}_Y \left(\frac{\hat{\Pi}_j \hat{\rho} \hat{\Pi}_j}{p_j} \right) \tag{2}$$

- $p_j(\mathrm{Tr}(\hat{\Pi}_j \hat{\rho} \hat{\Pi}_j)) \rightarrow$ Probability of getting a state of X given j^{th} state of Y.
- $\hat{\Pi}_j(|j\rangle\langle j|) \rightarrow$projection operator.
- **General definition of entropy**:

$$S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\ln\hat{\rho} = -\sum_i \sigma_i \ln \sigma_i \tag{3}$$

- **Defining Discord**:

$$\delta(X:Y) = \mathcal{I}(X:Y) - \mathcal{J}(X:Y) \tag{4}$$

$$\delta(X:Y)(\text{for classical systems}) = 0; \delta(X:Y)(\text{for quantum systems}) > 0 \tag{5}$$

Collapse/Decoherence models studied-Master Eqs

- **Continuous Spontaneous Localization (CSL)** [P. Pearle, Phys. Rev. A **39**, 1989, G. C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A **42**, 1990.] :

  1)Basic properties: non-linear, non-unitary, stochastic, amplifies.

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] - \frac{\lambda_{\mathrm{CSL}}}{2r_C^3\pi^{3/2}m_0^2}\int d\mathbf{x}[\hat{M}(\mathbf{x}), [\hat{M}(\mathbf{x}), \hat{\rho}(t)]] \tag{6}$$

  2) Standard Values[1]:

  $\lambda_{\mathrm{GRW}} = 10^{-17}\mathrm{s}^{-1},\ r_C = 10^{-7}\mathrm{m}^{-1};\ \lambda_{\mathrm{Adler}} = 10^{-8}\mathrm{s}^{-1},\ r_C = 10^{-7}\mathrm{m}^{-1}$

  [1] [A. Bassi and G. C. Ghirardi, 2003; A. Bassi, E. Ippoliti and S.L. Adler, 2005]

- **Gravity Induced Decoherence (GID)-The Diósi model**: Difference in self-gravitational energy of an object in **two possible mass configurations**→ decoherence between possible states.

$$\frac{\partial\rho(x,y,t)}{\partial t} = \frac{i\hbar}{2m}(\nabla_x^2 - \nabla_y^2)\rho(x,y,t) - \frac{1}{\hbar}(U(x-y) - U(0))\rho(x,y,t) \tag{7}$$

where

$$U(x,y) = -\frac{Gm^2}{R'}\left(\frac{6}{5} - \frac{1}{2}\frac{|x-y|^2}{R'^2}\right) \quad \text{for} \quad |x-y| \ll R' \tag{8}$$

[L. Diósi, P.L.A. **120** (1987)]

- **Environmental Decoherence (ED)** [E. Joos and H.D. Zeh, Z. Phys. B **59** (1985) 223.]:

1) Effect of **environment** (dust, photons, etc.) surrounding the system on measurement. 2)**Destroys interference** among alternatives → converting **pure quantum states into statistical mixture**.

$$\frac{\partial\rho(x,y,t)}{\partial t} = \frac{i\hbar}{2m}(\nabla_x^2 - \nabla_y^2)\rho(x,y,t) - \Gamma(x-y)^2\rho(x,y,t) \tag{9}$$

3) Γ →a parameter→ includes the effects due to **scattering, emission, absorption of ambient photons and collision with gas molecules**.

4) **Explicit forms of** Γ [O. Romero-Isart, Phys. Rev. A **84** (2011) 052121]:

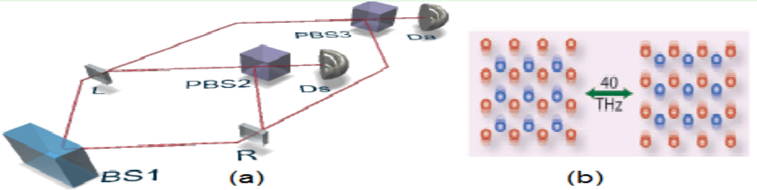
$$\Gamma_{sc} = \frac{8!8\zeta(9)cR'^6}{9\pi}\left(\frac{k_BT}{\hbar c}\right)^9\left[\text{Re}\left(\frac{\epsilon-1}{\epsilon+2}\right)\right]^2; \quad \Gamma_{coll} = \frac{8\sqrt{2\pi}\zeta(3)}{3\zeta(3/2)}\sqrt{m_{gas}}\frac{R'^2P}{\hbar^2}(k_BT)^{1/2}\ldots \tag{10}$$

Experimental Set-Up-Brief Description

[K. C. Lee, et al., Science **334** (2011) 1253]

Step 1. Generate **motional entanglement**$\rightarrow$two separate macroscopic diamond crystals pumped simultaneously (**ultra short pulses**)$\rightarrow$producing $|\Psi\rangle = |\Psi_L\rangle|\Psi_R\rangle$ $\rightarrow$ creating phonon, hence **Stokes photon** via spontaneous Raman scattering.

Step 2. Study excitations of optical phonon mode$\rightarrow$**Detecting Stokes and anti-Stokes photons at the two detectors**$\rightarrow$ macroscopic entanglement created and preserved within experimental time scale



Use discord as a means to ask: To what extent duration of entanglement be enhanced, ED be reduced $\rightarrow$ can probe an interesting domain of λ_{CSL}, $GID_{Diósi}$?

Basic Methodology

- Study discord between two diamond crystals (**left(L), right(R)**) for above set-up in presence of **mechanisms responsible for collapse/decoherence**.
- $|0_{L/R}\rangle$, $|1_{L/R}\rangle \rightarrow$ **ground** (rest) and **excited** (oscillating) phonon (crystal) states.
- **Basis**: $|0_L, 0_R\rangle$, $|1_L, 0_R\rangle$, $|0_L, 1_R\rangle$, $|1_L, 1_R\rangle$.
- **Initial condition/wavefunction** [S. Belli, et al., Phys. Rev. A **94**, 2016.]:

$$\Psi_0(t) = \frac{1}{\sqrt{2}}|1_{\mathrm{L}}, 0_{\mathrm{R}}\rangle + \frac{1}{\sqrt{2}}|0_{\mathrm{L}}, 1_{\mathrm{R}}\rangle \tag{11}$$

Basic Methodology *contd*...

- Study the effect of **CSL, GID, ED** on **each crystal**/subsystem→through evolution of ρ.
- Obtain expressions for entropies (**von-Neumann, joint, conditional**)→ expression for **discord as a function of time.**
- Use experimental values for different parameters→ **compare discord for CSL and GID models graphically**→find experimental time where **detection of either CSL or GID** becomes possible.

[Shreya Banerjee, Sayantani Bera and Tejinder P. Singh, Phys. Lett. A **380** (2016)]

Quantum Discord in the presence of CSL

What CSL does?

• Tries to **prevent superposition of two different matter distributions** (rest/oscillating) • Make each crystal **collapse into either of these states**.

• Rewriting master eq for present set-up:

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}(t)] - 2\eta[\hat{q}^L,[\hat{q}^L,\hat{\rho}_t]] - 2\eta[\hat{q}^R,[\hat{q}^R,\hat{\rho}(t)]] \tag{12}$$

$$\text{where, } \eta = \lambda_{\mathrm{CSL}}\frac{N^2}{d^2}\Gamma_\perp\left(\frac{R}{\sqrt{2}\,r_C}\right)\left[1 - e^{-\frac{d^2}{4\,r_C^2}}\right] \tag{13}$$

• $\hat{q}$ →position operators • η →the function of the **mass distribution for the two sublattices (same)** • each sublattice→ cylinder of radius R, width d.

- Expression for **quantum discord** as a function of time:

$$\delta(L:R) = S(\hat{\rho}_R) - S(\hat{\rho}) + \sum_j p_j S(\hat{\Pi}_j \hat{\rho}_L \hat{\Pi}_j)$$
$$= -\ln\left(\frac{1}{2}\right) + \sum_i \sigma_i \ln \sigma_i - \frac{1}{4}\left(1 + e^{-2\Lambda t}\right) \ln\left|\frac{(1 + e^{-2\Lambda t})}{2}\right|$$
$$- \frac{1}{4}\left(1 - e^{-2\Lambda t}\right) \ln\left|\frac{(1 - e^{-2\Lambda t})}{2}\right| \quad (14)$$

- where **the decay/ collapse rate** is $\boxed{\Lambda_{CSL} = \frac{2\eta\hbar}{3\omega m_0}}$
- Quantum discord in the presence of GID/ED: Expression for δ →**same as in CSL** with Λ substituted by $\Lambda_{Diosi}/\Lambda_{env}$ where

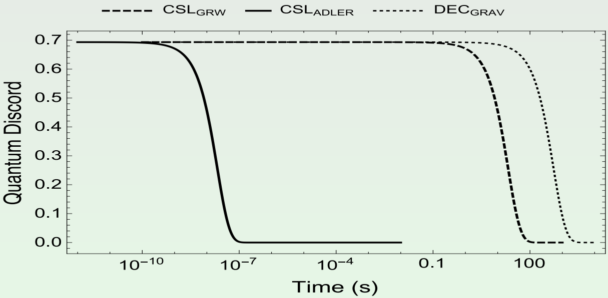
$$\boxed{\Lambda_{Diosi} = \frac{Gm^2}{6\omega R'^3 m_0};\ \Lambda_{env} = \frac{\Gamma\hbar}{3m_0\omega}}$$

[Shreya Banerjee, Sayantani Bera and Tejinder P. Singh, Phys. Lett. A **380** (2016)]

Decay rate constants for different models

Parameters	CSL_{GRW}	CSL_{Adler}	Gravity induced decoherence
$\omega(s^{-1})$	10^{13}	10^{13}	10^{13}
$m(kg)$	10^{-11}	10^{-11}	10^{-11}
N	5×10^{14}	5×10^{14}	5×10^{14}
$R(\mu m)$	1.3427	1.3427	–
$d(mm)$	0.25	0.25	–
$R'(\mu m)$	–	–	6.97
$\lambda_{CSL}(s^{-1})$	10^{-17}	10^{-8}	–
$r_C(m)$	10^{-7}	10^{-7}	–
$\Lambda_{CSL/Diosi}(s^{-1})$	0.0050103	0.50103×10^{7}	0.00019659

INFERENCES



• Discord for **pure Schrödinger evolution** ($\Lambda = 0$)→ system in **pure state** with $\delta = \ln 2 \rightarrow$ **cannot reduce quantum discord**. • **CSL, GID** $\rightarrow$ **effective in reducing quantum discord to zero** • Detection of **CSL**$_{\text{Adler}}$, **CSL**$_{\text{GRW}}$ and GID$_{\text{Diósi}}$ is possible if $t > 10^{-7}$s, 100s and 1500 s.

Bound on CSL parameter using quantum discord

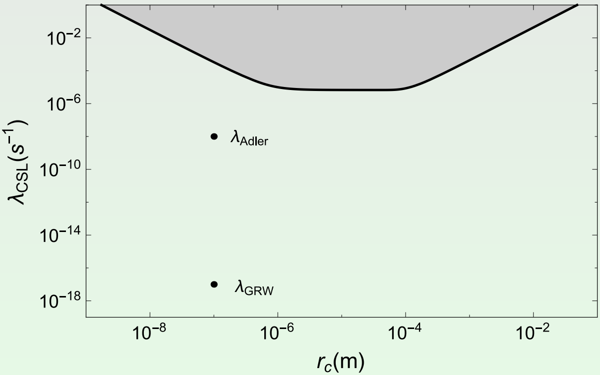
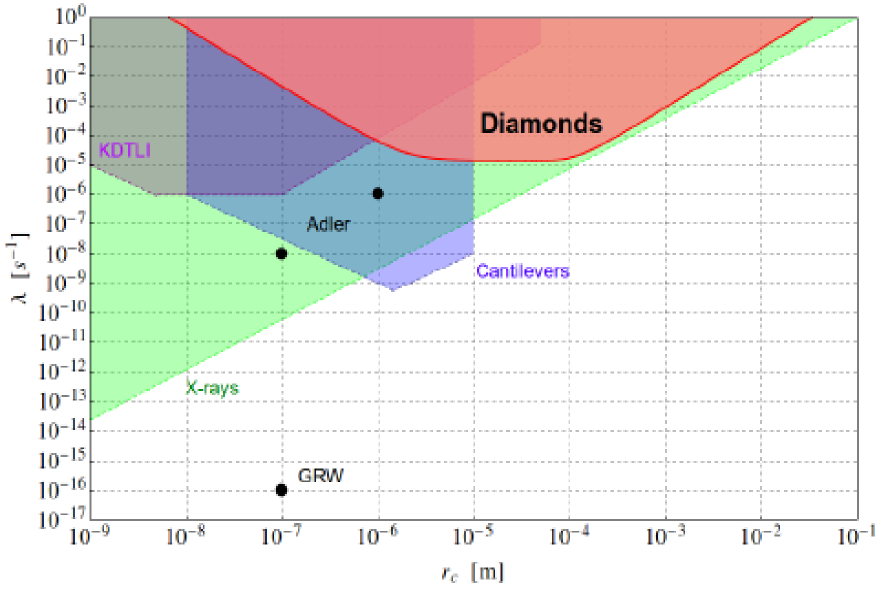


Figure: Figure shows the allowed $\lambda_{CSL} - r_C$ parameter space. We get an upper bound on $\Lambda_{CSL} < 10^{12} s^{-1}$; for $r_C = 10^{-5}$cm, upper bound on $\lambda_{CSL} \approx 10^{-3}\mathrm{s}^{-1}$. Result matches with S. Belli, et al., Phys. Rev. A **94**, 2016.

Summary

- Propose **quantum discord as a tool to compare and contrast collapse model (CSL) and fundamental decoherence** for an experimentally demonstrated macroscopic entanglement.
- Need to increase experimental time significantly in order to detect any of these effects → In particular, detection of **CSL**$_{\text{GRW}}$ and GID seems to be very challenging.
- Get an upper bound on $\Lambda_{CSL} < 10^{12} s^{-1}$; for $r_C = 10^{-5}$cm, $\lambda_{CSL} < 10^{-3}\text{s}^{-1}$.
- **Future Work**:1) apply to experiments where EID is important. 2) employ a discord analysis to the **inflationary universe** [J. Martin, V. Vennin, Phys. Rev. D **93**, 2016.].



Diamonds
KDTLI
Adler
Cantilevers
X-rays
GRW
λ [s^{-1}]
r_c [m]
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}