Information retrieval from black holes

Sumanta Chakraborty

IACS

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Outline

1. QFT in curved spacetime.
2. Effect of non-vacuum states.
3. The information paradox.
4. Can information be retrieved by quantum correlations?

Main References

1. Kinjalk Lochan, **Sumanta Chakraborty** and T. Padmanabhan, arXiv:1604.04987.
2. Kinjalk Lochan, **Sumanta Chakraborty** and T. Padmanabhan, arXiv:1603.01964.
3. Kinjalk Lochan and T. Padmanabhan, arXiv:1507.06402.

QFT in Flat Spacetime

- In quantum field theory the basic entity is the field. Excitation of these fields are the particles in momentum space.
- These particles are relativistic and have definite momentum.
- One of the basic problem that quantum field theory in *flat* spacetime addresses is as follows:

Sample

- Given an initial state ($t \rightarrow -\infty$) consisted of an electron with momentum $\mathbf{k_1}$ and a photon of momentum $\mathbf{k_2}$ what is the scattering cross section to a final state ($t \rightarrow \infty$) consisted of an electron with momentum $\mathbf{k_3}$ and a photon of momentum $\mathbf{k_4}$.

The spherical lion

- We will assume that these quantum fields are in some background field, which can be adequately described using classical equations.
- We will work exclusively with massless scalar field, such that the Lagrangian has only a kinetic term.
- The scalar field can be expanded in the Fourier space in terms of mode functions $u_{\mathbf{k}}(x)$ with creation and annihilation operators.

Scalar Field Expansion

$$\hat{\phi}(t,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(\hat{a}_{\mathbf{k}} u_{\mathbf{k}} + \hat{a}^{\dagger}_{\mathbf{k}} u^{*}_{\mathbf{k}} \right)$$

- For flat Minkowaski spacetime the mode functions $u_{\mathbf{k}}(x)$ are just plane waves, while in general curved spacetime the Klein-Gordan equation do not necessarily have plane wave solutions.

The observer dependence and particles

- Under general coordinate transformation two such mode functions: $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ will mix up.

Bogoliubov Transformation

$$v_{\mathbf{k}} = \int dV_{\mathbf{k}'} \left(\alpha_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}'} + \beta_{\mathbf{k}\mathbf{k}'} u^*_{\mathbf{k}'}\right)$$

- The scalar field can also be expanded in terms of the mode functions $v_{\mathbf{k}}$ with the respective creation and annihilation operators $b_{\mathbf{k}}$ and $b^\dagger_{\mathbf{k}}$ respectively.
- As a consequence the "a" vacuum will contain "b" particles.

Particles

$$\langle 0_a | N_{b,\mathbf{k}} | 0_a \rangle = \int dV_{\mathbf{k}'} |\beta_{\mathbf{k}\mathbf{k}'}|^2$$

What about excited states?

- What about number of "b"-particles in excited states of "a"-vacuum?
- We will concentrate on single particle excitations.

Single particle excitation

$$|\psi_a\rangle = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} f(\omega)\hat{a}^\dagger_\omega |0_a\rangle$$

- The number of "b" particles in the "a" particle excited state over and above the vacuum becomes

Correction

$$N_\Omega^{\rm Correction} = \left| \int_0^\infty \frac{d\bar{\omega}}{\sqrt{4\pi\bar{\omega}}} \alpha^*_{\Omega\bar{\omega}} f(\bar{\omega}) \right|^2 + \left| \int_0^\infty \frac{d\omega'}{\sqrt{4\pi\omega'}} \beta_{\Omega\omega'} f(\omega') \right|^2$$

You can go in but you can not come out

- As a massive star collapses and neither the degeneracy pressure of electron or the proton-neutron plasma is able to balance gravity, a black hole is born.
- It loses all possible information. Remains — mass, charge and angular momenta — no hair.
- One way membrane, one can throw things in but nothing comes out — Null surface.
- The line element for the background classical gravity corresponds to,

Schwarzschild

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$

Penrose Diagram

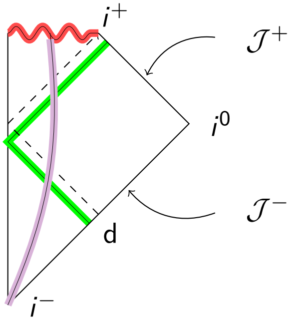
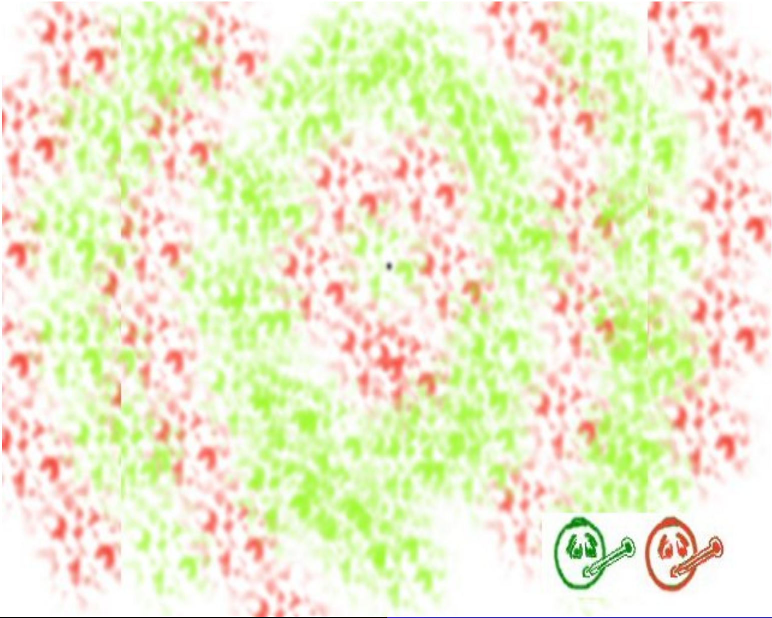


Figure: Penrose Diagram For Schwarzschild collapse

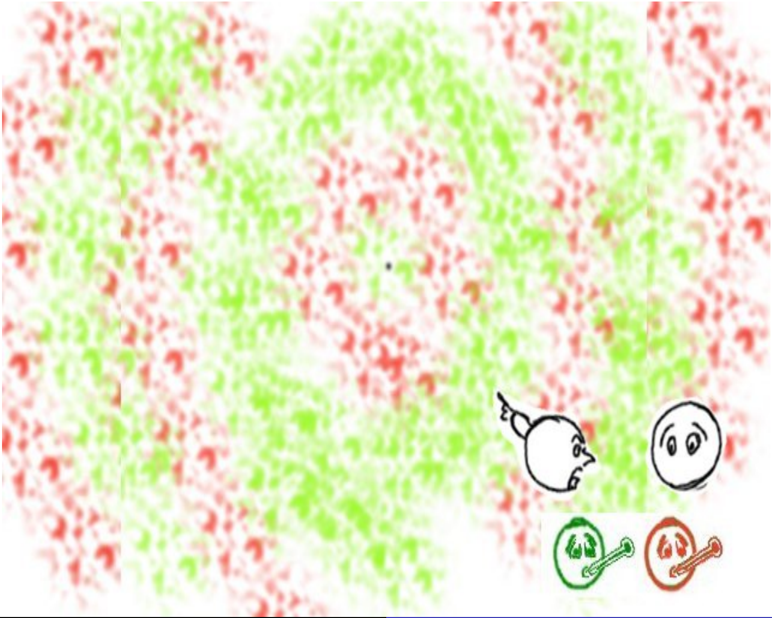
What is the Paradox?

- Let us throw a cup of tea into the black hole — only change, change of mass.
- However, in principle one can go inside the event horizon and with singularity resolution (if she knows quantum gravity) recovers what has been thrown inside.
- But the black hole evaporates thermally. Thus all the information about the cup of tea is lost, as the thermal radiation has a single information about temperature and hence mass.
- However a Planck mass remnant might be there, since semi-classical approximations break down.
- But the Planck sized remnant cannot hold all the information that has gone in the black hole.
- Hence the real paradox.

Only Thermal



Can there be a remnant?



Planck Size Remnant

Versions of the paradox

- I will be interested in information retrieval — From the spectrum at late times to tell what has fallen into the black hole.
- For the Schwarzschild spacetime vacuum defined for a scalar field before the collapse and at late times are different, due to geometry change.
- Hence the Bogoliubov coefficients are non-zero.

Coefficients

$$\alpha_{\Omega\omega} = \frac{1}{2\pi\kappa}\sqrt{\frac{\Omega}{\omega}}\exp\left[\frac{\pi\Omega}{2\kappa}\right]\exp\left[\frac{i\Omega}{\kappa}\log\frac{\omega}{C}\right]\Gamma\left[-\frac{i\Omega}{\kappa}\right]$$

$$\beta_{\Omega\omega} = -\frac{1}{2\pi\kappa}\sqrt{\frac{\Omega}{\omega}}\exp\left[-\frac{\pi\Omega}{2\kappa}\right]\exp\left[\frac{i\Omega}{\kappa}\log\frac{\omega}{C}\right]\Gamma\left[-\frac{i\Omega}{\kappa}\right]$$

Throw a particle

- Assume that we throw a particle in the black hole. Since all matter fields are inherently quantum this should be taken as an excited quantum state. In particular an one particle state

One particle

$$|\psi\rangle_{\rm in} = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} f(\omega)\hat{a}^\dagger_\omega |0_a\rangle$$

- The single particle mode we have thrown into the black hole at late times will lead to non-zero frequency cross-correlations, such that

Non-vacuum correlations

$$_{\rm in}\langle\psi|\hat{N}_{\Omega_1\Omega_2}|\psi\rangle_{\rm in} = \frac{1}{4\pi}\left[A(\Omega_1)A(\Omega_2)^* + B(\Omega_1)B(\Omega_2)^* + c.c.\right]$$

Hide and seek with $f(\omega)$

- We are working with a different operator.

Another Operator

$$\hat{N}_{\Omega_1\Omega_2} = \hat{b}^{\dagger}_{\Omega_1}\hat{b}_{\Omega_2}e^{-i(\Omega_1-\Omega_2)t} + \hat{b}^{\dagger}_{\Omega_2}\hat{b}_{\Omega_1}e^{i(\Omega_1-\Omega_2)t}$$

- The information about initial state can be obtained through $A(\Omega)$ and $B(\Omega)$. They depend on Fourier transformation $F(\Omega)$ of the original function $f(\omega)$.

Dependence on initial state

$$A(\Omega) = e^{-\frac{\pi\Omega}{2\kappa}}\frac{\sqrt{\Omega}}{2\pi\kappa}\Gamma\left[-i\frac{\Omega}{\kappa}\right]F\left(\frac{\Omega}{\kappa}\right)e^{-i\Omega t}$$

$$B(\Omega) = e^{\frac{\pi\Omega}{2\kappa}}\frac{\sqrt{\Omega}}{2\pi\kappa}\Gamma\left[-i\frac{\Omega}{\kappa}\right]F^{*}\left(-\frac{\Omega}{\kappa}\right)e^{-i\Omega t}$$

A derived quantity

- Given the frequency correlation $N_{\Omega_1\Omega_2}$, using the off diagonal components, one can construct,

Construction

$$\mathcal{D}(\Omega_1\Omega_2) = N_{\Omega_1\Omega_2} + \left(\frac{i}{\Omega_1 - \Omega_2}\right)\frac{\partial N_{\Omega_1\Omega_2}}{\partial t}$$

- This, through $N_{\Omega_1\Omega_2}$ depends on $F(\Omega)$. Inverse Fourier transform would yield the initial state $f(\omega)$.

Fourier Transform

$$f(\omega) = \frac{1}{2\pi\kappa}\int_{-\infty}^{\infty} d\Omega\ F\left(\frac{\Omega}{\kappa}\right) e^{-i\frac{\Omega}{\kappa}\ln\omega}$$

Extracting Information

- For real initial state $\Rightarrow f(\omega) = f^*(\omega)$.
- Properties of Fourier transform $\Rightarrow F(\Omega/\kappa) = F^*(-\Omega/\kappa)$.
- This leads to full information recovery.

Information

$$S_{\Omega_1\Omega_2} \equiv \frac{4\pi^3\kappa^2}{\sqrt{\Omega_1\Omega_2}} \frac{\mathcal{D}_{\Omega_1\Omega_2} e^{i(\Omega_1-\Omega_2)t}}{\Gamma\left[-i\frac{\Omega_1}{\kappa}\right]\Gamma\left[i\frac{\Omega_2}{\kappa}\right]\cosh\left(\frac{\pi(\Omega_1+\Omega_2)}{2\kappa}\right)}$$
$$= F\left(\frac{\Omega_1}{\kappa}\right)F^*\left(\frac{\Omega_2}{\kappa}\right)$$

- Real initial state can be fully reconstructed out of the correlations in the outgoing modes.

Knowledge about initial state

- Thus knowledge about the correction $N_{\Omega_1\Omega_2}$ will enable us to compute $F(\Omega/\kappa)$, the Fourier transform of $f(\omega)$.
- Complete information about $F(\Omega/\kappa)$ requires some symmetries — real initial state.
- Having obtained $F(\Omega/\kappa)$ one can apply inverse Fourier transform to get $f(\omega)$.
- Even though shown for a single particle state it works for multi-particle states as well.
- **Future Outlook:** We have only used two point correlation functions. More information may be retrieved by using higher order correlations. In these cases we might not have to assume any particular symmetry in order to retrieve all the information.

Summary

- Brief idea about quantum fields in curved spacetime, Schwarzschild solution and non-uniqueness of vacuum has been presented.
- Treating matter fields to be fundamentally quantum, from the non-vacuum distortions of the thermal spectrum in Schwarzschild spacetime we have derived information about initial state.
- The diagonal elements of the frequency correlator leads to complete information recovery in case of real and symmetric distribution, while off-diagonal elements requires the state to be real only.
- The calculation has been done for two-point function, higher-point functions might give more information.

Thank You