Ancilla Assisted

Quantum Measurements

T. S. Mahesh

Indian Institute of Science Education and Research, Pune

IISER PUNE

PhD students:

Abhishek Shukla ………… IISER Pune

Soumya S. Roy ………….. IISER Pune (Uni. Southampton, UK)

Koteswara Rao ………….. IISc Bangalore (Uni. Dortmund, Germany)

Undergraduate students:

Hemant Katiyar ………… IISER Pune (IQC, Canada)

Sharad Joshi ………….. IISER Pune (Uni. Bristol, UK)

Collaborators:

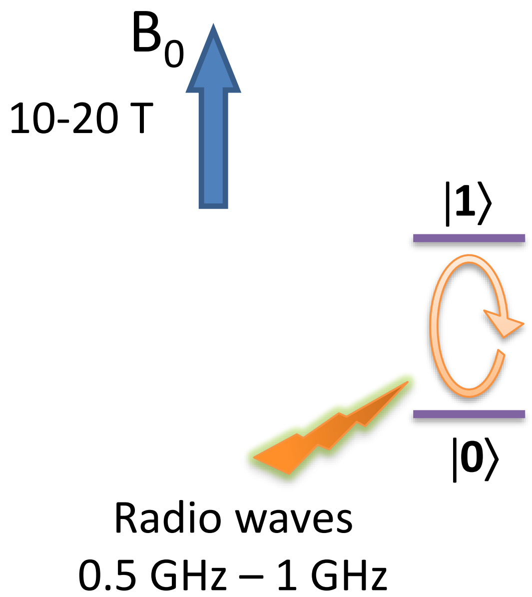
Vikram Athalye ………… Cummins College, Pune

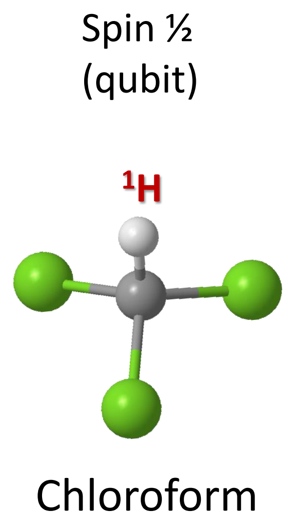
H. S. Karthik ………….. RRI, Bangalore

Prof. Usha Devi ………….. Bangalore Uni.

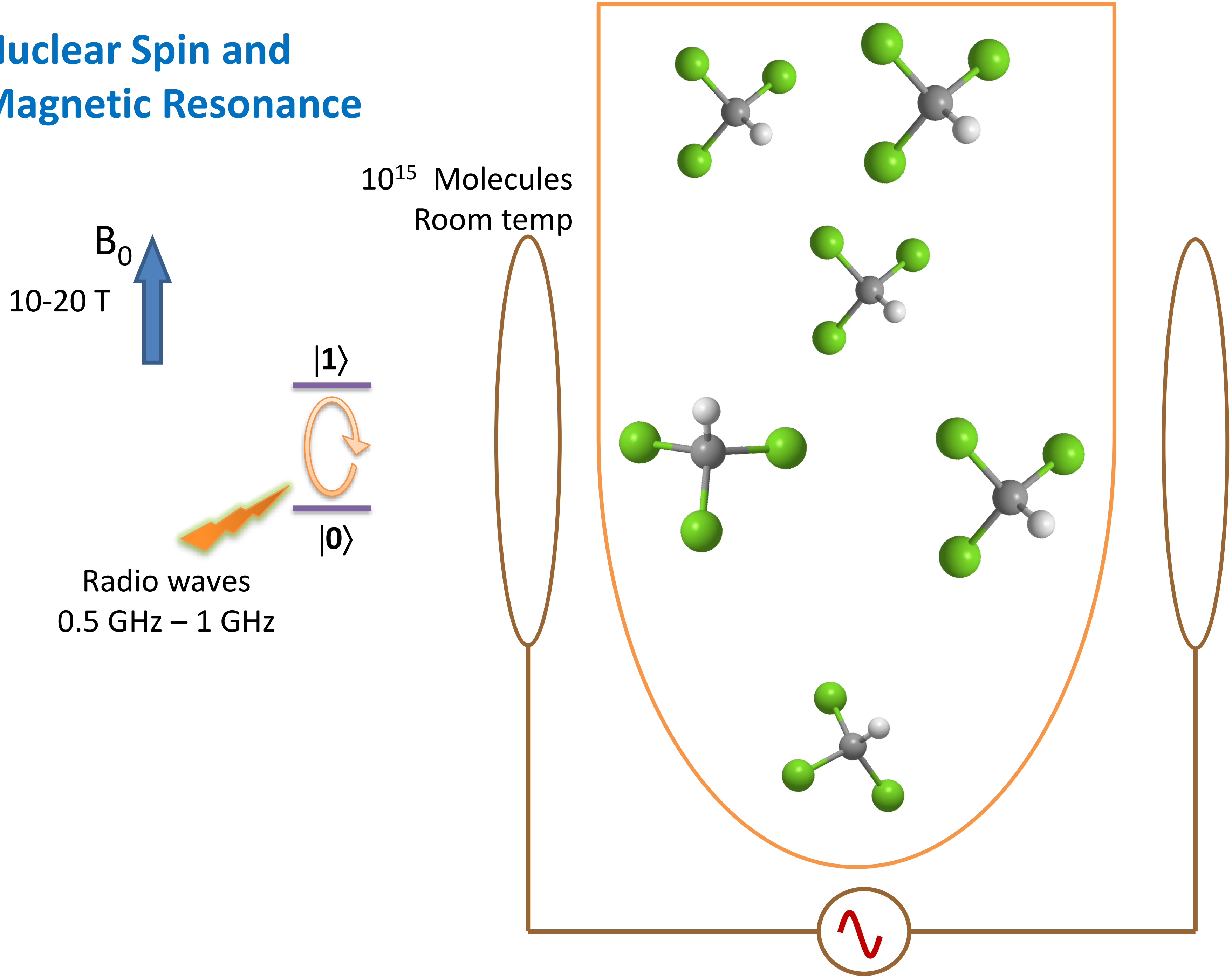
Prof. Rajagopal ………….. Inspire Inst. USA

Nuclear Spin and Magnetic Resonance





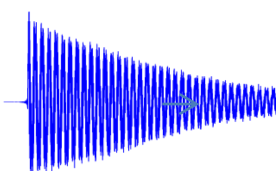
Nuclear Spin and Magnetic Resonance

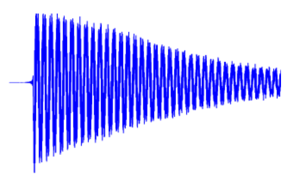


NMR Signal

Phase-sensitive ensemble detection
(Quadrature detection)

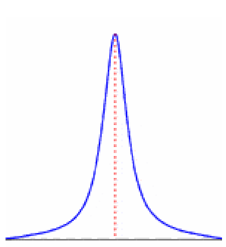
Real part	Imag part
$\langle\sigma_x\rangle = \mathrm{Tr}[\,\rho\,\sigma_x\,]$	$\langle\sigma_y\rangle = \mathrm{Tr}[\,\rho\,\sigma_y\,]$

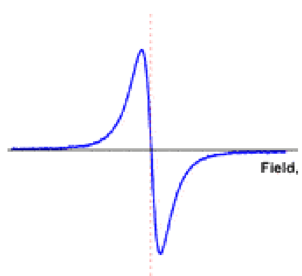


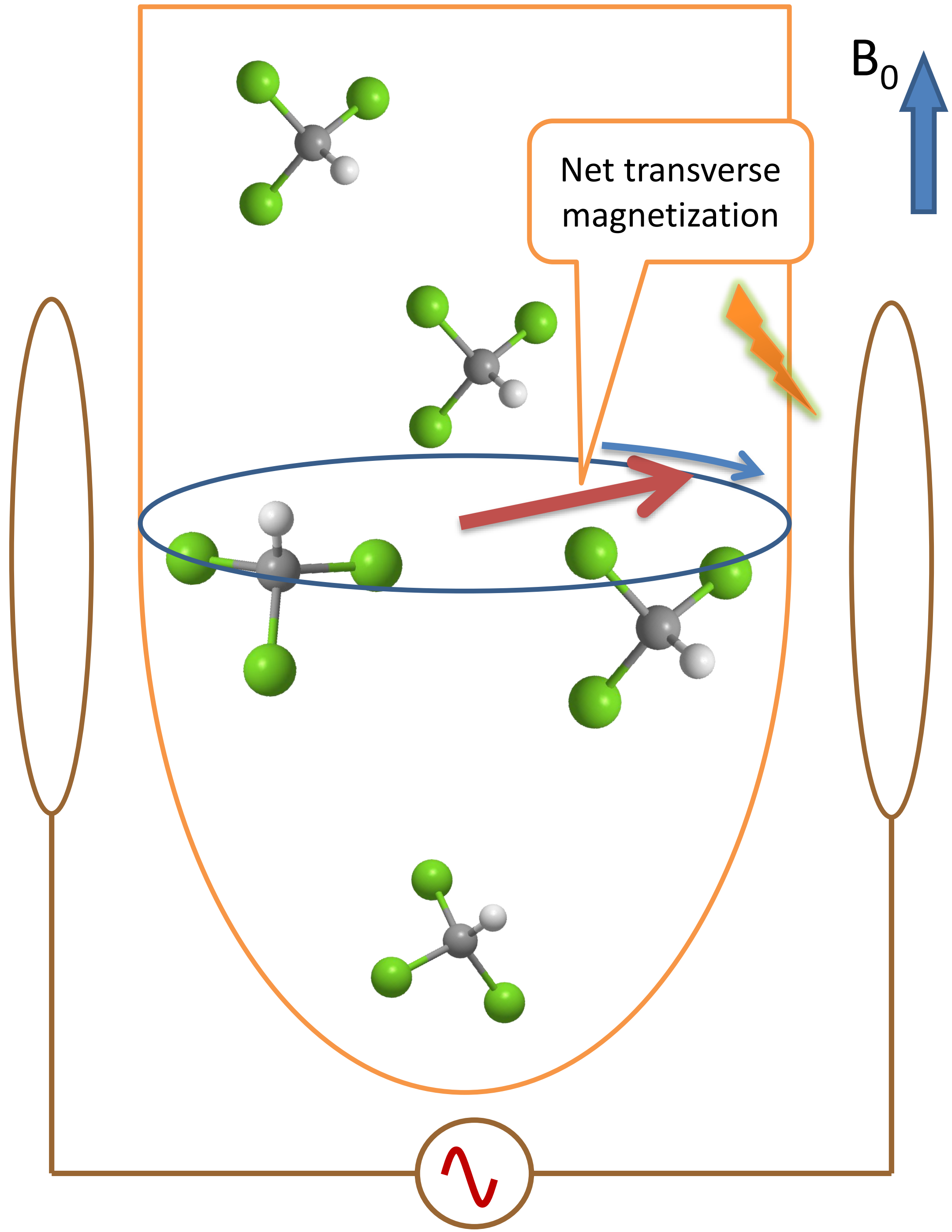


time →　　time →

Fourier
transform







Plan

Expectation values

Joint Probabilities

State Tomography

Process Tomography

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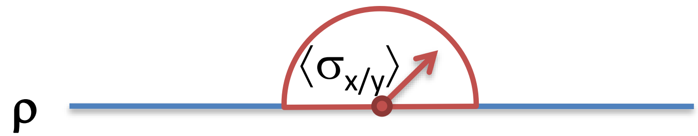
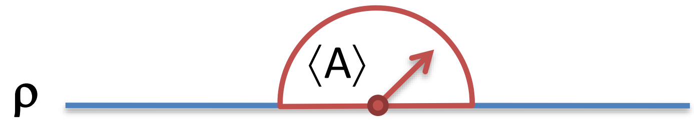
Expectation values

Std. NMR:

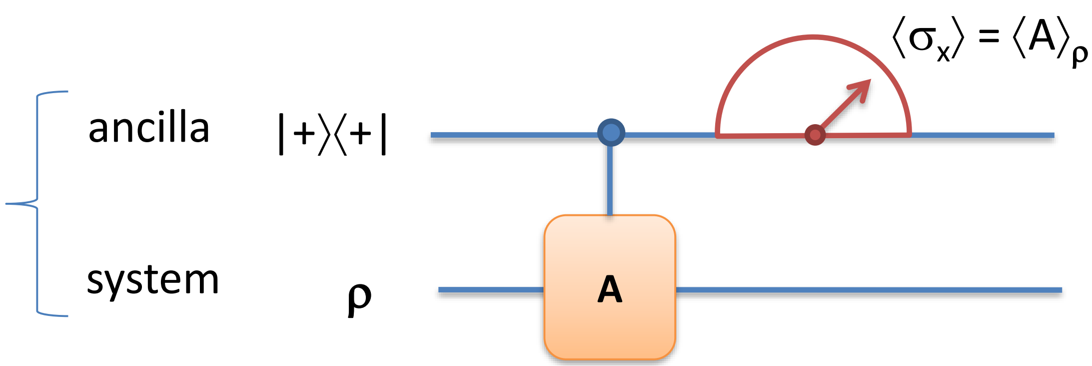


Problem:



Solution: (i) Expectation value of a unitary & Hermitian operator
(Moussa et al, PRL 2010)

Interacting qubits



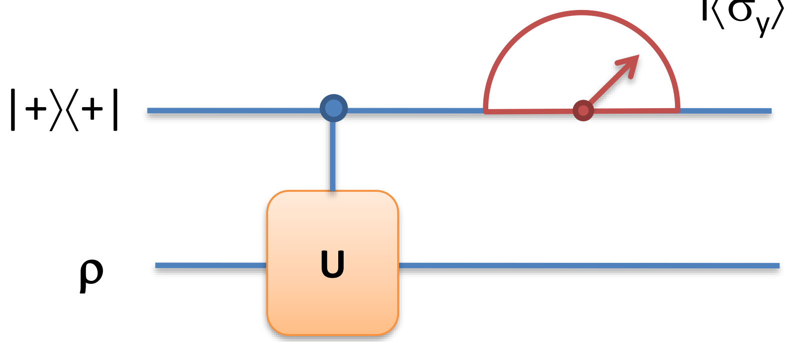
Expectation values

(S. Joshi et al, PRA 2014)

(ii) Expectation value of a general unitary

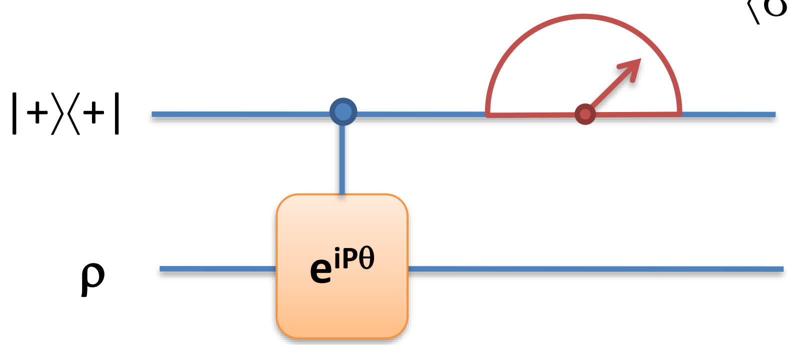
$$\langle \sigma_x \rangle = (\langle U \rangle_\rho + \langle U^\dagger \rangle_\rho)/2$$

$$i\langle \sigma_y \rangle = (\langle U \rangle_\rho - \langle U^\dagger \rangle_\rho)/2$$



(iii) Expectation value of a projector

$$\langle \sigma_x \rangle + i \langle \sigma_y \rangle = 1 + \langle P \rangle_\rho \, (e^{i\theta} - 1)$$

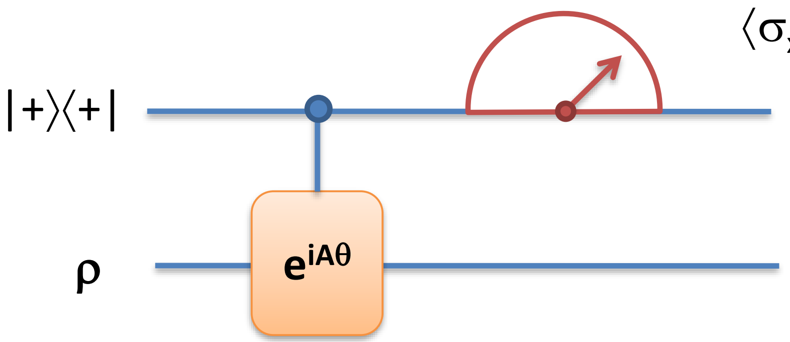


Expectation values

(S. Joshi et al, PRA 2014)

(iv) Expectation value of a diagonal Hermitian observable

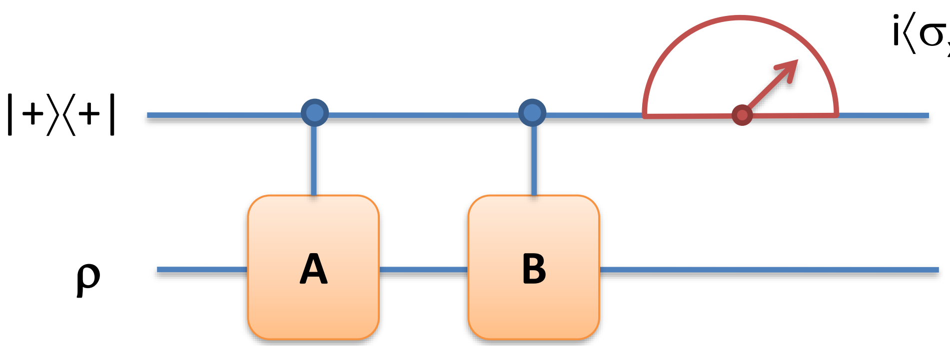
$\langle \sigma_x \rangle + i \langle \sigma_y \rangle = e^{i \langle A \rangle \theta}$



(v) Expectation value of noncommuting unitaries

$\langle \sigma_x \rangle = (\, \mathrm{tr}[BA\rho] + \mathrm{tr}[\rho A^\dagger B^\dagger]\,)/2$

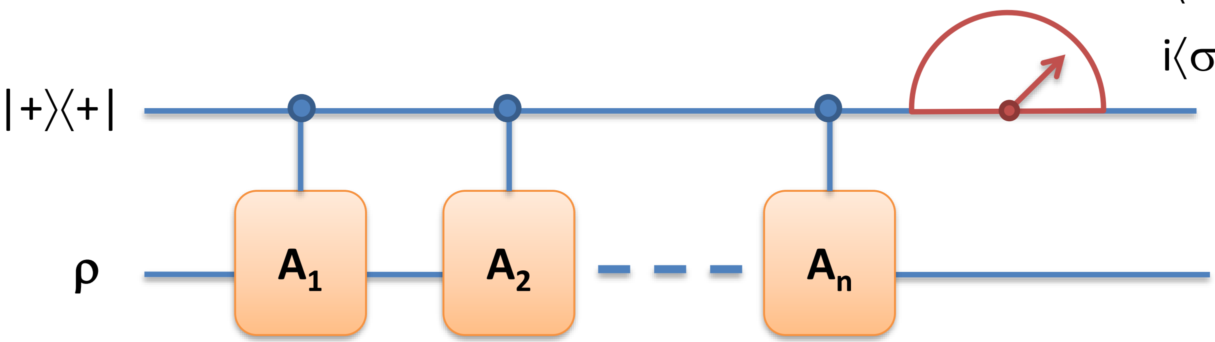
$i\langle \sigma_x \rangle = (\, \mathrm{tr}[BA\rho] - \mathrm{tr}[\rho A^\dagger B^\dagger]\,)/2$



(Katiyar et al, PRA 2013)

Expectation values

(vi) Expectation value of commuting Hermitian unitaries

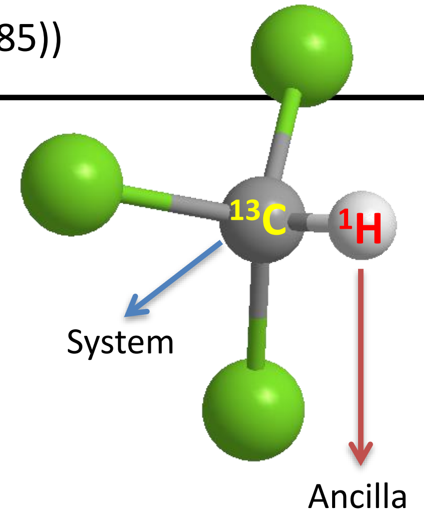


$\langle \sigma_x \rangle = \langle A_1 A_2 \ldots A_n \rangle_\rho$

$i\langle \sigma_y \rangle = 0$

(Moussa et al, PRL 2010)

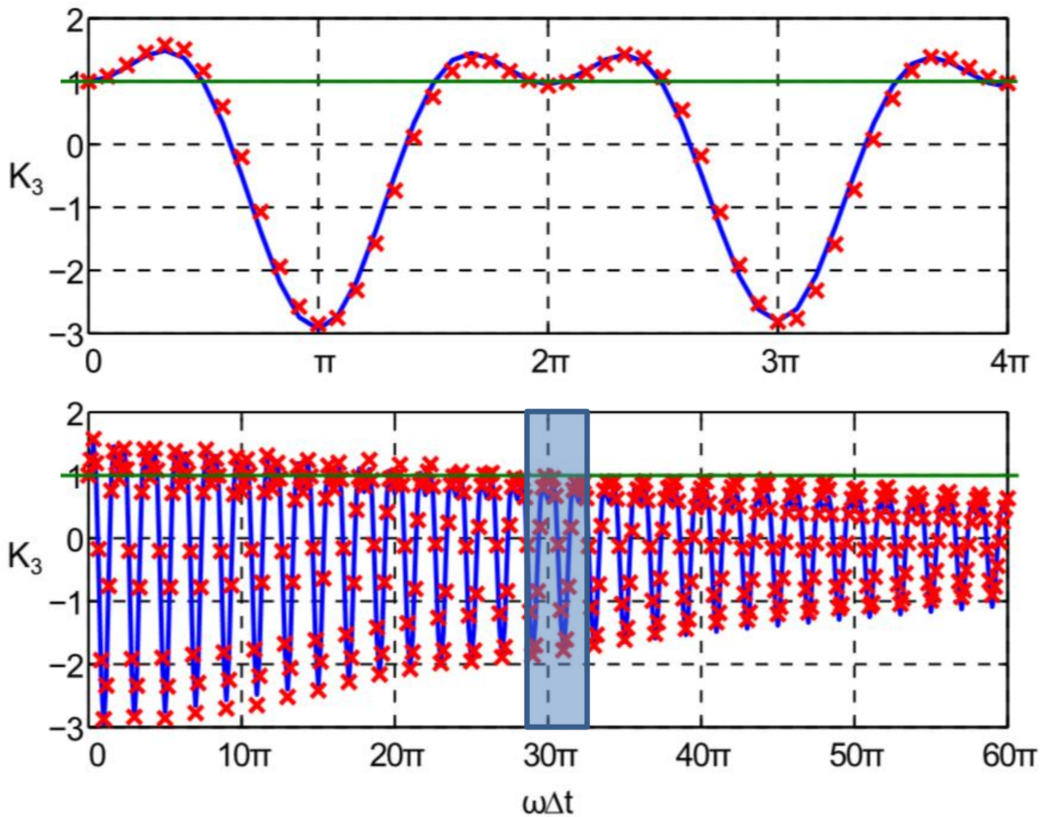
Application: Leggett-Garg inequality (Leggett & Garg, PRL (1985))



Roy et al, PRL 2011

Macrorealistic bound

$K_3 < 1$



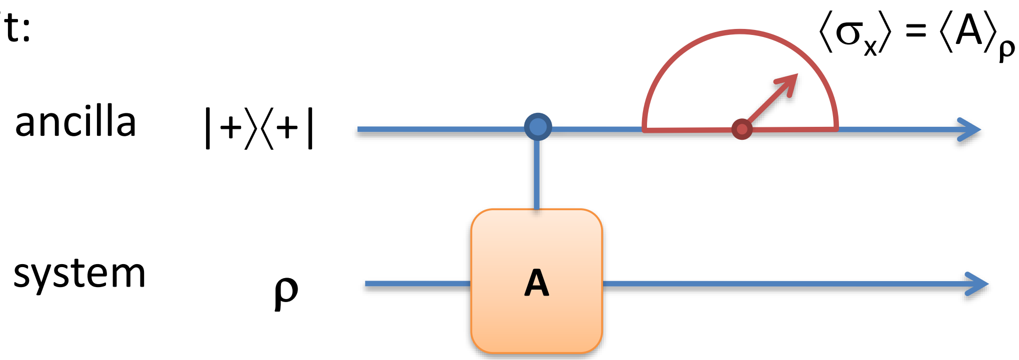
LGI
violation

LGI
satisfaction

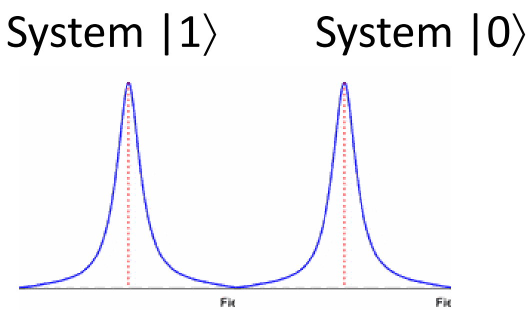
Post-selection

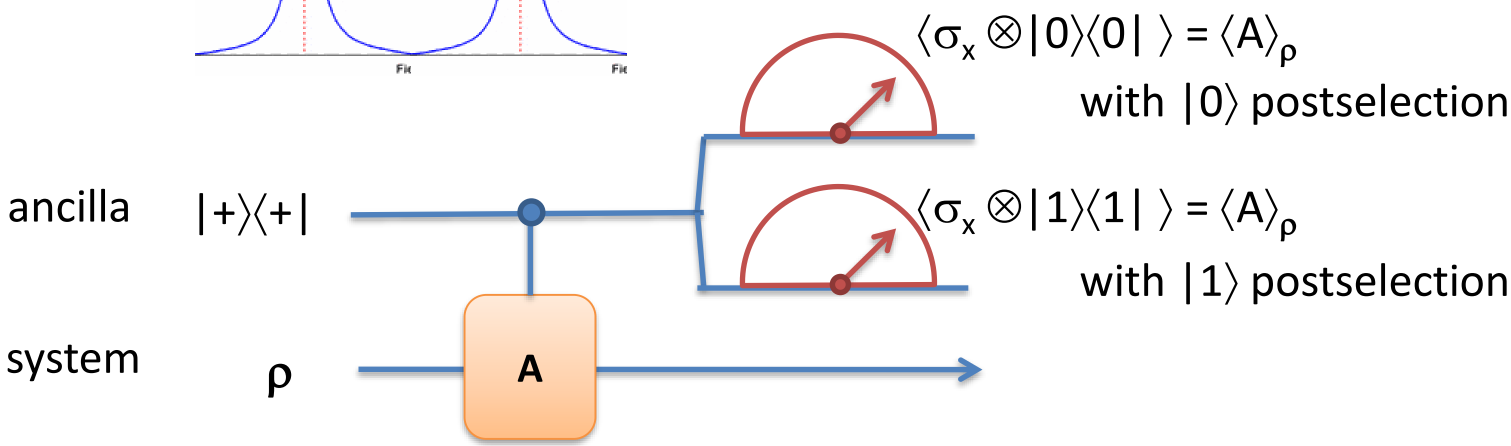
Long et al, Sci. Rep. 2014

Moussa circuit:



Ancilla spectrum:





Plan

Expectation values

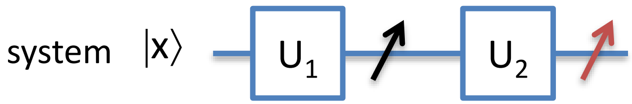
Joint Probabilities

State Tomography

Process Tomography

Noninvasive measurements (NIM)

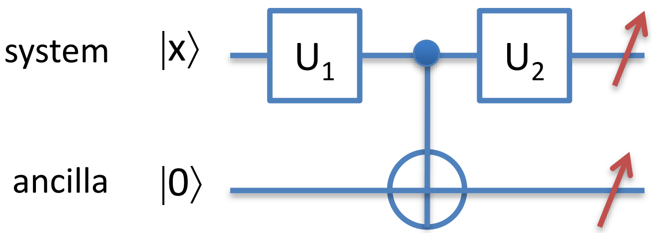
Problem:



Joint-probabilities ?

p(0,0), p(0,1), p(1,0), p(1,1) ?

Solution:



p(0,0)			
	p(0,1)		
		p(1,0)	
			p(1,1)

Joint-probabilities in diagonal density matrix !!

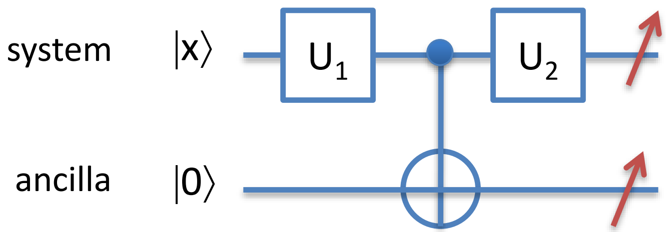
But ancilla is being flipped by the system !!

So not quite noninvasive

Noninvasive measurements via Negative Results

Knee et al, Nat. Comm. 2012

Expt 1 (Same as before)

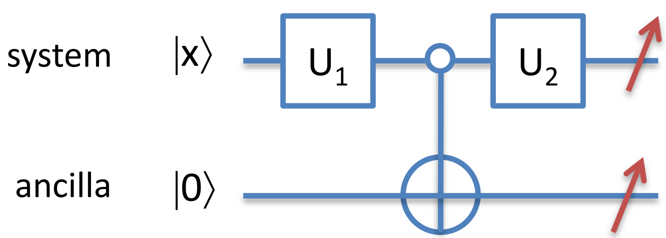


if system is 0
ancilla unaffected

p(0,0)			
	p(0,1)		
		Discard	

Joint-probabilities

Expt 2 (With anti-CNOT)



if system is 1
ancilla unaffected

Discard			
		p(1,0)	
			p(1,1)

Joint-probabilities

(A) Entropic LGI

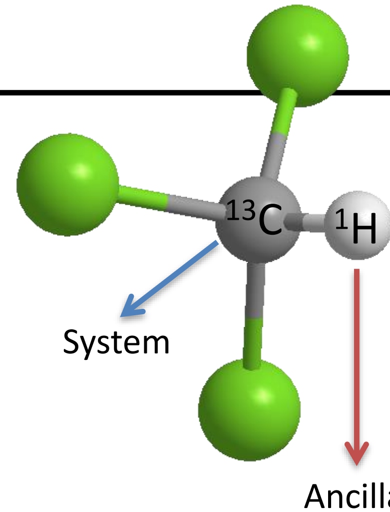
Karthik et al, PRA (2013)

System state: $\mathbb{1}/2$

Q_1 Q_2 Q_3

0 τ 2τ 3τ time

$Q_i \rightarrow q_i = \pm 1$



Dynamical observable : $S_z(\tau) = U S_z U^{\dagger}$, where $U = \exp(i\omega\tau S_x)$

<u>Procedure</u>:

Extract $p(q_i,q_j)$ via NIM

↓

Compute conditional probabilities using Bayes theorem

$p(q_j|q_i) = p(q_i,q_j)/p(q_i)$

↓

Compute conditional entropies

$H(Q_2|Q_1) = H(Q_3|Q_2) = H(\theta/2)$

$H(Q_3|Q_1) = H(\theta)$

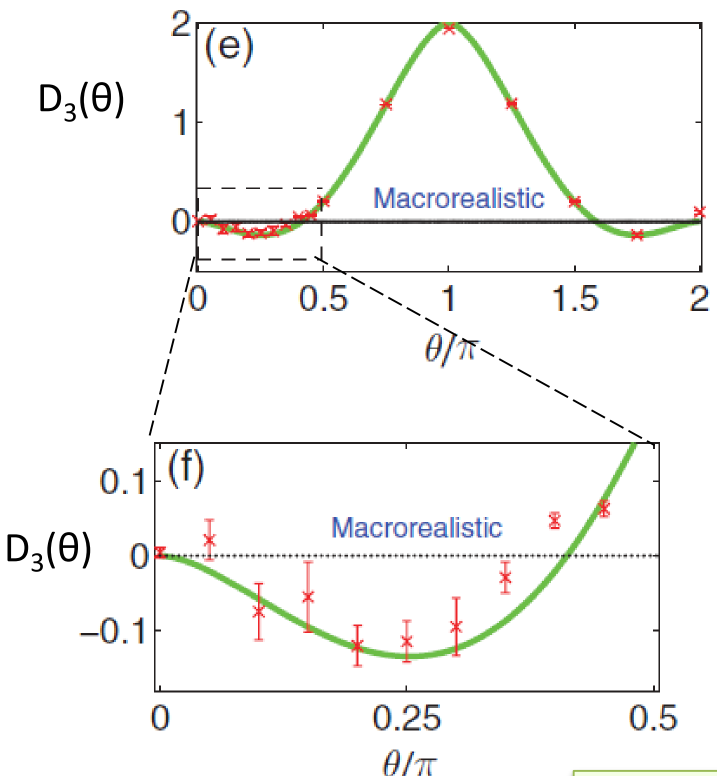
↓

Information deficit,

$D_3(\theta) = 2\,[H(\theta/2) - H(\theta)]$

≥ 0 for macrorealism

<u>Result</u>:



Hemant, Abhishek et al, PRA 052102 (2013)

(B) Probabilities and Moments

Let $\{X_i\}$ is a set of observables assuming dichotomic values $\{x_i = \pm 1\}$.

Moments are linear combinations of joint probabilities

$$\mu_{n1,n2,\ldots,nk} = \langle X_1^{n1} X_2^{n2} \ldots X_k^{nk} \rangle = \sum_{\{x_k\}} x_1^{n1} x_2^{n2} \ldots x_k^{nk} P(x_1,x_2,\ldots,x_k)$$

In classical probability theory: Moments are invertible,

i.e., we can extract probabilities by moments

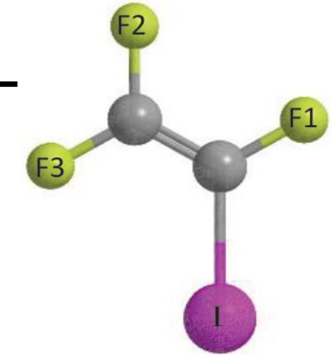
$$P(x_1,x_2,\ldots,x_k) = \sum_{\{n1,n2,\ldots,nk\}} x_1^{n1} x_2^{n2} \ldots x_k^{nk} \mu_{n1,n2,\ldots,nk}$$

In Quantum probability theory: Are moments invertible?

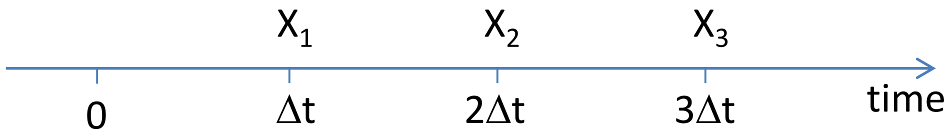
Can we extract probabilities by moments?

- "NO" in general

(B) Probabilities and Moments



$\hat{\rho}_{\rm in} = \mathbb{1}/2$



Hamiltonian $\hat{H} = \frac{1}{2}\hbar\,\omega\sigma_x$

Evolution $\hat{U}(t_i) = e^{-i\,\sigma_x\,\omega\,t_i/2} = \hat{U}_i$

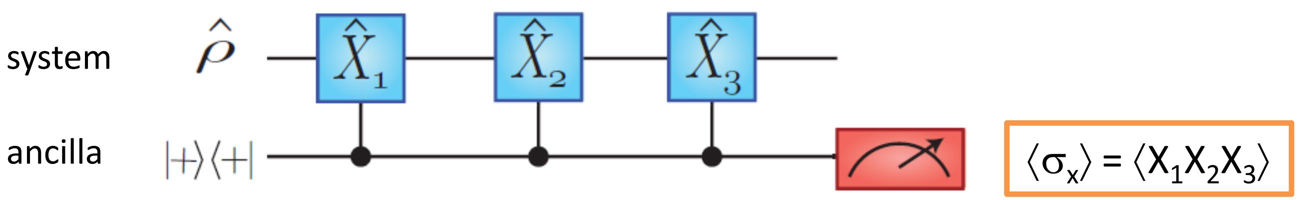
Karthik et al,
PRA, 052118 (2013)

Observables
$$\begin{cases} \hat{X}_1 = \sigma_z, \\ \hat{X}_2 = \sigma_z(\Delta t) = \sigma_z\ \cos(\omega\Delta t) + \sigma_y\ \sin(\omega\Delta t), \\ \hat{X}_3 = \sigma_z(2\Delta t) = \sigma_z\ \cos(2\omega\Delta t) + \sigma_y\ \sin(2\omega\Delta t) \end{cases}$$

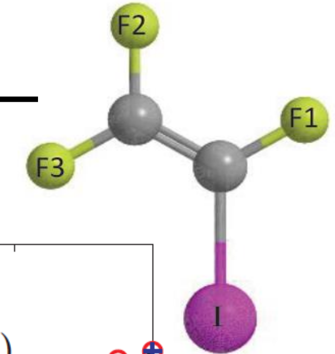
Measurement of moments by Moussa protocol:

Moussa et al, PRA (2010)

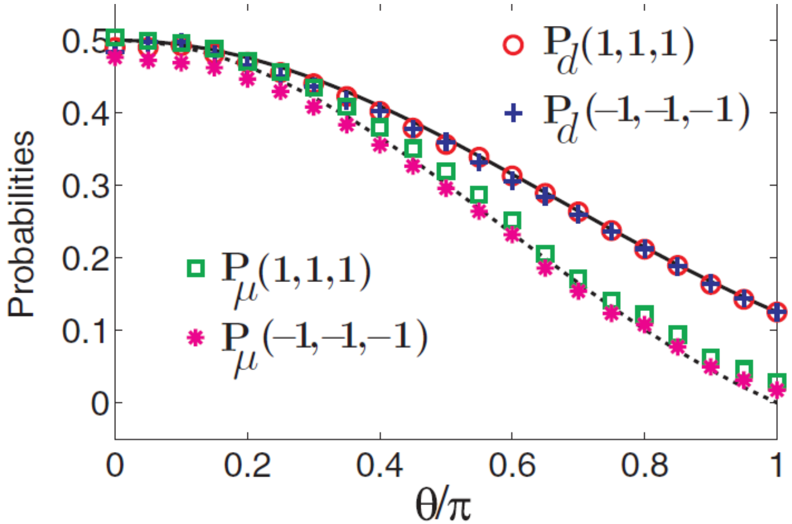
$\mu_{n1,n2,n3} = \langle X_1^{n1}\, X_2^{n2}\, X_3^{n3} \rangle$

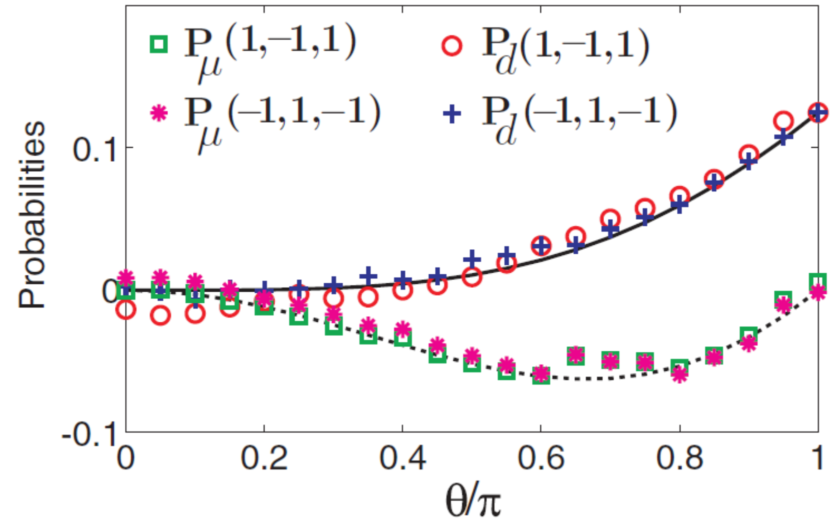


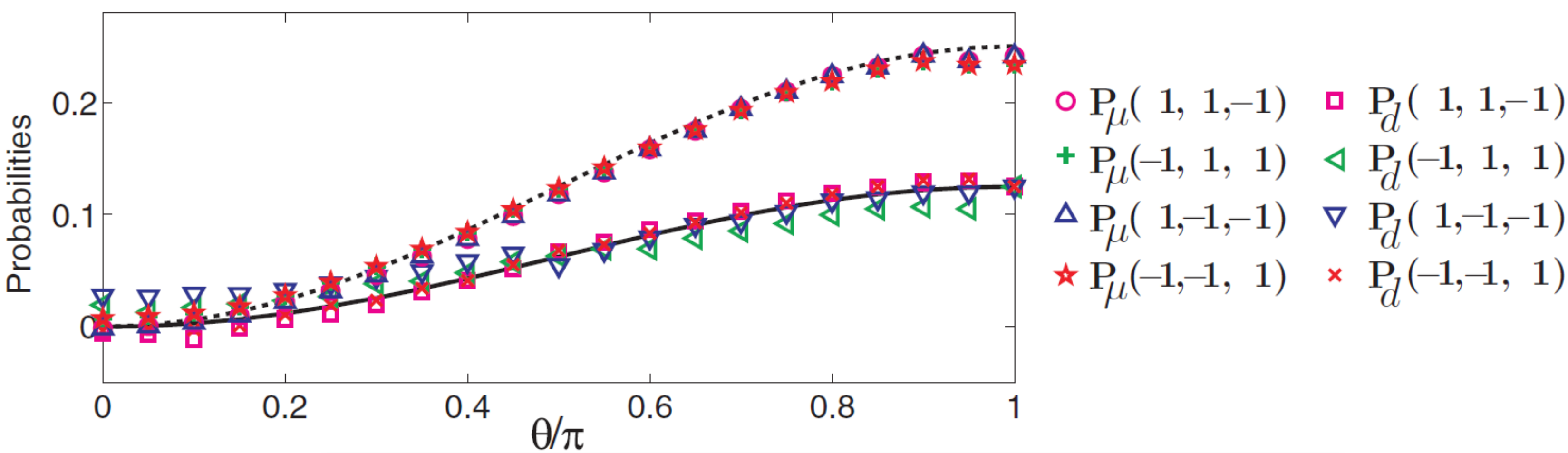
(B) Probabilities and Moments



Karthik et al,
PRA, 052118 (2013)







In Quantum probability theory moments are NOT invertible

i.e., We CAN NOT extract probabilities from moments !!

Plan

Expectation values

Joint Probabilities

State Tomography

Process Tomography

Quantum State Tomography: 1-qubit

The most general 1-qubit quantum state:

purity-factor
depends on state preparation,
temperature, energy gap etc.
(Reference experiment)

Constant
background-population

$$\rho = \frac{1}{2}\left[\mathbb{1} + \varepsilon_{purity} \underbrace{(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)}_{\text{Incompatible observables}} \right]$$

two measurements for three unknowns

ρ needs to be prepared at least two times

Quantum State Tomography: 1-qubit

The most general 1-qubit quantum state:

purity-factor
depends on state preparation,
temperature, energy gap etc.
(Reference experiment)

Constant
background-population

$$\rho \;=\; \frac{1}{2}\,[\,\underbrace{\mathbb{1}}_{\text{Constant background-population}} + \underbrace{\varepsilon_{purity}}_{\text{purity-factor}}\,\overbrace{(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)}^{\text{Incompatible observables}}\,]$$

In NMR: Transverse components of

precessing magentization

- measured by time-separated ensemble detections

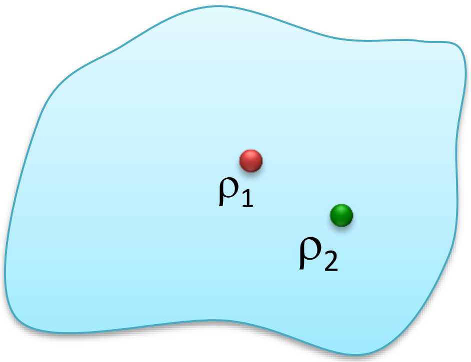
- quadrature detection

ρ can be measured in a SINGLE experiment

Fidelity (Correlation)

Measure of overlap b/w two density matrices:

$$\boxed{F = \frac{\mathrm{tr}(\rho_1 \rho_2)}{[\mathrm{tr}(\rho_1{}^2)\ \mathrm{tr}(\rho_2{}^2)\ ]^{1/2}}}$$



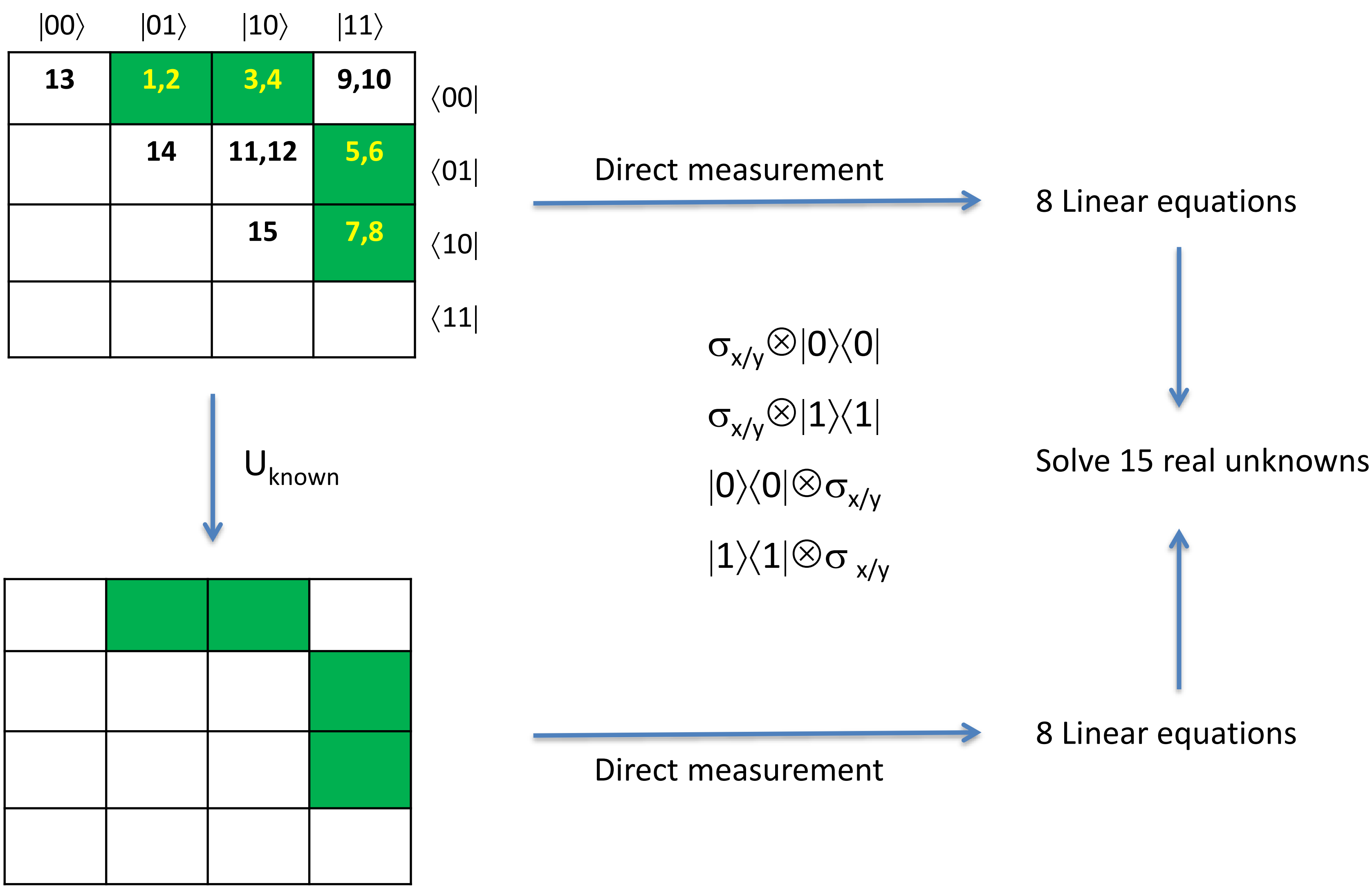
$$\rho_1 = \mathbb{1} + \varepsilon_1\ \mathrm{n}_1 \cdot \sigma$$

$$\rho_2 = \mathbb{1} + \varepsilon_2\ \mathrm{n}_2 \cdot \sigma$$

$$F = \frac{1+\varepsilon_1\ \varepsilon_2\ (\mathrm{n}_1 \cdot \mathrm{n}_2)}{[\ (1+\varepsilon_1{}^2)\ (1+\varepsilon_2{}^2)\ ]^{1/2}}$$

for a single qubit

Quantum State Tomography: 2-qubits



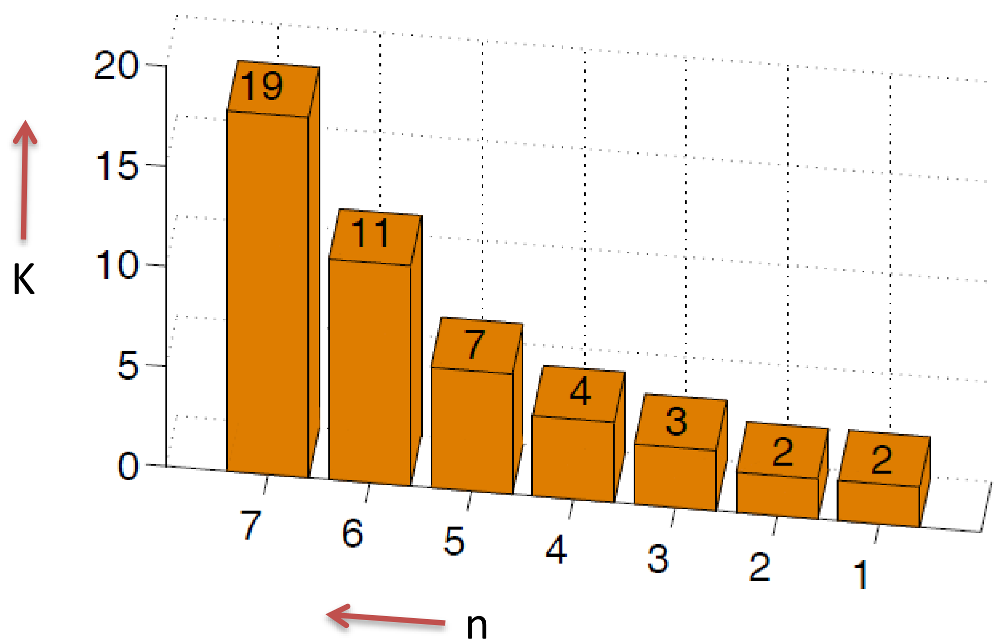
Atleast two independent experiments are needed !!

Quantum State Tomography: n-qubits

$N = 2^n$

Minimum number of experiments

$$K = \frac{\text{total number of unknowns}}{\text{total number of direct observables}} = \left\lceil \frac{N^2 - 1}{nN} \right\rceil \sim N/n$$

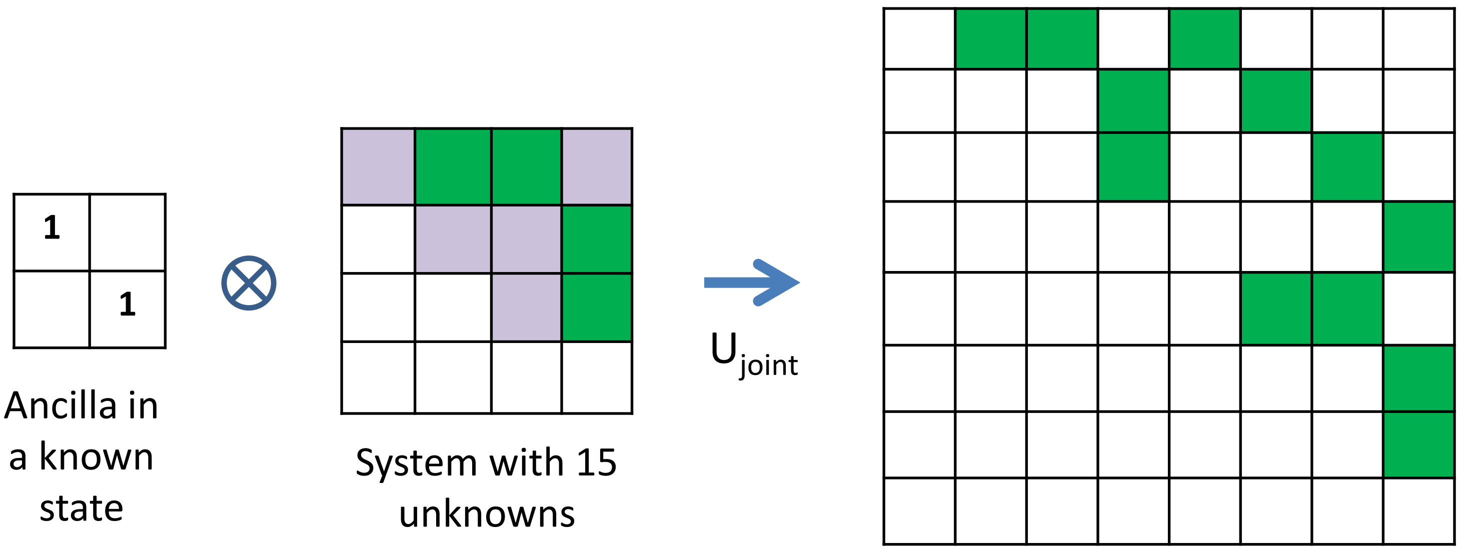


Grows Exponentially with n

Ancilla-Assisted Quantum State Tomography (AAQST)

Ancilla qubits lead to a larger number of direct observables

P. V. Nieuwenhuizen et. al.
PRL 2004



Ancilla in a known state

System with 15 unknowns

Joint-space with 12+12 direct observables !

Minimum number of experiments

$$K = \frac{\text{total number of unknowns}}{\text{total number of direct observables}} = \lceil 15/24 \rfloor = 1$$

Complete characterization of a quantum state by a SINGLE-joint measurement !!

Advantage: repeated state preparations avoided

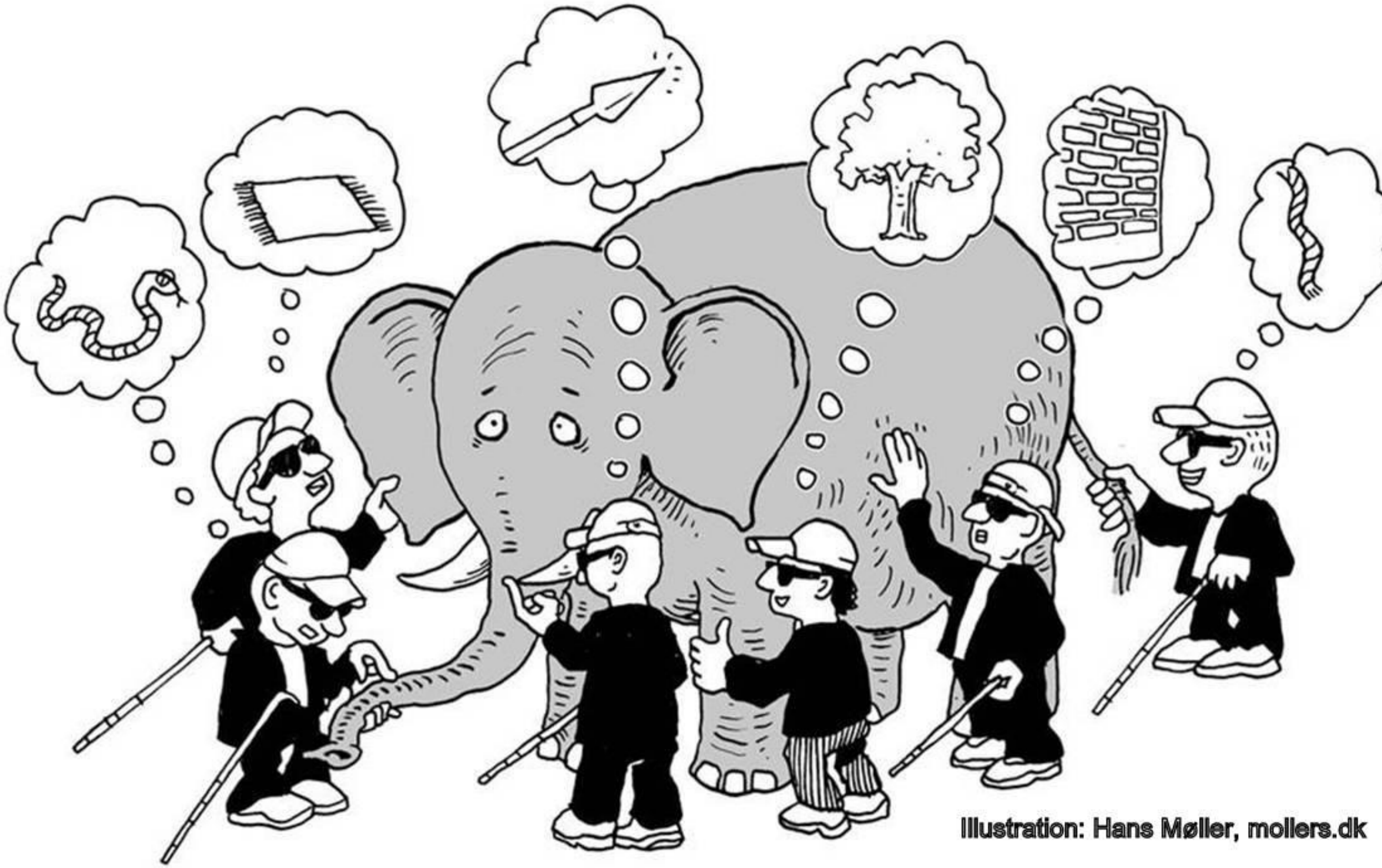


Illustration: Hans Møller, mollers.dk

QST Vs AAQST

Minimum number of experiments:

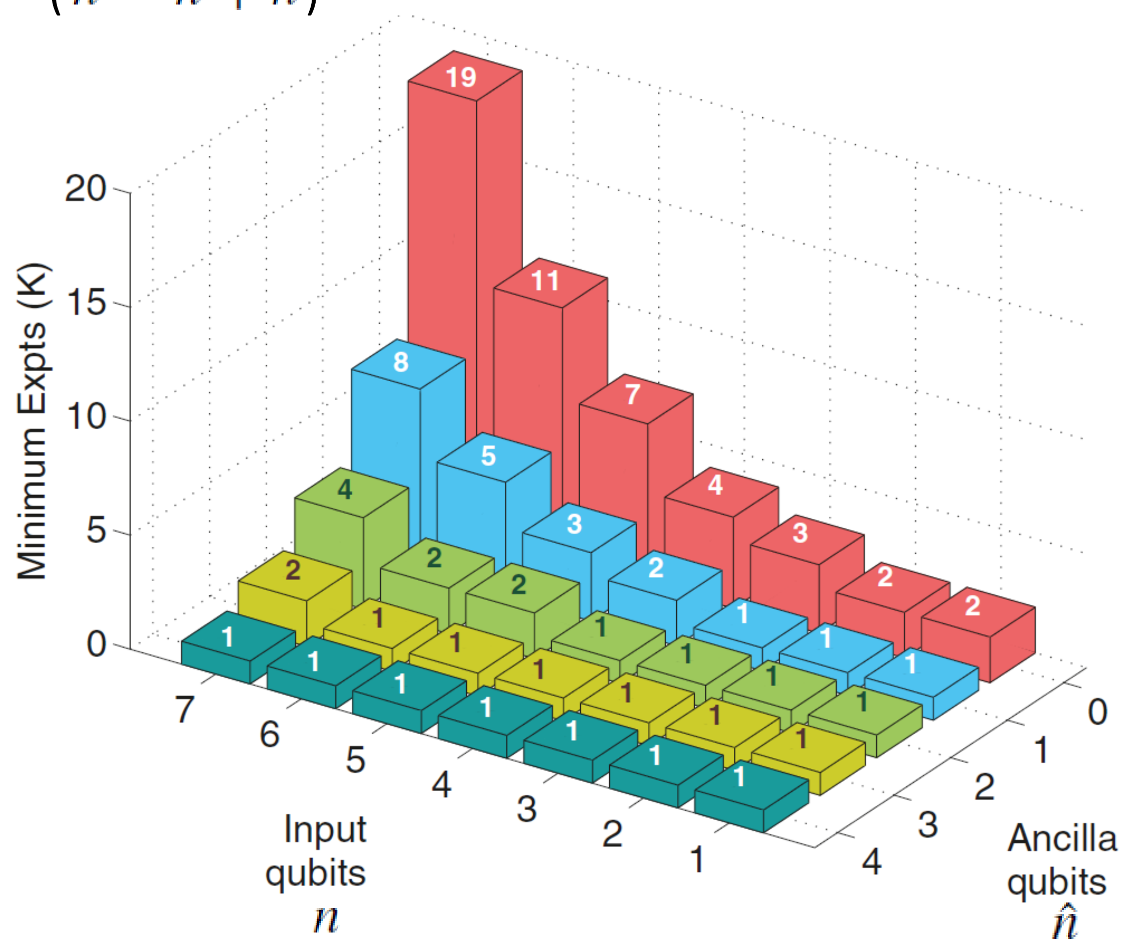
AAQST
$$K = \left\lceil \frac{N^2 - 1}{\tilde{n}\tilde{N}} \right\rceil \leq K = \left\lceil \frac{N^2 - 1}{nN} \right\rceil$$
QST

$(\tilde{n} = n + \hat{n})$

with sufficient number of ancilla qubits, i.e.,

$$\tilde{n}\tilde{N} \geqslant (N^2 - 1)$$

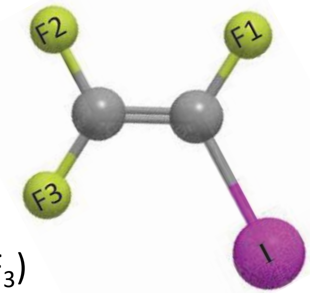
K=1 !!



Results (1-spin ancilla and 2-spin system)

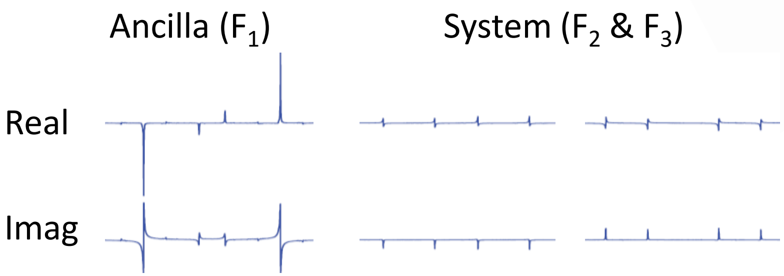
Abhishek Shukla et al, PRA 2013

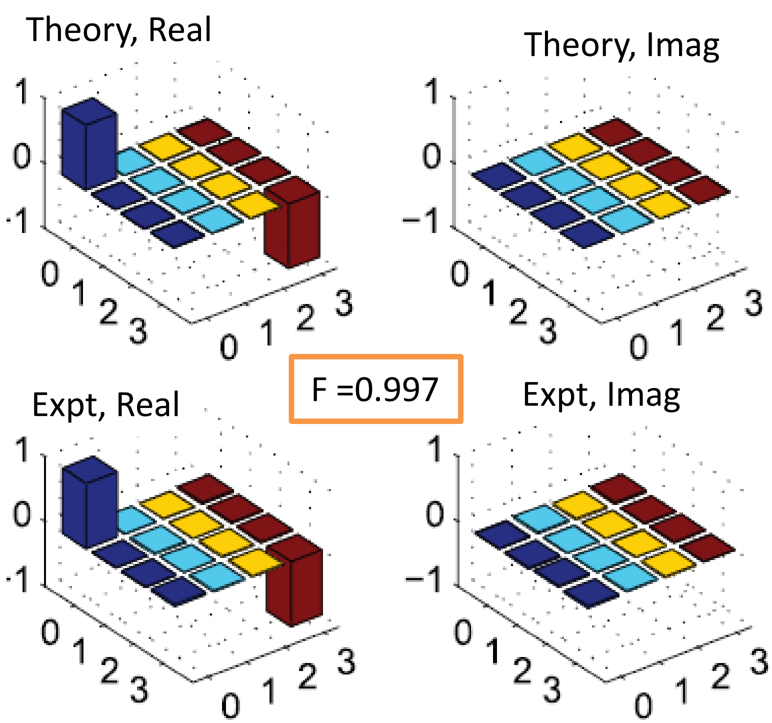
Trifluoroiodoethylene



State 1

$\rho_1 = \sigma^2_z + \sigma^3_z$





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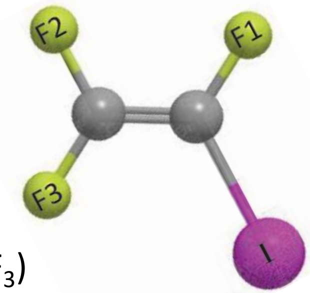
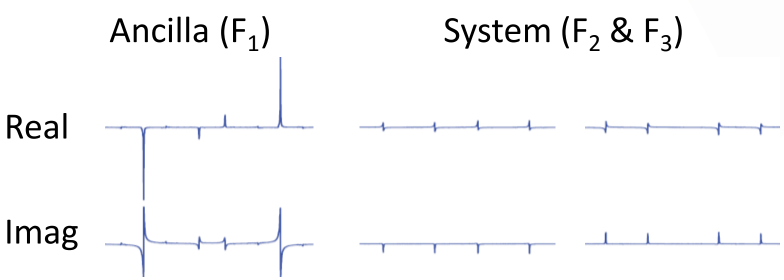


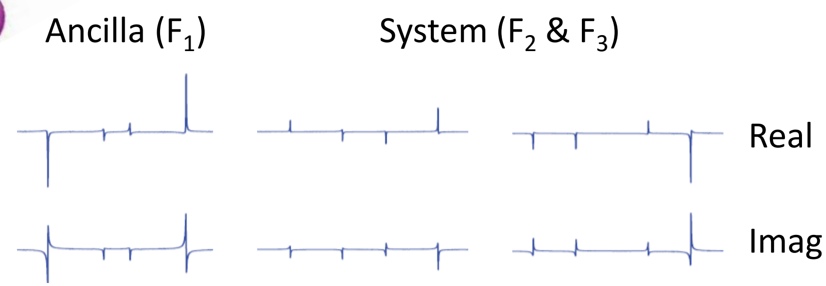
State 1

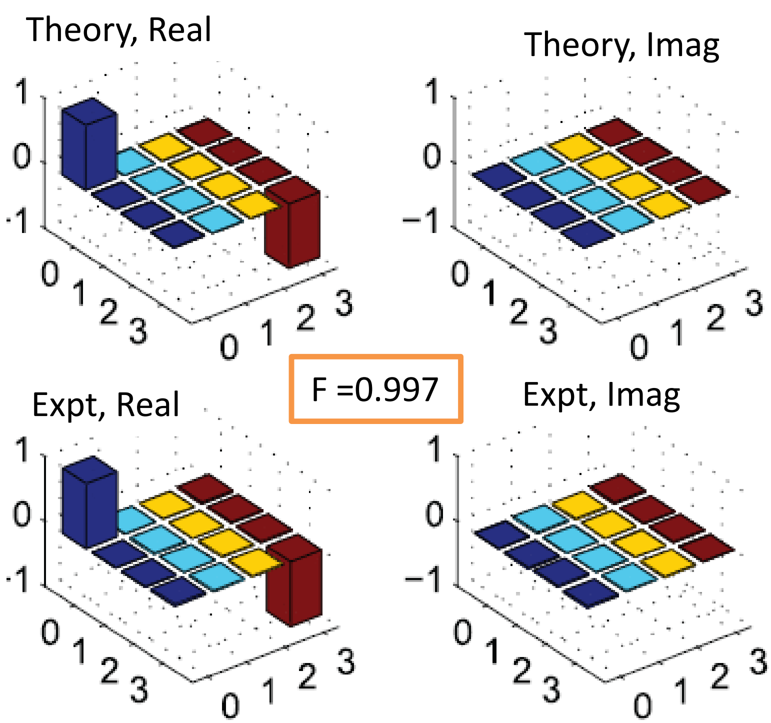
$\rho_1 = \sigma^2_z + \sigma^3_z$

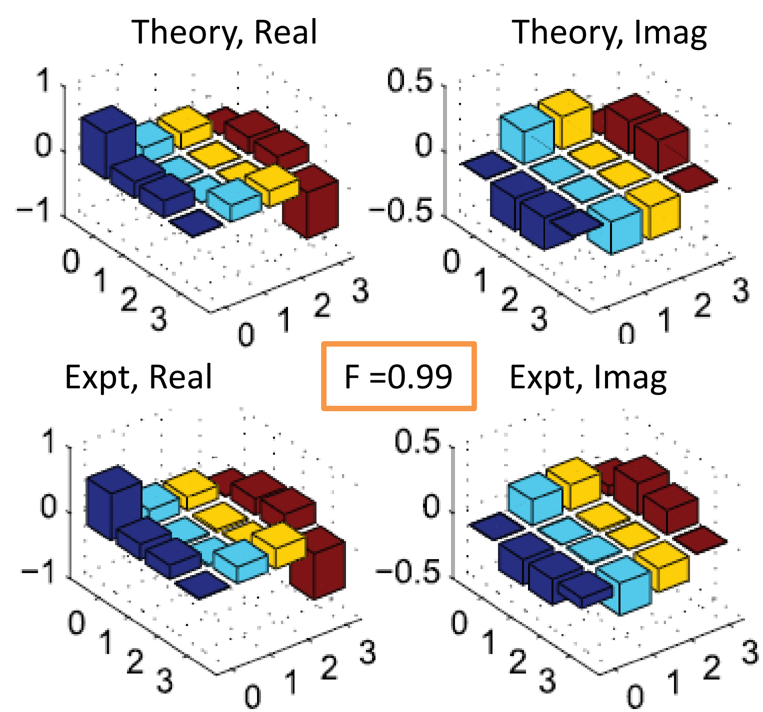
State 2

$\rho_2 = \sigma^2_x\sigma^3_x - \sigma^2_y\sigma^3_y + \sigma^2_z + \sigma^3_z$





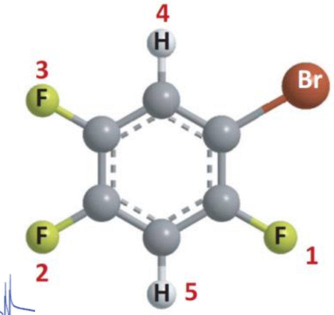




2-spin ancilla and 3-spin system

Abhishek Shukla et al, PRA 2013

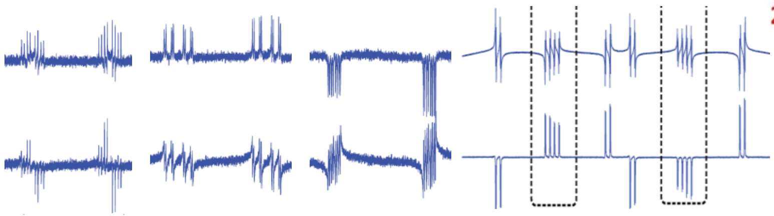
Bromotrifluorobenzene



State 1

$\rho_1 = \sigma^1{}_z + \sigma^2{}_z + \sigma^3{}_z$

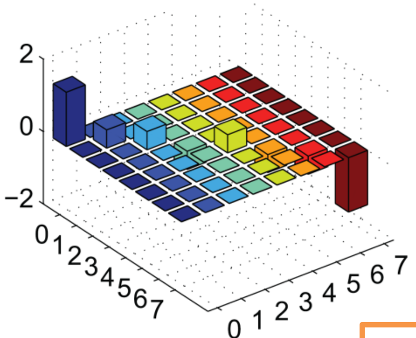
System: F_1, F_2 & F_3 ancilla: H_1 & H_2



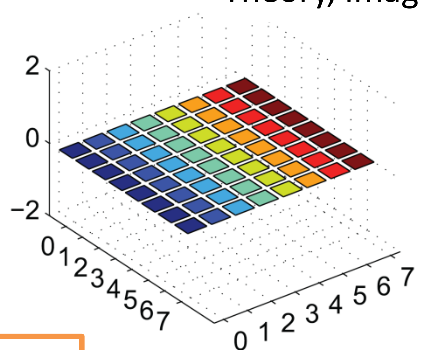
Real

Imag

Theory, Real

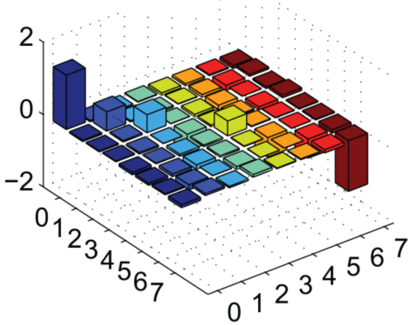


Theory, Imag

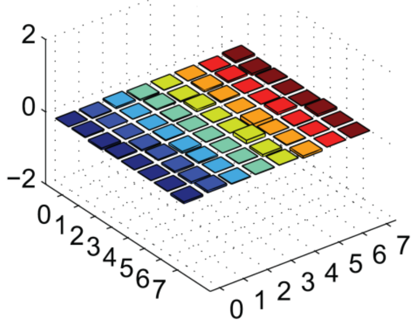


F =0.98

Expt, Real



Expt, Imag

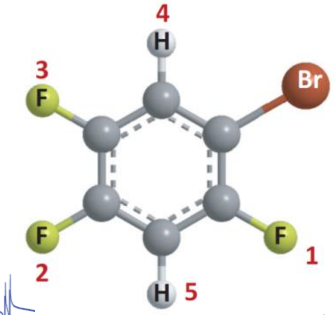


2-spin ancilla and 3-spin system

Abhishek Shukla et al, PRA 2013

$$\left(\frac{\pi}{2}\right)_x^F \tau_0 (\pi)_x^H \tau_0 \left(\frac{\pi}{2}\right)_y^{F_1}$$

Bromotrifluorobenzene

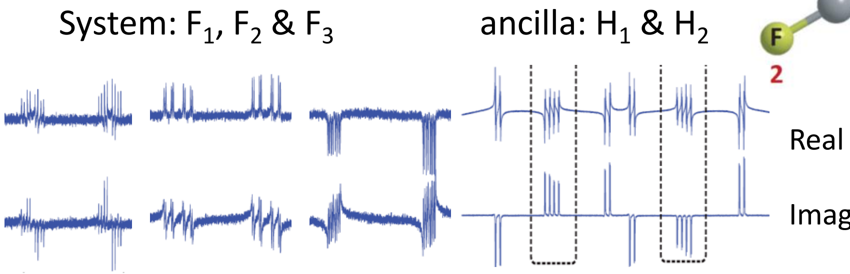


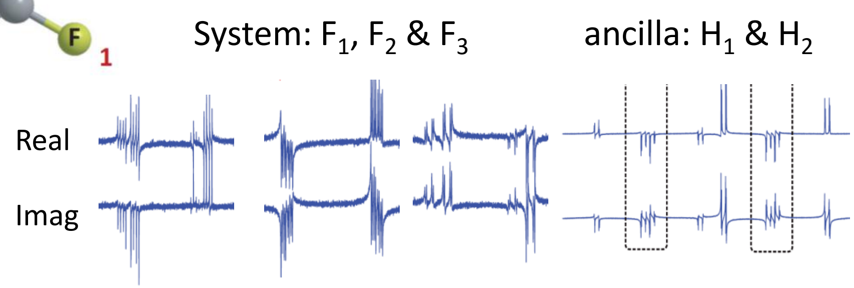
State 1

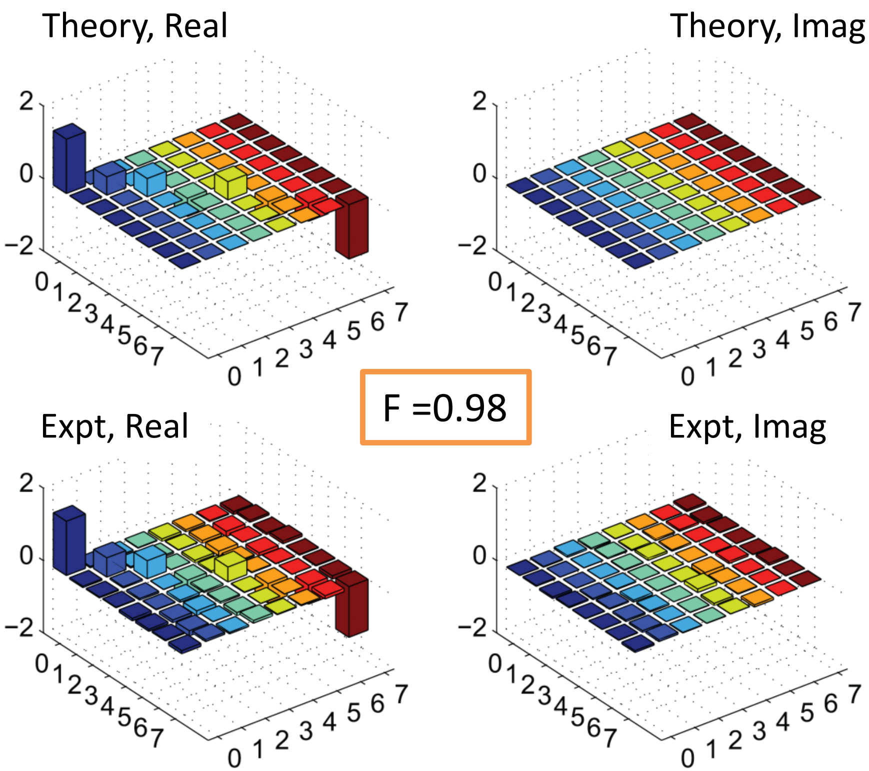
$\rho_1 = \sigma^1{}_z + \sigma^2{}_z + \sigma^3{}_z$

State 2

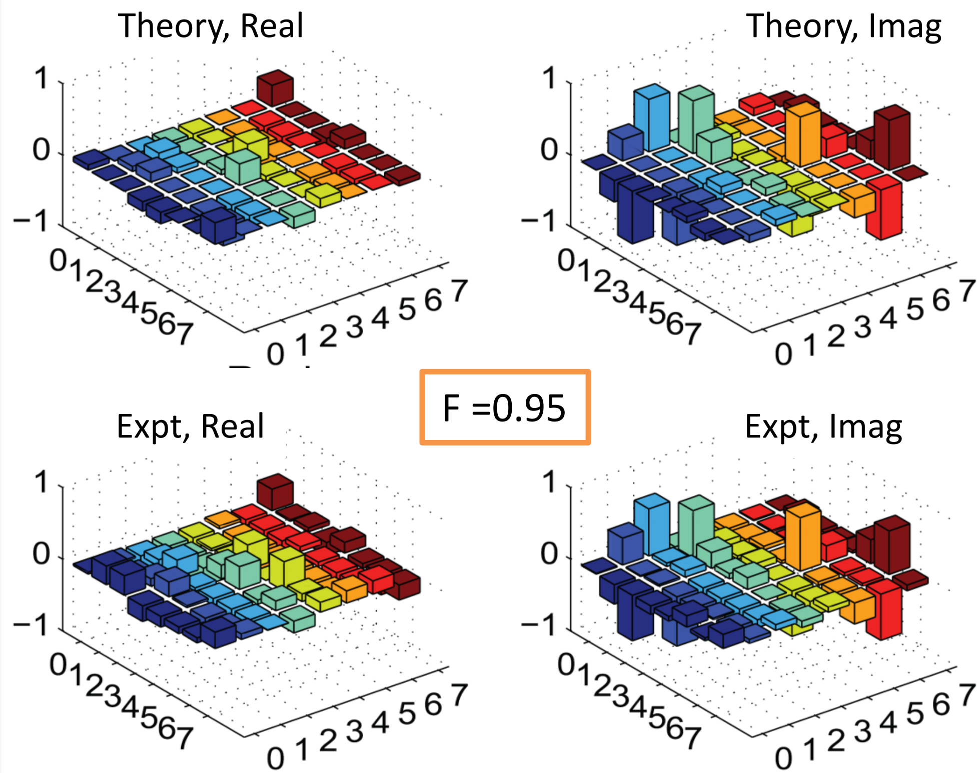
$\rho_2 = U \rho_1 U^\dagger$







F =0.98



F =0.95

Plan

Expectation values

Joint Probabilities

State Tomography

Process Tomography

Quantum Process Tomography

A general quantum process maps a quantum state to a quantum state

Superoperator

$$\rho \xrightarrow{\varepsilon} \rho'$$

Linear map

χ matrix representation

$$\varepsilon(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

fixed set of operators

completely characterizes the process

<u>Why Quantum Process Tomography?</u>

To characterize unknown processes.

To see the effect of imperfect control fields

To understand the effect of decoherence

Quantum Process Tomography

Standard procedure:

Apply the process independently on a set of input states and characterize the output states

Compute χ matrix based on the output states and the fixed set of basis operators

1 $$\varepsilon(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

basis states

2 $$\tilde{E}_m \rho_j \tilde{E}_n^\dagger = \sum_k \beta_{jk}^{mn} \rho_k$$

computed from $\{ \tilde{E}_m \}$ and $\{ \rho_j \}$

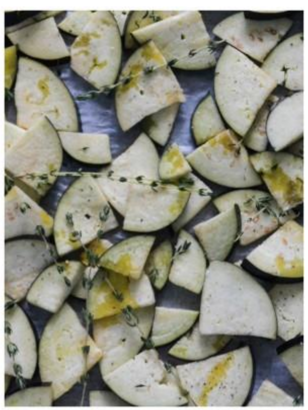
3 $$\varepsilon(\rho_j) = \sum_k \lambda_{jk} \rho_k = \sum_k \sum_{mn} \beta_{jk}^{mn} \chi_{mn} \rho_k$$

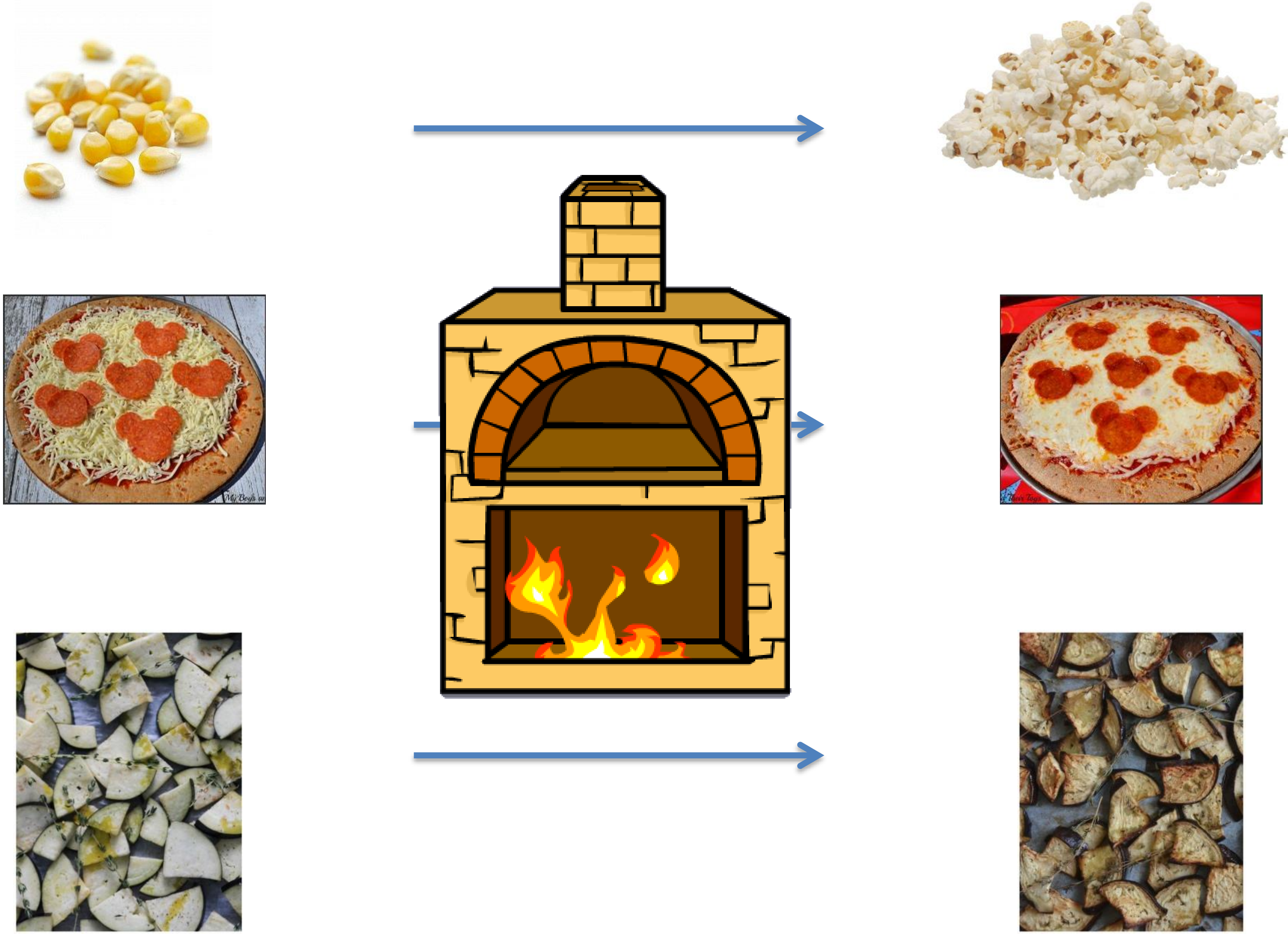
obtained from output states

4 $$\boxed{\beta\chi = \lambda}$$

Chuang et al,
J. Mod. Optics, 1997

Process?





Quantum Process Tomography: 1-Qubit

input → output

$$\rho_1 = \begin{bmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{\mathcal{E}} \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{13} & \lambda_{14} \end{bmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{1k}\}$$

$$\rho_2 = |+\rangle\langle+| - i\,|-\rangle\langle-| - (1-i)\mathbb{1}/2 = \begin{bmatrix} 0 & \mathbf{1} \\ 0 & 0 \end{bmatrix} \xrightarrow{\mathcal{E}} \begin{bmatrix} \lambda_{21} & \lambda_{22} \\ \lambda_{23} & \lambda_{24} \end{bmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{2k}\}$$

$$\rho_3 = |+\rangle\langle+| + i\,|-\rangle\langle-| - (1+i)\mathbb{1}/2 = \begin{bmatrix} 0 & 0 \\ \mathbf{1} & 0 \end{bmatrix} \xrightarrow{\mathcal{E}} \begin{bmatrix} \lambda_{31} & \lambda_{32} \\ \lambda_{33} & \lambda_{34} \end{bmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{3k}\}$$

$$\rho_4 = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{bmatrix} \xrightarrow{\mathcal{E}} \begin{bmatrix} \lambda_{41} & \lambda_{42} \\ \lambda_{43} & \lambda_{44} \end{bmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{4k}\}$$

$$\boxed{\chi = \beta^{-1}\lambda}$$

Total: 6 QST (each needs 2 expts)

↓

$\mathcal{E}$ needs to be applied 12 times !
typically takes an hour

Chuang et al,
J. Mod. Optics, 1997

Ancilla Assisted Process Tomography (AAPT):

Altepeter et al, PRL 2003

Ancilla-system joint space

$$\frac{1}{2}\begin{array}{cccc c} 00 & 01 & 10 & 11 & \\ \left[\begin{matrix} 1 \\ 0 \\ 0 \\ 1 \end{matrix}\right. & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \left.\begin{matrix} 1 \\ 0 \\ 0 \\ 1 \end{matrix}\right] & \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix} \end{array} \xrightarrow{1^{(A)}\otimes\mathcal{E}^{(S)}} \frac{1}{2}\begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{21} & \lambda_{22} \\ \lambda_{13} & \lambda_{14} & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \lambda_{32} & \lambda_{41} & \lambda_{42} \\ \lambda_{33} & \lambda_{34} & \lambda_{43} & \lambda_{44} \end{bmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST (Requires multiple experiments)}} \{\lambda_{jk}\}$$

$$\boxed{\chi = \beta^{-1}\lambda}$$

Single-Scan Process Tomography (SSPT):

Abhishek Shukla et al, PRA (in press)

AAPT + AAQST → SSPT

Additional ancilla

Ancilla-System joint space

$$\begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\mathbb{I}^{A1} \otimes \varepsilon^{S}} \frac{1}{4} \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{21} & \lambda_{22} & 0 & 0 & 0 & 0 \\ \lambda_{13} & \lambda_{14} & \lambda_{23} & \lambda_{24} & 0 & 0 & 0 & 0 \\ \lambda_{31} & \lambda_{32} & \lambda_{41} & \lambda_{42} & 0 & 0 & 0 & 0 \\ \lambda_{33} & \lambda_{34} & \lambda_{43} & \lambda_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{11} & \lambda_{12} & \lambda_{21} & \lambda_{22} \\ 0 & 0 & 0 & 0 & \lambda_{13} & \lambda_{14} & \lambda_{23} & \lambda_{24} \\ 0 & 0 & 0 & 0 & \lambda_{31} & \lambda_{32} & \lambda_{41} & \lambda_{42} \\ 0 & 0 & 0 & 0 & \lambda_{33} & \lambda_{34} & \lambda_{43} & \lambda_{44} \end{bmatrix}$$

AAQST

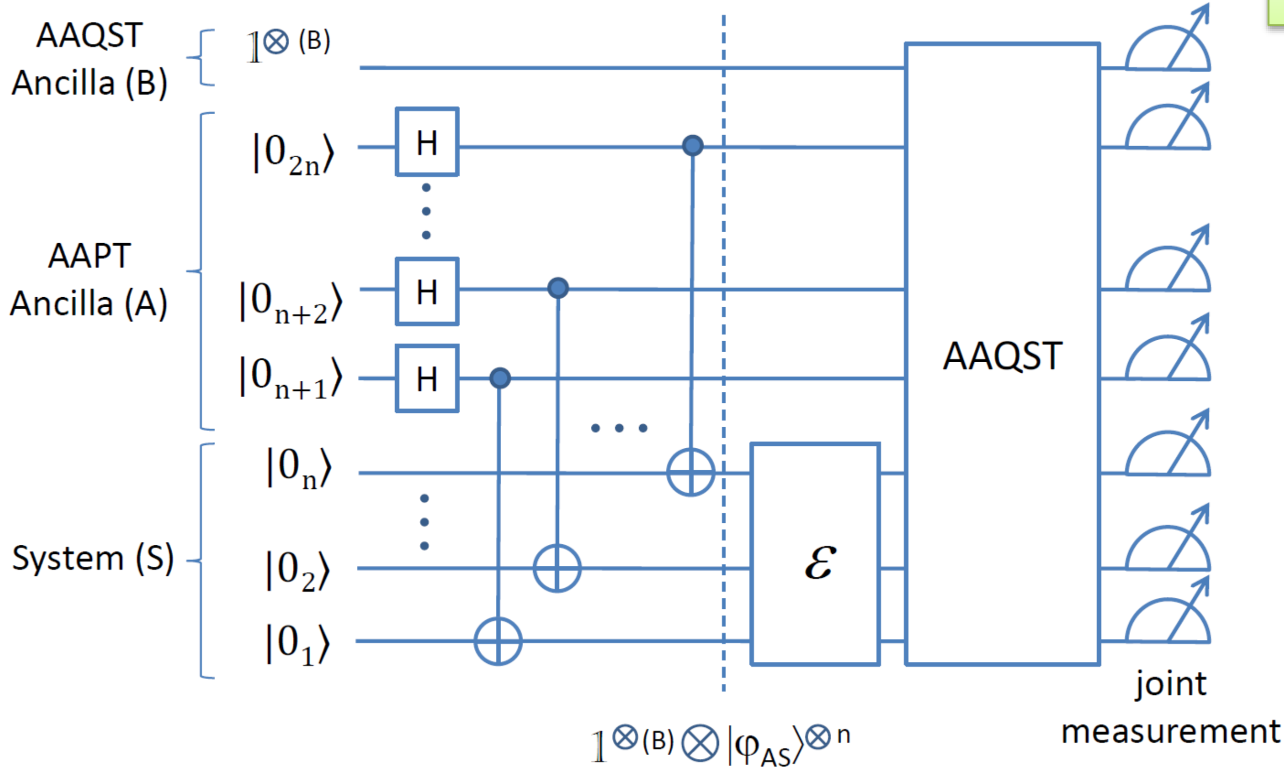
Single scan !!

$$\boxed{\chi = \beta^{-1}\lambda}$$

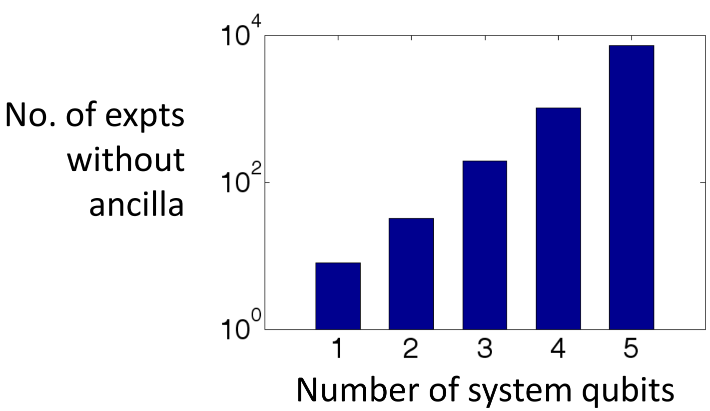
- Process tomography in seconds !
- Single application of process !
- Useful for characterizing dynamic / random processes

SSPT of n-qubits: circuit

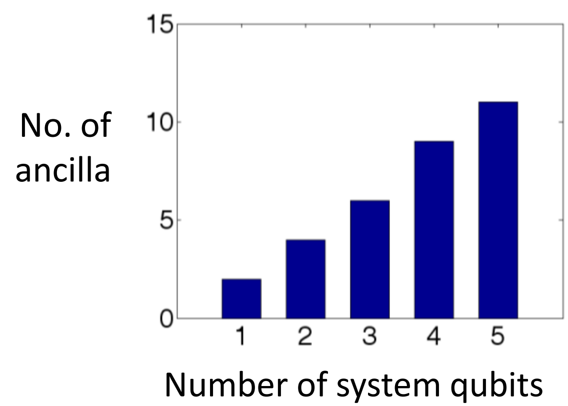
Abhishek Shukla et al, PRA (in press)



Standard method:

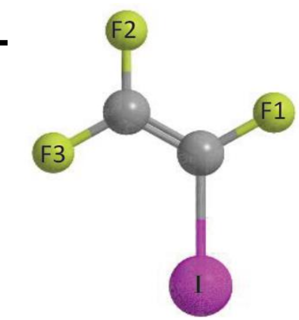


SSPT:

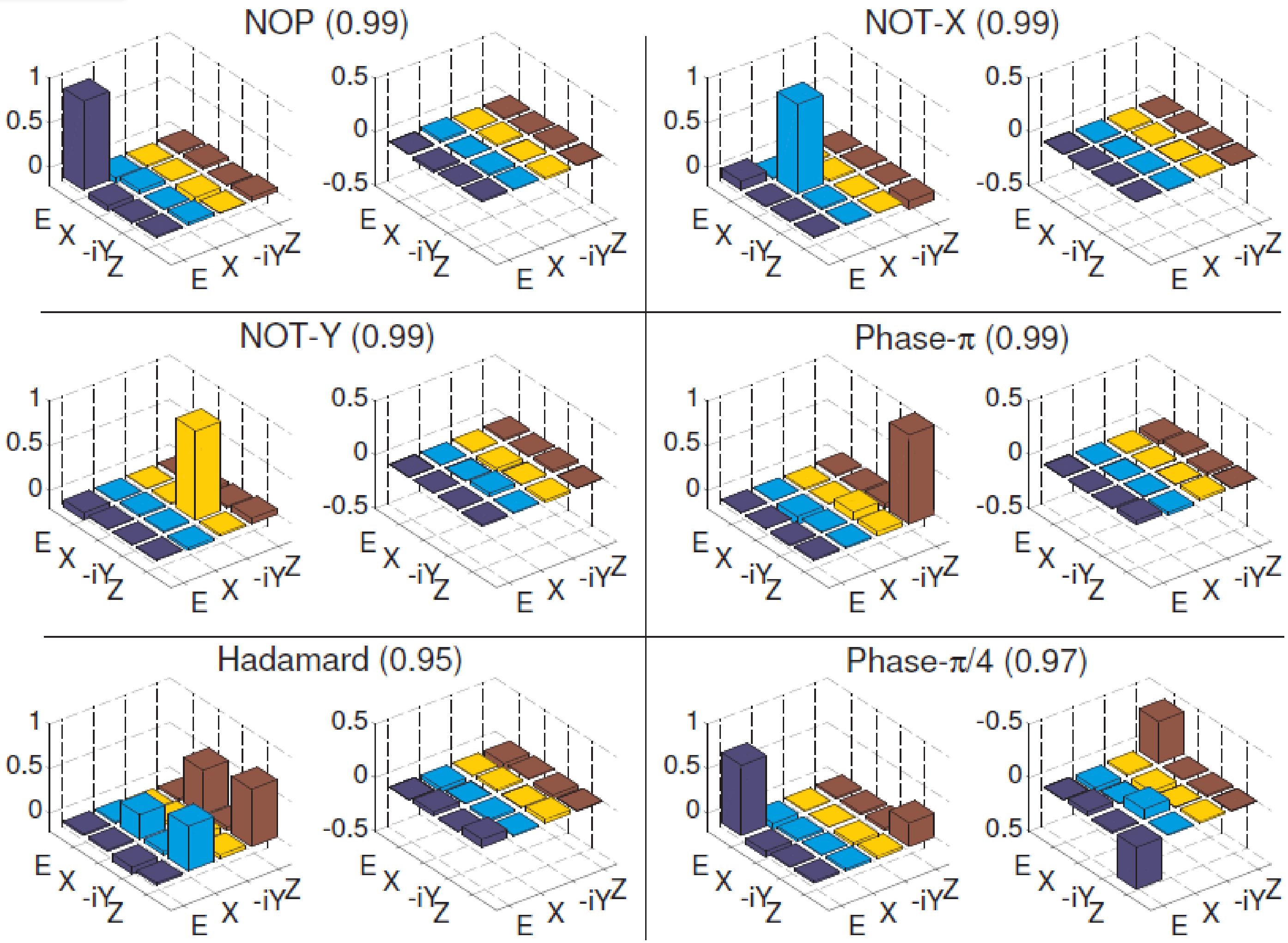


Experimental 1-qubit SSPT Results

Trifluoroiodoethylene

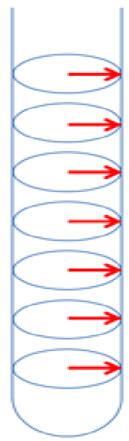


Abhishek Shukla et al,
PRA (in press)



Twirling process

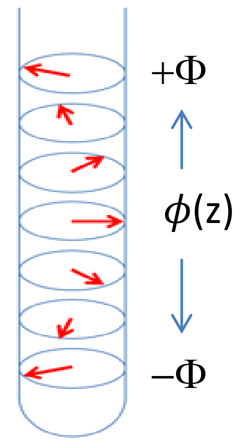
$$\rho_{\rm in} = \begin{array}{|c|c|}\hline \rho_{00} & \rho_{01} \\ \hline & \rho_{11} \\ \hline \end{array} \xrightarrow{\varepsilon_{\rm twirl}} \begin{array}{|c|c|}\hline \rho_{00} & \\ \hline & \rho_{11} \\ \hline \end{array} = \rho_{\rm out}$$



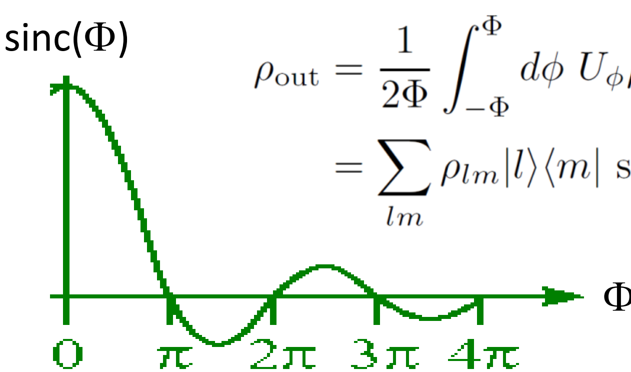
All in phase

Gradient Pulse (G_z)

$$U_\phi = \exp\left(-i\frac{\phi}{2}\sum_{j=1}^{n}\sigma_{jz}\right)$$



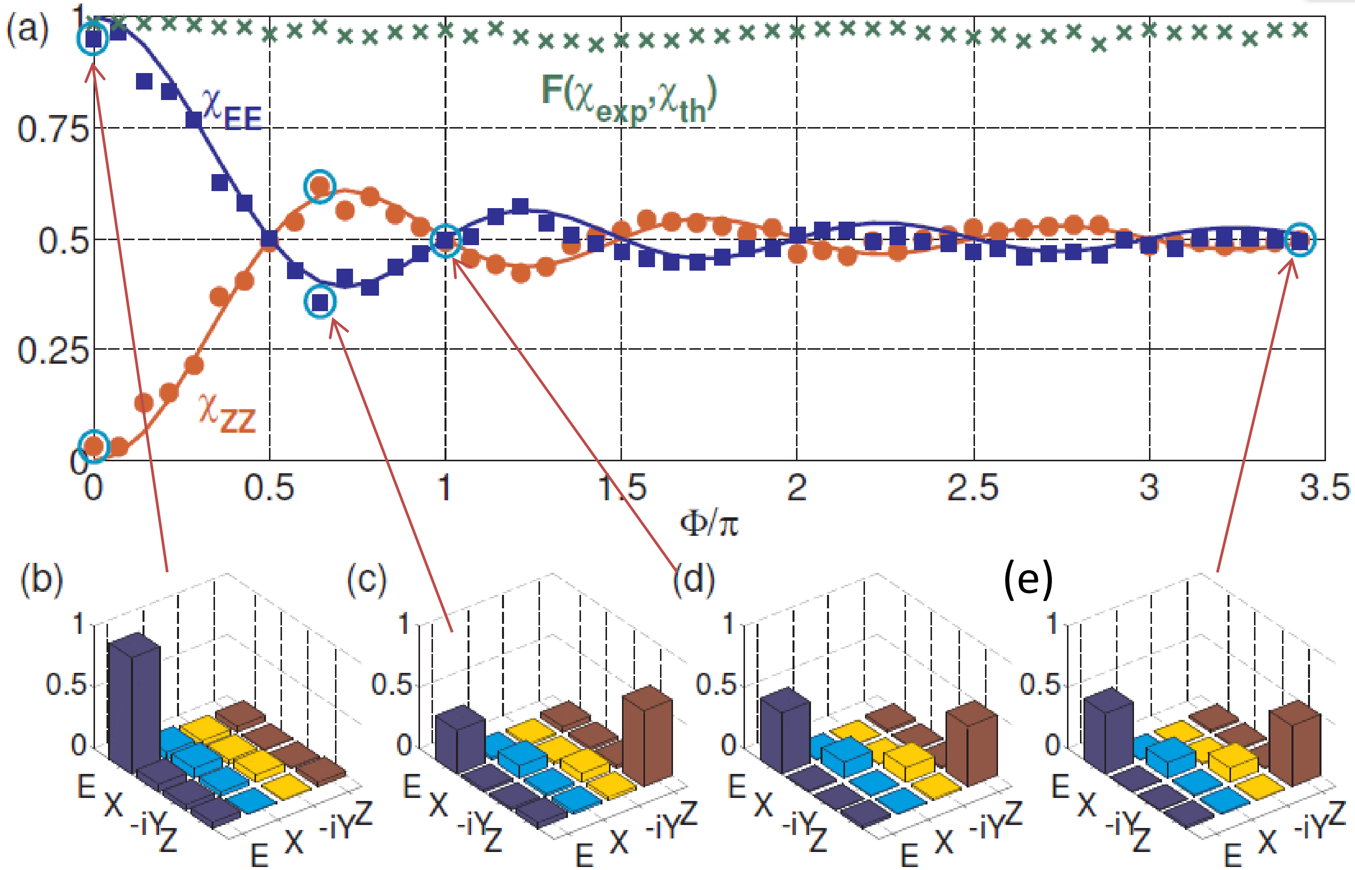
Dephased (twirling)
due to gradient



$$\begin{aligned}\rho_{\rm out} &= \frac{1}{2\Phi}\int_{-\Phi}^{\Phi} d\phi\; U_\phi \rho_{\rm in} U_\phi^\dagger \\ &= \sum_{lm} \rho_{lm} |l\rangle\langle m| \;\mathrm{sinc}(q_{lm}\Phi)\end{aligned}$$

Tracking a twirl via SSPT: Experimental results

Abhishek Shukla et al, PRA (in press)



Main advantages of SSPT:

1. Ultrafast: Time taken:

   In the above case, SSPT ~ 4 minutes (QPT ~ Hours)

2. SSPT is the only way for characterizing a

   dynamical (or random or irreproducible) process

Summary

Expectation values

Joint Probabilities

State Tomography

Process Tomography

- Expectation values for a variety of operators by generalized Moussa protocol
- Post-selection

Summary

Expectation values

Joint Probabilities

State Tomography

Process Tomography

- Noninvasive measurements of joint probabilities

Summary

Expectation values

Joint Probabilities

State Tomography

Process Tomography

- Fast state tomography using ancilla qubits
- Single-shot state tomography

Summary

Expectation values

Joint Probabilities

State Tomography

Process Tomography

- Fast process tomography using ancilla qubits
- Single-shot process tomography
- Tomography of twirling process