An Investigation of the Influence of Gravity on Macroscopic Mechanical Quantum Superpositions

Miles Blencowe
Dartmouth College

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1769

fq(x) Foundational Questions Institute

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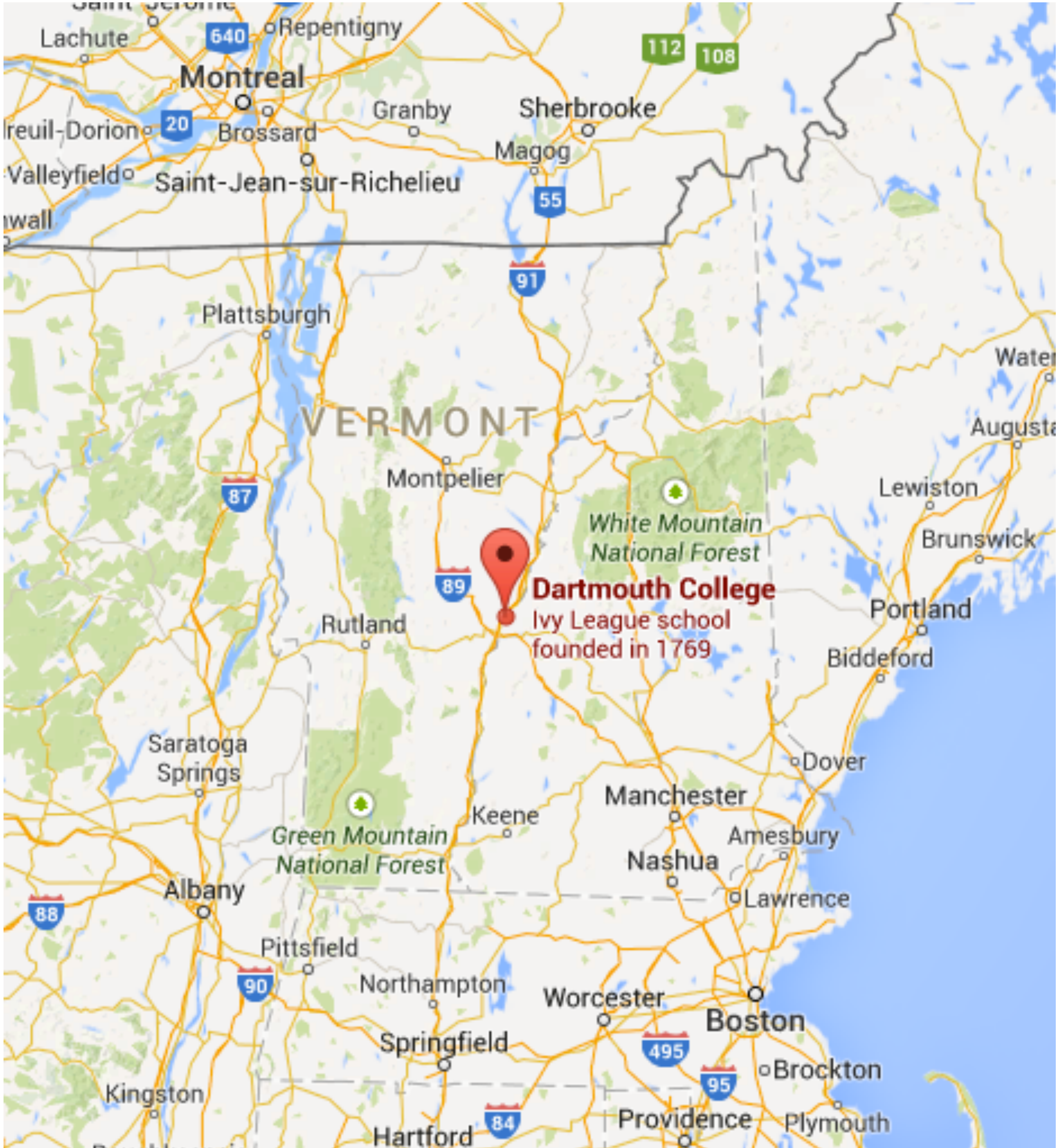
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Dartmouth College
Ivy League school
founded in 1769
Rutland
Portland
Biddeford
Saratoga
Springs
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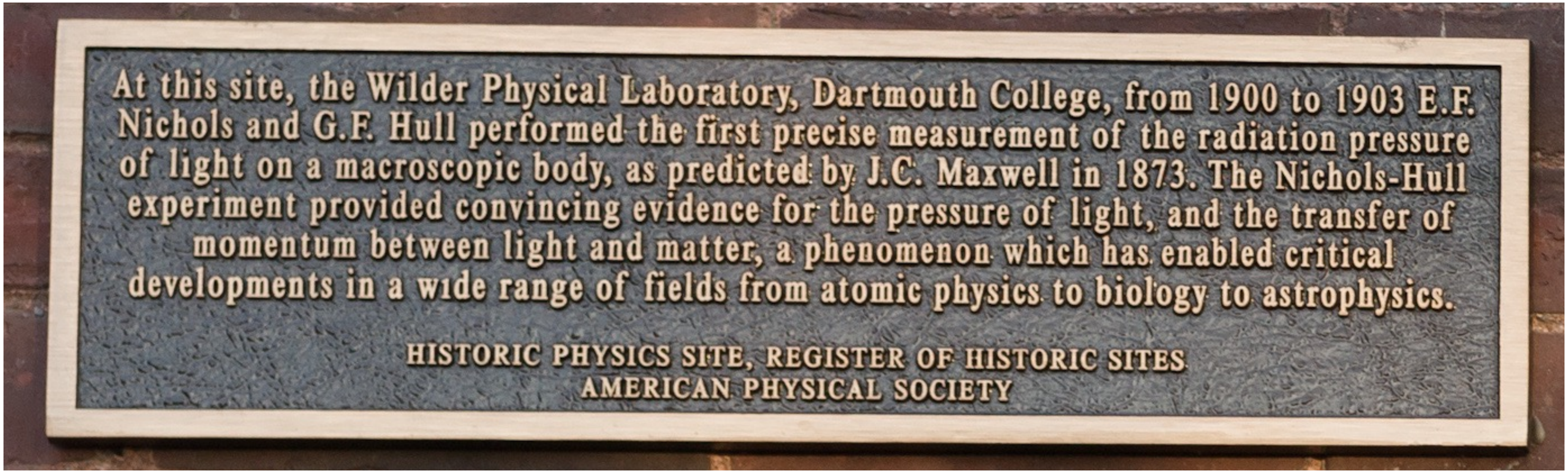
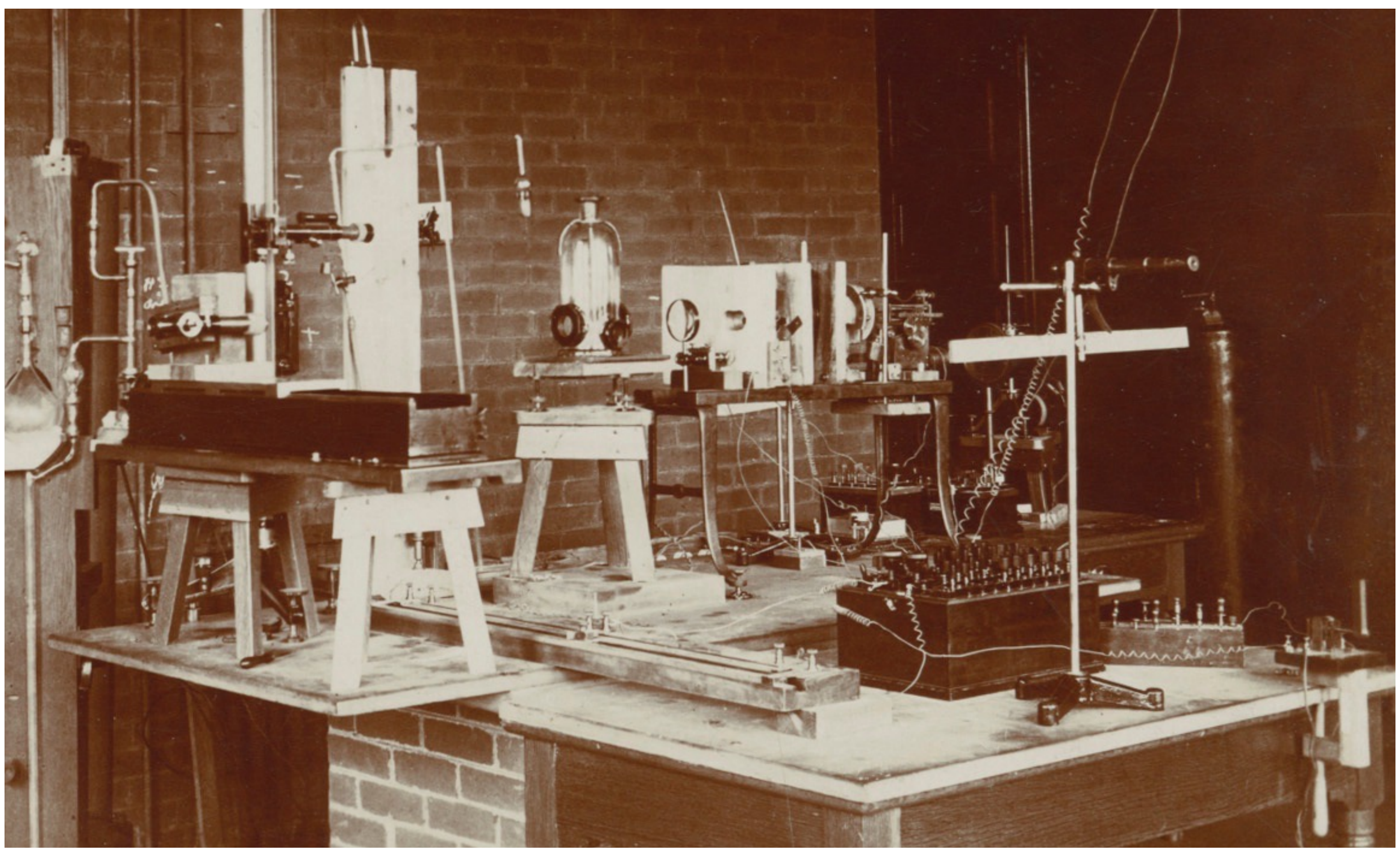

At this site, the Wilder Physical Laboratory, Dartmouth College, from 1900 to 1903 E.F. Nichols and G.F. Hull performed the first precise measurement of the radiation pressure of light on a macroscopic body, as predicted by J.C. Maxwell in 1873. The Nichols-Hull experiment provided convincing evidence for the pressure of light, and the transfer of momentum between light and matter, a phenomenon which has enabled critical developments in a wide range of fields from atomic physics to biology to astrophysics.
HISTORIC PHYSICS SITE, REGISTER OF HISTORIC SITES
AMERICAN PHYSICAL SOCIETY



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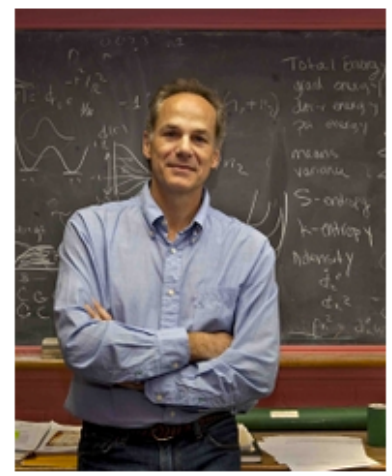

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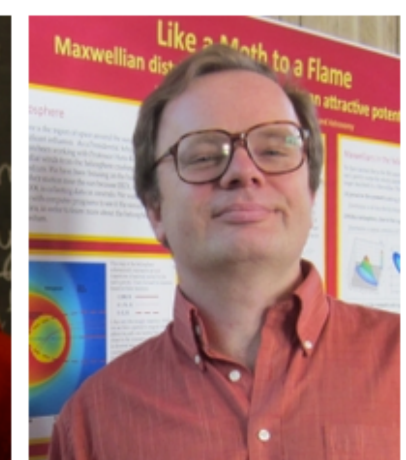

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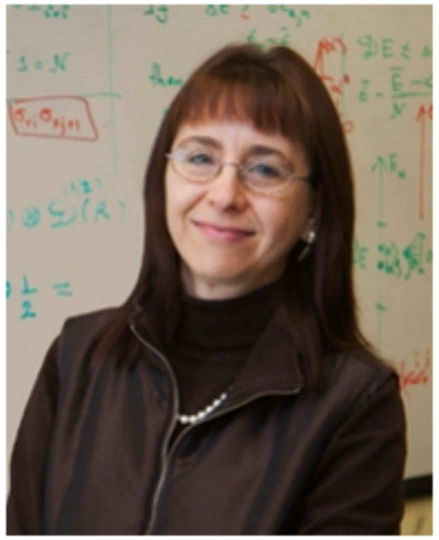

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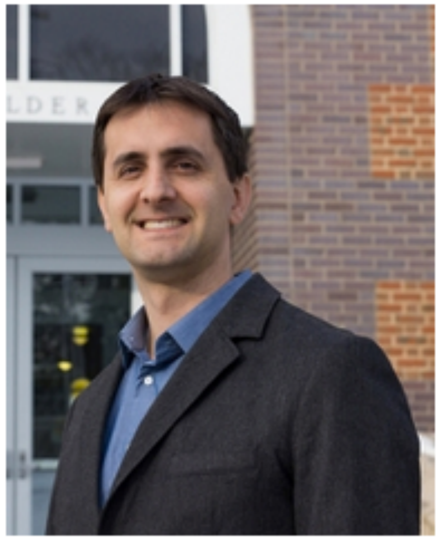


Kevin Wright
Assistant Professor of Physics and Astronomy

Current Conditions

-25°F
RealFeel®: -14°F

Humidity: 77%
Dew Point: -30°F
Cloud Cover: 35%
Visibility: 10 Miles

AccuWeather.com 5-Day Forecast

Tuesday	Wednesday	Thursday	Friday	Saturday
HI: 17°F	HI: 26°F	HI: 17°F	HI: 20°F	HI: 26°F
LO: 2°F	LO: -9°F	LO: -11°F	LO: -14°F	LO: -1°F

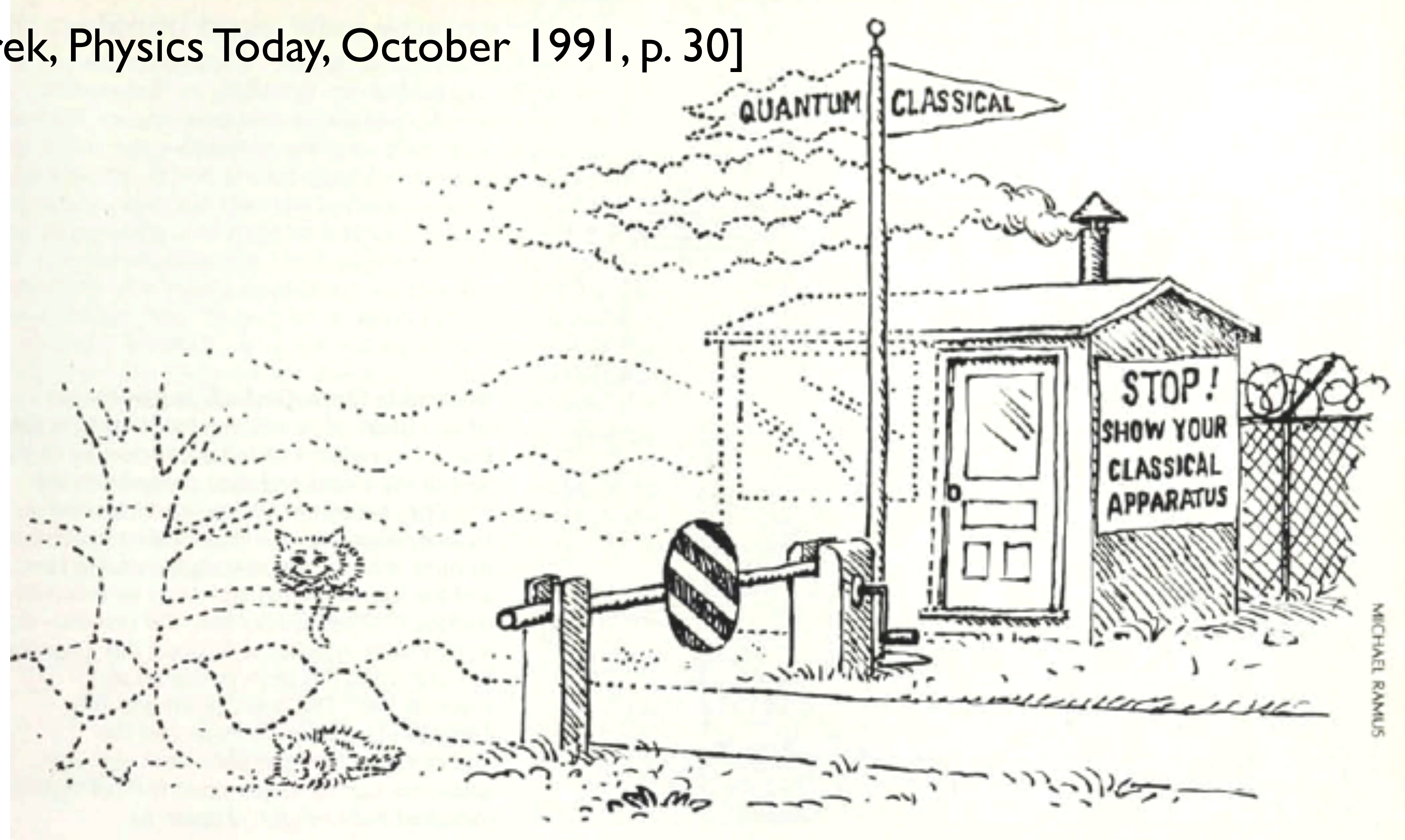
Forecast Details | Hour-By-Hour | 15-Day Forecast

(Feb 24, 2015)

Does quantum superposition principle apply at arbitrarily large mass/distance scales?

Does gravity place a fundamental limit on these scales?

[W. H.Zurek, Physics Today, October 1991, p. 30]

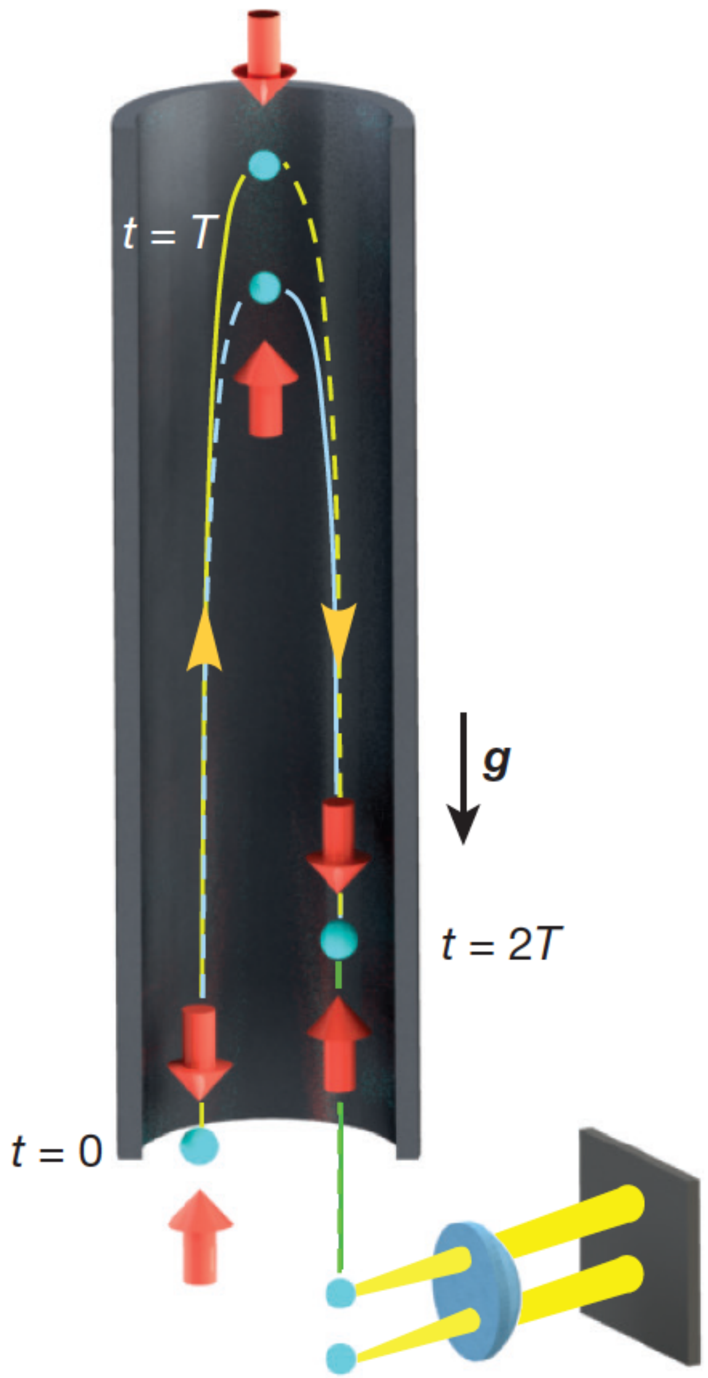


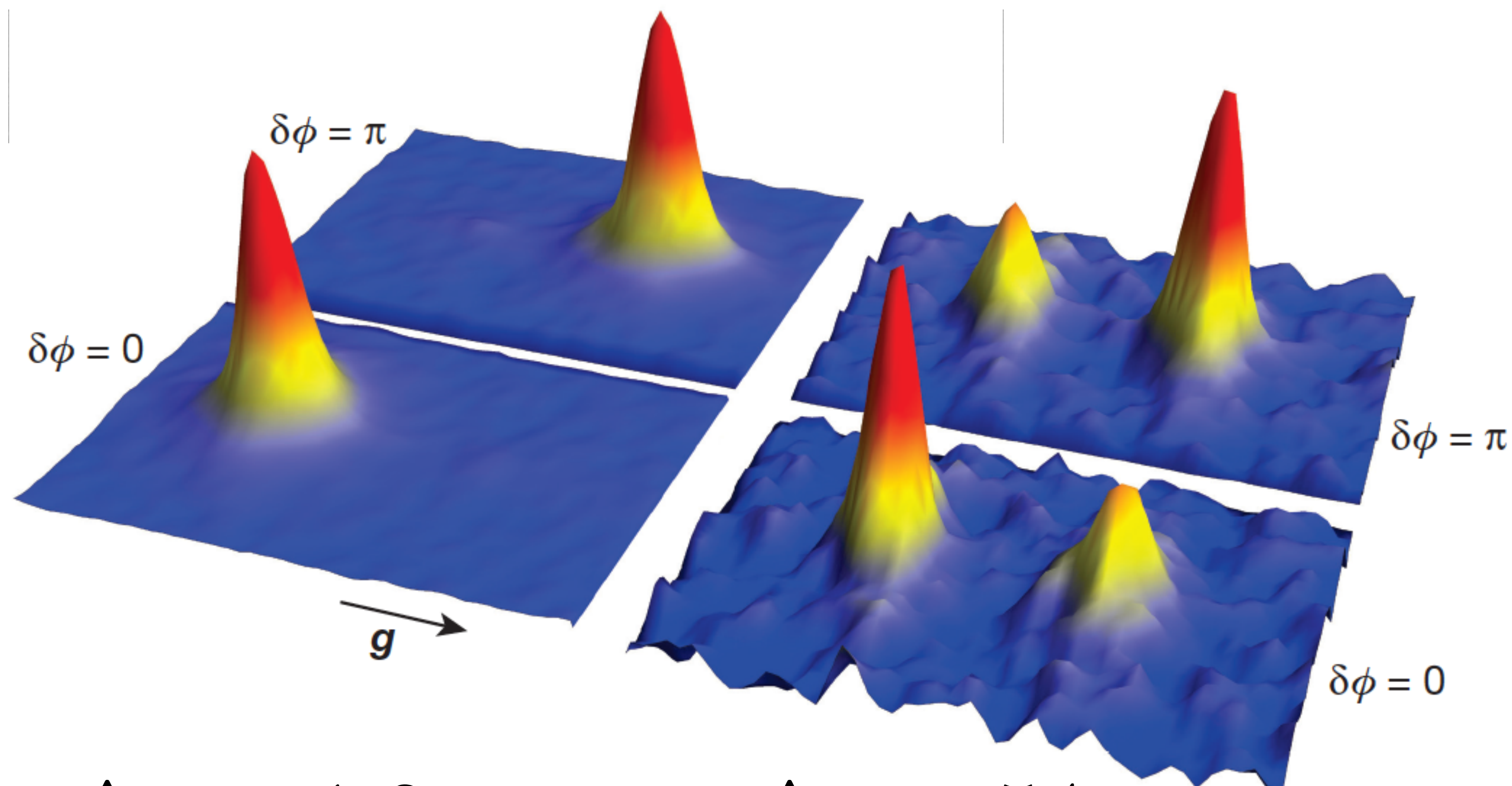
Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

Quantum superposition at the half-metre scale

T. Kovachy[1], P. Asenbaum[1], C. Overstreet[1], C. A. Donnelly[1], S. M. Dickerson[1], A. Sugarbaker[1], J. M. Hogan[1] & M. A. Kasevich[1]

[Nature **528**, 530 (2015)]





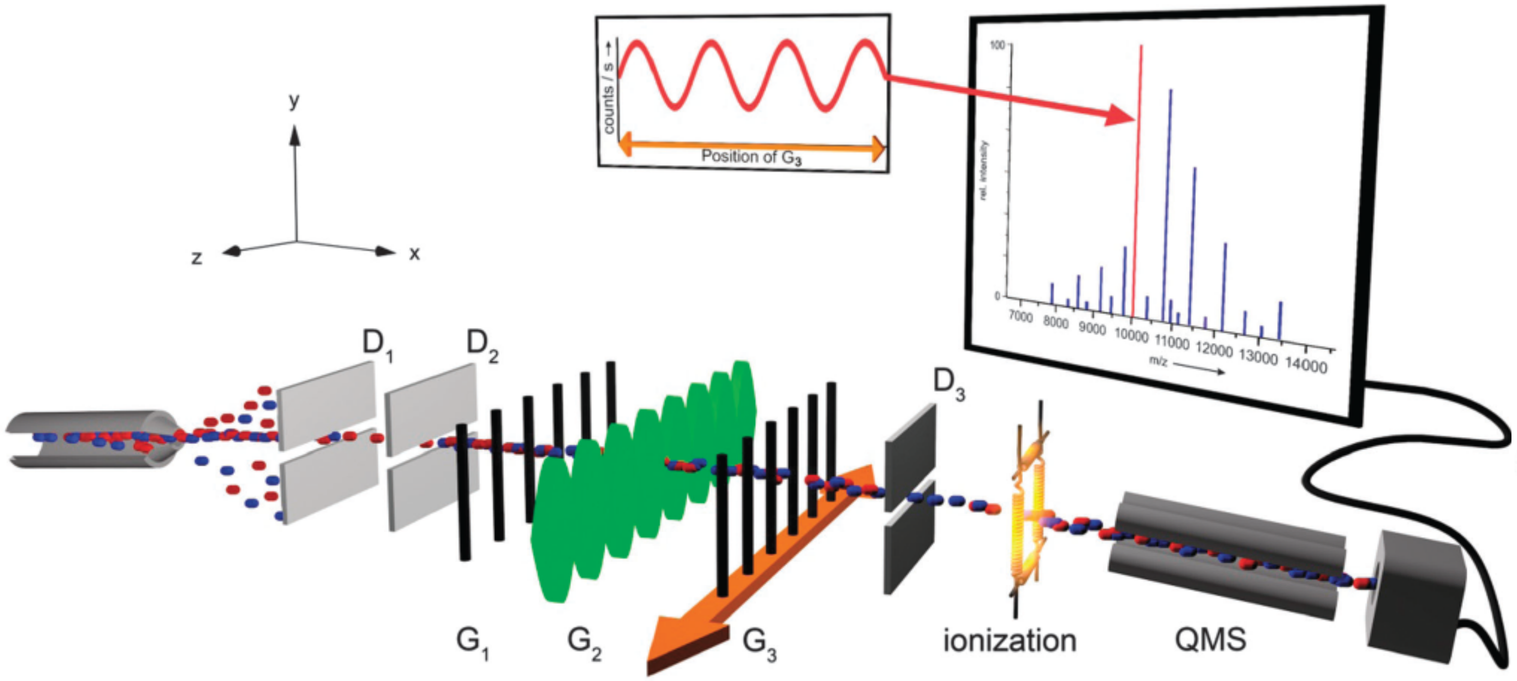
$\Delta z \approx 1.2$ cm $\qquad \Delta z \approx 54$ cm

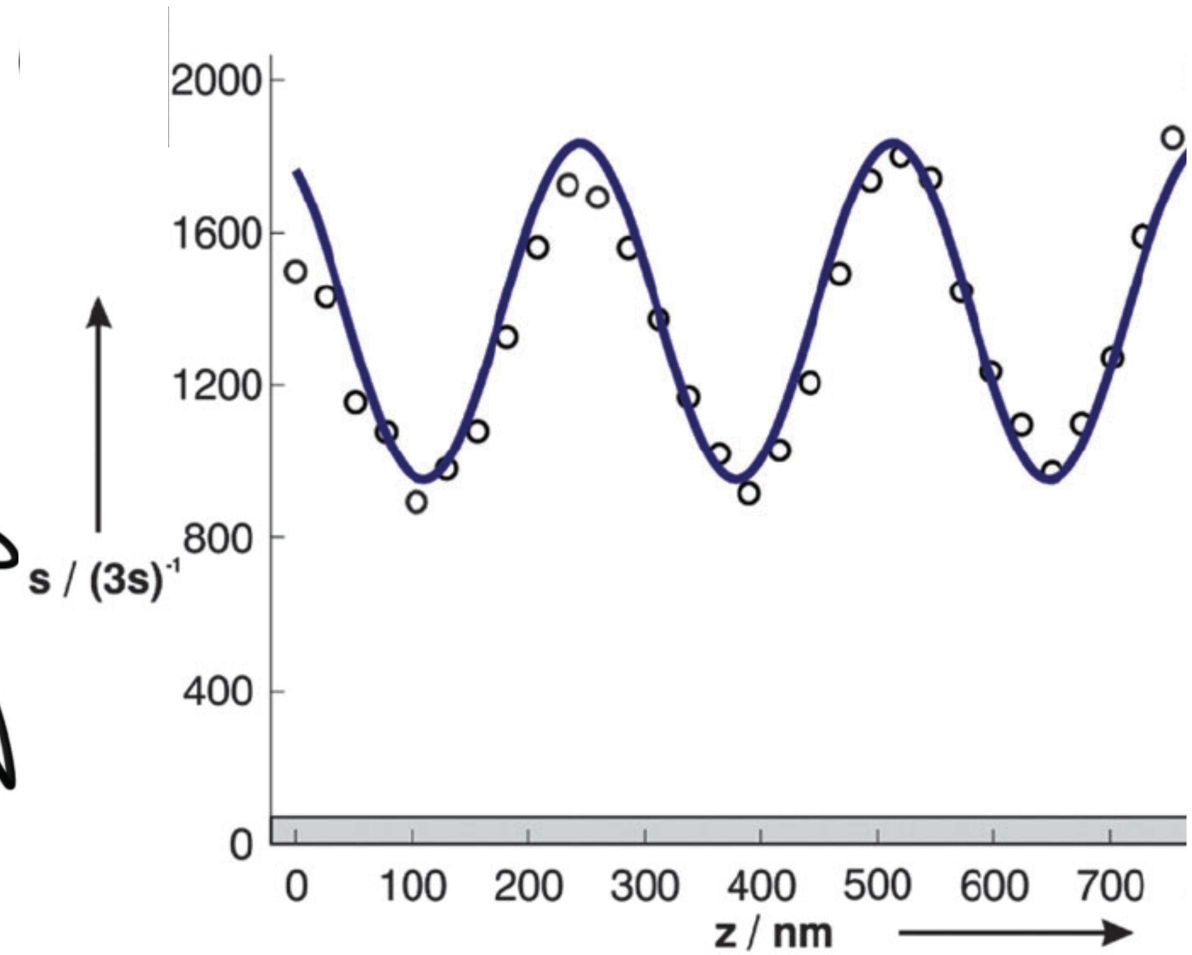
$$M \approx 87\, m_n$$

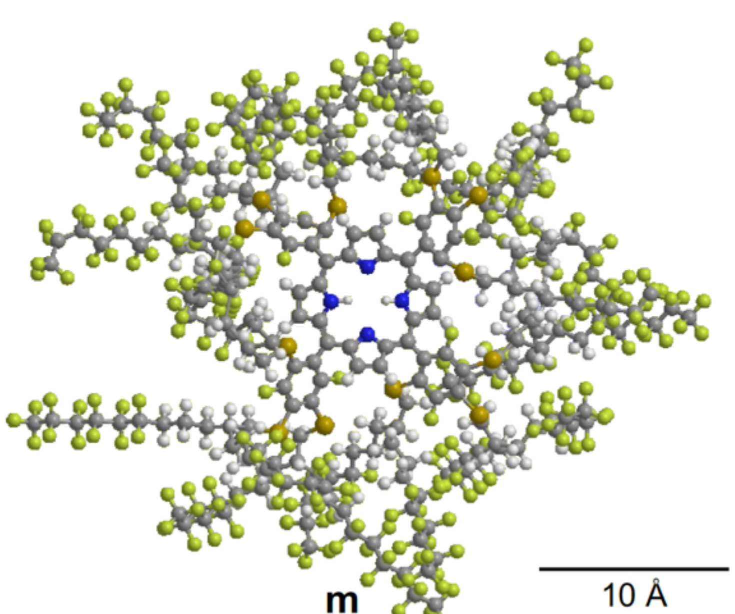
Matter–wave interference of particles selected from a molecular library with masses exceeding 10 000 amu

Sandra Eibenberger,[a] Stefan Gerlich,[a] Markus Arndt,*[a] Marcel Mayor*[bc] and Jens Tüxen[b]

[Phys. Chem. Chem. Phys., 15, 14696 (2013)]





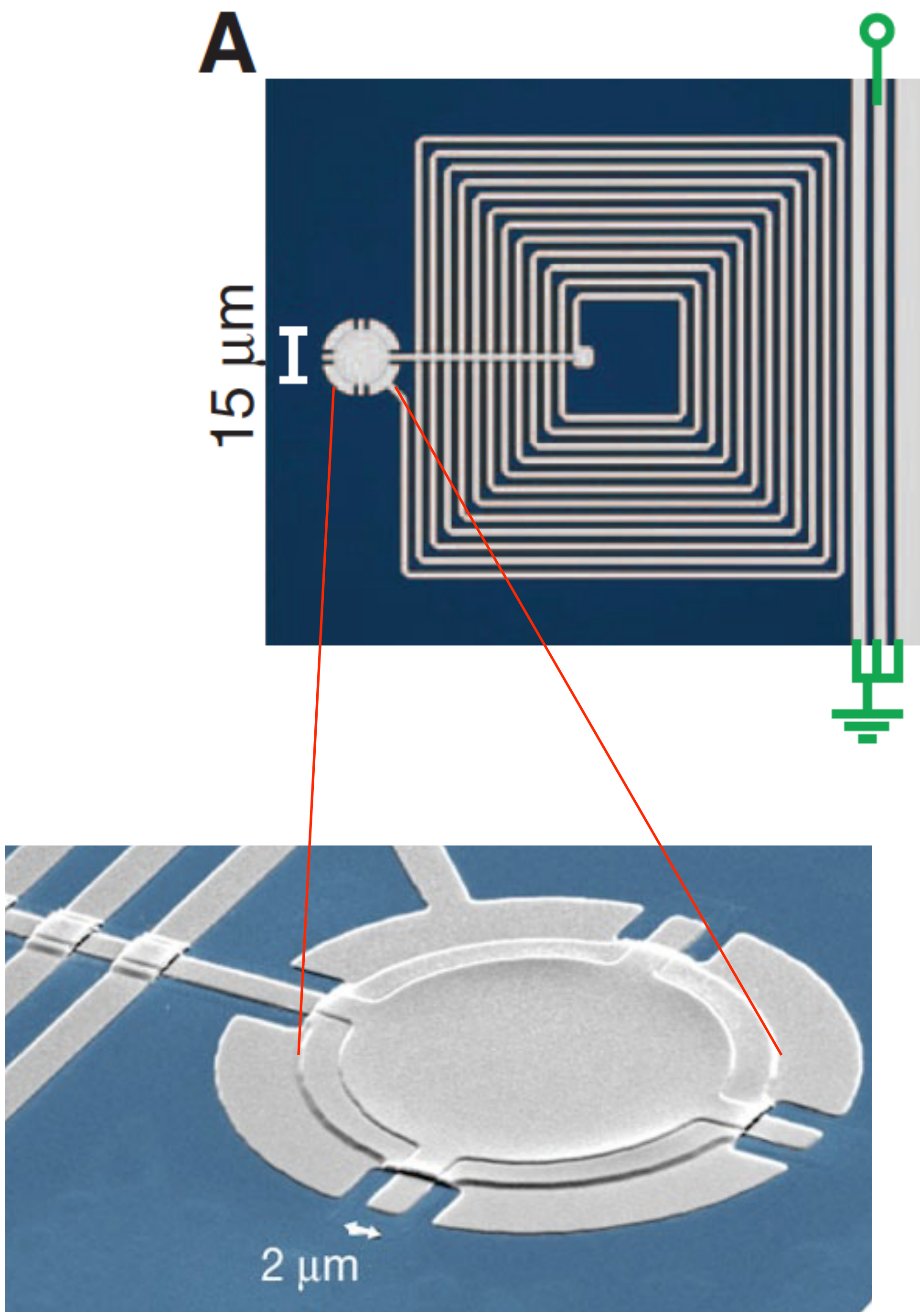


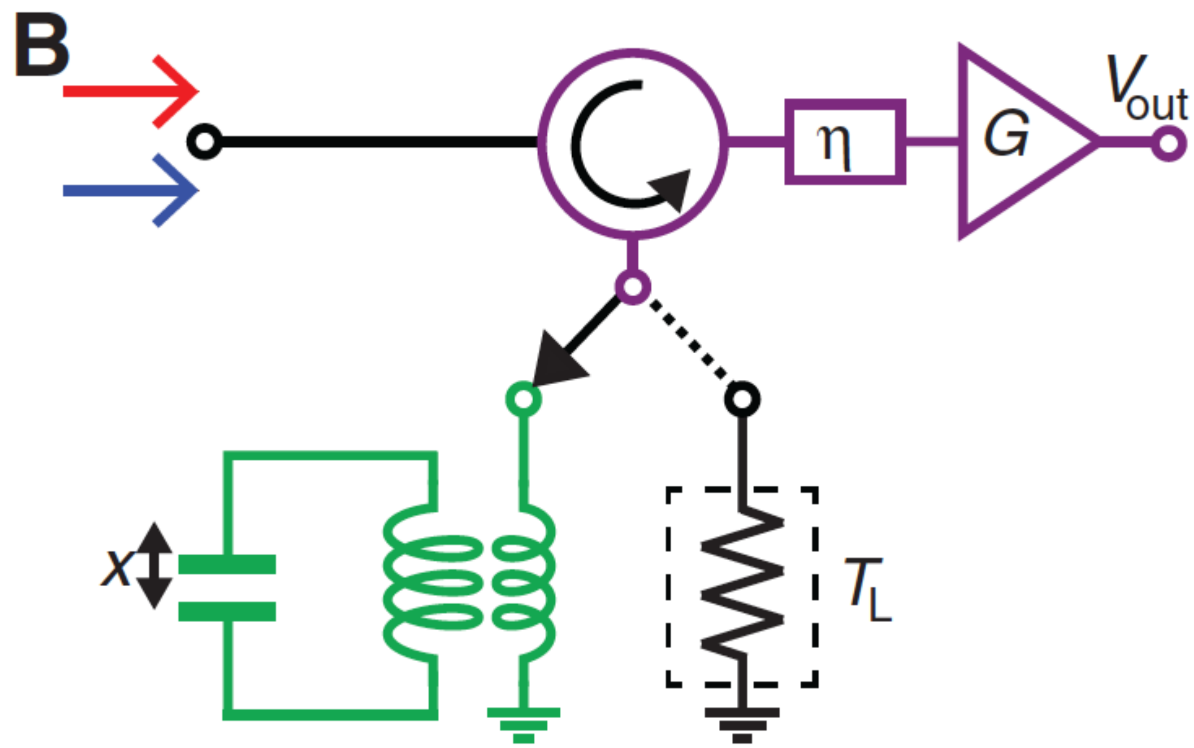
$$M \approx 10^4\, m_n \qquad \Delta z \approx 266\,\mathrm{nm}$$

Entangling Mechanical Motion with Microwave Fields

T. A. Palomaki,[1,2]* J. D. Teufel,[3] R. W. Simmonds,[3] K. W. Lehnert[1,2]

[Science **342**, 710 (2013)]





$$M \approx 10^{13}\, m_n \qquad \Delta z \sim 10\,\mathrm{fm}$$

VOLUME 34, NUMBER 23 PHYSICAL REVIEW LETTERS p. 1472 9 JUNE 1975

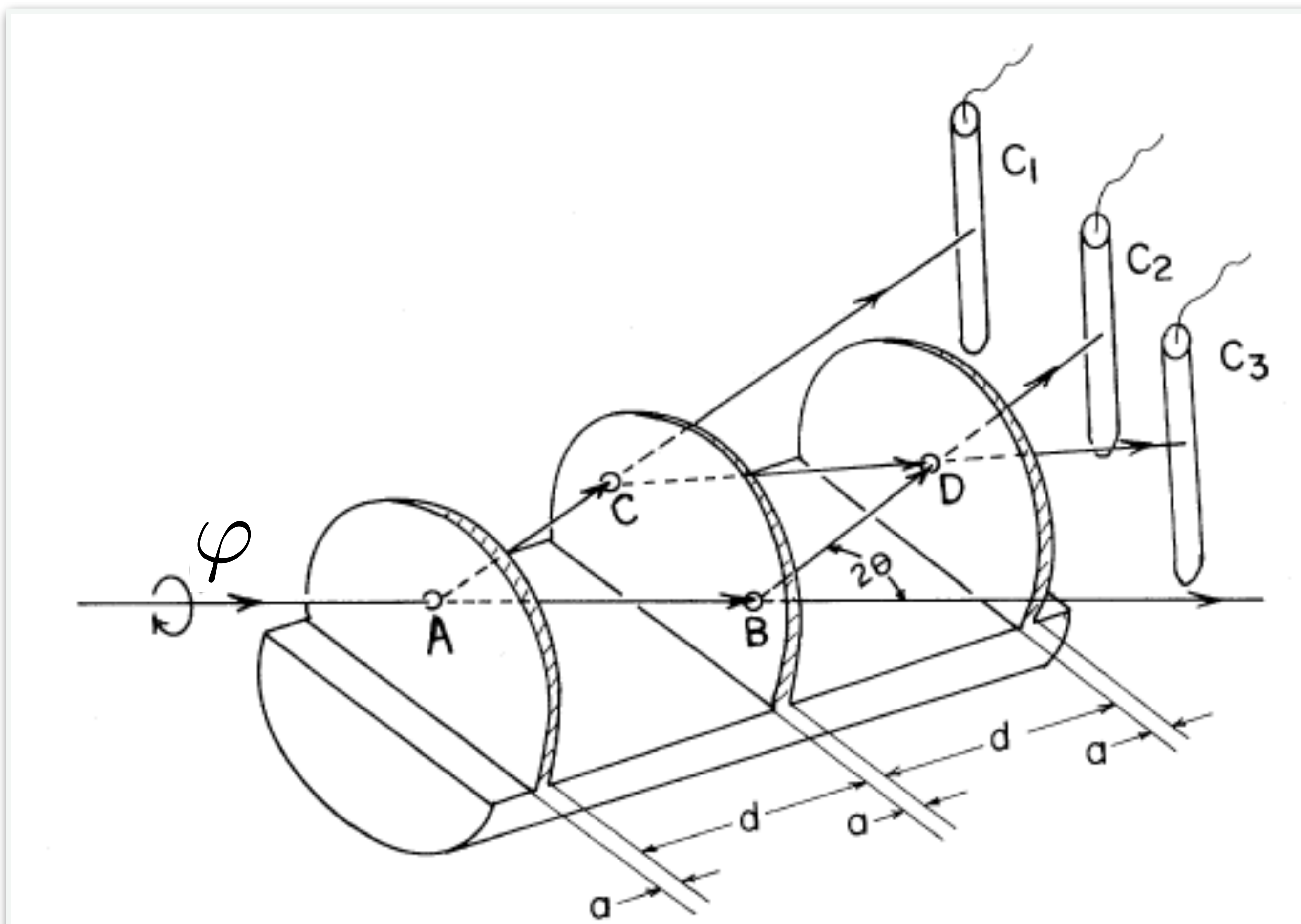
Observation of Gravitationally Induced Quantum Interference*

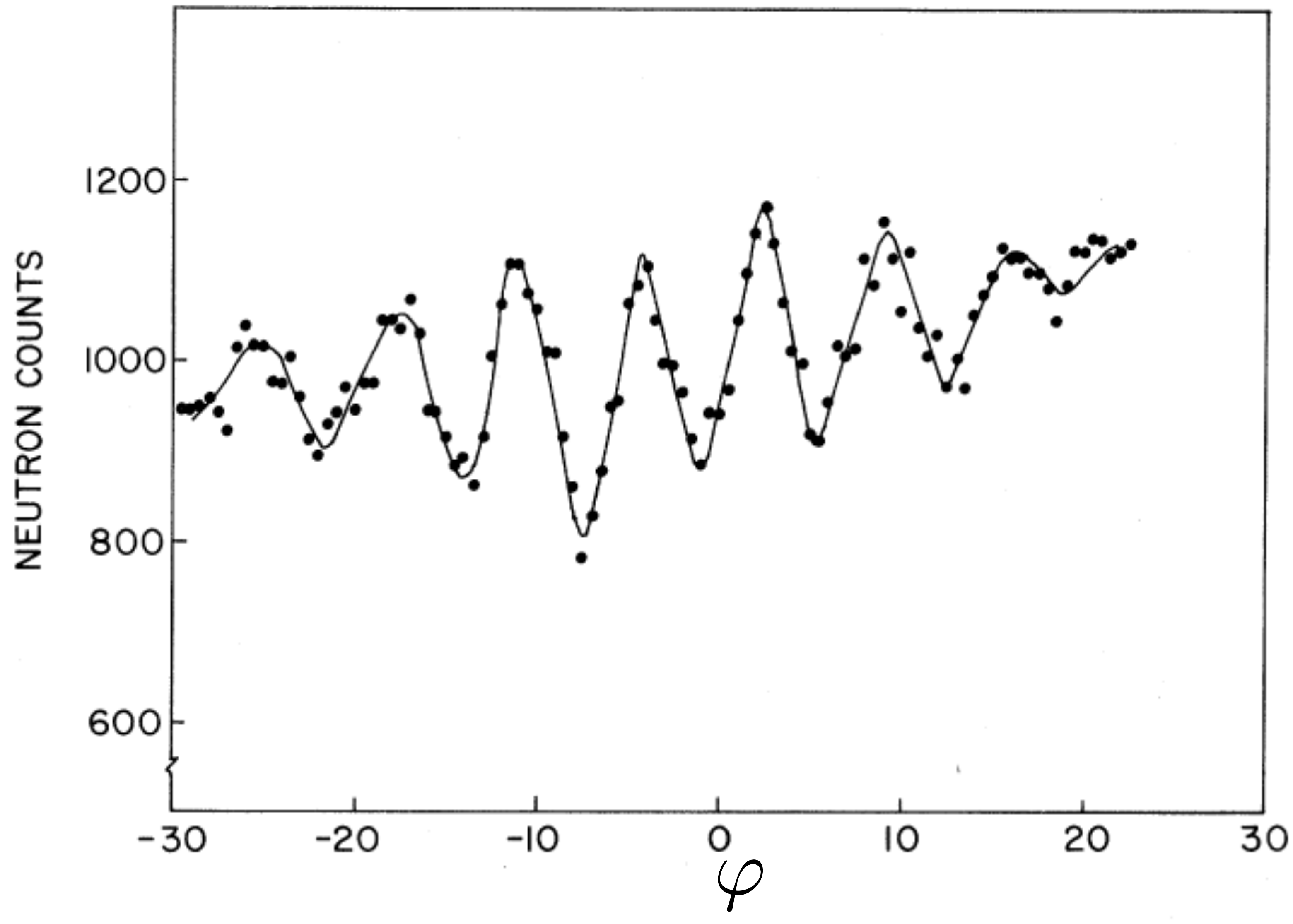
R. Colella and A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907

and

S. A. Werner
Scientific Research Staff, Ford Motor Company, Dearborn, Michigan 48121
(Received 14 April 1975)

We have used a neutron interferometer to observe the quantum-mechanical phase shift of neutrons caused by their interaction with Earth's gravitational field.





The difference count, $I_2 - I_3$, as a function of interferometer rotation angle

$$I_2 - I_3 = \alpha + \beta \cos\left(\frac{m_n g \sin\varphi A}{\hbar v}\right)$$

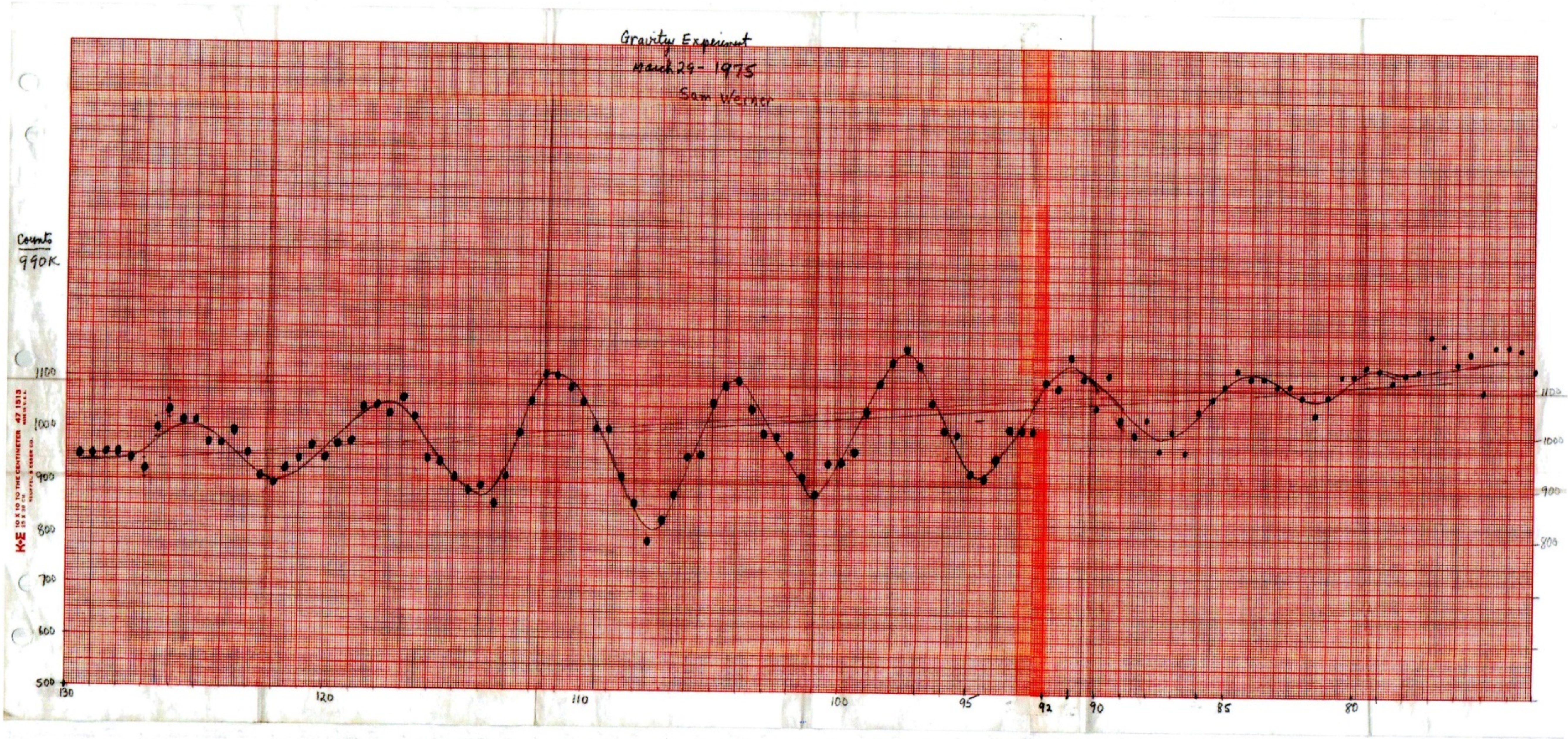
$\Delta\phi$

relative phase shift between interfering beams

α, β constants

A area enclosed by interfering beam paths m_n neutron mass v neutron velocity

First observation of Gravitationally-Induced Quantum Interference



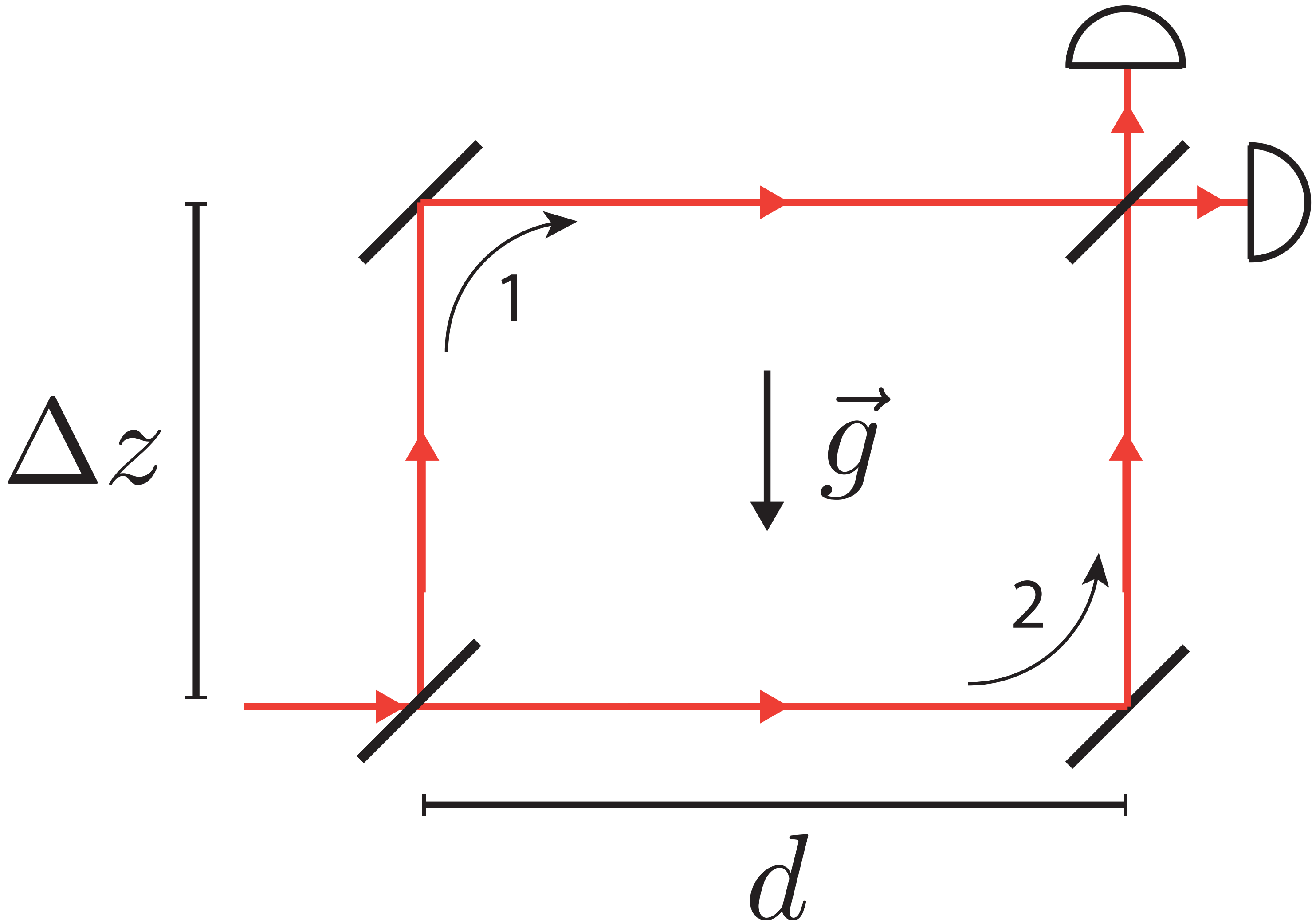
Original plot of COW experiment data taken at beam port F at the Ford Nuclear Reactor, Phoenix Memorial Laboratory, The University of Michigan, Ann Arbor.

Sam Werner, Albert Overhauser, Roberto Colella

March, 1975

A relativistic proper time formulation of the relative phase shift:

[D. M. Greenberger, Rev. Mod. Phys. **55**, 875 (1983)]



$$\Delta\phi = \frac{1}{\hbar}\int_{\mathrm{path}\,1} m_n c^2 d\tau - \frac{1}{\hbar}\int_{\mathrm{path}\,2} m_n c^2 d\tau = \frac{m_n c^2}{\hbar}\Delta\tau$$

m_n neutron rest mass $\qquad$ τ proper time

$$\Delta\phi = \frac{1}{\hbar}\int_{\mathrm{path}\,1} m_n c^2 d\tau - \frac{1}{\hbar}\int_{\mathrm{path}\,2} m_n c^2 d\tau \quad = \frac{m_n c^2}{\hbar}\Delta\tau$$

$$d\tau = \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\,dt/c\,; \quad g_{00} \approx -(1+2gz/c^2)\,; \quad g_{ij} = \delta_{ij}$$

$$m_n c^2 d\tau \approx \left(m_n c^2 + m_n g z - \frac{1}{2} m_n v^2\right) dt$$

$$\Delta\phi \approx \frac{m_n g \Delta z d}{\hbar v} = \frac{m_n g A}{\hbar v}$$

Does gravity place a fundamental limit on the quantum superposition principle?

$$\Delta\phi = \frac{1}{\hbar}\int_{\mathrm{path}\,1} Mc^2 d\tau - \frac{1}{\hbar}\int_{\mathrm{path}\,2} Mc^2 d\tau$$

Composite dynamical system: M fluctuates

Gravity is dynamical: proper time τ fluctuates

Heuristically:

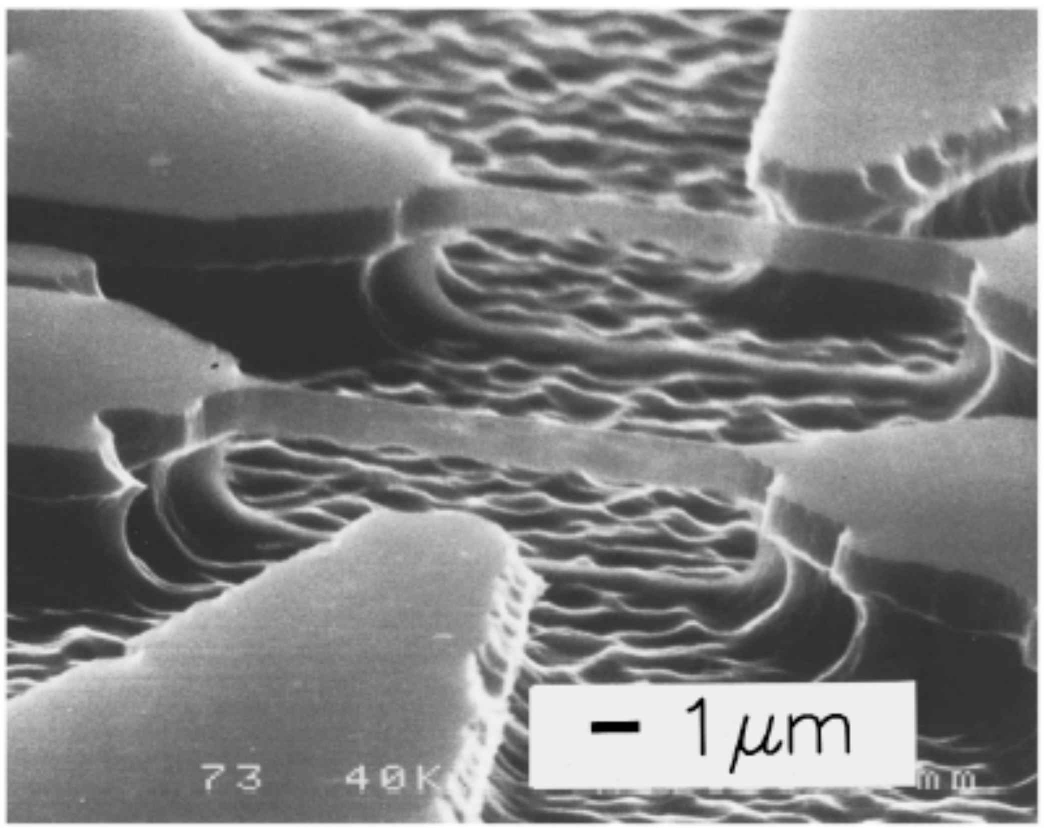
$$\cos(\Delta\phi) \to \langle\cos(\Delta\phi)\rangle = \underbrace{e^{-\langle\Delta\phi^2\rangle/2}}_{\text{decoherence}}\cos(\langle\Delta\phi\rangle)$$

Learning from mechanical quantum analogues (via the equivalence principle)…

Mesoscopic mechanical resonators as quantum noninertial reference frames

B. N. Katz,[1,*] M. P. Blencowe,[1] and K. C. Schwab[2,3]

[Phys. Rev. A **92**, 042104 (2015)]



vibration frequency:
$\Omega \sim 2\pi \times 100 \text{ MHz}$

amplitude: $X_0 \sim 1 \text{ nm}$

max acceleration:
$|\ddot{X}(\pi n/\Omega)| = \Omega^2 X_0 \sim 10^9 \text{ ms}^{-2}$

A particle attached to the vibrating surface experiences a g-force comparable to the surface gravity of a neutron star!

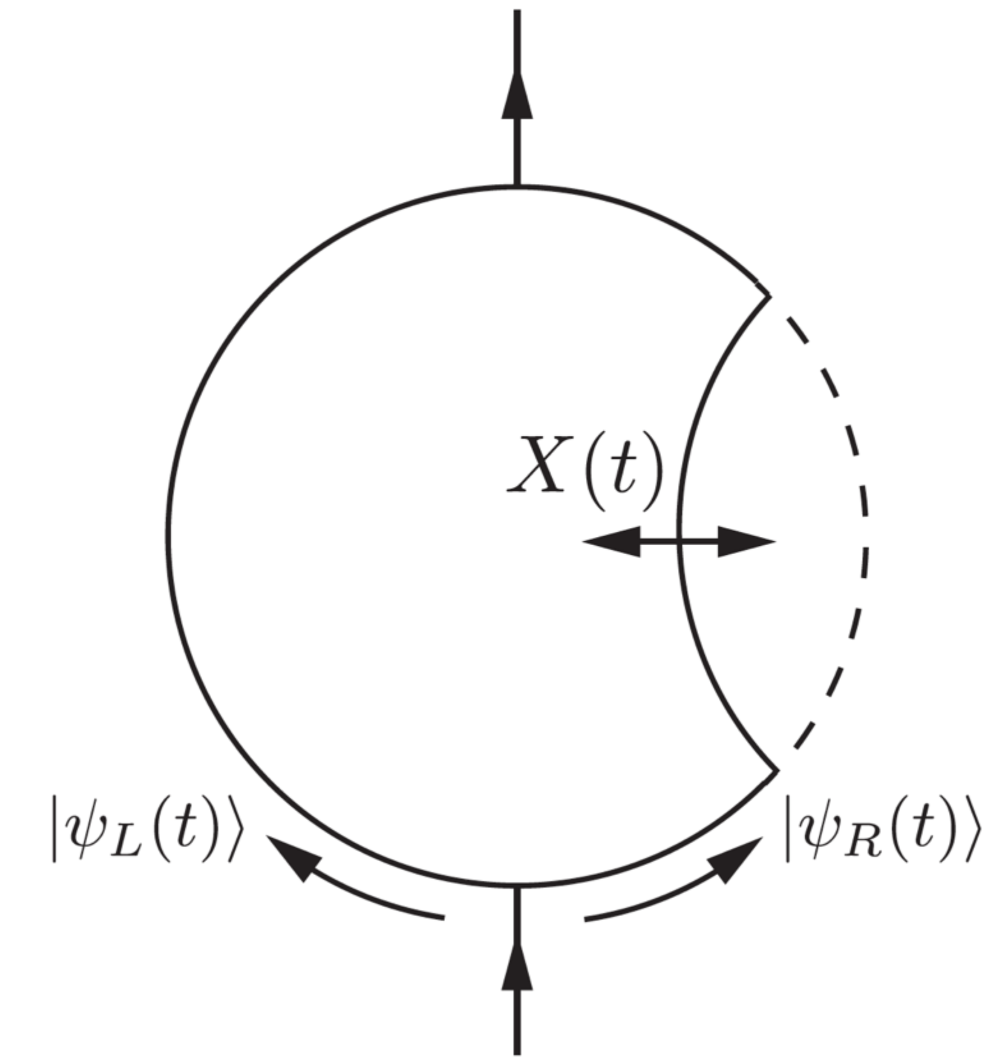


FIG. 1. Scheme of the two-path atom interferometer with noninertial, vibrating wire frame forming part of the right path.

Suppose vibrating wire frame in a quantum state: $\hat{X}(t)$

$\langle\cos(\Delta\phi)\rangle \to 0$ when atom (particle) dwell time in frame satisfies

$$T \gg \frac{\hbar}{\Delta E_{\mathrm{atom}}}$$

Uncertainty in atom's energy due to vibrating frame quantum uncertainty

Alternative, relativistic "twin paradox" formulation for decoherence due to quantum reference frame?

Formally,

$$\langle \cos(\Delta\phi) \rangle = \left\langle \mathcal{T} e^{-i\frac{mc^2}{\hbar}\left[\int_0^T dt (d\hat{\tau}_R/dt) - T\right]} \right\rangle$$

$\hat{\tau}_R$ quantum proper time operator

Now back to gravity…

Wish to quantitatively evaluate the effect of gravity on macroscopic mass/energy quantum superpositions

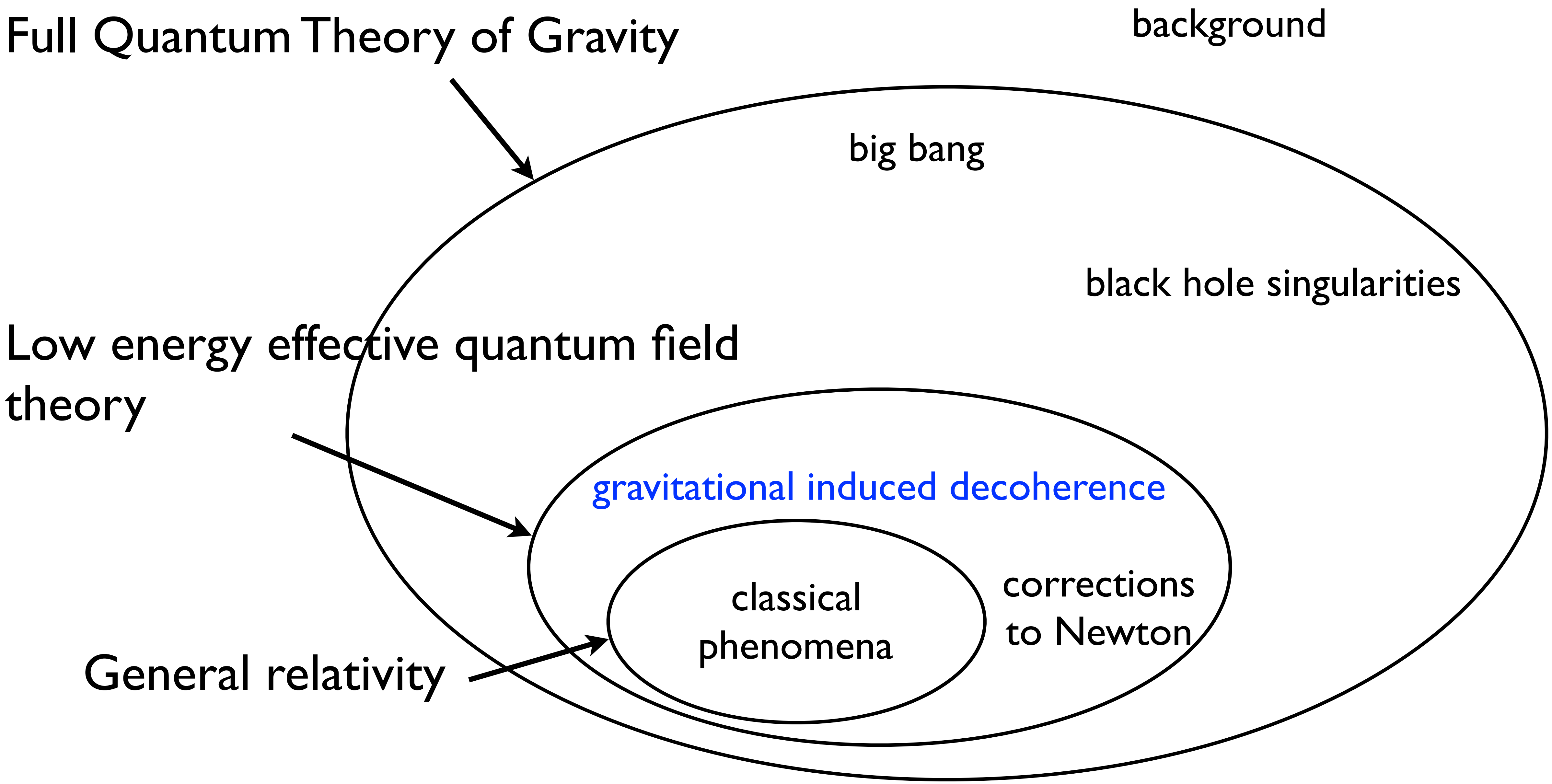
Philosophy: view standard, perturbative quantum field approach as an effective theory, valid at low energies.

[J.F. Donoghue, PRD **50**, 3874 (1994); arXiv:1209.3511]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\hat{g}_{\mu\nu} = g^{(0)}_{\mu\nu} + \hat{h}_{\mu\nu}$$

e.g., Minkowski or weak curvature background



Ultimate Decoherence Border for Matter-Wave Interferometry

Brahim Lamine,* Rémy Hervé, Astrid Lambrecht, and Serge Reynaud

[Phys. Rev. Lett. **96**, 050405 (2006)]

Effective Field Theory Approach to Gravitationally Induced Decoherence

M. P. Blencowe*

[Phys. Rev. Lett. 111, 021302 (2013)]

A master equation for gravitational decoherence: probing the textures of spacetime

C Anastopoulos[1] and B L Hu[2]

[Class. Quantum Grav. **30**, 165007 (2013)]

Universal decoherence due to gravitational time dilation

Igor Pikovski[1,2,3,4★], Magdalena Zych[1,2,5], Fabio Costa[1,2,5] and Časlav Brukner[1,2]

[Nature Phys. 11, 668 (2015)]

Consider massive scalar field (composite matter system) interacting with gravity (environment):

Action

$$S\left[g_{\mu\nu},\varphi\right] = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}R - \frac{1}{2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \left(\frac{mc}{\hbar}\right)^2\varphi^2\right]$$

where $\kappa = \sqrt{32\pi G/c^3}$ is the gravitational coupling constant.

Assume metric deviations from flat (Minkowski) spacetime are small: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

Expand action to second order in *h*: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

$$S\left[h_{\mu\nu},\varphi\right] \approx S_{\rm sys}\left[\varphi\right] + S_{\rm env}\left[h_{\mu\nu}\right] + S_{\rm int}\left[h_{\mu\nu},\varphi\right]$$

where

$$S_{\rm sys} = -\frac{1}{2}\int d^4x \left[\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \left(\frac{mc}{\hbar}\right)^2\varphi^2\right]$$

$$S_{\rm env} = \int d^4x \left(-\frac{1}{2}\partial^\alpha h^{\mu\nu}\partial_\alpha h_{\mu\nu} + \partial_\nu h^{\mu\nu}\partial^\alpha h_{\mu\alpha} - \partial_\mu h\partial_\nu h^{\mu\nu} + \frac{1}{2}\partial^\mu h\partial_\mu h\right)$$

$$S_{\rm int} = \int d^4x \left(\frac{\kappa}{2}T^{\mu\nu}(\varphi)h_{\mu\nu} + \cancel{\frac{\kappa^2}{4}U^{\mu\nu\alpha\beta}(\varphi)h_{\mu\nu}h_{\alpha\beta}}\right)$$

first order in *h*

second order in *h*: graviton scattering, emission and absorption (neglect)

energy momentum tensor of scalar field

Now wish to determine $\hat{\rho}_{\rm sys}(t)$, assuming gravity environment in a thermal equilibrium state.

Use closed time path integral approach:

$$\hat{\rho}_{\rm sys}[\varphi, \varphi', t]$$
$$= \int d\varphi_0 d\varphi_0' \hat{\rho}_{\rm sys}[\varphi_0, \varphi_0', 0] \int_{\varphi_0}^{\varphi} [d\varphi^+] \int_{\varphi_0'}^{\varphi'} [d\varphi^-] \exp\left\{ i \left(S_{\rm sys}[\varphi^+] - S_{\rm sys}[\varphi^-] + S_{\rm IF}[\varphi^+, \varphi^-] \right) / \hbar \right\}$$

$S_{\rm IF}$ is the "Feynman-Vernon influence action": gives affect of gravity environment on scalar matter system. Straightforward to evaluate for $S_{\rm int}$ neglecting quadratic in *h* term:

$$S_{\rm int} = \int d^4x \left(\frac{\kappa}{2} T^{\mu\nu}(\varphi) h_{\mu\nu} + \cancel{\frac{\kappa^2}{4} U^{\mu\nu\alpha\beta}(\varphi)} h_{\mu\nu} h_{\alpha\beta} \right)$$

Evaluate S_{IF} to lowest, quadratic order in κ with harmonic gauge fixing term inserted in S_{env}. Obtain Born-approximated master equation for the scalar system:

$$\partial_t \rho_{\rm sys}(t) = -i[H_{\rm sys}, \rho_{\rm sys}(t)]$$
$$- \int_0^t d\tau \int d\mathbf{r} d\mathbf{r}' \Big\{ N(\mathbf{r}-\mathbf{r}', \tau)\Big(2[T_{\mu\nu}(\mathbf{r}), [T^{\mu\nu}(\mathbf{r}', -\tau), \rho_{\rm sys}(t)]] - [T_\mu{}^\mu(\mathbf{r}), [T_\nu{}^\nu(\mathbf{r}', -\tau), \rho_{\rm sys}(t)]]\Big)$$
$$-iD(\mathbf{r}-\mathbf{r}', \tau)\Big(2[T_{\mu\nu}(\mathbf{r}), \{T^{\mu\nu}(\mathbf{r}', -\tau), \rho_{\rm sys}(t)\}] - [T_\mu{}^\mu(\mathbf{r}), \{T_\nu{}^\nu(\mathbf{r}', -\tau), \rho_{\rm sys}(t)\}]\Big)\Big\}$$

where the noise and dissipation kernels are

$$N(\vec{r}, t) = \left(\frac{\kappa}{4}\right)^2 \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{\omega} \cos\omega t[1 + 2n(\omega)]$$

$$\omega = ck$$

$$n(\omega) = \frac{1}{\exp(\hbar\omega/k_B T) - 1}$$

$$D(\vec{r}, t) = \left(\frac{\kappa}{4}\right)^2 \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{\omega} \sin\omega t$$

$$\kappa^2 \propto G$$

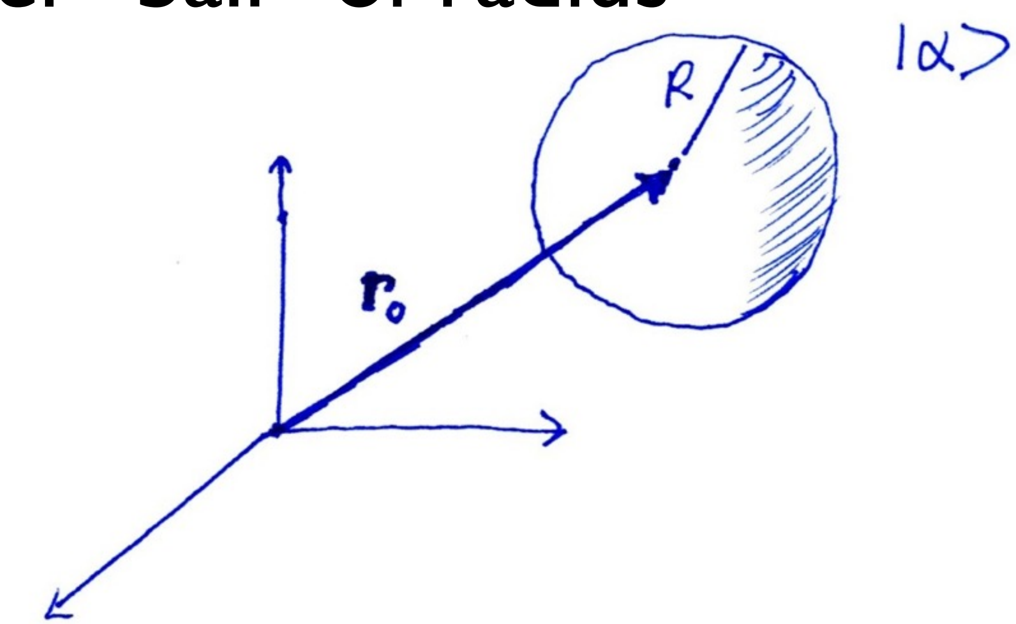
Construct initial quantum state for scalar field system:

Coherent state $$|\alpha\rangle = \exp\left[-\frac{1}{2}\int d\mathbf{k}|\alpha(\mathbf{k})|^2 + \int d\mathbf{k}\alpha(\mathbf{k})a^\dagger(\mathbf{k})\right]|0\rangle$$

where $\alpha(\mathbf{k}) = A_0 R^3 \sqrt{\frac{\omega_m(k)}{2}} e^{-i\mathbf{k}\cdot\mathbf{r}_0-(kR)^2/2}$; $\omega(k) = \sqrt{m^2c^4/\hbar^2 + k^2c^2}$

satisfies
$$\begin{aligned}\langle\alpha|\varphi(\mathbf{r})|\alpha\rangle &= A_0 e^{-(\mathbf{r}-\mathbf{r_0})^2/(2R^2)}\\ \langle\alpha|\dot{\varphi}(\mathbf{r})|\alpha\rangle &= 0\end{aligned}$$

Describes a stationary Gaussian matter "ball" of radius R, centered at $\mathbf{r_0}$:



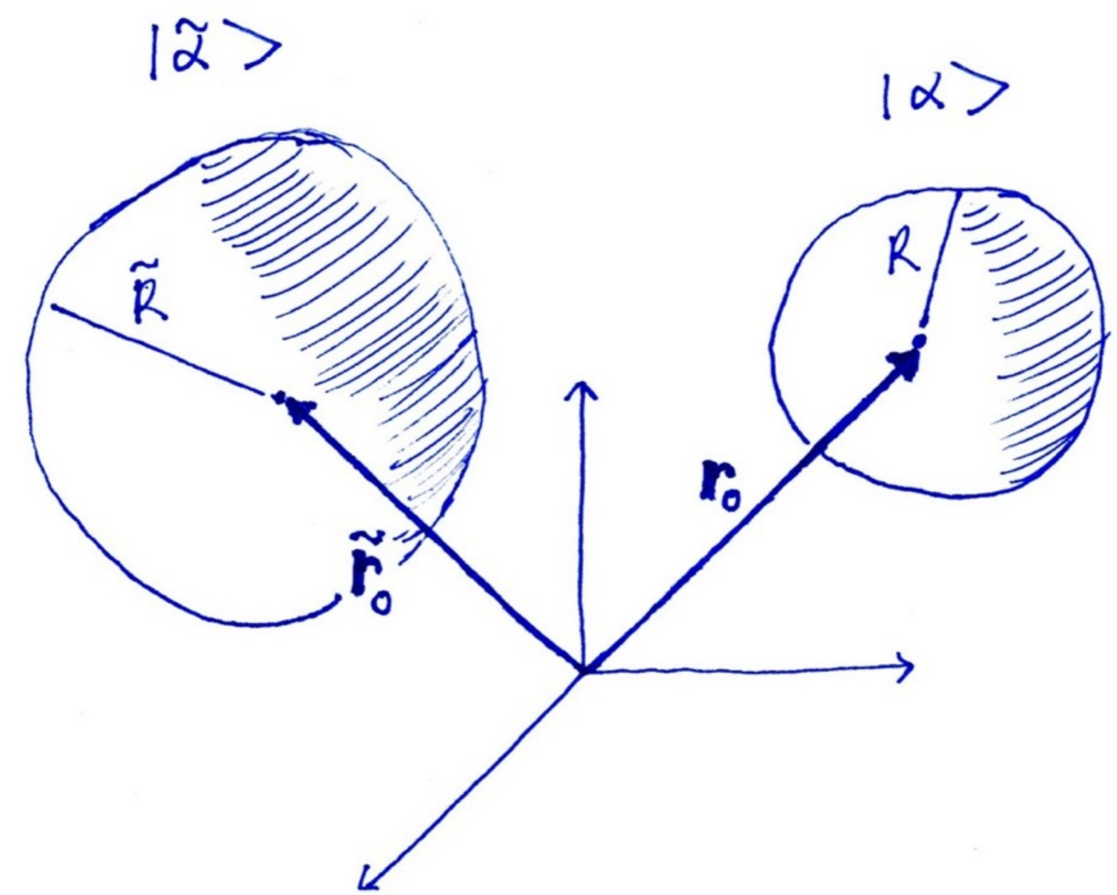
Consider ball radius $R \ggg \lambda_{\text{compton}} = \dfrac{\hbar}{mc} \sim 10^{-16}\ \text{m}$ for a nucleon mass.

Hence, rest mass is dominant energy content of ball: non-relativistic, approximately stationary with little dispersion.

Macroscopic initial superposition state: $\hat{\rho}_{\text{sys}}(0) = |\Psi(0)\rangle\langle\Psi(0)|$

where $|\Psi(0)\rangle = \dfrac{1}{\sqrt{2}}\left(|\alpha\rangle + |\tilde{\alpha}\rangle\right)$

$$\mathrm{Tr}\left(\hat{\rho}_{\text{sys}}(0)\hat{\varphi}(\mathrm{r})\right)$$



Noise part of master equation (responsible for decoherence) simplifies approximately to:

$$\partial_t \rho_{\rm sys}[\phi,\phi',t] = \cdots - \int_0^t d\tau \int d\mathbf{r} d\mathbf{r}' N(\mathbf{r}-\mathbf{r}',\tau)$$
$$\times \left[\frac{1}{2}m^2(\phi(\mathbf{r}))^2 - \frac{1}{2}m^2(\phi'(\mathbf{r}))^2\right]\left[\frac{1}{2}m^2(\phi(\mathbf{r}'))^2 - \frac{1}{2}m^2(\phi'(\mathbf{r}'))^2\right]\rho_{\rm sys}[\phi,\phi',t]$$

where have used the fact that $T_{00}(\phi) \approx \frac{1}{2}m^2\phi^2$ dominates in the non-relativistic, stationary limit.

Master equation expressed in field coordinate basis.

Yields the following decoherence time for the initial macroscopic “ball” superposition state:

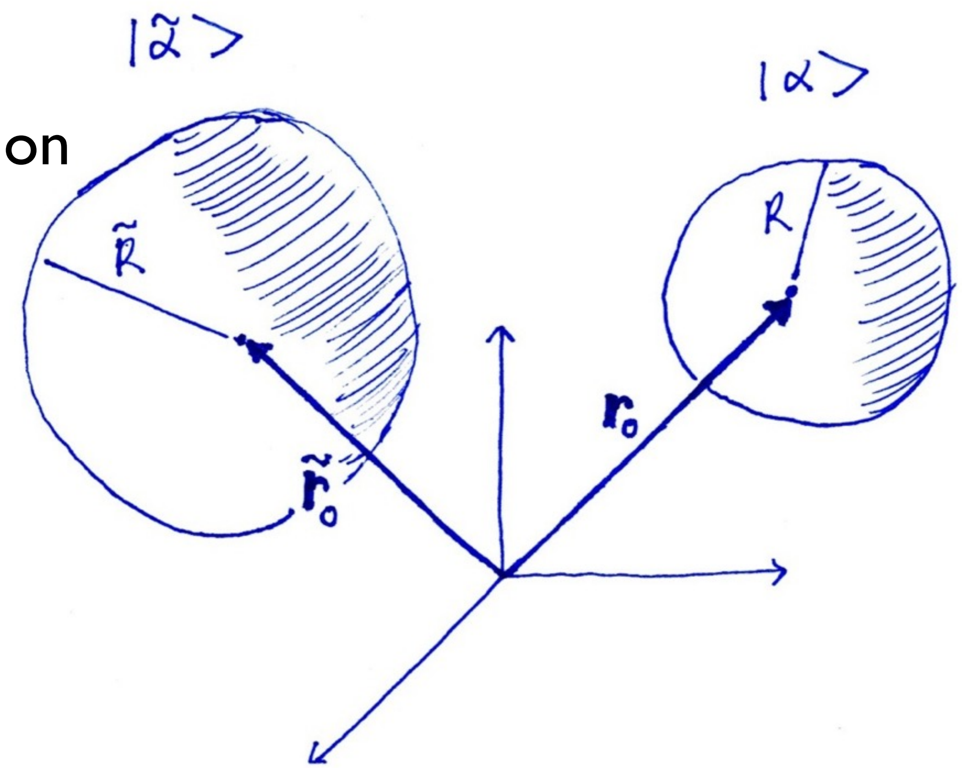
$$\hat{\rho}_{\mathrm{sys}}(0) = |\Psi(0)\rangle\langle\Psi(0)| \qquad |\Psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle + |\tilde{\alpha}\rangle\right)$$

Ball energies in the superposition

Gravitational environment temperature

$$\boxed{\tau_{\mathrm{decohere}}^{-1} = \frac{k_B T}{\hbar}\left(\frac{E-\tilde{E}}{E_P}\right)^2}$$

Planck energy

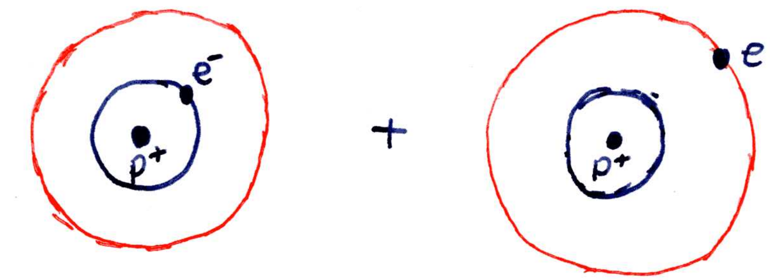


$t \gg \hbar/(k_B T)$ high temperature limit

Distinct energy superpositions decohere, not position.

See also: Anastopoulos and Hu, Class, Quantum Grav. **30**, 165007 (2013).

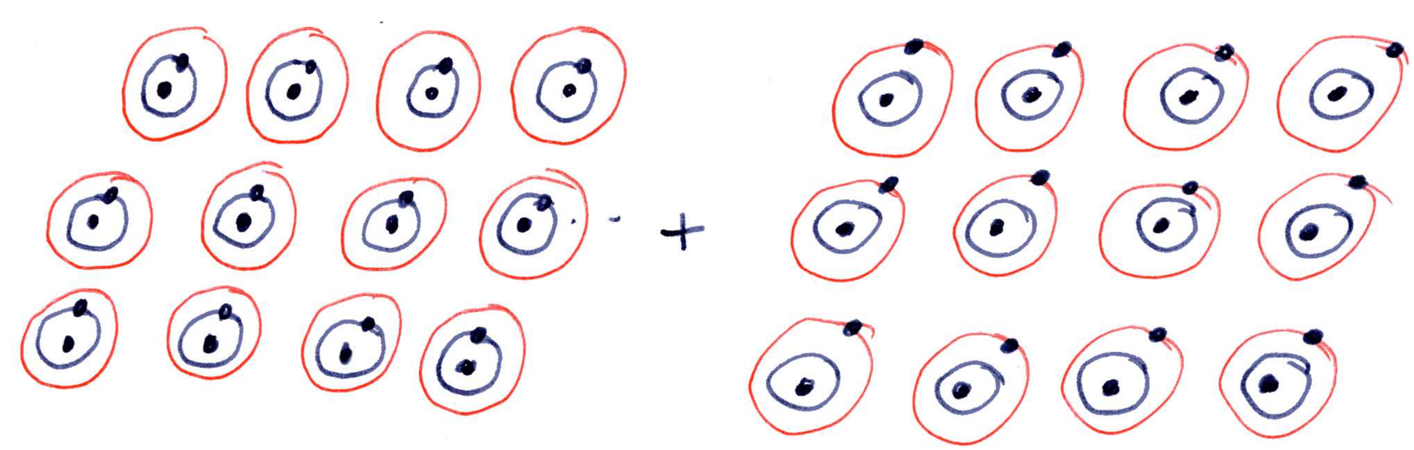
Some time scales: assume T~1K; eV atoms



- A hydrogen atom simultaneously in a ground and an excited electron state: decoherence rate ~ 10^{-45} secs^{-1}.

- A gram object where every atom is simultaneously in a ground and an excited state: decoherence rate ~ 10^{2} secs^{-1}.

- A kilogram object where every atom is simultaneously in a ground and an excited state: decoherence rate ~ 10^{8} secs^{-1}.



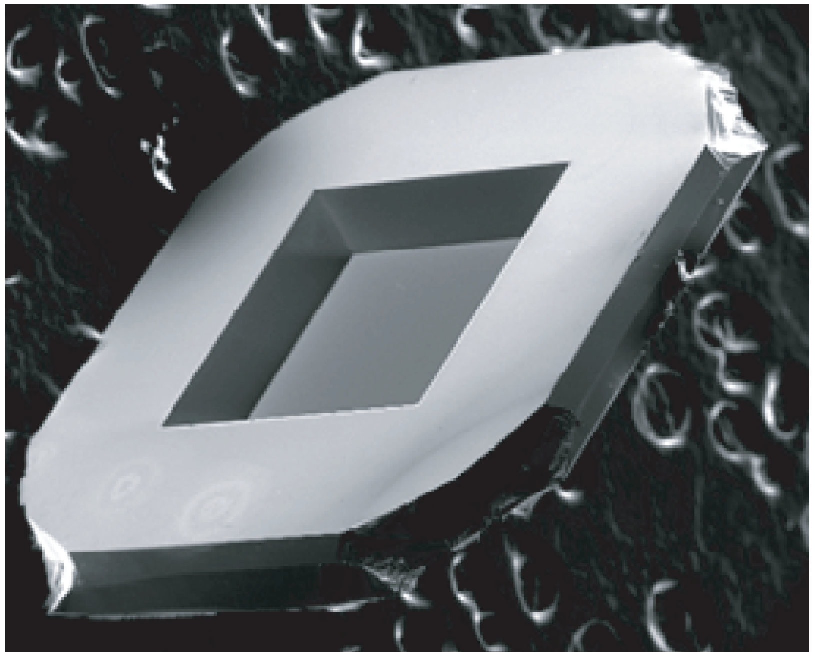
Back to mechanical analogues: the "phase damped oscillator"

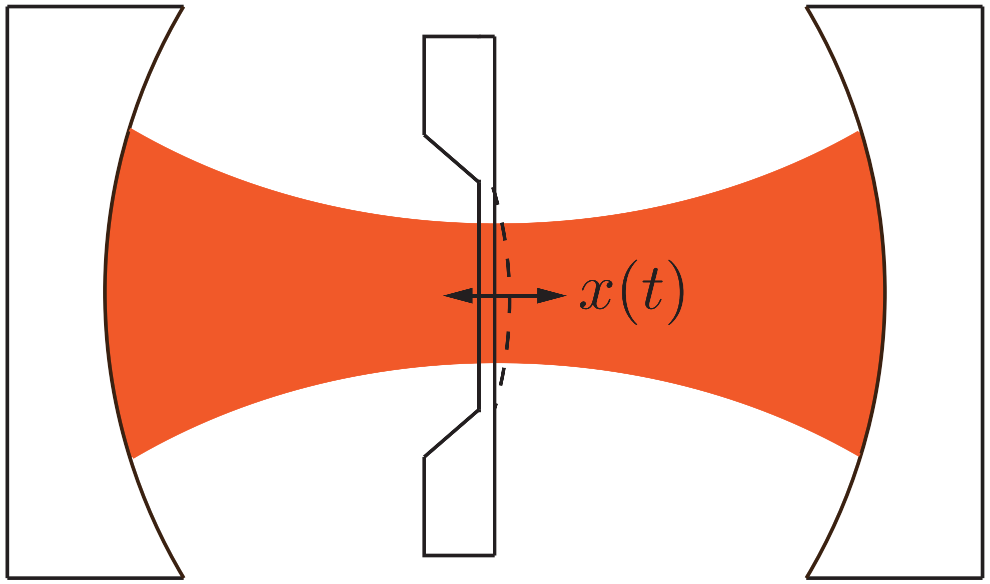
[C.W. Gardiner and P. Zoller, Quantum Noise (Springer 2000)]

$$H = \hbar\omega_0 a^\dagger a \left(1 + \sum_i \lambda_i \frac{q_i}{\Delta_i}\right) + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2\right)$$

$$\Delta_i = \sqrt{\hbar/(2m_i\omega_i)}$$

E.g., optomechanical system with dense mechanical mode spectrum





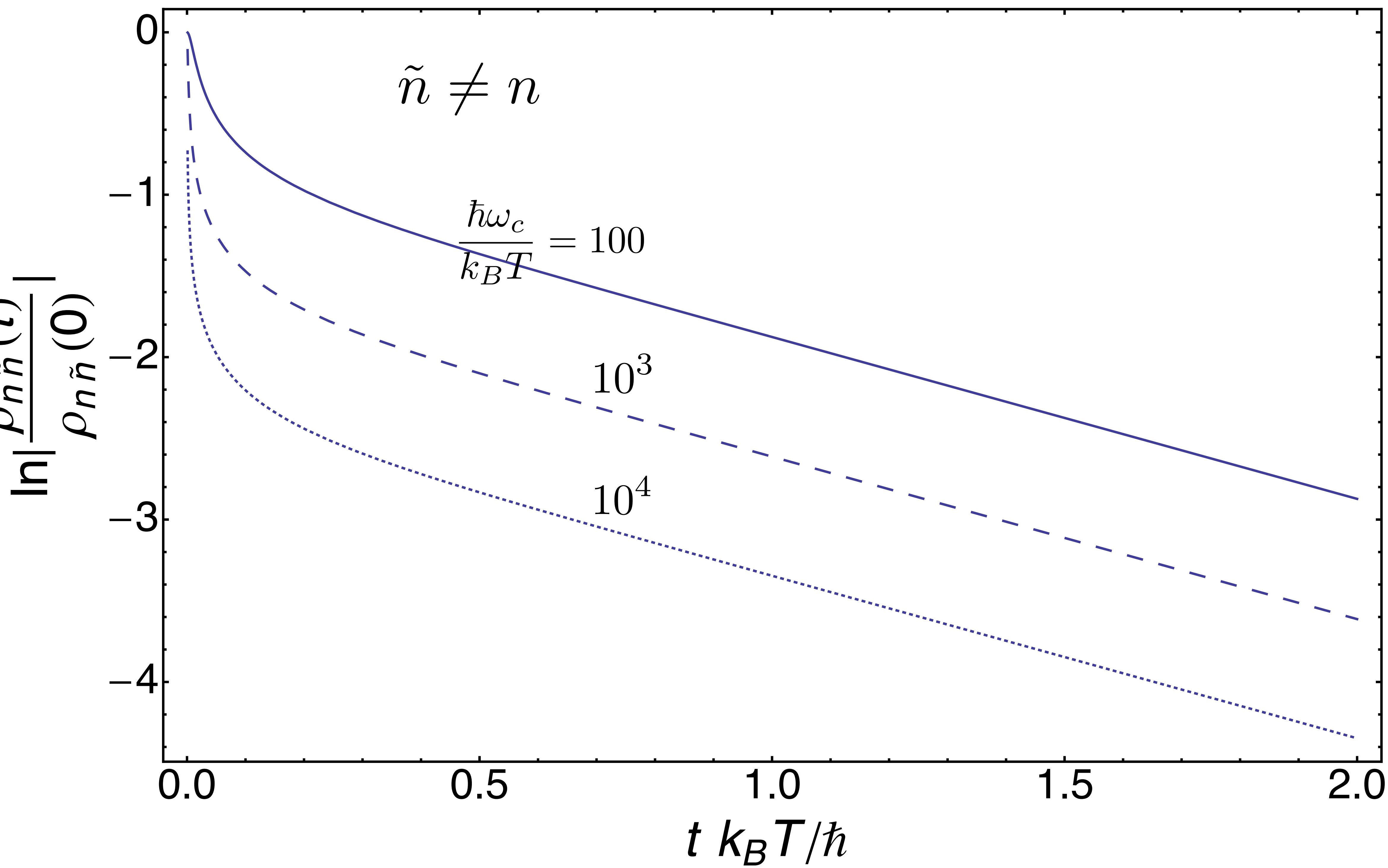
Exactly solvable; assume Ohmic bath spectral density

$$\pi \sum_i \lambda_i^2 \delta(\omega - \omega_i) = C\omega e^{-\omega/\omega_c}$$

Time evolution of oscillator system density matrix in high temperature limit $t \gg \hbar/(k_B T) \gg \omega_c^{-1}$:

$$\rho_{n\tilde{n}}(t) = \rho_{n\tilde{n}}(0) \exp\Bigg[-i\omega_0(n-\tilde{n})t + i\frac{C}{\pi}\omega_0^2(n-\tilde{n})(n+\tilde{n}+1)(\omega_c t - \pi/2) - \left(\frac{1}{\pi}\ln\left(\frac{\hbar\omega_c}{k_B T}\right) + \frac{1}{\pi}\ln\left(\frac{\sinh\pi}{\pi}\right) - 1\right)\left(\frac{E_n - E_{\tilde{n}}}{\hbar/\sqrt{C}}\right)^2 - \underline{\frac{k_B T}{\hbar}\left(\frac{E_n - E_{\tilde{n}}}{\hbar/\sqrt{C}}\right)^2 t} \Bigg]$$

$$\frac{\hbar\omega_c}{k_B T} \gg 1 \qquad\qquad E_n = n\hbar\omega_0$$



$\tilde{n} \neq n$
$\frac{\hbar\omega_c}{k_B T} = 100$
10^3
10^4
$\ln\left|\frac{\rho_{n\tilde{n}}(t)}{\rho_{n\tilde{n}}(0)}\right|$
$t\, k_B T/\hbar$
0
−1
−2
−3
−4
0.0
0.5
1.0
1.5
2.0

Open problems:

- Validity of the master equation approach (gauge issues, initial system-environment product state assumption etc…).

- Consequences of graviton emission/absorption for decoherence.

- Decoherence at low temperatures.

- Decoherence "on Earth," i.e., in presence of weak background gravitational field (breaking of translational symmetry may result in spatial superpositions decohering).

- Decoherence versus "actual" collapse (postulate of "undecidability" [R. Gambini *et al*., Found. Phys. **40**, 93 (2010)] allows for interpretation of actual outcome or event: effective field theory method can provide quantitative predictions for such outcomes).

Gravity is very weak, but then quantum coherence is very fragile!