Exploring Hidden Non-locality using Weak Interaction and Post-Selection

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Non-locality in the usual Bell scenario

- Non-locality of bipartite qubit state usually demonstrated in an archetypal scenario, where Alice and Bob make two dichotomic projective measurements each.

- Correlations between outcomes of measurements in this scenario are tested for non-locality using the Bell-CHSH type inequalities.

- Violations of these inequalities are *not* the necessary condition for showing non-locality of the state in question. In other words, satisfying these inequalities is *not* a guarantee that the state is local.

Non-locality beyond the usual Bell scenario:I

- Popescu(1994) suggested "more complicated experiments"(appropriate filtering) can reveal non-locality not captured in the standard Bell-CHSH scenario. -hidden nonlocality

- Local filtering before testing the correlations for non-locality ("hidden non-locality") is one such example. (Gisin(1996)....Brunner *et al* (2013))

Hidden Non-locality of bipartite qubit state-Gisin

• Bipartite qubit state shared between Alice and Bob

$$\rho(\lambda, \alpha) = \lambda P_{\alpha,\beta} + \frac{1}{2}(1-\lambda)(P_{++} + P_{--}) \tag{1}$$

• $P_{\alpha,\beta}$ is the projection operator for $|\psi_{\alpha,\beta}\rangle = \alpha|+-\rangle - \beta|-+\rangle$ and $P_{++/--}$ is the projector of $|++\rangle/|--\rangle$.
• $\rho(\lambda, \alpha)$ displays Bell-non-locality for projective measurement if and only if the corresponding M function is greater than 1.
• M function $M(\lambda, \alpha) = max\{(2\lambda - 1)^2 + 4\lambda^2\alpha^2\beta^2, 8\lambda^2\alpha^2\beta^2\}$
• If that maximum is obtained from the first term then $\lambda > \frac{1}{1+\alpha^2\beta^2}$. (To make sure first term is always greater than the second term we will always take $\lambda > \frac{1}{2-2\alpha\beta}$)

Hidden Non-locality of bipartite qubit state-Gisin

Consider filters in which one of the polarizations is fully transmitted and other partly reflected. The transmission matrices are given by

$$T_{\text{left}} = \begin{bmatrix} \sqrt{\beta/\alpha} & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

$$T_{\text{right}} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\beta/\alpha} \end{bmatrix} \tag{3}$$

$$\rho_{\text{filter}} = \frac{T_{\text{left}} \otimes T_{\text{right}} \rho(\lambda, \alpha) T_{\text{left}}^{\dagger} \otimes T_{\text{right}}^{\dagger}}{Tr(T_{\text{left}} \otimes T_{\text{right}} \rho(\lambda, \alpha) T_{\text{left}}^{\dagger} \otimes T_{\text{right}})} \tag{4}$$

$$= \frac{1}{2\lambda\alpha\beta + (1-\lambda)} (2\lambda\alpha\beta P_{\text{singlet}} + \frac{1}{2}(1-\lambda)(P_{\psi_{++}} + P_{\psi_{--}}) \tag{5}$$

Hidden Non-locality of bipartite qubit state-Gisin

• It can be shown that ρ_{filter} violates Bell-CHSH inequality for $\lambda > \frac{1}{1+2\alpha\beta(\sqrt{2}-1)}$

• One can choose λ in such way that before filtering it does not violate Bell-CHSH inequality but after filtering ρ_{filter} violates Bell-CHSH inequality .

Experimental test of Hidden Nonlocality

• Experimental test of hidden nonlocality of Gisin state has been carried out [Nature $409, 1014 - 1017$(22February 2001)] with photons.

• $\rho(\lambda, \alpha)$ is created by passing each of parametric down converted photons through thick birefringent crystals.

• Local filters with transmission coefficients $T_H = 1$ and $T_V = \epsilon$ is used in each of the arms.

• Hidden non-local states had been found for a range of ϵ and λ.

Filtering with a variant of weak measurement

• Standard Weak Measurement scenario entangles system and device state and then postselect a particular system state.

• We proposed a filtering scheme based on von Neumann type weak interactions between the system and a pointer in both the wings of a bipartite system and subsequent post-selection of system based on pointer values, which amounts to filtration.

• Choosing a system state corresponding to a particular value of detector state has been discussed as a variant of Weak measurement [PLA, $130, 323 - 329$,Nature $474, 188191$(09 June 2011)].

Filtering with a variant of weak measurement

- $\rho_{in} = \rho_A \otimes \rho_B \qquad \rho^s_{AB} = Tr_d[\rho_{in}]$

$\rho_A = \rho^s_A \otimes \rho^d_A$ where ρ^s_A is the system state corresponding to Alice and ρ^d_A is the corresponding detector state.

- Alice and Bob applies local unitary weak interactions U_A and U_B and they locally filter out system states corresponding to particular values of their local detectors. The corresponding local projectors of Alice and Bob are $\Pi_{A/B} = \mathbf{I} \otimes P_{A/B}$.

- The filtered system state can be operationally represented as

$$\rho^{filter}_{AB} = \frac{Tr_d[\Pi_A \otimes \Pi_B(U_A \otimes U_B \rho_A \otimes \rho_B U^\dagger_B \otimes U^\dagger_A)]}{Tr[\Pi_A \otimes \Pi_B(U_A \otimes U_B \rho_A \otimes \rho_B U^\dagger_B \otimes U^\dagger_A)]} \qquad (6)$$

$Tr_d[...]$ is the partial trace over the detector states corresponding to Alice and Bob.

Comparison between Gisin Filter and our proposed filter

• To stress the relevance of the proposed filtering operation comparison between the two filters have been made in the case of Gisin state $\rho(p, \alpha)$ given by Eq. [1]

• The unitary operators corresponding to the local interactions of Alice and Bob are $U_A = e^{-ig_a \sigma_z \otimes P_x} \approx (1 - ig_a \sigma_z \otimes P_x)$ and $U_B = e^{-ig_b \sigma_z \otimes P_y} \approx (1 - ig_b \sigma_z \otimes P_y)$.

• If we apply filtration process defined by Eq. [(6)] and compare the value of M function with Gisin type filtration process given by Eq. [4] then we get the following result.

Comparison between Gisin Filter and our proposed filter

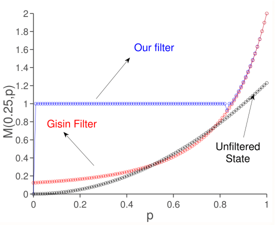

Figure: The value of M function is plotted against the parameter p for a choice of $\theta = 0.25$ for unfiltered states, (the values plotted are maximized over x, y, g_a, g_b)

Inequivalence of entanglement and nonlocality

• Dropping Bob's filter for simplicity, we see that the value of position x for which the entanglement is maximum (measured here by concurrence) will not usually coincide with the value for which the non-locality (measured by the value of Bell-CHSH operator) is maximum.

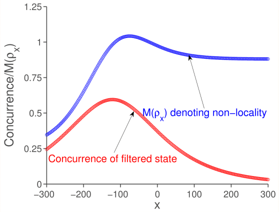

Figure: Fixing $\theta = 0.25$ and $p = 0.89$ for Werner-like states,we plot the value of concurrence and M-function for filtered states characterized by post-selection parameter x

Future Studies

• To find out whether increasing the strength of interaction can make the proposed filter stronger than Gisin filter.

• Obtaining hidden nonlocality by observing violation of I_{3322} inequality for filtered state.

• Find out set of state where the proposed filter can extract nonlocality but Gisin filter does not.


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