Probing incompatibility between Quantum Mechanics and Macrorealism for large spin and for large mass

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Fundamental Quantum Questions relevant to the Theme of this talk

1. To what extent it is possible to test the realist models of QM behaviour of single particles?

2. To what extent it is possible to test QM in the macrolimit? What are the limits of observability of quantum effects in the macro-domain?

3. What is the relationship between the weird behaviour of the microscopic world described by quantum physics and the everyday macroscopic world of human experience? Under what conditions do classicality emerge out of quantum mechanics (QM)?

Plan of the Talk

- Meaning of the notion of *Macrorealism* (MR) as used in this line of study, incorporating the notion of *Noninvasive Measurability*.
- Significance of testing the notion of MR vis-à-vis QM.
- Different necessary conditions of MR - *Leggett-Garg inequality* (LGI), *Wigner's form of LGI* (WLGI), and the *No-Signalling in Time* (NSIT) condition - formulated in terms of *time-separated* joint probabilities/correlation functions corresponding to successive measurement outcomes for a system.
- Study of QM violation of MR in two different contexts:

  (a) For *large spin* system in uniform magnetic field subjected to *coarse-grained measurements*.

  (b) For appropriately measured oscillating nano-objects of mass $\sim 10^6 - 10^9$ amu prepared in the *Schrödinger Coherent State* (the most "classical-like" of all quantum states) of a linear harmonic oscillator.
- Implications of these studies and outlook, seeking to probe the macrolimits of the Quantum World in conjunction with the testing of the notion of Realism.

The notion of Macrorealism

- *Macrorealism* is the conjunction of the notions of *Realism* and *Noninvasive Measurability (NIM)* which seems a natural point of view based on everyday experience.
- *Realism*: At *any* instant, even when not measured, a system is in a *definite* one of the available states and all its observable properties have *definite values*.
- *Noninvasive Measurability (NIM)*: It is possible, at least in principle, to *determine which* of the states the system is in, corresponding to the premeasured values of its properties, *without affecting* the state or the system's subsequent evolution.
- For the *class of realist models* envisaged by these assumptions, Realism implies NIM in the context of Negative result Measurement (NRM).

  This class does *not* include, for example, Bohm's realist model.

Significance of studying QM predicted testable violation of MR

- Violation of a necessary condition of MR can be used as a tool for revealing *nonclassicality* in a context that is usually thought to be entailing classical behaviour.

  The first example discussed in this Talk will illustrate this feature and compare the efficacy of the different necessary conditions of MR in this context.

- Violation of a necessary condition of MR can be invoked for ruling out a class of realist models, besides displaying nonclassicality in a classical-like context

  Experimental setup for demonstrating this dual significance necessarily requires an unambiguous implementation of *Negative Result Measurement*. This is also required to ensure the relevance of the expt. tests of MR in the micro-regimes.

  This will be illustrated by the second example of this Talk, proposing a feasible experimental setup in a hitherto unexplored context.

Various necessary conditions proposed for testing macrorealism

1. *Leggett-Garg Inequality (LGI)*: [A. J. Leggett and A. Garg, *Phys. Rev. Lett.* **54**, 857 (1985)]

   Derived as a testable algebraic consequence of the deterministic form of Macrorealism.

2. *Wigner's form of LGI(WLGI)*: [D. Saha, S. Mal, P. K. Panigrahi, D. Home, *Phys. Rev. A* **91**, 032117 (2015)]

   Derived as a testable algebraic consequence of the probabilistic form of Macrorealism.

3. *No-Signalling in Time (NSIT)*: [J. Kofler and C. Brukner, *Phys. Rev. A* **87**, 052115 (2013)]

   This condition is formulated as a statistical version of NIM to be satisfied by any macrorealist theory. Violation of NSIT implies violation of NIM at an individual macrorealist level.

Leggett-Garg Inequality (LGI)

- We consider temporal evolution for a two state system where the available states are, say, 1 and 2.
- Let Q(t) be on observable quantity such that, whenever measured, it is found to take a value+1 (-1) depending on whether the system is in 1 (2). Considering values of Q at three subsequent times $t_1 < t_2 < t_3$, it follows that

$$Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3) = +1 \text{ or } -3$$

whence one obtains for the 'grand' ensemble average

$$\langle Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3)\rangle_G \leq 1$$

Now, dividing the whole ensemble of runs into three subensembles, S_1, S_2, and S_3, consider measurement of Q on each subensemble of runs at the times (t_1, t_2) for S_1, (t_2, t_3) for S_2 and (t_3, t_1) for S_3 corresponding to the same initial state at $t = 0$.

One can then use the following deterministic consequence of the assumptions of realism and NIM:

For any set of runs corresponding to the *same initial state* at, say, $t = 0$, any individual $Q(t_i)$ has the same definite value, irrespective of the pair in which it occurs, i.e., the value of $Q(t_i)$ in any pair *does not* depend on whether *any prior* or *subsequent measurement* has been made on the system.

Leggett-Garg Inequality (LGI)

It then follows that

$$\langle Q(t_1)Q(t_2)\rangle_{S_1} + \langle Q(t_2)Q(t_3)\rangle_{S_2} - \langle Q(t_1)Q(t_3)\rangle_{S_3} \leq 1$$

where the grand ensemble average has been replaced by the respective subensemble averages. The impossibility of 'backward causation' is also assumed here.

The above inequality can be written as

$$C \equiv C_{12} + C_{23} - C_{13} \leq 1 \quad (1)$$

where the temporal correlation

$$C_{ij} = \langle Q(t_i)Q(t_j)\rangle$$

LHS of the inequality (1) is an experimentally measurable quantity. This is the Leggett-Garg inquality imposing macrorealist constraint on the temporal correlations pertaining to any two level system.

Wigner's form of Leggett-Garg inequality (WLGI)

Here again consider temporal evolution of a two state system where the available states are, say, 1 & 2 and consider measurement of Q at t_1, t_2 and t_3 ($t_1 < t_2 < t_3$).

Here the notion of realism implies the *existence of overall joint probabilities* $\rho(Q_1, Q_2, Q_3)$ pertaining to different combinations of definite values of outcomes for the relevant measurements.

The assumption of NIM implies that the probabilities of such outcomes would be *unaffected by measurements*. Hence, by appropriate marginalization, the observable probabilities can be obtained.

For example, the observable joint probability $P(Q_2+, Q_3-)$ of obtaining the outcomes +1 and -1 for the sequential measurements of Q at the instants t_2 and t_3, respectively, can be written as

$$P(Q_2+, Q_3-) = \sum_{Q_1=\pm 1} \rho(Q_1, +, -)$$

Similarly writing the other measurable marginal joint probabilities $P(Q_1-, Q_3-)$ and $P(Q_1+, Q_2+)$, we get

$$P(Q_1+, Q_2+) + P(Q_1-, Q_3-) - P(Q_2+, Q_3-) = \rho(+,+,+) + \rho(-,-,-) \quad (2)$$

Then invoking *non-negativity of the overall joint probabilities* occurring on the RHS of the above equation, the following form of WLGI is obtained in terms of three pairs of two-time joint probabilities.

$$P(Q_2+, Q_3-) - P(Q_1+, Q_2+) - P(Q_1-, Q_3-) \leq 0$$

Similarly, other forms of WLGI involving any number of pairs of two-time joint probabilities can be derived by using various combinations of the observable joint probabilities.

No-Signalling in Time (NSIT)

Statement: The measurement outcome statistics for any observable at any instant is *independent* of whether any prior measurement has been performed.

Consider a system whose time evolution occurs between two possible states. Probability of obtaining the outcome $+1$ for the measurement of a dichotomic observable Q at an instant, say, t_2 *without* any earlier measurement being performed, is denoted by $P(Q_2 = +1)$.

NSIT requires that $P(Q_2 = +1)$ should remain *unchanged* even when an earlier measurement is made at t_1

$$P(Q_2 = +1) = P(Q_1 = +1, Q_2 = +1) + P(Q_1 = -1, Q_2 = +1)$$

The idea of Negative Result Measurement (NRM)

- Consider a two-state system whose available states are, say, 1 (Q = +1) and 2 (Q = -1).

- The measuring apparatus be such that if $Q(t_1)$ is, say, $+1$, the probe is *triggered*, while if $Q(t_1) = -1$, it is *not*.

  The results of *those postselected runs* are used for which $Q(t_1) = -1$, followed by the measurement of Q at $t_2 \rightarrow$ These results used for determining the joint probabilities $P{-}{+}(t_1, t_2)$ and $P{-}{-}(t_1, t_2)$.

- One can then use a complementary setup so that for a value of $Q(t_1) = -1$ the probe is *triggered, while for* $Q(t_1) = +1$, it is *not*.

  In this case, the results of *those postselected runs* are used for which $Q(t_1) = +1$, followed by the measurement of Q at $t_2 \rightarrow$ These results used for determining $P{+}{-}(t_1, t_2)$ and $P{-}{-}(t_1, t_2)$.

The idea of Negative Result Measurement (NRM)

- For interpreting the experimental violation of any macrorealist condition involving NRM, if one sticks to Realism in the sense defined earlier, then the possibility of a state being affected by the NRM procedure, in which the measurement information is essentially obtained when *no interaction occurs*, cannot be accommodated unless the notion of 'state' is appropriately defined with a realist model (e.g., Bohm model).

  Thus, in the context of NRM and the realist models considered within the framework of MR, NIM is argued to be a natural corollary of the notion of realism
- Empirical violation of the macrorealist condition observed using NRM would therefore rule out a class of realist models within the framework of MR.
- Two experimental claims to date using NRM for loophole-free implementation of NIM: G. Knee et al., *Nature Communications* 3, 606 (2012) → Spin-bearing phosphorus impurities in silicon sample.

  C. Robens et al. *Physical Review X* 5, 011003 → Quantum Walks in a lattice having cesium atoms.

  Criticisms persist about the possible loopholes in the claim of implementing NRM in the above mentioned experiments.

QM violation of MR for large spin

The first work presented in this Talk - *QM violation of Macrorealism for large spin and its robustness against Coarse-grained measurements*

Collaboration with Shiladitya Mal and Debarshi Das, Physical Review A (in press)

Backdrop

Emergence of Classicality from QM

- One of the suggested views (Born, Berry,....) is that the *limits of observability* of quantum effects due to *measuremental imprecision* determine the nature of emergence of classicality, arguing it to be particularly relevant for higher-dimensional systems.

  Rigorous scrutiny of this approach in recent years using *multilevel spin systems:*

- For *unrestricted measurement accuracy* involving projections onto individual levels (so that the consecutive eigenvalues can be resolved), QM violation of MR persists for arbitrary large spin. C. Budroni and C. Emary, Phys. Rev. Lett. 113, 050401 (2014)

  Similar result also holds for the QM violation of Local Realism (LR) for the entangled systems; D. Home and A. S. Majumdar, Phys. Rev. A 52, 4959 (1995); A. Cabello, PRA 65, 062105 (2002).

- On the other hand, arguments have been put forward to justify the emergence of Classicality in the large spin limit, essentially if measurements are *coarse-grained* - J. Kofler and C. Brukner, *"Classical World arising out of Quantum Physics under the Restriction of Coarse-Grained Measurements"* Phys. Rev. Lett. 99, 180403 (2007); PRL 101, 090403 (2008).

  Similar contention also made for entangled systems in the large dimension limit arguing *disappearance of QM violation of LR* for *Coarse-grained measurements* - S. Raeisi et al., Phys. Rev. Lett, 107, 250401 (2011).

QM violation of MR for large spin

Motivation

No study yet probing emergence of classicality for higher dimensional quantum systems by modelling coarse-graining of measurements in a very general way for varying ability of resolving eigenvalues and by including fuzziness of measurement of each eigenlevel.

The key result obtained

Our study reveals that by employing QM violation of MR as a tool classicality does *not* emerge in large limit of spin, whatever be the *unsharpness* and degree of *coarse-graining of the measurements*. For this purpose, employing the different necessary conditions of MR (LGI, WLGI and NSIT), their relative efficacy in demonstrating non-classicality is assessed – NSIT is found to be most effective in

Specifying the Hamiltonian, initial condition and measurement times

- Consider a QM spin j system in a uniform magnetic filed of magnitude B_0 along the x direction. The relevant Hamiltonian is ($\hbar = 1$)

$$H = \Omega J_x$$

  where $\Omega \rightarrow$ angular precession frequency ($\propto B_0$), $J_x \rightarrow$ x component of spin angular momentum.

- We initialize the system so that at t=0, the system is in the state $|-j;j\rangle$; where $|m;j\rangle$ denotes the eigen state of J_z operator with eigenvalue m.

- Consider measurements of Q at times t_1, t_2 & t_3 ($t_1 < t_2 < t_3$) & set the measurement times as $\Omega t_1 = \pi$ and $\Omega(t_2 - t_1) = \Omega(t_3 - t_2) = \frac{\pi}{2}$

- Now, consider the following form of 3-term LGI:

$$K_{LGI} = C_{12} + C_{23} - C_{13} \leq 1$$

  The following form of 3-term WLGI:

$$K_{WLGI} = P(Q_2+, Q_3+) - P(Q_1-, Q_2+) - P(Q_1+, Q_3+) \leq 0$$

  and the following form of NSIT:

$$K_{NSIT} = P(Q_3 = -1) - [P(Q_2 = +1, Q_3 = -1) + P(Q_2 = -1, Q_3 = -1)] = 0$$

Modelling coarse grained measurement in an arbitrary spin system

Considering measurements of spin-z component (J_z) observable in a spin-j system, the outcomes of J_z measurements are denoted by m, m takes the values $-j$, $-j+1$, $-j+2$, ..., $+j$. For modelling coarse grained measurement through appropriate dichotomization, different number of measurement outcomes are clubbed together into two groups, the grouping scheme being characterized by a particular value of x.

Let Q be such an observable that
$Q = -1$ for $m = -j, ..., -j + x$ (No. of outcomes in this group $= x + 1$),
$Q = +1$ for $m = -j + x + 1, ..., +j$ (No. of outcomes in this group $= 2j - x$), where $0 \leq x \leq$ integer part (j) and x being integer.

• The asymmetry in the number of measurement outcomes clubbed together into two groups characterizes the biasness of coarse graining of the measurement outcomes.

• The asymmetry decreases and, hence, the biasness of coarse graining of the measurement outcomes decreases with an increasing value of x.

• $x =$ integer part (j), signifying equal number of outcomes in the two groups denotes the grouping scheme corresponding to the most unbiased coarse graining of the measurement outcomes.

Modelling coarse grained measurement in an arbitrary spin system

• In this modelling, clubbing of the measurement outcomes into two groups makes the measurement coarse grained. However, the boundary between the two groups of outcomes is assumed to be precise which, in general, is *not* true in the realization of the macrolimit.

• Employing, in conjunction, *unsharp measurement* corresponding to each eigenvalue makes this boundary also imprecise.

• Thus, by simultaneously clubbing the different measurement outcomes together and by invoking unsharp measurement one can capture in a more *general way* what is entailed by the coarse graining of the measurement outcomes.

Modelling fuzziness of a measurement through sharpness parameter in a two-level system

Consider measurements involving a two-state system with states $|A\rangle$ and $|B\rangle$. Note that for the ideal measurement of the dichotomic observable $Q = |A\rangle\langle A| - |B\rangle\langle B|$, the respective probabilities of the outcomes ± 1 and the way a measurement affects the observed state are determined by the projection operators $P_{\pm}$ onto the state $|A\rangle$ ($|B\rangle$).

In order to capture the effect of fuzziness or imprecision involved in a measurement, in the formalism of unsharp measurement, a parameter (λ) known as the sharpness parameter is introduced to characterize non-idealness of a measurement by defining what is referred to as the effect operator given by

$$F_{\pm} = \lambda P_{\pm} + (1 - \lambda)\mathbb{I}/2$$

where $\mathbb{I} = |A\rangle\langle A| + |B\rangle\langle B|$. It is clear that $(1 - \lambda)$ amount of white noise is present in the measurement, where $0 < \lambda \leq 1$ and $F_{\pm}$ are mutually commuting Hermitian operators with non-negative eigenvalues; $F_{+} + F_{-} = \mathbb{I}$.

For an unsharp measurement pertaining to an initial state ρ_0, the probability of an outcome, say, $+1$ is given by $tr(\rho_0 F_{+})$ for which the post-measurement state is given by $(\sqrt{F_{+}}\rho_0\sqrt{F_{+}})/tr(\rho_0 F_{+})$.

Modelling unsharp measurement in an arbitrary spin system

Consider measurements of spin-z component (J_z) observable in a spin-j system. In the formalism of POVM, to characterize the non-idealness of a measurement, the effect operators are given by

$$F_m = \lambda P_m + (1-\lambda)\frac{\mathbb{I}}{d}$$

where λ is the sharpness parameter ($0 < \lambda \leq 1$);

P_m is the projector $|m;j\rangle\langle m;j|$, where $|m;j\rangle$ is the eigenvector of J_z operator with eigenvalue m;

$\mathbb{I}$ is the identity operator,

d is the dimension of the system (for spin j system, $d = 2j+1$)

Tool for calculation

- Let $R = e^{-i\frac{\pi}{2}J_x}$, the time evolution operators are as follow:

$$U(t_1 - 0) = e^{-i\pi J_x} = R^2$$

$$U(t_2 - 0) = R^3$$

$$U(t_2 - t_1) = R$$

$$U(t_3 - t_2) = R$$

$$U(t_3 - t_1) = R^2$$

So here the time evolution operators have the form of rotation operators about x axis.

- We, therefore, take the help of *Wigner's D Matrix* to evaluate the matrix elements of the rotation operators in arbitrary spin system.

- It is a square matrix of dimension $2j + 1$.

Summary of the key Results

Considering projective measurement $(\lambda = 1)$

• For projective measurement, for any j (including for arbitrarily large value), QM violation of LGI does not occur for *unbiased* coarse graining of the measurement outcomes. On the other hand, for this type of coarse graining of measurement outcomes, QM violations of WLGI and NSIT persist for any j.

j	Magnitude of the QM violation of LGI for unbiased coarse grained measurement (x = integer part(j))	Magnitude of the QM violation of WLGI for unbiased coarse grained measurement (x = integer part(j))	Magnitude of the QM violation of NSIT for unbiased coarse grained measurement (x = integer part(j))
10	No violation	0.20	0.48
20	No violation	0.21	0.49
30	No violation	0.22	0.50

• For projective measurement, for any j (including for arbitrarily large value), QM violation of any necessary condition of MR decreases with increasing values x, i.e., with decreasing biasness of coarse graining of the measurement outcomes.

j	Magnitude of the QM Violation of					
	LGI for		WLGI for		NSIT for	
	$x = 10$	$x = 20$	$x = 10$	$x = 20$	$x = 10$	$x = 20$
40	1.52	1.32	0.76	0.66	0.76	0.66
60	1.61	1.46	0.81	0.73	0.81	0.72
80	1.67	1.53	0.83	0.77	0.83	0.76
100	1.70	1.58	0.85	0.79	0.85	0.79

Summary of the key Results

Considering unsharp measurement ($0 < \lambda \leq 1$)

• The results show that for the most *unbiased* coarse graining of the measurement outcomes, i.e., when the number of outcomes in the two groups becomes almost equal (equal for half integer spin system and difference of outcomes is 1 for integer spin system), then the QM violation of NSIT persists for *any* degree of fuzziness of the measurement; the QM violation of WLGI persists up to a *certain* degree of fuzziness of the measurement, while the QM violation of LGI *does not* occur in this case even for sharp measurement.

j	Range of λ for which the QM violation of WLGI persists for unbiased coarse grained measurement (x = integer part(j))	Range of λ for which the QM violation of NSIT persists for unbiased coarse grained measurement (x = integer part(j))
10	$(0.78, 1]$	$(0, 1]$
20	$(0.76, 1]$	$(0, 1]$
30	$(0.75, 1]$	$(0, 1]$

Summary of the key Results

Considering unsharp measurement ($0 < \lambda \leq 1$)

• For a fixed value of spin and fixed biasness of coarse graining of the measurement outcomes (fixed j and fixed x), QM violations of the different necessary conditions of MR decrease with decreasing values of λ, i.e. increasing fuzziness of the measurement.

• For fixed and finite values of j and x, the range of λ for which the QM violation of MR persists is the maximum for NSIT, followed by WLGI and then LGI.

The range of λ for which the QM violation of NSIT persists is always $(0, 1]$ for any value of j and x.

Essence of the Results

These results signify that, in the limit of arbitrarily large spin system, for the most *unbiased* coarse graining of the measurement outcomes, i.e., when the number of outcomes in the two groups becomes almost equal, even then classicality does *not* emerge within QM for *any* degree of fuzziness of the measurement. This is best illustrated through the QM violation of NSIT, followed by that of WLGI.

In this work we have not considered the *coarse graining of measurement times*; H. Jeong et al. Phys. Rev. Lett. 112, 010402 (2014).

It remains an open question to what extent the coarse graining of measurement times, together with coarse-graining of measurement outcomes, would affect the above results for LGI, WLGI as well as

Linear Harmonic Oscillator

The second work presented in this Talk

System considered - *Linear Harmonic Oscillator (LHO).*

Collaboration with Sougato Bose (Univ. College London) and Shiladitya Mal.

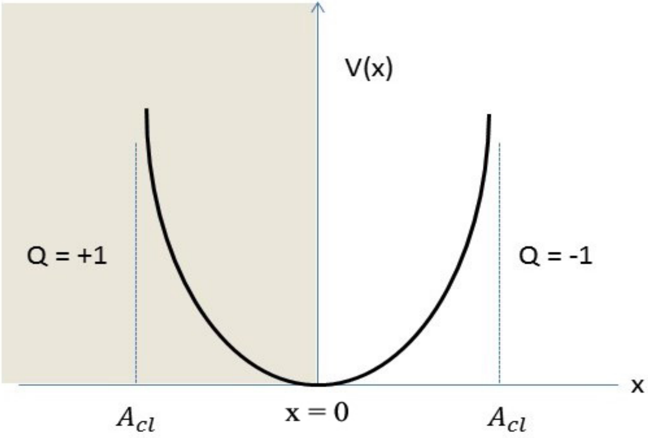
Twin objectives of the proposed expt. setup using nano-objects of large mass, $10^6 - 10^9$ *amu.*

1. *Loophole-free implementation* of *NIM* through *NRM* for empirically testing the class of realist models defined within the notion of MR.
2. Showing *nonclassicality* of the *Schrödinger Coherent State* persisting up to the macrolimit.

   LHO considered in our example is *not* coupled with any auxiliary quantum system, and involves *continuous variables.* Therefore, for probing MR,*discretization* is invoked by considering measurement of the following type:

   Coarse-grained spatial measurement determining *which one of the halves* of the region, the oscillating particle is *in* at any given instant.

THE SETUP



SCHEMATIC DESCRIPTION OF OUR WORK

Linear Harmonic Oscillator

- Initial wavepacket is

$$\psi(x,0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_0}} \exp\left(-\frac{x^2}{4\sigma_0^2} + \frac{ip_0x}{\hbar}\right) \tag{3}$$

- Here one considers measuring localization of the particle. If the particle is found in the region between $x \to -\infty$ and $x = 0$, then the measurement outcome is denoted by $+1$. If the particle is found in the region between $x = 0$ and $x \to \infty$, then the outcome is denoted by -1.
- The above mentioned condition is satisfied by defining the following measurement operator

$$\hat{O} = \int_{-\infty}^{0} |x\rangle\langle x|dx - \int_{0}^{\infty} |x\rangle\langle x|dx \tag{4}$$

PROPERTIES OF THE OBSERVABLE $\widehat{O}$

- The observable $\widehat{O}$ has two eigenstates having eigenvalues $+1$ and -1 respectively. For the eigenvalue $+1$, we have the corresponding eigenstate defined by

$$\hat{O}\int_{-\infty}^{0}\langle x|\psi\rangle|x\rangle dx = +1\int_{-\infty}^{0}\langle x| \tag{5}$$

- For the eigenvalue -1 we have the corresponding eigenstate defined by

$$\hat{O}\int_{0}^{\infty}\langle x|\psi\rangle|x\rangle dx = -1\int_{0}^{\infty}\langle x|\psi\rangle|x\rangle dx \tag{6}$$

TIME EVOLUTION

- Initial wave packet is evolved by the following propagator

$$K(x', t'; x, t) = \sqrt{\frac{m\omega}{2\pi i\hbar \sin\omega(t - t')}} \quad (7)$$

$$\exp\left(\frac{im\omega}{2\hbar \sin\omega(t - t')}\left((x'^2 + x^2)\cos\omega(t - t') - 2xx'\right)\right)$$

- Wave packet at the instant t is given by

$$\psi(x, t) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_t}} \exp\left(-\frac{\alpha(t) + \beta x + \gamma(t)x^2}{4\hbar\sigma_0\sigma_t}\right) \quad (8)$$

TIME EVOLUTION

- $\alpha(t), \beta, \gamma(t)$ are given by

$$\begin{aligned}
\alpha(t) &= 2i\sigma_0^2\langle x(t)\rangle \\
\beta &= 2ip_0\sigma_0^2 \\
\gamma(t) &= \hbar\cos\omega t + 2im\omega\sigma_0^2\sin\omega t \\
\langle x(t)\rangle &= \frac{p_0}{m\omega}\sin\omega t \\
\sigma_t &= \frac{i\hbar\sin\omega t + 2m\omega\sigma_0^2\cos\omega t}{2m\omega\sigma_0.}
\end{aligned} \tag{9}$$

- Note that $p_0/m\omega = A_{Cl}$ is the amplitude of the corresponding classical oscillation.

MEASUREMENT RESULTS AT TIME t

- Probability at time t of finding the particle in the region between $x \to -\infty$ and $x = 0$ is given by

$$P_+(t) = \int_{-\infty}^{0} |\psi(x,t)|^2 dx = \frac{1}{2}\left(1 - Erf(\frac{\langle x(t)\rangle}{\sqrt{2}|\sigma_t|})\right) \qquad (10)$$

- Probability at time t of finding the particle in the region between $x = 0$ and $x \to \infty$ is given by

$$P_-(t) = \int_{0}^{\infty} |\psi(x,t)|^2 dx = \frac{1}{2}\left(1 + Erf(\frac{\langle x(t)\rangle}{\sqrt{2}|\sigma_t|})\right) \qquad (11)$$

ERROR FUNCTION

- Error function is defined as

$$Erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-z^2) dz \tag{12}$$

- Few properties of error function are

$$Erf(\infty) = 1 \tag{13}$$

$$Erf(-t) = -Erf(t) \tag{14}$$

POST-MEASUREMENT STATE AT TIME t

- When the particle is found at the instant t in the region between $x \to -\infty$ and $x = 0$, the *post-measurement state* is given by

$$|\psi_{+}^{PM}(t)\rangle = \int_{-\infty}^{0} \psi(x', t)|x'\rangle dx' \quad (15)$$

- When the particle is found at the instant t in the region between $x = 0$ and $x \to \infty$, the *post-measurement state* is given by

$$|\psi_{-}^{PM}(t)\rangle = \int_{0}^{\infty} \psi(x', t)|x'\rangle dx' \quad (16)$$

FURTHER EVOLUTION OF THE STATE AFTER 1st MEASUREMENT

- If $+1$ result is obtained at, say, $t = t_1$, then the post-measurement state under the harmonic oscillator potential evolves into the following state at the instant $t = t_2$

$$|\psi_{+}^{PM}(t_2)\rangle = \int_{-\infty}^{\infty} K(x', t_1; x, t_2)\psi(x', t_1)_{+}^{PM}|x'\rangle dx' \qquad (17)$$

- If -1 result is obtained at, say, $t = t_1$, then the post-measurement state under the harmonic oscillator potential evolves into the following state at the instant $t = t_2$

$$|\psi_{-}^{PM}(t_2)\rangle = \int_{-\infty}^{\infty} K(x', t_1; x, t_2)\psi(x', t_1)_{-}^{PM}|x'\rangle dx' \qquad (18)$$

JOINT PROBABILITIES AFTER THE 2nd MEASUREMENT

- Conditional Probability of finding the particle in the region between $x \to -\infty$ and $x = 0$ at the instant t_2 when $\pm$ result for the measurement of the localization operator $\widehat{0}$ has been obtained at the instant t_1 is given by

$$P_{\pm/+}(t_1, t_2) = \int_{-\infty}^{0} |\psi(x, t_2)_{\pm}^{PM}|^2 dx \tag{19}$$

- Similarly, the Conditional Probability of finding the particle in the region between $x = 0$ and $x \to \infty$ at the instant t_2 when $\pm$ result for the measurement of the localization operator $\widehat{0}$ has been obtained at the instant t_1 is given by

$$P_{\pm/-}(t_1, t_2) = \int_{0}^{\infty} |\psi(x, t_2)_{\pm}^{PM}|^2 dx \tag{20}$$

TEMPORAL CORRELATION FUNCTIONS

- The *temporal correlation function*, say, C_{12} occurring in the Leggett-Garg inequality is given by

$$C_{12} = P_{++}(t_1, t_2) - P_{+-}(t_1, t_2) + P_{--}(t_1, t_2) - P_{-+}(t_1, t_2) \quad (21)$$

- where $P_{++}(t_1, t_2)$ is the *joint probability* of finding the measurement outcomes $+1$, $+1$ at the respective times t_1 and t_2; similarly, $P_{+-}(t_1, t_2)$, $P_{--}(t_1, t_2)$, and $P_{-+}(t_1, t_2)$ denote the corresponding *joint probabilities*. Thus, by evaluating these *joint probabilities*, one can calculate the quantity C_{12}.

  In a similar way, the other temporal correlation functions C_{23}, C_{34}, C_{14} occurring in the *4-term LGI* can also be calculated, thereby checking the validity of the *4-term LGI*

$$|C_{12} + C_{23} + C_{34} - C_{14}| \leq 2$$

SCHRÖDINGER COHERENT STATE

Taking $\sigma_0 = \sqrt{\frac{\hbar}{2m\omega}}$ in $\psi(x, 0)$ corresponds to Schrödinger Coherent State.

- Probability density of this time-evolved state is given by

$$|\psi(x,t)|^2 = \sqrt{\frac{m\omega}{\hbar\pi}} \exp\left(-m\omega \frac{(x - \frac{p_0}{m\omega}\sin\omega t)^2}{\hbar}\right) \tag{22}$$

- The probability density of this wave packet oscillates *without spreading* or *changing shape* with its *peak* following classical motion and $\Delta x \Delta p = \hbar/2$. Hence coherent state is regarded as the "best possible" *quasi-classical* quantum description of the motion of a linear harmonic oscillator.

SALIENT FEATURES OF CALCULATIONAL RESULTS

In our setup, the key parameters are p_0, ω where p_0 is the initial peak momentum (expectation value of momentum corresponding to the initial wave packet) and ω is the angular frequency of the corresponding classical oscillation. Suitably choosing p_0, ω and by appropriate tuning of $t, \Delta t$, the QM violation of LGI for a given mass (m) may be shown.

In the calculational results we present, using the 4-term LGI, p_0 and ω are throughout chosen such that the corresponding classical amplitude of oscillation $(A_{Cl} = p_0/m\omega)$ ranges from 10^{-4} m to 10^{-10} m, $\Delta t = 2.4 \times 10^{-6}$ s and $t_1 = 1.5 \times 10^{-6}$ s where $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \Delta t$, with the time period (T) of oscillation $T = 3.14 \times 10^{-6}$ s corresponding to $\omega = 2 \times 10^6$ Hz.

QUANTUM VIOLATION OF LGI FOR LARGER MASSES

Tuning p_0 appropriately one can find violation of LGI at large masses.

$\omega = 2 \times 10^6$ Hz. $\sigma_0 = \sqrt{\frac{\hbar}{2m\omega}}$; $A_{Cl} = \frac{p_0}{m\omega}$

MASS (amu)	$\sigma_0(m)$	$p_0(Kgm/s)$	$A_{Cl}(m)$	C
10	3.9×10^{-8}	10^{-24}	10^{-4}	2.62
10^3	3.9×10^{-9}	10^{-23}	10^{-5}	2.58
10^6	1.2×10^{-10}	10^{-21}	10^{-6}	2.50
10^8	1.2×10^{-11}	10^{-20}	10^{-7}	2.54
10^{10}	1.2×10^{-12}	10^{-21}	10^{-10}	2.70

v_0 ranges from 10^2 m/s (for m = 10 amu), 10 m/s (for $m = 10^3$ amu), 2 m/s (for $m = 10^6$ amu) to 10^{-4} m/s (for $m = 10^{10}$ amu), 10^{-8} m/s (for $m = 10^{20}$ amu).

QUANTUM VIOLATION OF LGI WITH CHANGING MASS FOR A FIXED INITIAL PEAK MOMENTUM

- Here the key parameters p_0, ω are kept fixed and $t_1, \Delta t$ are appropriately chosen to maximize quantum violation of LGI.

MASS(amu)	$\sigma_0(m)$	$A_{Cl}(m)$	C
10^2	1.2×10^{-8}	10^{-5}	2.8
10^3	3.8×10^{-9}	10^{-6}	2.74
10^4	1.2×10^{-9}	10^{-7}	2.65
10^6	1.2×10^{-10}	10^{-9}	1.56

- $\omega = 2 \times 10^6$ Hz, $p_0 = 3.3 \times 10^{-24} kgm/s, v_0 = 2 \times 10^2 \mathrm{m/s}$
  $T = 3.14 \times 10^{-6}$s, $A_{Cl} = p_0/m\omega$,
  $t_1 = 1.5 \times 10^{-6}$s, $\Delta t = 2.4 \times 10^{-6}$s.

Some salient points

- If for given values of m and ω, p_0 is increased, the corresponding A_{Cl} is also increased. Then the QM value of C is found to be gradually decreasing; eventually $C < 2$ for appropriately large A_{Cl}, for example, for $m = 10^3$ amu, $\omega = 2 \times 10^6$ Hz, $C < 2$ for $A_{Cl} > 10^{-4}$ m with $v_0 \gtrsim 10^2$ m/s.
- Comparing the calculational results with that using 3-term LGI, WLGI and NSIT, we find that for a given mass, the values of v_0 and A_{Cl} required to show significant violation of MR are optimal using 4-term LGI.
- Thus, in contrast to the first example discussed in this talk, the use of NSIT has *no* special advantage in this second example.
- On the other hand, a very recent expt. test of MR for the magnetic flux trapped in a superconducting ring [G. C. Knee et al. *Nature Communications* 7, 13253 (2016)] requires NSIT to show violation of MR for the low visibility of measurement for which LGI is *not* violated.

3-term LGI results

- $K_L = C_{12} + C_{23} - C_{13}, \omega = 1.2 \times 10^6 Hz$.
  $t_1 = 0.476T, \Delta t = 0.723T, T = 5.24 \times 10^{-6} s$.

M(amu)	σ(m)	v_0(m/s)	A_{Cl}(m)	K_L
10^6	1.5×10^{-10}	2×10^{-3}	1.6×10^{-8}	1.5
10^{10}	1.5×10^{-12}	2×10^{-5}	1.6×10^{-11}	1.3

WLGI results

- $K_W = p(Q_2^+ Q_3^-) - p(Q_1^+ Q_2^+) - p(Q_1^- Q_3^-), \omega = 1.2 \times 10^6 Hz$. ϵ_{max}/σ indicates above which no violation obtained. $t_1 = 0.006T, \Delta t = 0.755T, T = 5.24 \times 10^{-6} s$.

M(amu)	σ(m)	v_0(m/s)	A_{Cl}(m)	K_W
10^6	1.5×10^{-10}	2×10^{-3}	1.6×10^{-8}	0.47
10^{10}	1.5×10^{-12}	2×10^{-4}	1.6×10^{-10}	0.46

NSIT results

- $K_N = p(Q_2^+) - p(Q_1^+Q_2^+) - p(Q_1^-Q_2^+), \omega = 1.2 \times 10^6 Hz$. $t_1 = 0, \Delta t = T/2, T = 5.24 \times 10^{-6} s$.

M(amu)	σ(m)	v_0(m/s)	A_{Cl}(m)	K_N
10^6	1.5×10^{-10}	2×10^{-3}	1.6×10^{-8}	0.5
10^{10}	1.5×10^{-12}	2×10^{-5}	1.6×10^{-11}	0.5

The Proposed Setup

Experimental aspects of the proposed scheme

- The system considered is a nano-object of mass, say, 10^6 amu trapped by laser fields that generate a harmonic well of $\omega \sim 10^6$ Hz [Y. Bateman et al. *Nature Communications* 5, 4788 (2014)].
- Damping and decoherence effects are *negligible* for such a system in the experimental time-scale of $1/\omega(\sim 10^{-6} s)$ where the typical decoherence time is 1 - 10 ms for optically levitated oscillating objects.
- The positions of the optically levitated masses can be observed with extremely high spatial resolution by means of photo-diodes using the interferometric (phase sensitive) detection of light scattered from the objects.

  This enables detection of positions with *sub-Angstrom resolutions*, while the detecting time-window to achieve such resolutions is much *smaller* than the experimental time-scale $1/\omega$ so that the measurements can be regarded instantaneous.
- By criss-crossing, say, the $x > 0$ domain of the well with several such scattering light fields whose intensity is ensured to fall to zero *sharply* at $x = 0$, in case the nano-object *fails* to scatter light, then its state will be projected to the eigenstate of its localization within the $x < 0$ domain of the well.

The Proposed Setup

- Imprecision involved in realizing such coarse-grained measurement would lie in *differentiating* sharply the presence of the object in the $x < 0$ compared to in the $x > 0$ domain of the harmonic well which is also critical for implementing ideal NRM.
- Interestingly, in this procedure, the robustness of implementing this *differentiation* is found to be the *same* of the order of σ_0 around $x = 0$, for 4 term or 3-term LGI, WLGI as well as for NSIT.
- The setup proposed seems to be promising to provide a procedure for a convincing realization of NRM that can be scaled up to large masses of optically levitated nano-objects.

To conclude......

A glimpse is provided of the line of studies that probe conceptual aspects of QM incompatibility with MR, its implications, and the empirical testing of MR with increasing rigour using various necessary conditions of MR whose relative efficacy depends upon the experimental context.

This research direction, apart from providing powerful tests of nonclassicality, complements probing of the QM incompatibility with LR towards providing greater constraints on the various possible realist models of the physical world.