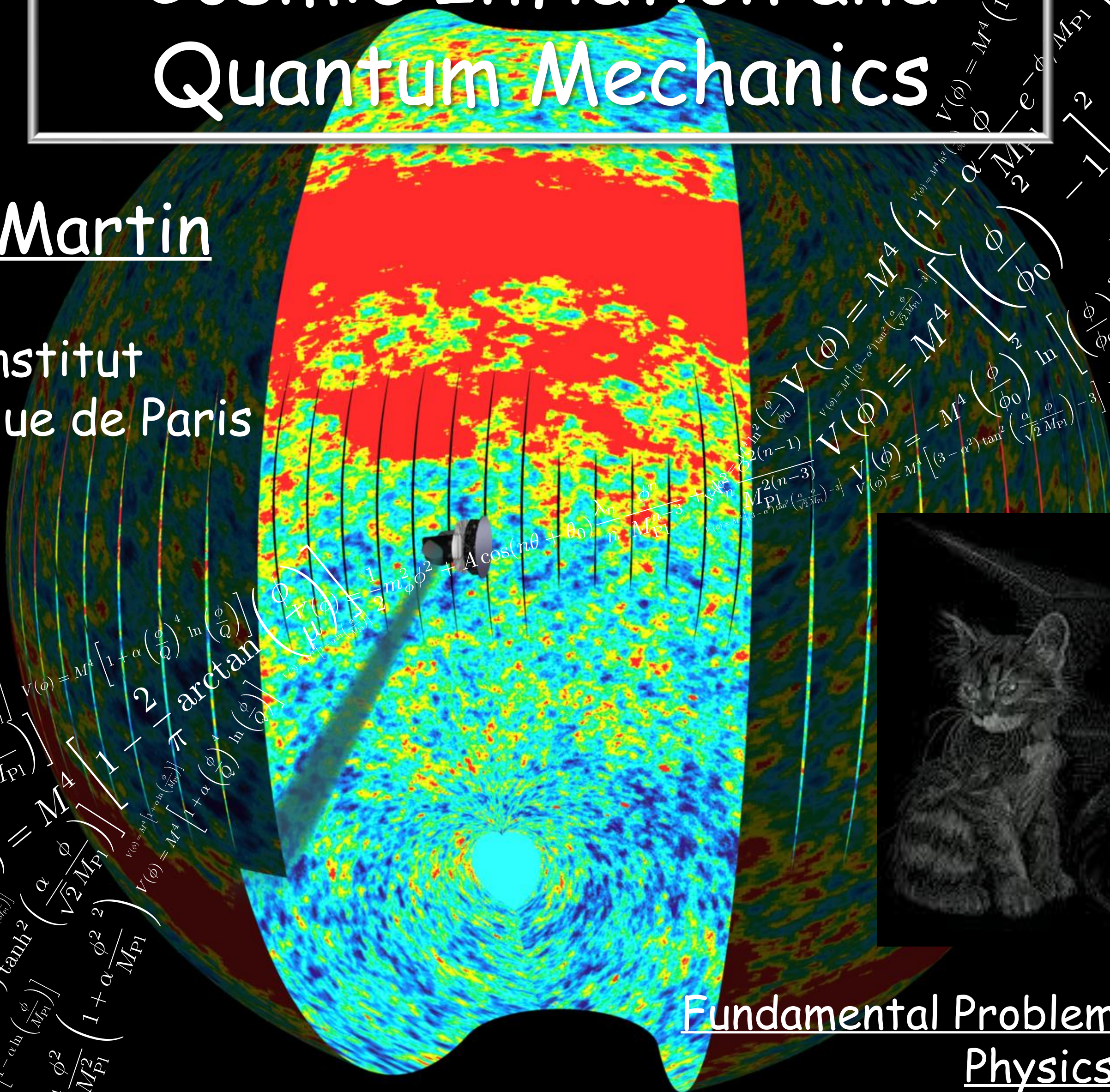
Cosmic Inflation and Quantum Mechanics

Jerome Martin

CNRS/Institut d'Astrophysique de Paris



Fundamental Problems of Quantum Physics

ICTS, Bengalore

December 05- 10, 2016

Outline

- The theory of cosmic inflation in brief: basic principles & observational status
- Cosmological fluctuations of quantum-mechanical origin
- Quantum Mechanics in the sky? Can we show that inflationary perturbations are of quantum-mechanical origin?
- Conclusions

Outline

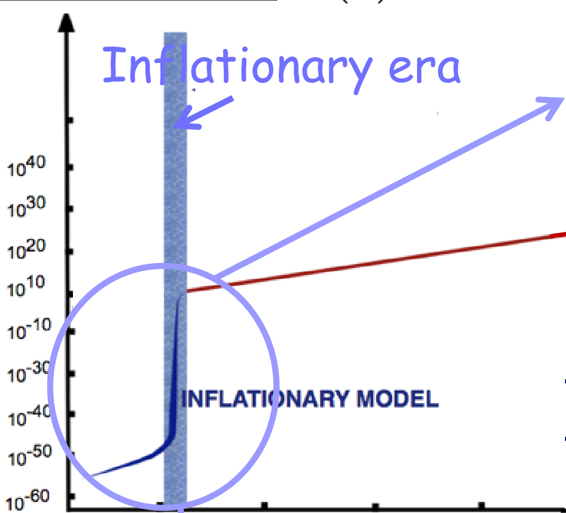
- The theory of cosmic inflation in brief: basic principles & observational status

- Cosmological fluctuations of quantum-mechanical origin

- Quantum Mechanics in the sky? Can we show that inflationary perturbations are of quantum-mechanical origin?

- Conclusions

Scale factor $a(t)$



$$a \propto \exp(Ht)$$

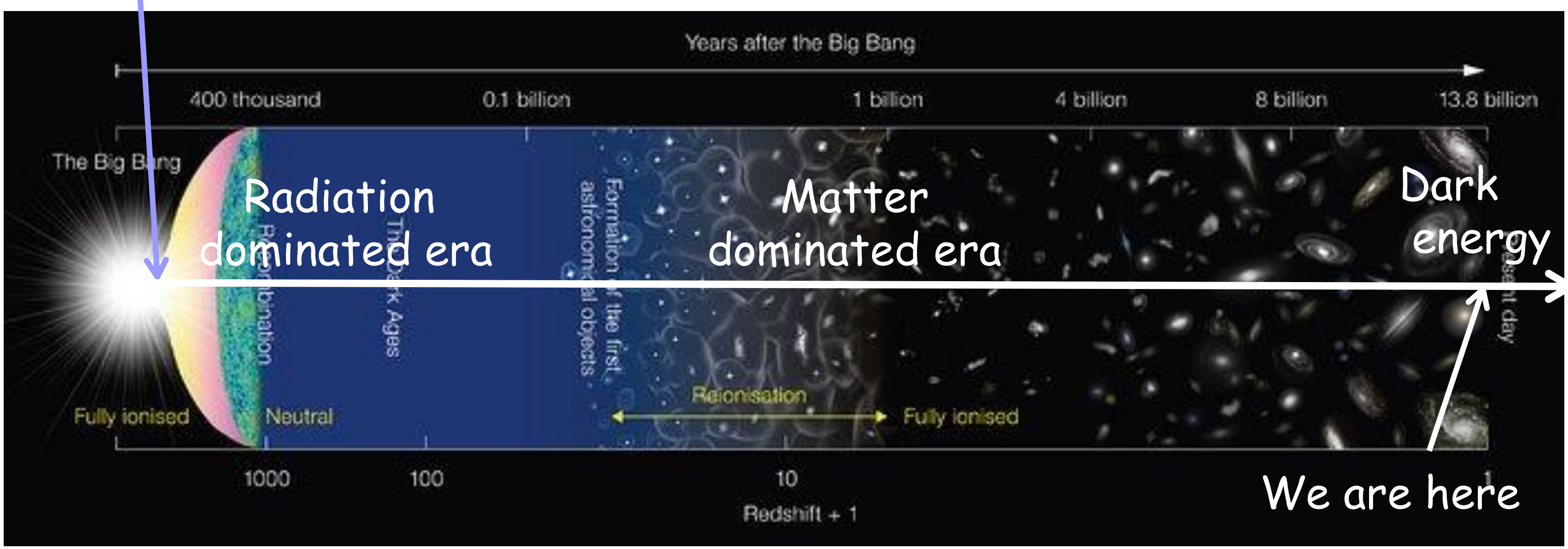
$$\ddot{a} > 0$$

Standard evolution: Hot Big Bang model

$$\ddot{a} < 0$$

Inflation is a phase of <u>accelerated, quasi exponential, expansion</u> taking place in the very <u>early Universe</u>. It solves the puzzles of the standard model of cosmology.

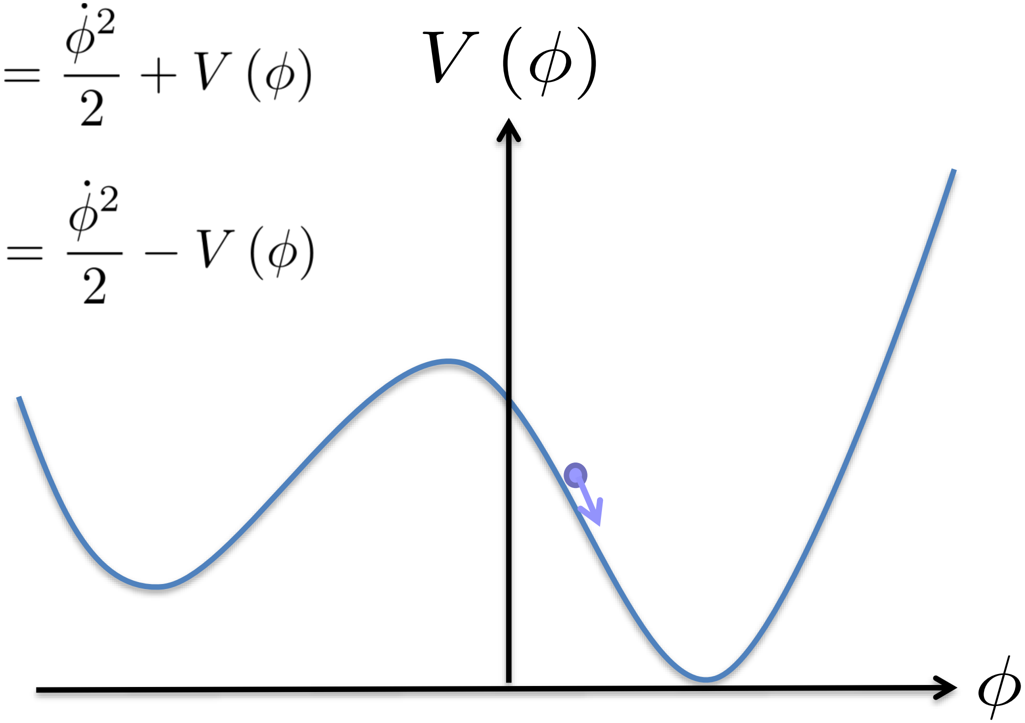
time



Inflation is (usually) realized with one (or many) scalar field(s)

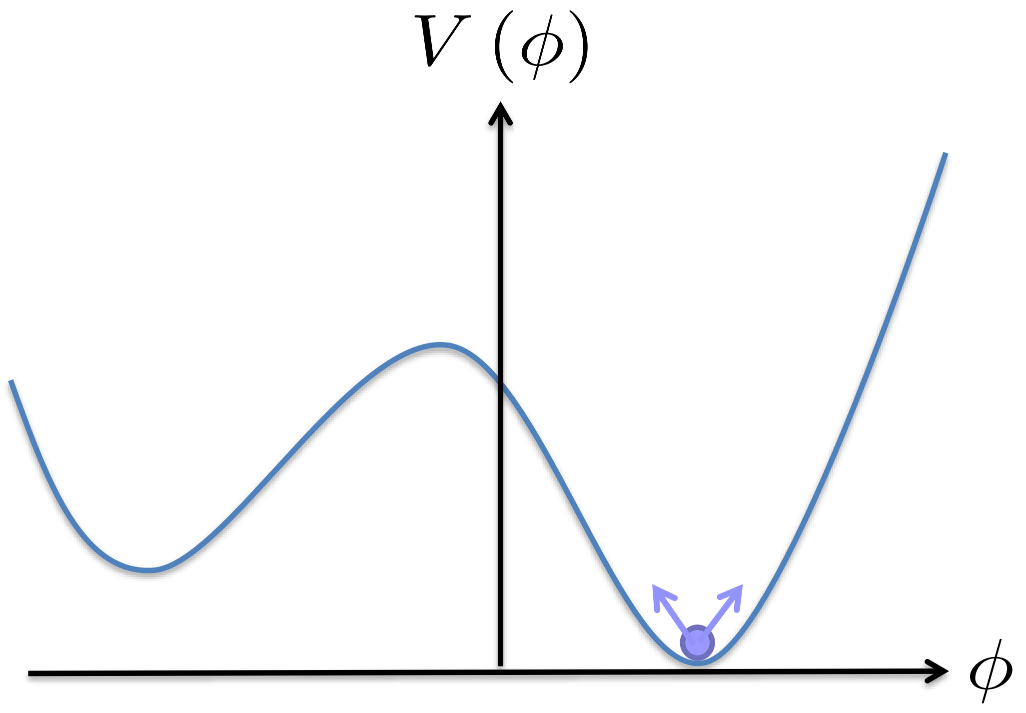
$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$



If the scalar field moves slowly (the potential is flat), then pressure is negative which, in the context of GR, means accelerated expansion and, hence, inflation takes place.

Inflation (usually) stops when the field reaches the bottom of the potential



The field oscillates, decays and the decay products thermalize …Then the radiation dominated era starts …

One important scale in the problem, the Hubble parameter or the Hubble radius

$$H = \frac{\dot{a}}{a}, \qquad \ell_{\mathrm{H}} = \frac{1}{H}$$

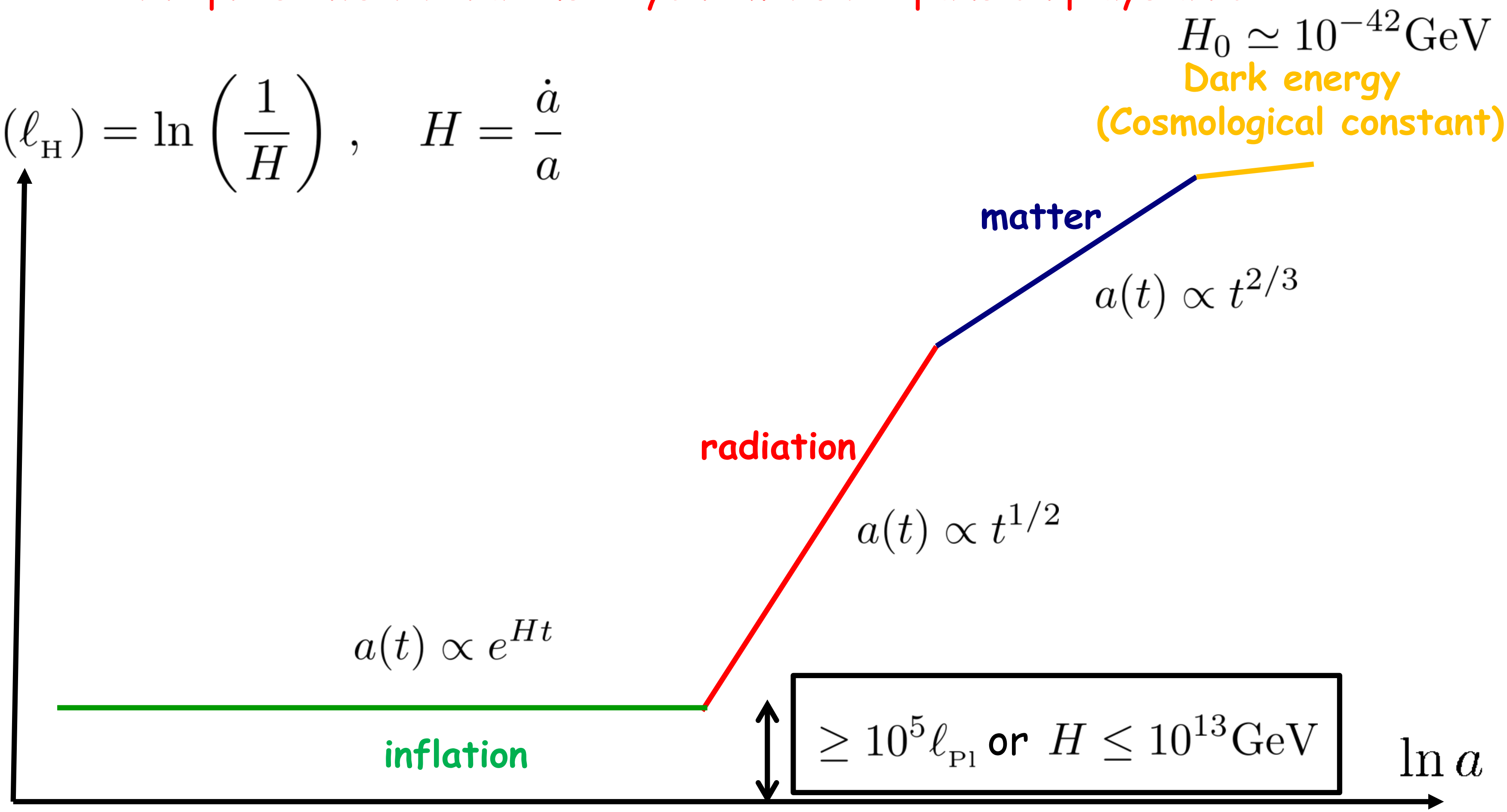
it represents the radius beyond which expansion plays a role

One important scale in the problem, the Hubble parameter or the Hubble radius

$$H = \frac{\dot{a}}{a}, \quad \ell_{\rm H} = \frac{1}{H}$$

it represents the radius beyond which expansion plays a role

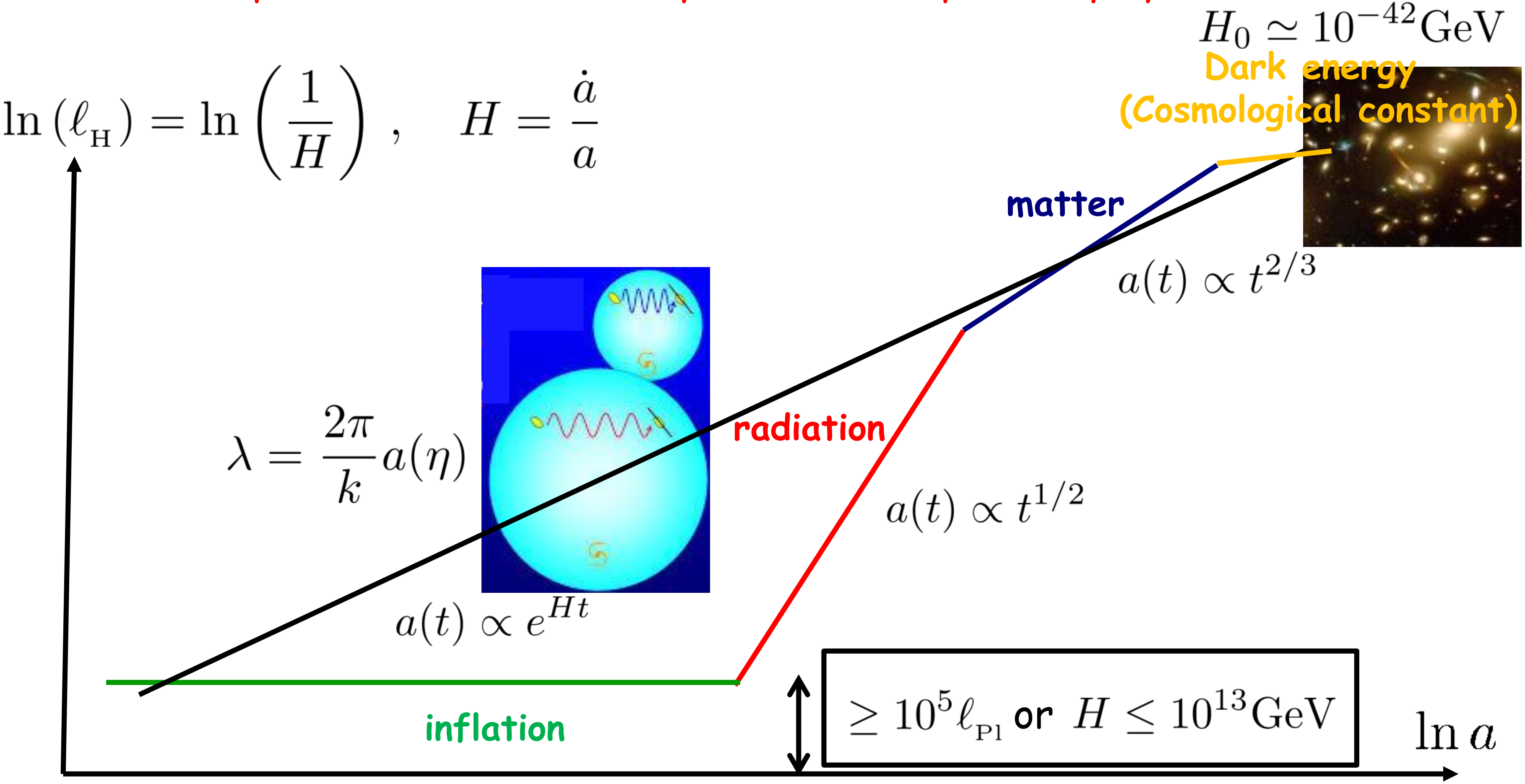
$$\ln\left(\ell_{\rm H}\right) = \ln\left(\frac{1}{H}\right), \quad H = \frac{\dot{a}}{a}$$

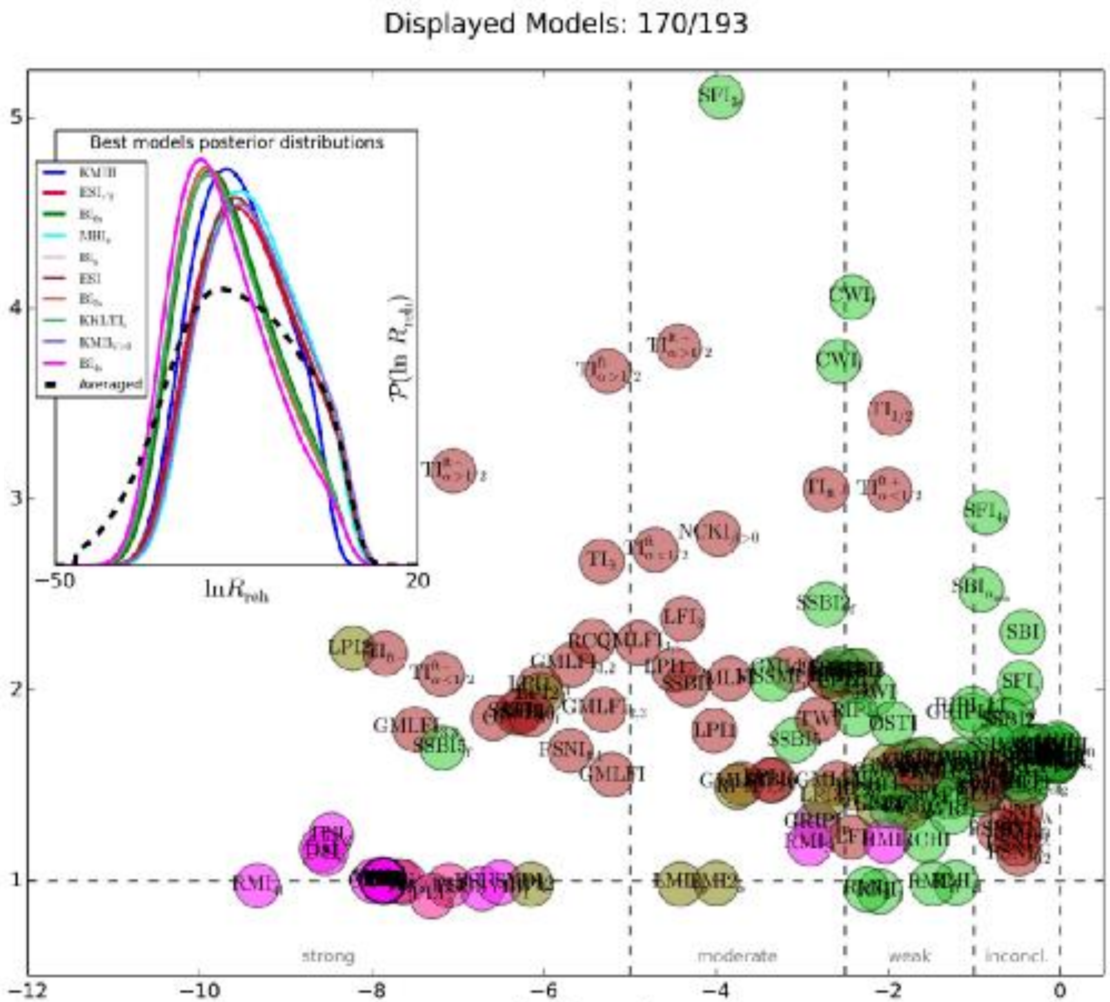


One important scale in the problem, the Hubble parameter or the Hubble radius

$$H = \frac{\dot{a}}{a}, \quad \ell_{\rm H} = \frac{1}{H}$$

it represents the radius beyond which expansion plays a role





Constraints on reheating

Model performance $\ln(\mathcal{E}/\mathcal{E}_{\rm best})$

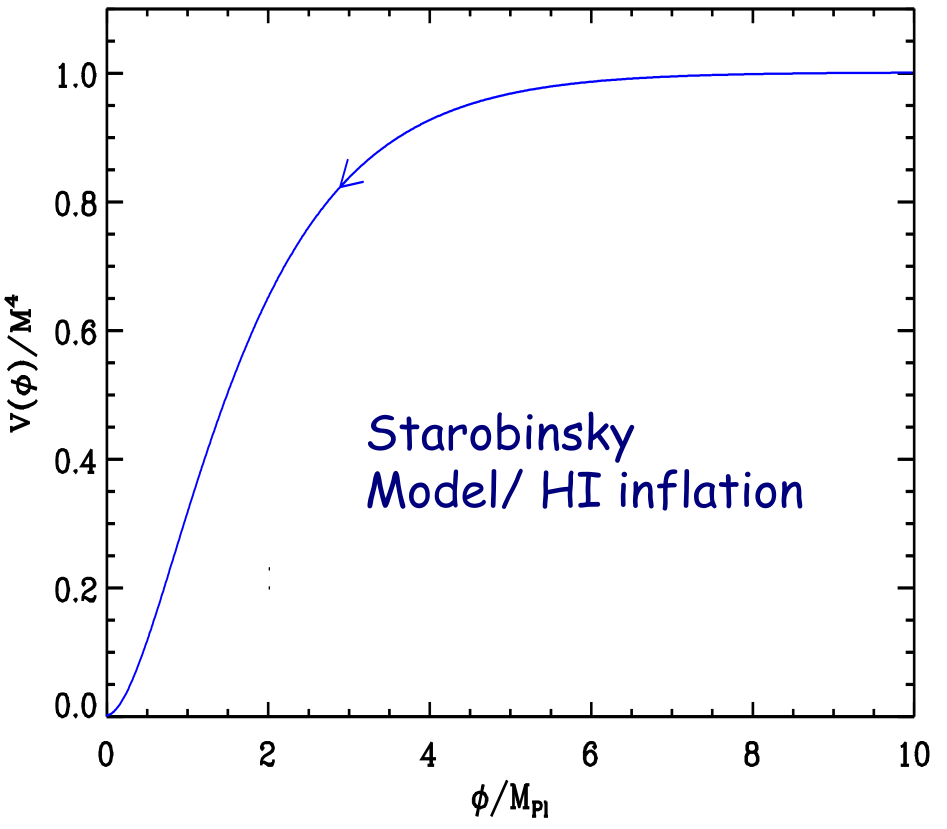
No constraint on reheating

J. Martin, C. Ringeval & V. Vennin, Phys. Dark Univ. 5-6 (2014), 75, arXiv:1303.3787

J. Martin, C. Ringeval & V. Vennin, Phys. Rev. Lett. 114 (2015), 081303, arXiv:1410.7958

Plateau inflationary models are the winners!

J. Martin, C. Ringeval R. Trotta & V. Vennin, JCAP1403 (2014), 039, arXiv:1312.3529



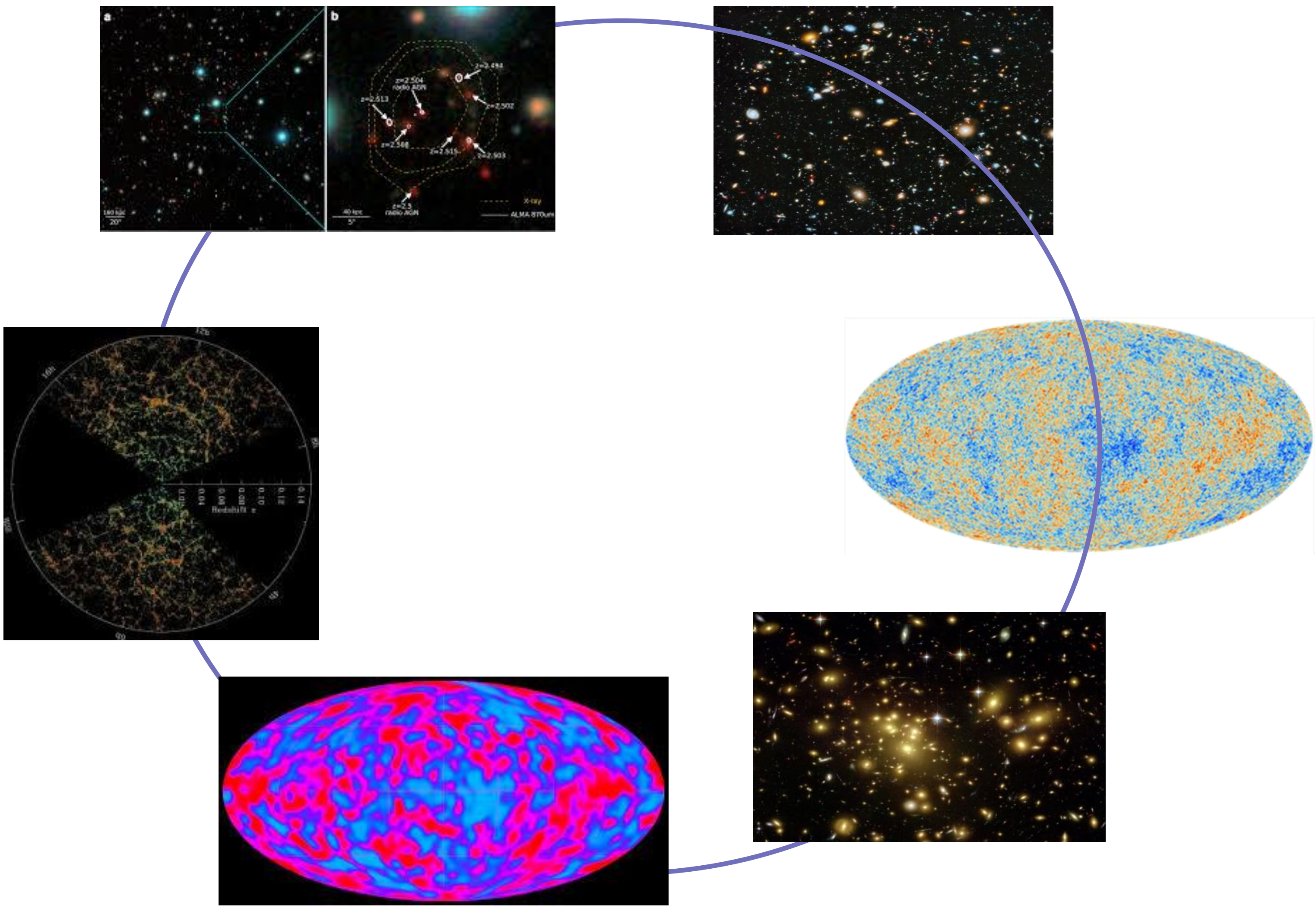
$$V(\phi) = M^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)^2$$

Outline

- The theory of cosmic inflation in brief: basic principles & observational status
- Cosmological fluctuations of quantum-mechanical origin
- Quantum Mechanics in the sky? Can we show that inflationary perturbations are of quantum-mechanical origin?
- Conclusions

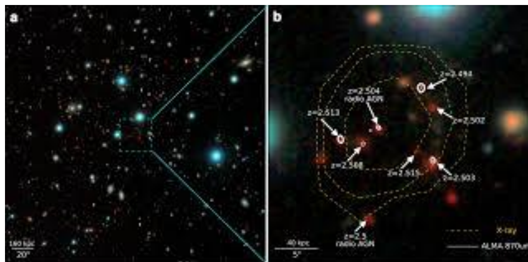
Structure formation

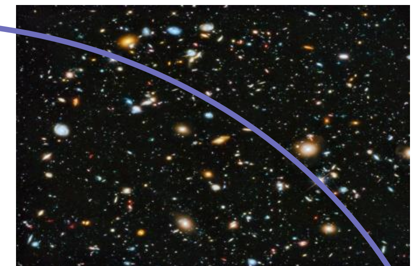
<u>There are inhomogeneities (structures=galaxies, galaxy clusters, CMB anisotropies etc …) in the Universe …</u>

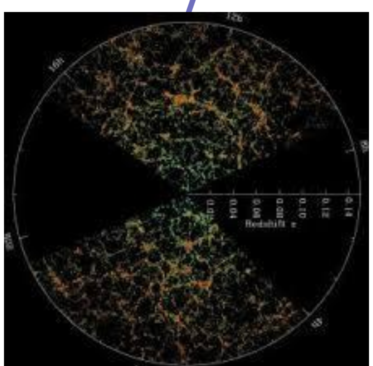


Structure formation

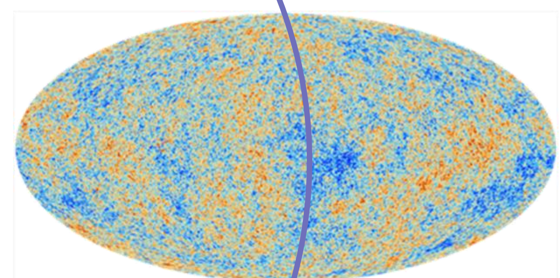
There are inhomogeneities (structures=galaxies, galaxy clusters, CMB anisotropies etc …) in the Universe …

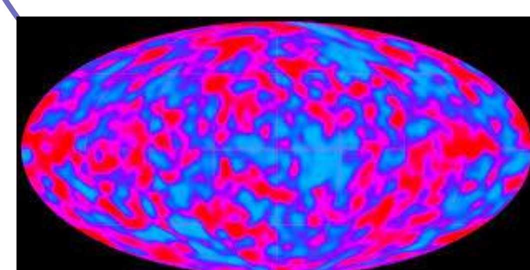






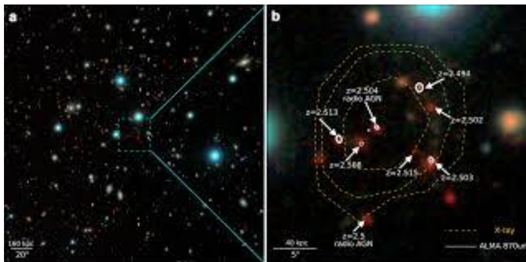
How do they grow?

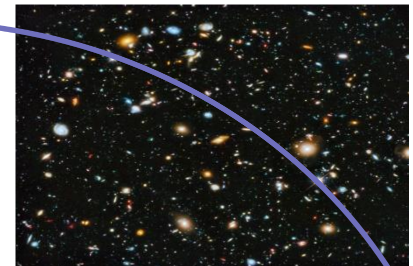


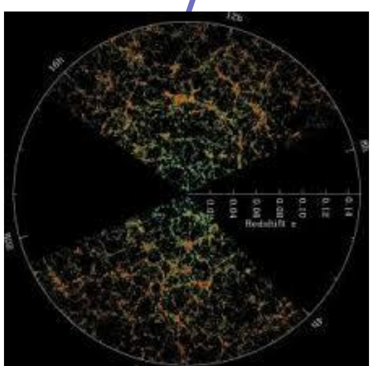


Structure formation

There are inhomogeneities (structures=galaxies, galaxy clusters, CMB anisotropies etc …) in the Universe …

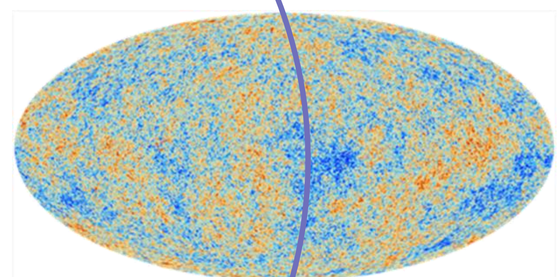


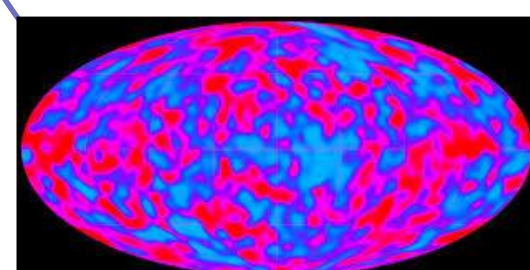




How do they grow?

What is their origin?

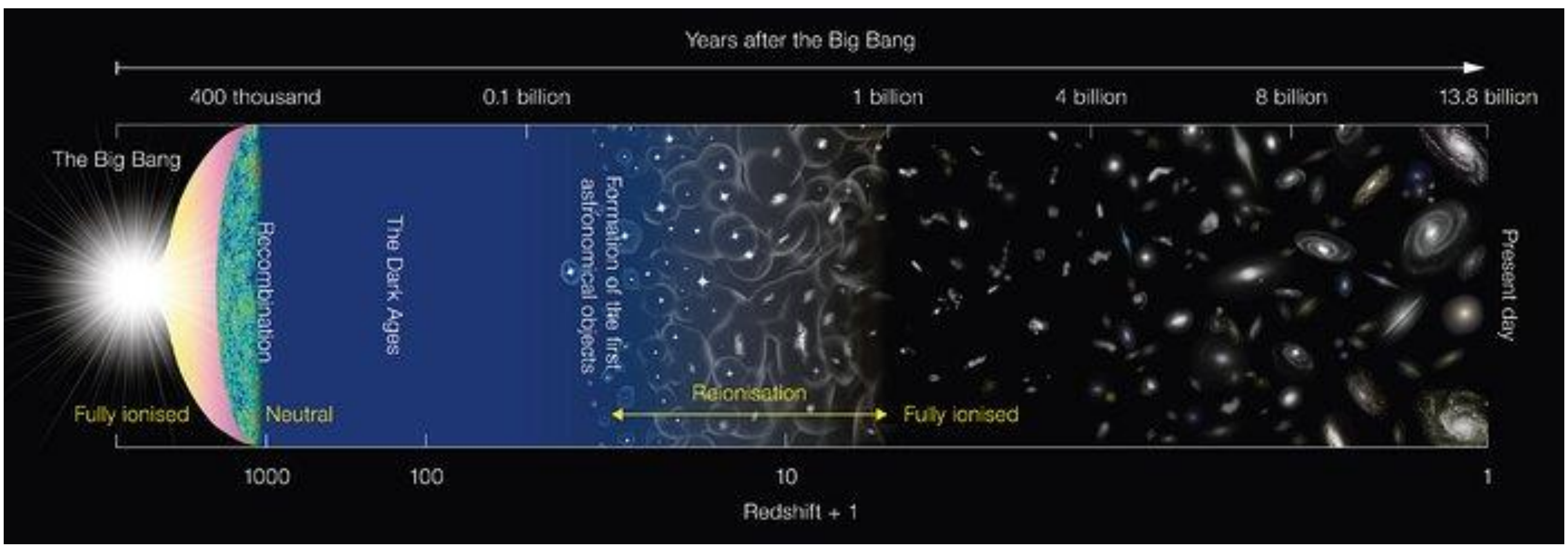




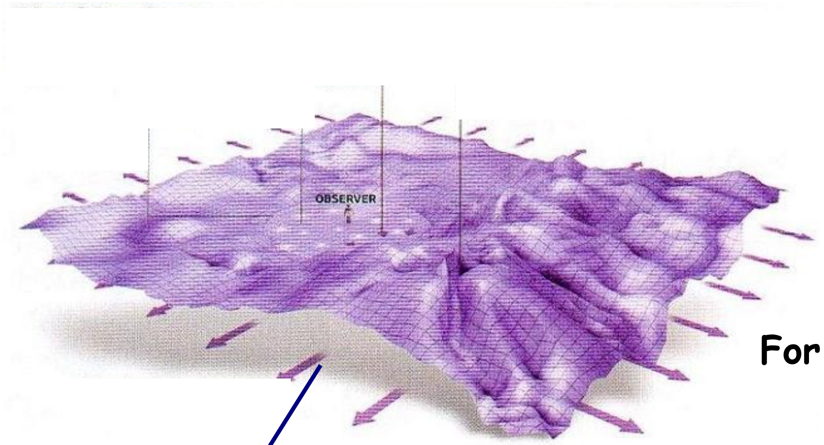
How do they grow?

Gravitational instability

$$\frac{\delta\rho}{\rho} \simeq 10^{-5} \longrightarrow \frac{\delta\rho}{\rho} \simeq 1$$

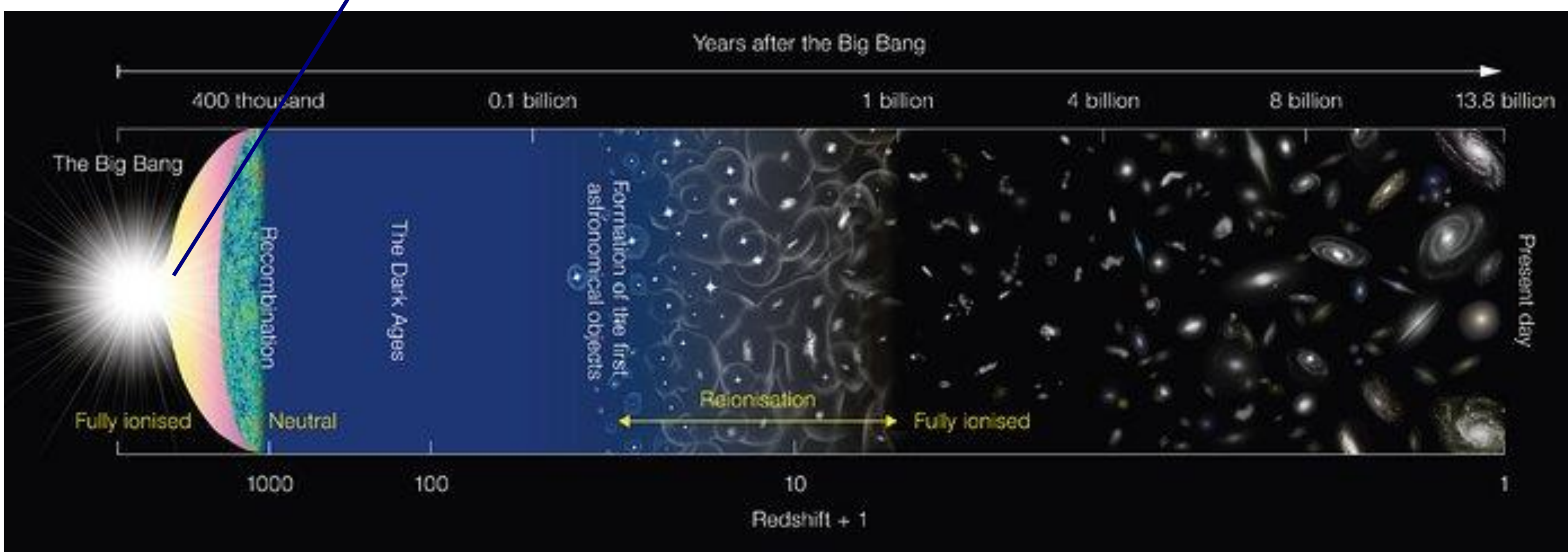


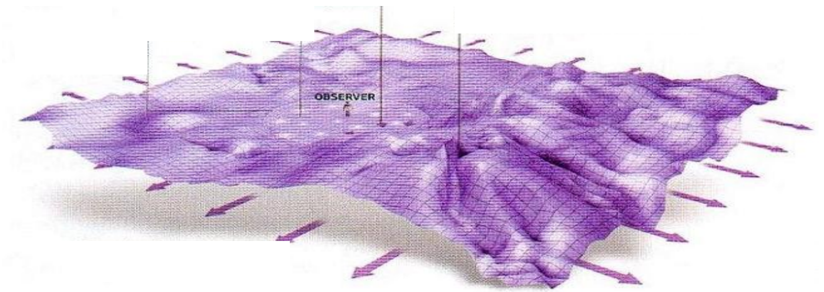
What is their origin?



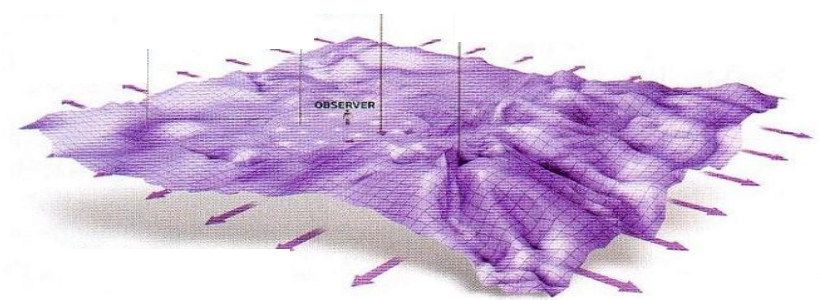
Quantum fluctuations during inflation

For a review, see J. Martin, Lect. Notes Phys. 669 (2005), 199, hep-th/040611





1- In the early Universe, the fluctuations are small and, hence, a linear approach is possible

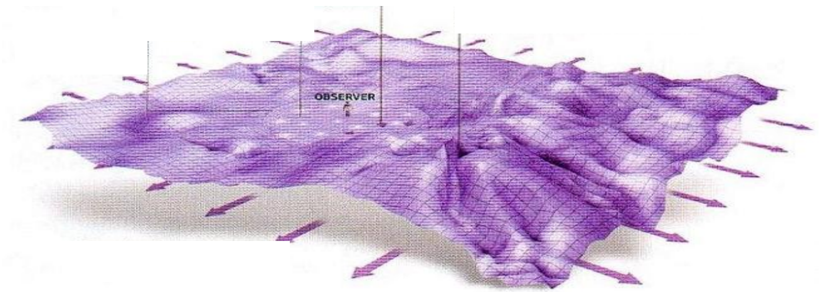


1- In the early Universe, the fluctuations are small and, hence, a linear approach is possible

2- One perturbs the inflaton field and the metric

$$\phi(\eta) \longrightarrow \phi(\eta) + \delta\phi(\eta, \mathbf{x})$$

$$a(\eta) \longrightarrow a(\eta)\left[1 + \Phi_{\rm B}(\eta, \mathbf{x})\right]$$



1- In the early Universe, the fluctuations are small and, hence, a linear approach is possible

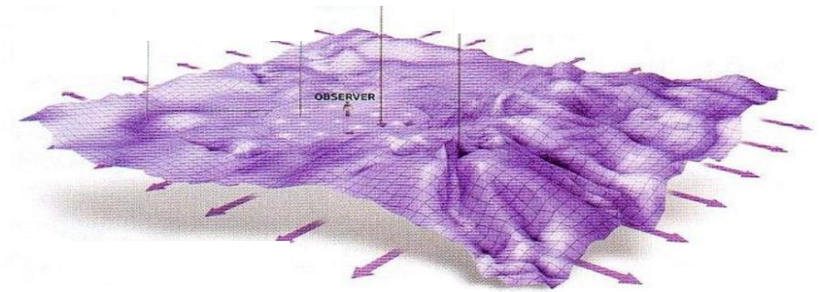
2- One perturbs the inflaton field and the metric

$$\phi(\eta) \longrightarrow \phi(\eta) + \delta\phi(\eta, \mathbf{x})$$

$$a(\eta) \longrightarrow a(\eta)\left[1 + \Phi_{\mathrm{B}}(\eta, \mathbf{x})\right]$$

3- The fluctuations of the system Gravity + Inflaton field can be described by a single variable, the so-called Mukhanov-Sasaki variable

$$v(\eta, \mathbf{x}) = a(\eta)\left(\delta\phi + \frac{\phi'}{a'/a}\Phi_{\mathrm{B}}\right) = \frac{1}{(2\pi)^{3/2}}\int \mathrm{d}\mathbf{k}\, v_{\mathbf{k}}(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}}$$

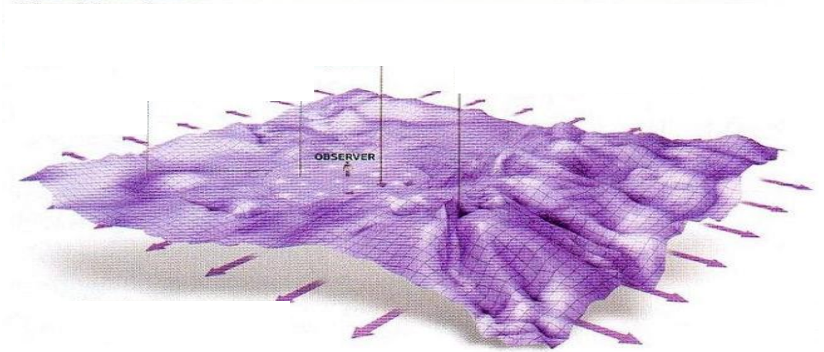


4- Expansion of the action GR + scalar field (inflaton field) leads to

$$H = \int \mathrm{d}^3\mathbf{k} \left\{ p_{\mathbf{k}} p_{\mathbf{k}}^* + \underbrace{\left[k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} \right]} v_{\mathbf{k}} v_{\mathbf{k}}^* \right\}$$

$$\omega^2(k,\eta) \equiv k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} = \mathcal{F}(k, a, a', a'', a''', a'''')$$

$$\epsilon_1 = 2 - \frac{a''a}{a'^2}$$

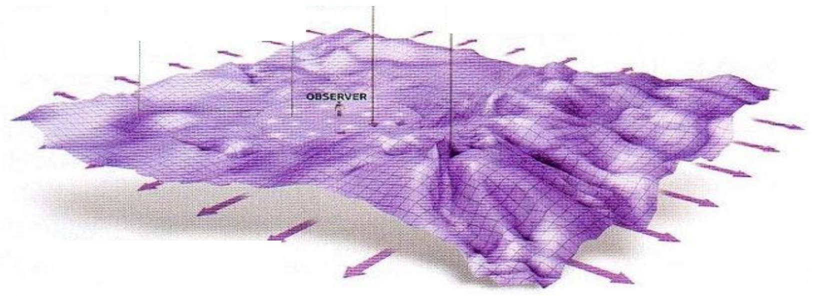


4- Expansion of the action GR + scalar field (inflaton field) leads to

$$H = \int \mathrm{d}^3\mathbf{k} \left\{ p_{\mathbf{k}} p_{\mathbf{k}}^* + \left[k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} \right] v_{\mathbf{k}} v_{\mathbf{k}}^* \right\}$$

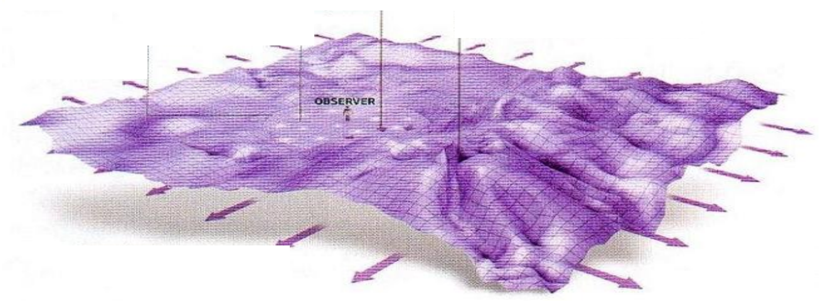
5- The equation of motion is that of a parametric oscillator, the time dependence of the frequency being given by the background

$$v_{\mathbf{k}}''(\eta) + \omega^2(k,\eta) v_{\mathbf{k}}(\eta) = 0$$



6- Finally the fluctuations are quantized in the standard fashion, namely

$$\Psi\left[v(\eta,\mathbf{x})\right]=\prod_{\mathbf{k}}\Psi\left(v_{\mathbf{k}}^{\mathrm{R}}\right)\Psi\left(v_{\mathbf{k}}^{\mathrm{I}}\right)\longrightarrow\ i\frac{\partial}{\partial\eta}\Psi\left(v_{\mathbf{k}}\right)=-\frac{\partial^2}{\partial v_{\mathbf{k}}^2}\Psi\left(v_{\mathbf{k}}\right)+\omega^2(k,\eta)\Psi\left(v_{\mathbf{k}}\right)$$



6- Finally the fluctuations are quantized in the standard fashion, namely

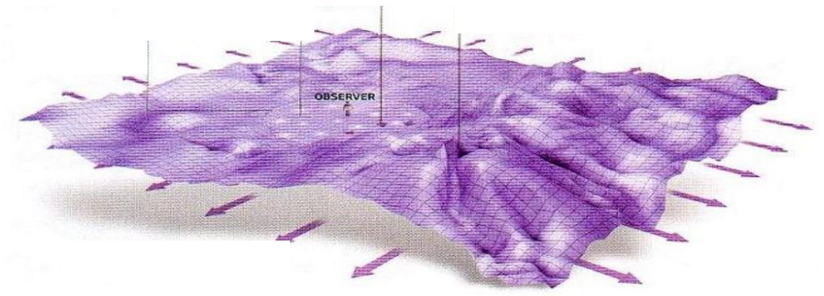
$$\Psi\left[v(\eta,\mathbf{x})\right]=\prod_{\mathbf{k}}\Psi\left(v_{\mathbf{k}}^{\mathrm{R}}\right)\Psi\left(v_{\mathbf{k}}^{\mathrm{I}}\right)\longrightarrow i\frac{\partial}{\partial\eta}\Psi\left(v_{\mathbf{k}}\right)=-\frac{\partial^2}{\partial v_{\mathbf{k}}^2}\Psi\left(v_{\mathbf{k}}\right)+\omega^2(k,\eta)\Psi\left(v_{\mathbf{k}}\right)$$

7- The quantum state of the system is a Gaussian state with a time-dependent variance, the time dependence being determined by the dynamics of the backgound

$$\Psi\left(v_{\mathbf{k}}\right)=\left[\frac{2\Re\Omega_{\mathbf{k}}(\eta)}{\pi}\right]^{1/4}e^{-\Omega_{\mathbf{k}}(\eta)v_{\mathbf{k}}^2}$$

with

$$\left\{\begin{array}{l}\Omega_{\mathbf{k}}(\eta)=-\dfrac{i}{2}\dfrac{f_{\mathbf{k}}'}{f_{\mathbf{k}}}\\ f_{\mathbf{k}}''+\omega^2(k,\eta)f_{\mathbf{k}}=0\end{array}\right.$$

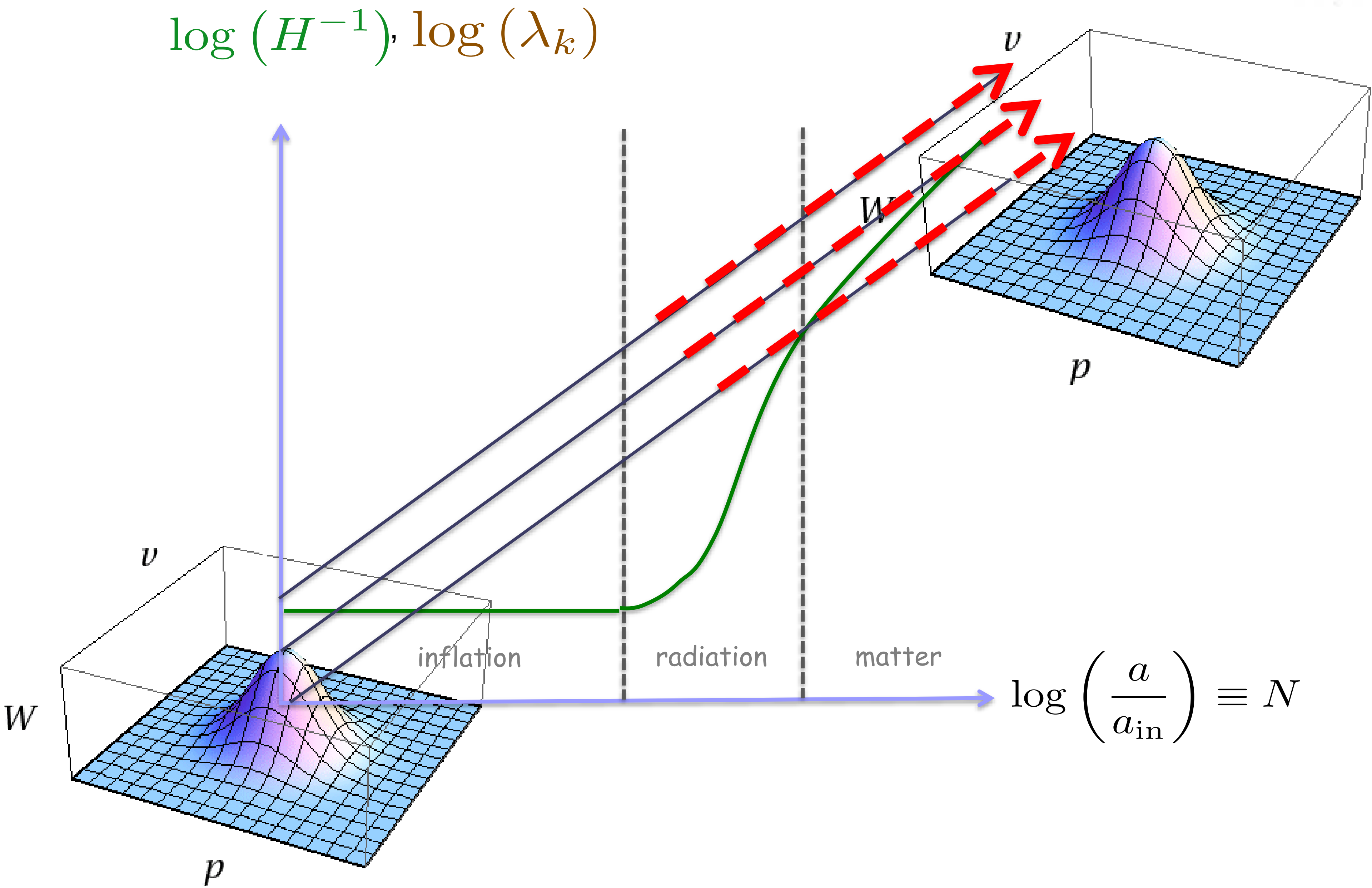


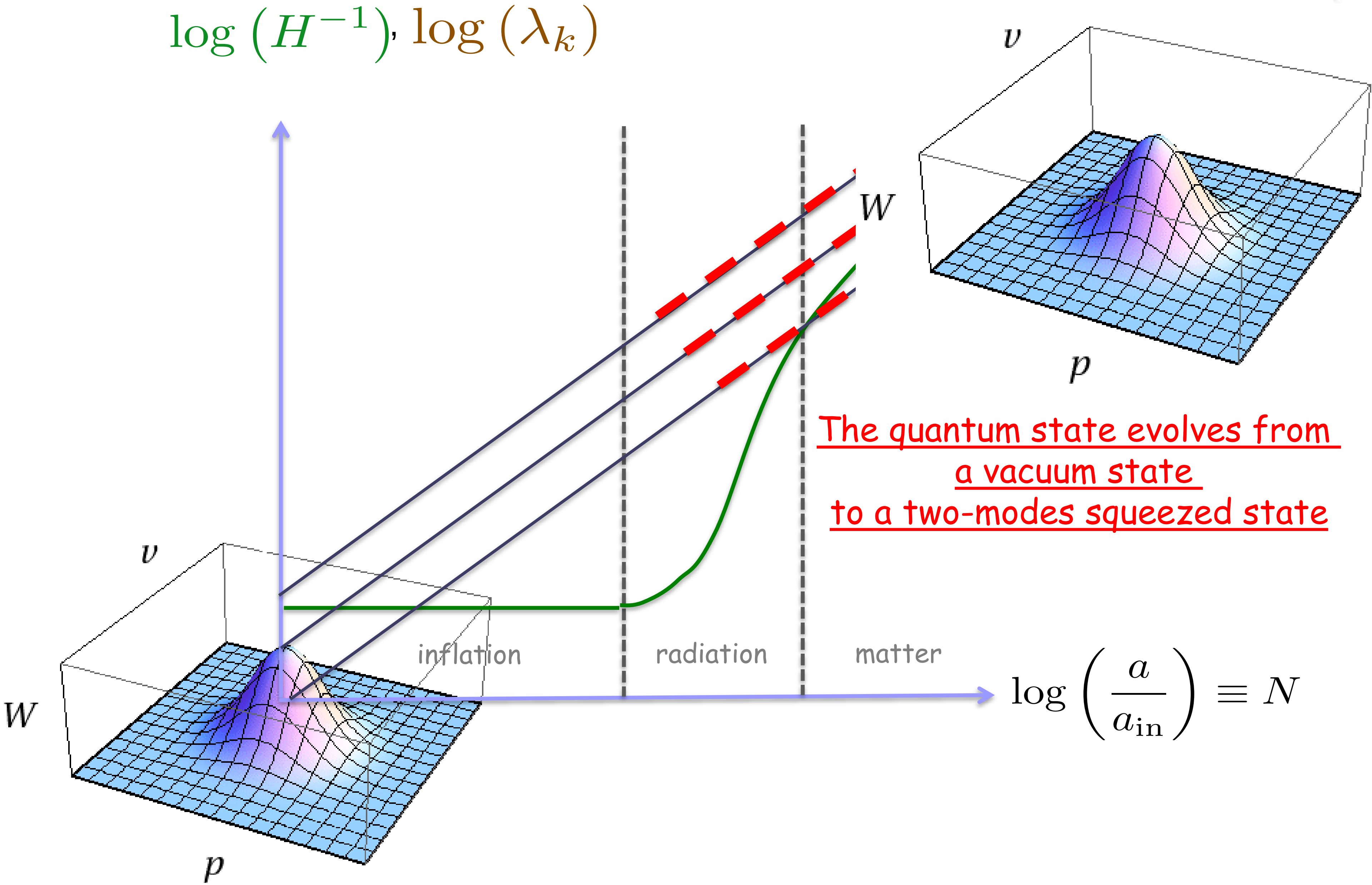
8- Initially, at the beginning of inflation, the quantum state is the vacuum state

$$\Psi\left(v_{\mathbf{k}}\right)=\left[\frac{2\Re\Omega_{\mathbf{k}}(\eta)}{\pi}\right]^{1/4}e^{-\Omega_{\mathbf{k}}(\eta)v_{\mathbf{k}}^{2}}$$

$$\lim_{k/(aH)\to\infty}\Omega_{\mathbf{k}}(\eta)=\frac{k}{2}$$

Perturbations of quantum-mechanical origin





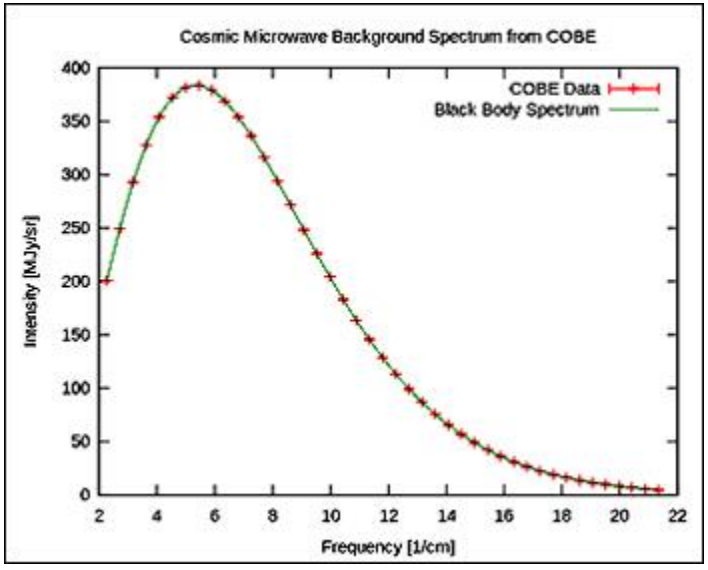
The quantum state evolves from
a vacuum state
to a two-modes squeezed state

The cosmological two-mode squeezed state is (very!) strongly squeezed

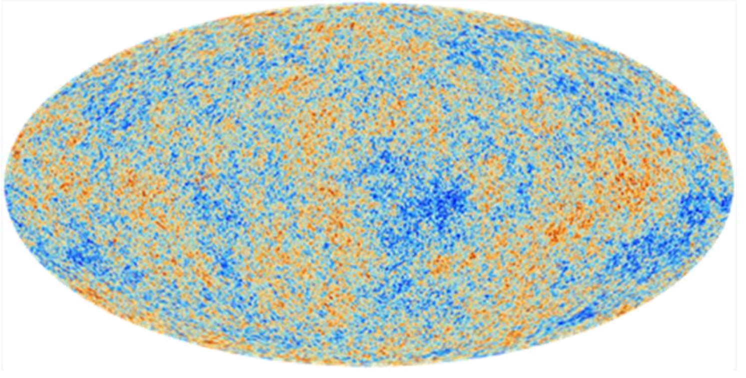
$$|\Psi_{\mathbf{k}}\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{+\infty} e^{2in\varphi_k} (-1)^n \tanh^n r_k |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

CMB is the most accurate black body ever produced in Nature

CMB anistropy is the strongest squeezed state ever produced in Nature



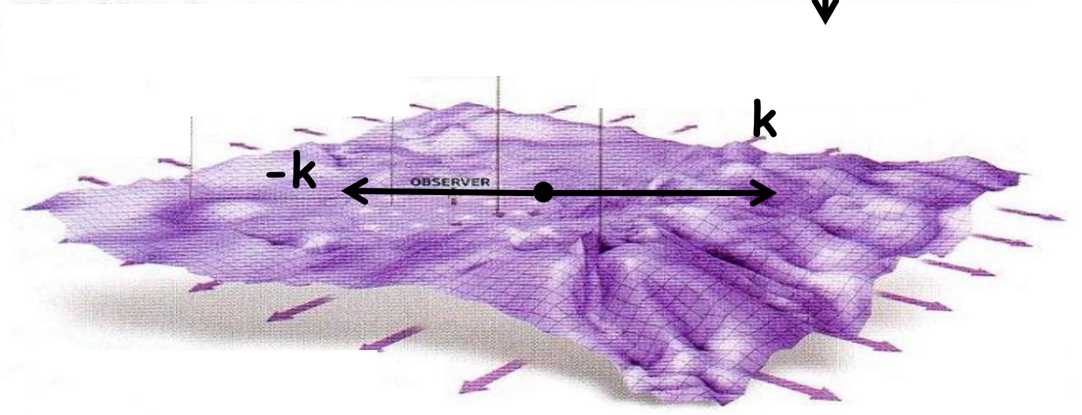
$$r_k = \mathcal{O}\left(10^2\right)$$



In the Heisenberg picture, this corresponds to creation of particle out of the vacuum (with opposite momenta), thanks to the dynamical background

$$\hat{H} = \int \mathrm{d}^3\mathbf{k} \left[\frac{k}{2}\left(\hat{c}_{\mathbf{k}}\hat{c}^{\dagger}_{\mathbf{k}} + \hat{c}_{-\mathbf{k}}\hat{c}^{\dagger}_{-\mathbf{k}}\right) - \frac{i}{2}\frac{(a\sqrt{\epsilon_1})'}{a\sqrt{\epsilon_1}}\left(\hat{c}_{\mathbf{k}}\hat{c}_{-\mathbf{k}} - c^{\dagger}_{-\mathbf{k}}c^{\dagger}_{\mathbf{k}}\right)\right]$$

The pump source vanishes if space-time is not dynamical, a'=0



This is similar to the Schwinger effect: interaction of a quantum field with a classical source

J. Martin, Lect. Notes Phys. 738 (2008), 195
arXiv:0704.3540

Schwinger effect

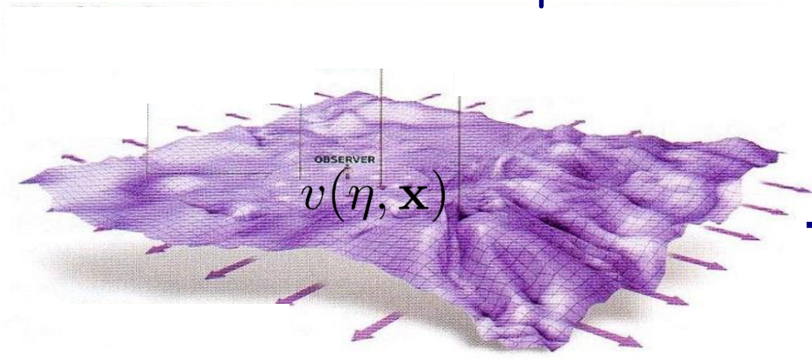
- Electron and positron fields
- Classical electric field
- Amplitude of the effect controlled by E

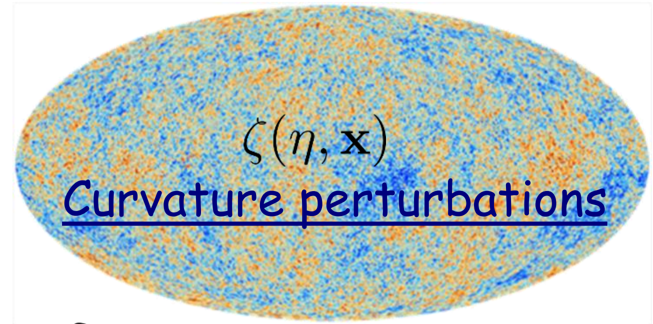
Inflationary cosmological perturbations

- Inhomogeneous gravity field
- Background gravitational field: scale factor
- Amplitude controlled by the Hubble parameter H

See also dynamical Schwinger effect, dynamical Casimir effect etc …

Inflation is phenomenologically very successful





$$\frac{v}{M_{\rm Pl}\, a(\eta)\sqrt{2\epsilon_1}} = \zeta \sim \frac{\delta T}{T}$$

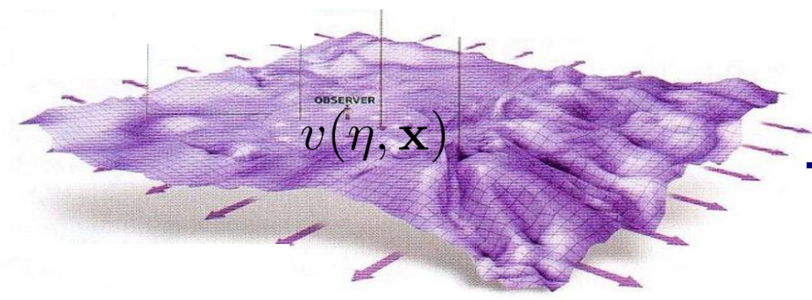
Two-point correlation function

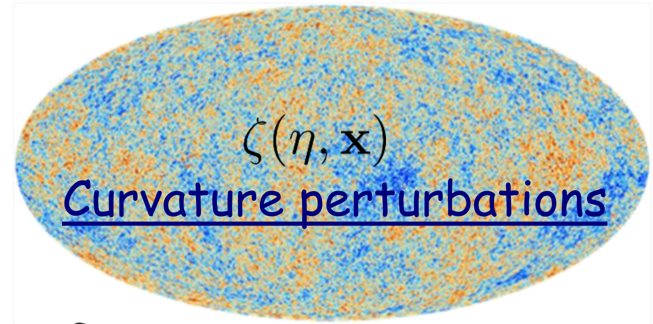
$$\left\langle \hat{\zeta}(\eta, \mathbf{x})\hat{\zeta}(\eta, \mathbf{x}+\mathbf{r})\right\rangle = \int_0^{+\infty} \frac{\mathrm{d}k}{k} P_\zeta(k) \frac{\sin(kr)}{kr}$$

$$P_\zeta(k) = A_{\rm S}\left(\frac{k}{k_{\rm P}}\right)^{n_{\rm S}-1}$$ with $n_{\rm S} \simeq 1$

Perturbations of quantum-mechanical origin

Inflation is phenomenologically very successful





$$\frac{v}{M_{\mathrm{Pl}}\, a(\eta)\sqrt{2\epsilon_1}} = \zeta \sim \frac{\delta T}{T}$$

Two-point correlation function

$$\left\langle \hat{\zeta}(\eta, \mathbf{x})\hat{\zeta}(\eta, \mathbf{x}+\mathbf{r}) \right\rangle = \int_0^{+\infty} \frac{\mathrm{d}k}{k} P_\zeta(k) \frac{\sin(kr)}{kr}$$

$$P_\zeta(k) = A_{\mathrm{S}} \left(\frac{k}{k_{\mathrm{P}}}\right)^{n_{\mathrm{S}}-1} \quad \text{with} \quad n_{\mathrm{S}} \simeq 1$$

Planck data (2015)

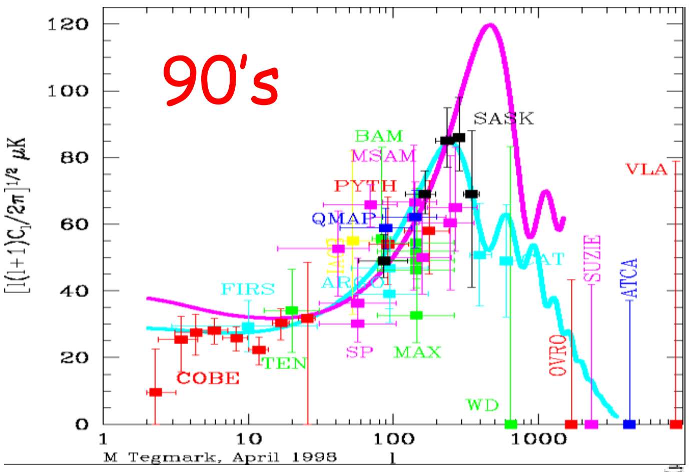
$$n_{\mathrm{S}} = 0.9655 \pm 0.0062$$

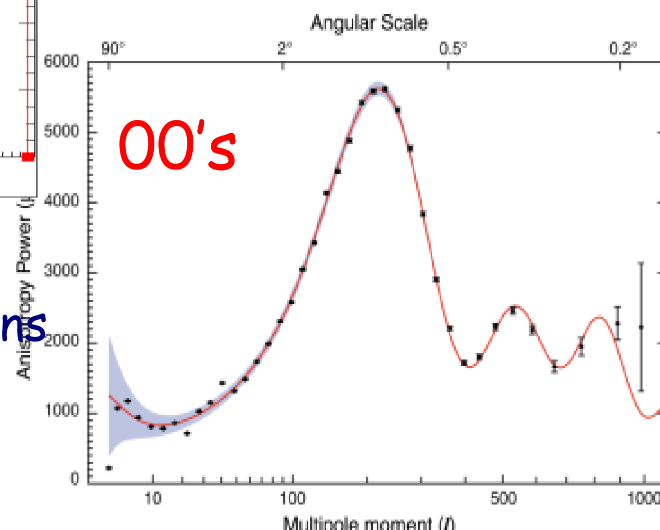
$$\ln\left(10^{10} A_{\mathrm{S}}\right) = 3.08 \pm 0.03$$

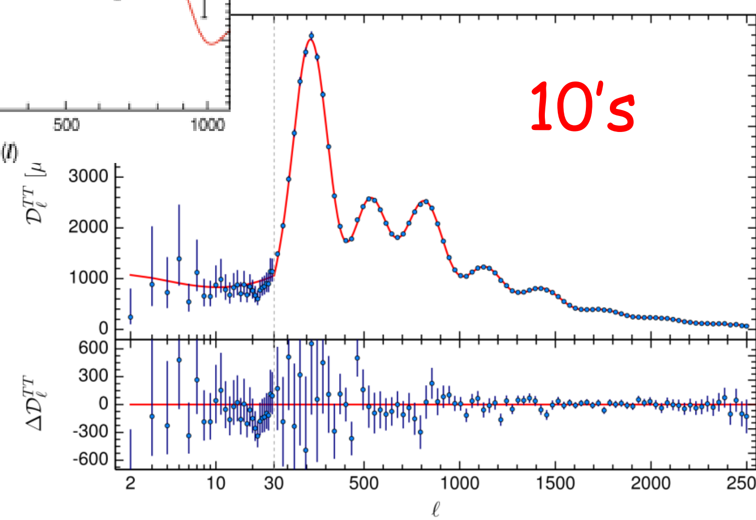
J. Martin, Astrophys. Space Sci, 45 (2016), 41, arXiv:1502.05733

Inflation is phenomenologically very successful

$$\left\langle 0 \left| \frac{\delta T}{T}(\vec{e}_1)\frac{\delta T}{T}(\vec{e}_2) \right| 0 \right\rangle = \sum_{\ell=2}^{+\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$







- The curvature perturbations is constant on large scales

$$\zeta' \simeq e^{-N_*} \simeq e^{-50}$$

- The constancy of curvature perturbations is responsible for the oscillatory pattern

Outline

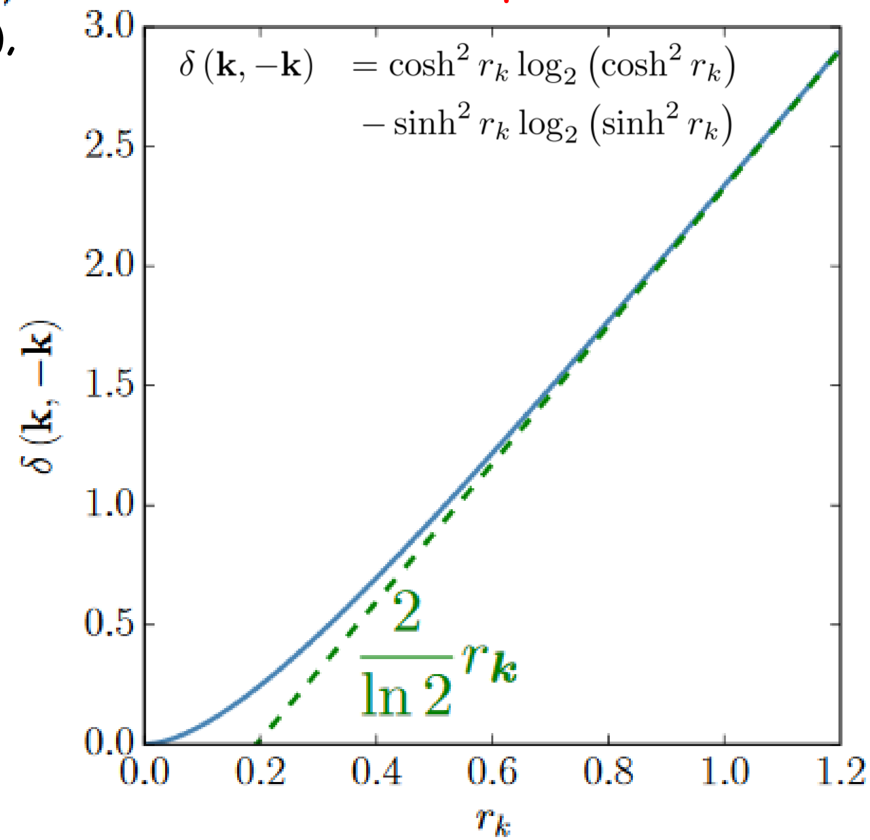
- The theory of cosmic inflation in brief: basic principles & observational status
- Cosmological fluctuations of quantum-mechanical origin
- Quantum Mechanics in the sky? Can we show that inflationary perturbations are of quantum-mechanical origin?
- Conclusions

Discord of a two-mode squeezed state

Discord: find a way to calculate the mutual information which coincides for classical correlations but may differ for quantum systems

H. Ollivier & W. Zurek, Phys. Rev. Lett. 88 (201), 017901.

For two-mode squeezed states



J. Martin & V. Vennin, PRD93 (2016), 1023505, arXiv:1510.04038

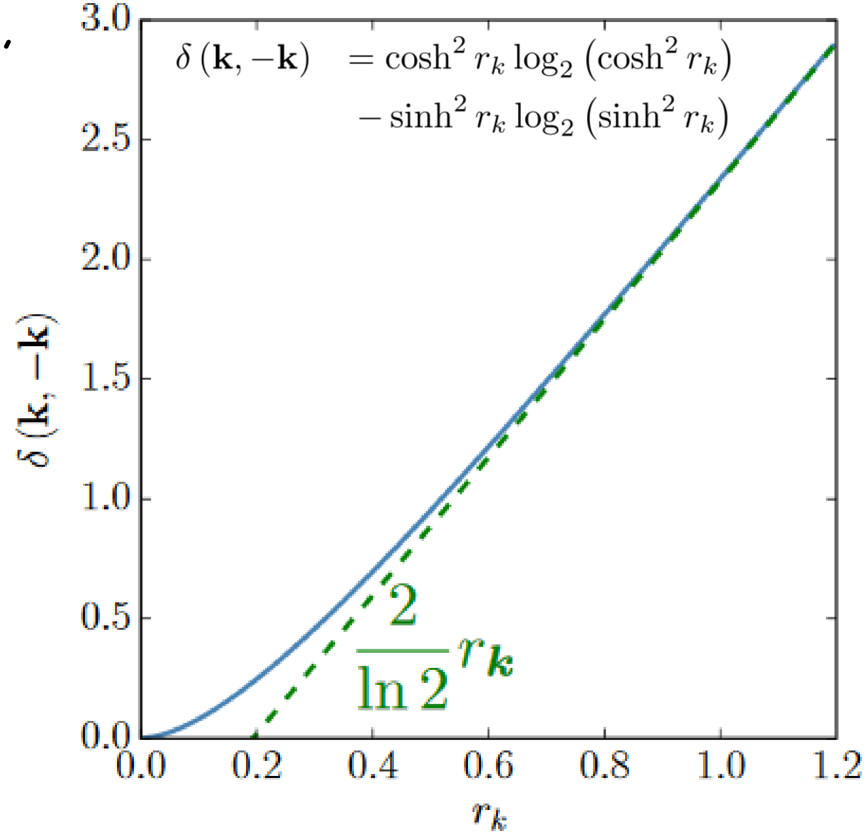
So there should be large quantum correlations (entanglement) in the sky and revealing thenm would be a mean to find a signature of the quantum mechanical origin of the perturbations

Discord of a two-mode squeezed state

Discord: find a way to calculate the mutual information which coincides for classical correlations but may differ for quantum systems

H. Ollivier & W. Zurek, Phys. Rev. Lett. 88 (201), 017901.

For two-mode squeezed states



J. Martin & V. Vennin, PRD93 (2016), 1023505, arXiv:1510.04038

But, in many instances, the quantum correlations are hidden in the decaying mode of the perturbations

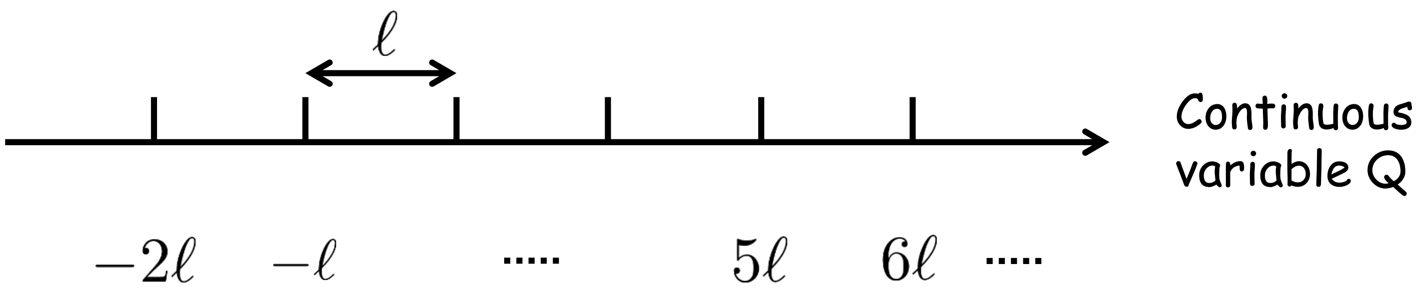
Can we observe quantum correlations in a cosmological (CHSH) Bell experiments?

How to construct a spin operator out of a continuous variable

Can we observe quantum correlations in a cosmological (CHSH) Bell experiments?

How to construct a spin operator out of a continuous variable

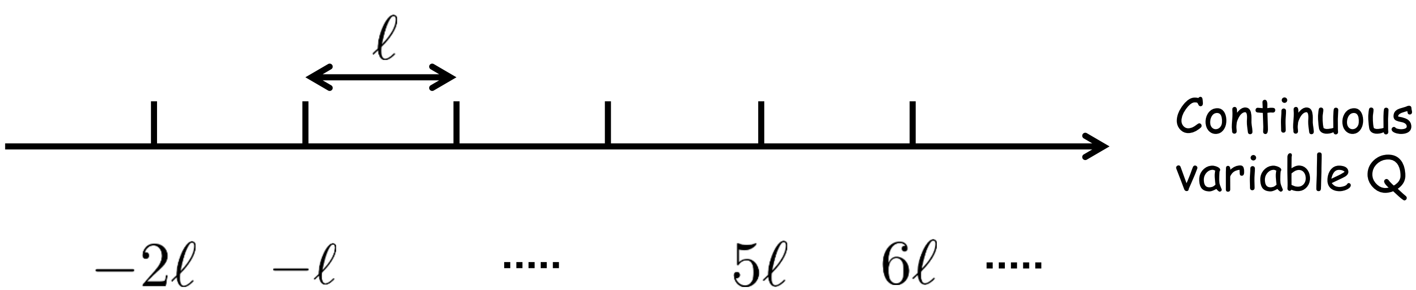
1- Coarse grain the continuous variable



Can we observe quantum correlations in a cosmological (CHSH) Bell experiments?

How to construct a spin operator out of a continuous variable

1- Coarse grain the continuous variable



2- Introduce the operator

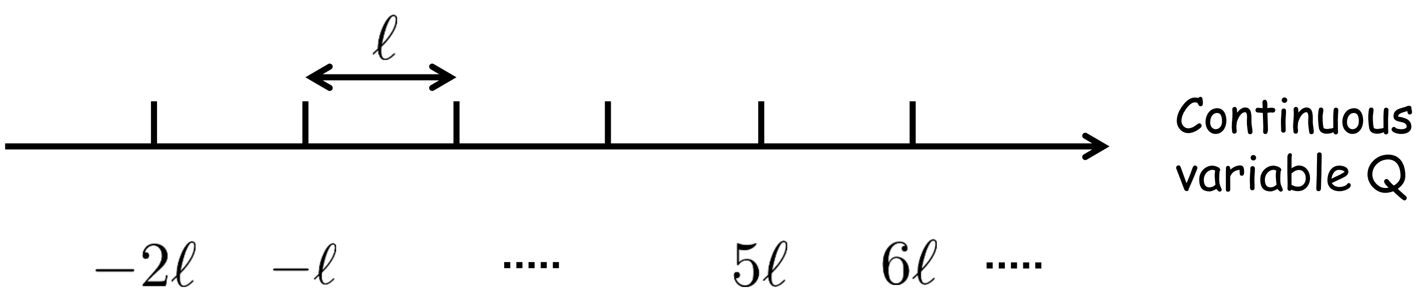
$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}Q |Q\rangle\langle Q|$$

J. A Larsson, PRA 70 (2004), 022102, quant-ph/0310140

Can we observe quantum correlations in a cosmological (CHSH) Bell experiments?

How to construct a spin operator out of a continuous variable

1- Coarse grain the continuous variable



2- Introduce the operator

$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}Q |Q\rangle\langle Q|$$

$$\hat{S}_z^2(\ell) = 1$$

It is a spin!

Can we observe quantum correlations in a cosmological (CHSH) Bell experiments?

How to construct a spin operator out of a continuous variable

3- The other components are introduced with

$$\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}Q|Q\rangle\langle Q+\ell|$$

$$\downarrow$$

$$\hat{S}_x(\ell) = \hat{S}_+(\ell) + S_+^\dagger(\ell)$$

$$\hat{S}_y(\ell) = -i\left[\hat{S}_+(\ell) - S_+^\dagger(\ell)\right]$$

J. A Larsson, PRA 70 (2004), 022102, quant-ph/0310140

Bell's operator for two-mode squeezed state (1 is k and 2 is -k)

$$\begin{aligned}\hat{B}(\ell) \quad &= \left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]+\left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]\\ &+\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]-\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]\end{aligned}$$

Classically

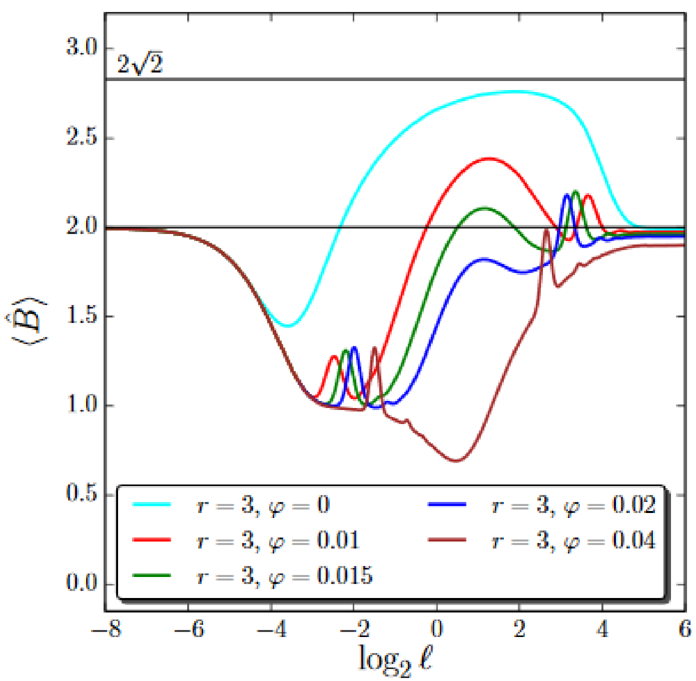
$$-2 \le B \le 2$$

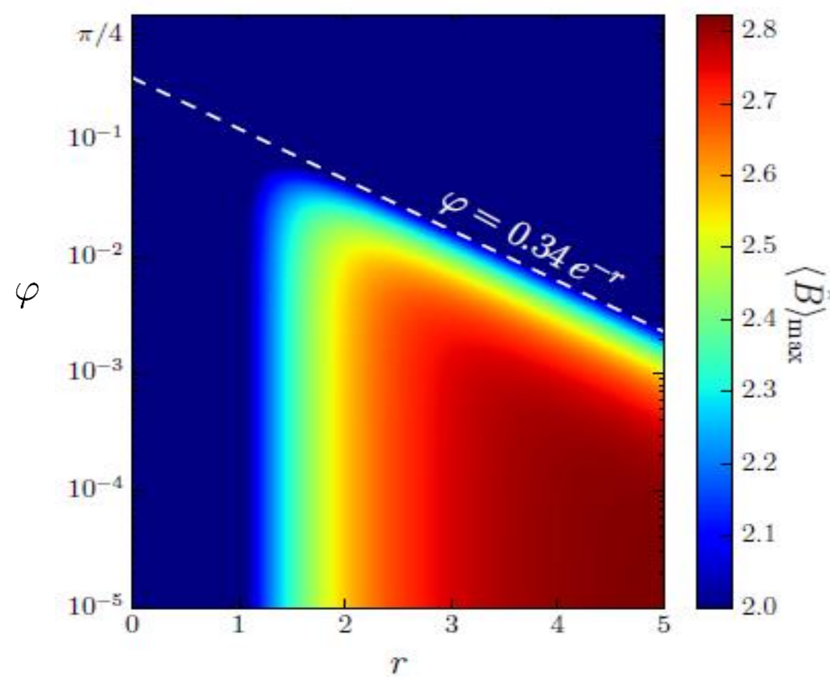
Quantum-mechanically

$$\left\langle \hat{B}(\ell)\right\rangle = \left\langle \Psi_{2-\mathrm{squeezed}} \left| \hat{B} \right| \Psi_{2-\mathrm{squeezed}} \right\rangle$$

Bell's operator for two-mode squeezed state (1 is k and 2 is -k)

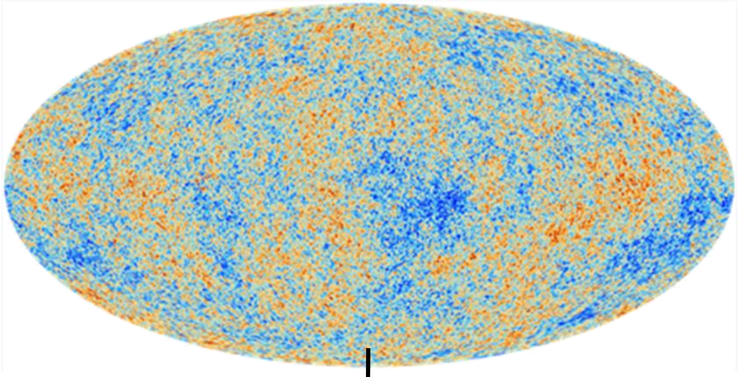
$$\hat{B}(\ell) = \left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]+\left[\mathbf{n}\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]$$
$$+\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]-\left[\mathbf{n}'\cdot\hat{\mathbf{S}}^{(1)}(\ell)\right]\otimes\left[\mathbf{m}'\cdot\hat{\mathbf{S}}^{(2)}(\ell)\right]$$





J. Martin & V. Vennin, PRA93 (2016), 062117,
arXiv:1605.02944

How can we measure the fictitious spin?

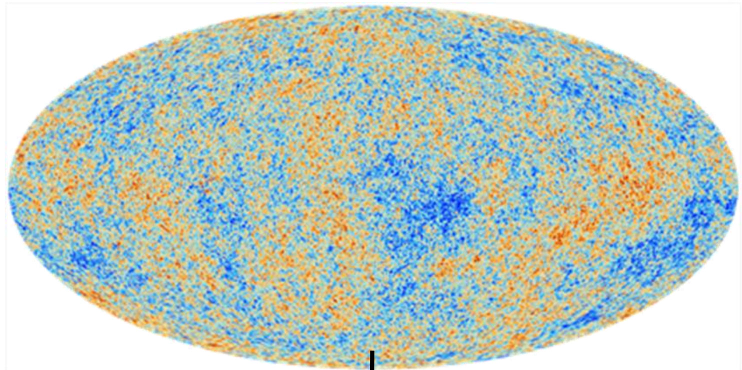


$$Q \sim v_{\mathbf{k}}$$

Find n such that $n\ell < v_{\mathbf{k}} < (n+1)\ell$

$$S_z(\ell) = (-1)^n$$

How can we measure the fictitious spin?



$$Q \sim v_{\mathbf{k}}$$

Find n such that $n\ell < v_{\mathbf{k}} < (n+1)\ell$

$$S_z(\ell) = (-1)^n$$

But this also requires a measurement of the x-component of the spin which means measuring p~ Q' ~ e^{-N_*} that is to say the decaying mode …

Outline

- The theory of cosmic inflation in brief: basic principles & observational status

- Cosmological fluctuations of quantum-mechanical origin

- Quantum Mechanics in the sky? Can we show that inflationary perturbations are of quantum-mechanical origin?

- Conclusions

Conclusions

Recap

- According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which a discordant and entangled state

Recap

- According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which a discordant and entangled state

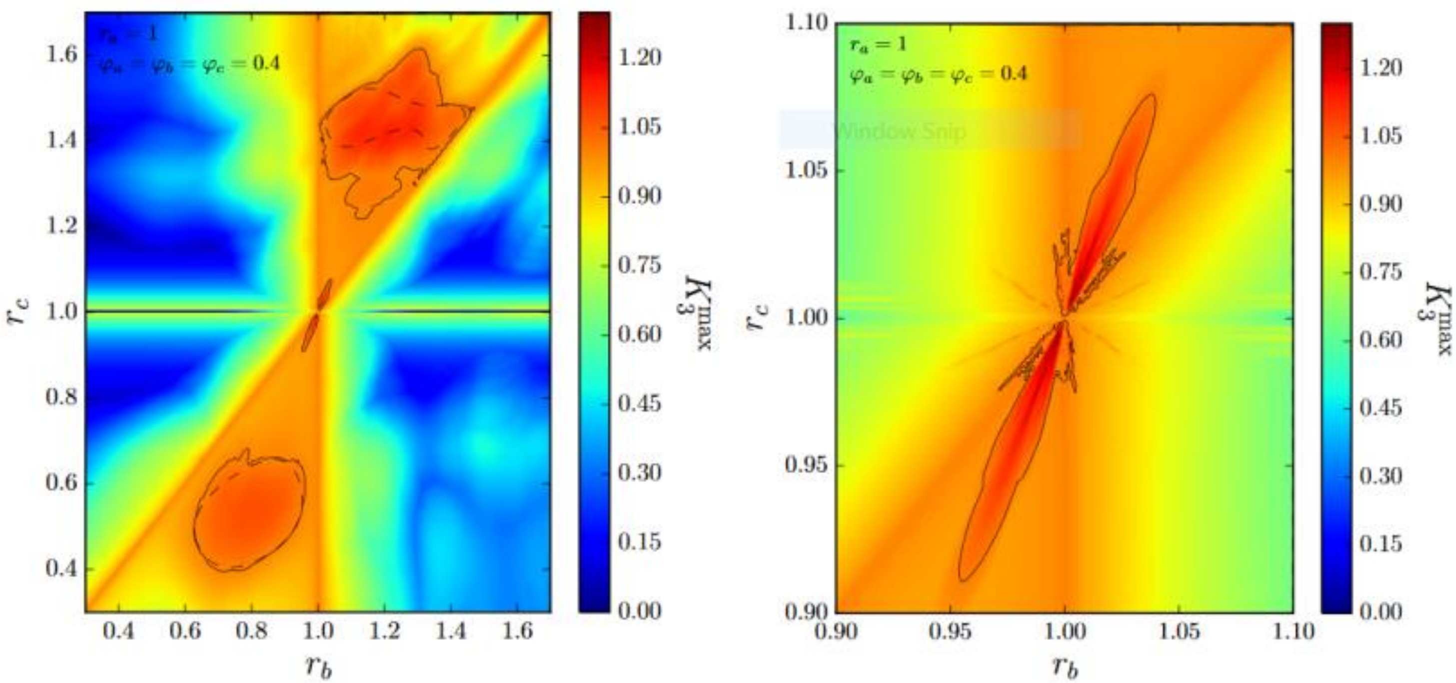
- However, a quantum mechanical signature in the sky seems to be hidden in the decaying mode

Recap

- According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which a discordant and entangled state
- However, a quantum mechanical signature in the sky seems to be hidden in the decaying mode
- Ways out?? Measuring the z-component only but at different times?

Recap

Measure the z-component only but at different times= Leggett-Garg inequalities



J. Martin & V. Vennin, PRA94 (2016), 052135, arXiv:1611.0185

The Leggett-Garg inequalities are violated for a two-mode squeezed state

Recap

- According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which a discordant and entangled state

- However, a quantum mechanical signature in the sky seems to be hidden in the decaying mode

- Ways out?? Measuring the z-component only but at different times?

- Inflation can also be used to test theories beyond QM such as CSL, de Broglie Bohm formulation etc …

P. Canate, P. Pearle & D. Sudarsky, PRD87 (2013), 104024, arXiv:1211.3463

S. Das, S. Sahu, S. Banerjee & TP Singh, PRD90 (2014), 043503, arXiv:1404.5740

J. Martin, V. Vennin & P. Peter, PRD86 (2012), 103524, arXiv:1207.2086

Recap

- According to cosmic inflation, the CMB fluctuations are placed in a strongly two- mode squeezed state which a discordant and entangled state

- However, a quantum mechanical signature in the sky seems to be hidden in the decaying mode

- Ways out?? Measuring the z-component only but at different times?

- Inflation can also be used to test theories beyond QM such as CSL, de Broglie Bohm formulation etc …

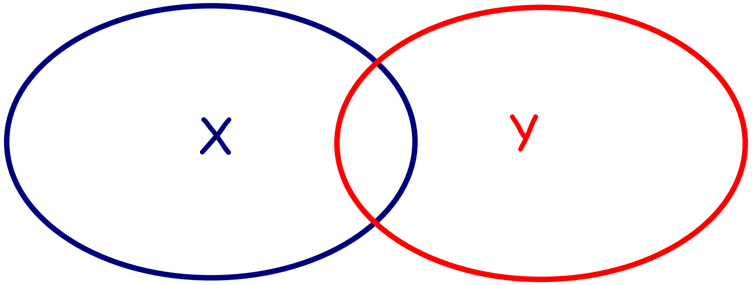
  P. Canate, P. Pearle & D. Sudarsky, PRD87 (2013), 104024, arXiv:1211.3463

  S. Das, S. Sahu, S. Banerjee & TP Singh, PRD90 (2014), 043503, arXiv:1404.5740

  J. Martin, V. Vennin & P. Peter, PRD86 (2012), 103524, arXiv:1207.2086

- Take away message: inflation is not only a successful scenario of the early Universe, it is also a very interesting playground for foundational issues of quantum mechanics

$$J(X,Y) \equiv S(X) - S(X|Y)$$



1) The bipartite system is characterized by the pdf: $p(X = x, Y = y)$

2) The "individual" pdf's are the marginalized pdf's :

$$p(X = x) = \sum_y p(X = x, Y = y)$$

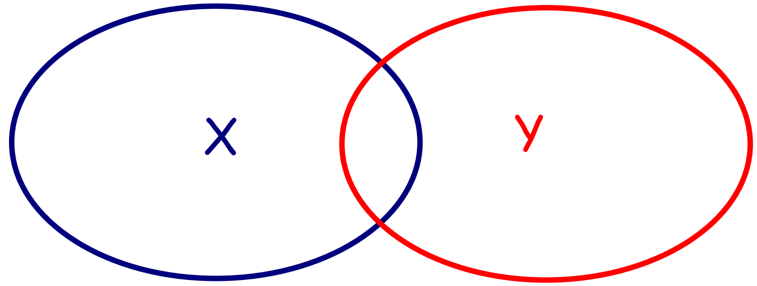
$$p(Y = y) = \sum_x p(X = x, Y = y)$$

3) Conditional entropy is defined by:

$$S(X|Y = b) = \sum_x p(X = x|Y = b) \ln [p(X = x|Y = b)]$$

$$S(X|Y) = \sum_b p(Y = b) S(X|Y = b)$$

Quantum discord



1) The bipartite system is characterized by the density matrix: $\hat{\rho}(X,Y)$

2) Then, one measures an observable of the system Y, characterized by the operator

$$\mathrm{Id}(X)\otimes\hat{\Pi}_j(Y) \quad \text{with} \quad \sum_j \hat{\Pi}_j(Y)=1$$

The state of the system becomes $\hat{\rho}(X,Y)\rightarrow\hat{\rho}(X,Y)\dfrac{\hat{\mathrm{Id}}(X)\otimes\hat{\Pi}_j(Y)}{p_j}$

$$p_j=\mathrm{Tr}\left[\hat{\rho}(X,Y)\hat{\mathrm{Id}}(X)\otimes\hat{\Pi}_j(Y)\right]$$

3) If we only have access to X:

$$\hat{\rho}\left(X;\hat{\Pi}_j\right)=\mathrm{Tr}\left[\hat{\rho}(X,Y)\frac{\hat{\mathrm{Id}}(X)\otimes\hat{\Pi}_j(Y)}{p_j}\right]$$

$$\longrightarrow \quad J(X,Y)=S\left[\hat{\rho}(X)\right]-\sum_j p_j S\left[\hat{\rho}\left(X;\hat{\Pi}_j\right)\right]$$

Can we reproduce the date with a "classical" state?

Can we reproduce the date with a "classical" state?

A non-discordant state has necessarily the following form

$$\hat{\rho}_{\mathrm{cl}} = \bigotimes_{\mathbf{k}} \sum_{n,m} p_{nm}(\mathbf{k}) \hat{\rho}_n(\mathbf{k}) \otimes \hat{\rho}_m(-\mathbf{k})$$

Can we reproduce the date with a "classical" state?

A non-discordant state has necessarily the following form

$$\hat{\rho}_{\rm cl} = \bigotimes_{\mathbf{k}} \sum_{n,m} p_{nm}(\mathbf{k}) \hat{\rho}_n(\mathbf{k}) \otimes \hat{\rho}_m(-\mathbf{k})$$

Is there a choice of p_{nm}'s which leads to the same correlations functions?

Classical state

Can we reproduce the date with a "classical" state?

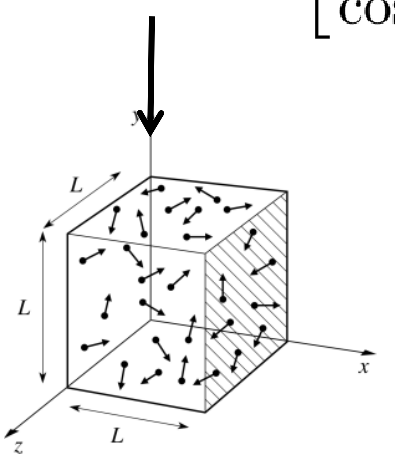
A non-discordant state has necessarily the following form

$$\hat{\rho}_{\rm cl} = \bigotimes_{\mathbf{k}} \sum_{n,m} p_{nm}(\mathbf{k}) \hat{\rho}_n(\mathbf{k}) \otimes \hat{\rho}_m(-\mathbf{k})$$

Is there a choice of p_{nm}'s which leads to the same correlations functions?

$$p_{nm} = \left(1 - e^{\beta_k}\right) e^{-\beta_k n} \delta(n-m)$$

$$\beta_k = -\ln\left[\frac{\cosh(2r_k) + \sinh(2r_k)\cos(2\varphi_k) - 1}{\cosh(2r_k) + \sinh(2r_k)\cos(2\varphi_k) + 1}\right]$$



$$\hat{\rho}_{\rm cl} = \left(1 - e^{\beta_k}\right) \sum_{n=0}^{+\infty} e^{-\beta_k n} |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle\langle n_{\mathbf{k}}, n_{-\mathbf{k}}|$$

$$\rightarrow \langle \hat{\zeta}_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle_{\rm cl} = \langle \hat{\zeta}_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle$$

One reproduces exactly the power spectrum of curvature perturbations

Can we reproduce the date with a "classical" state?

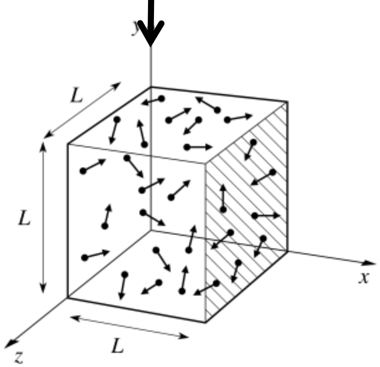
A non-discordant state has necessarily the following form

$$\hat{\rho}_{\rm cl} = \bigotimes_{\mathbf{k}} \sum_{n,m} p_{nm}(\mathbf{k}) \hat{\rho}_n(\mathbf{k}) \otimes \hat{\rho}_m(-\mathbf{k})$$

Is there a choice of p_{nm}'s which leads to the same correlations functions?

$$p_{nm} = \left(1 - e^{\beta_k}\right) e^{-\beta_k n} \delta(n-m)$$

$$\beta_k = -\ln\left[\frac{\cosh(2r_k) + \sinh(2r_k)\cos(2\varphi_k) - 1}{\cosh(2r_k) + \sinh(2r_k)\cos(2\varphi_k) + 1}\right]$$



$$\hat{\rho}_{\rm cl} = \left(1 - e^{\beta_k}\right) \sum_{n=0}^{+\infty} e^{-\beta_k n} |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle\langle n_{\mathbf{k}}, n_{-\mathbf{k}}|$$

$$\langle \hat{\zeta}_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle_{\rm cl} = \langle \hat{\zeta}_{\mathbf{k}} \hat{\zeta}_{\mathbf{k}} \rangle$$

One reproduces exactly the power spectrum of curvature perturbations

Can we reproduce the date with a "classical" state?

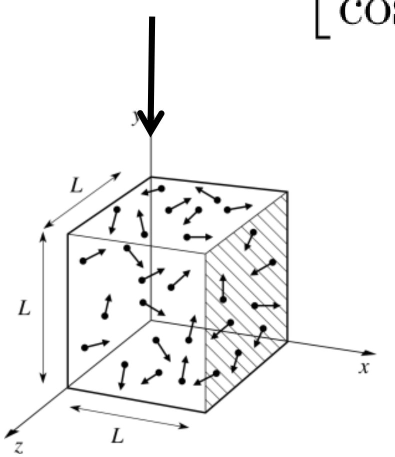
A non-discordant state has necessarily the following form

$$\hat{\rho}_{\rm cl} = \bigotimes_{\mathbf{k}} \sum_{n,m} p_{nm}(\mathbf{k}) \hat{\rho}_n(\mathbf{k}) \otimes \hat{\rho}_m(-\mathbf{k})$$

Is there a choice of p_{nm}'s which leads to the same correlations functions?

$$p_{nm} = \left(1 - e^{\beta_k}\right) e^{-\beta_k n} \delta(n-m)$$

$$\beta_k = -\ln\left[\frac{\cosh(2r_k) + \sinh(2r_k)\cos(2\varphi_k) - 1}{\cosh(2r_k) + \sinh(2r_k)\cos(2\varphi_k) + 1}\right]$$

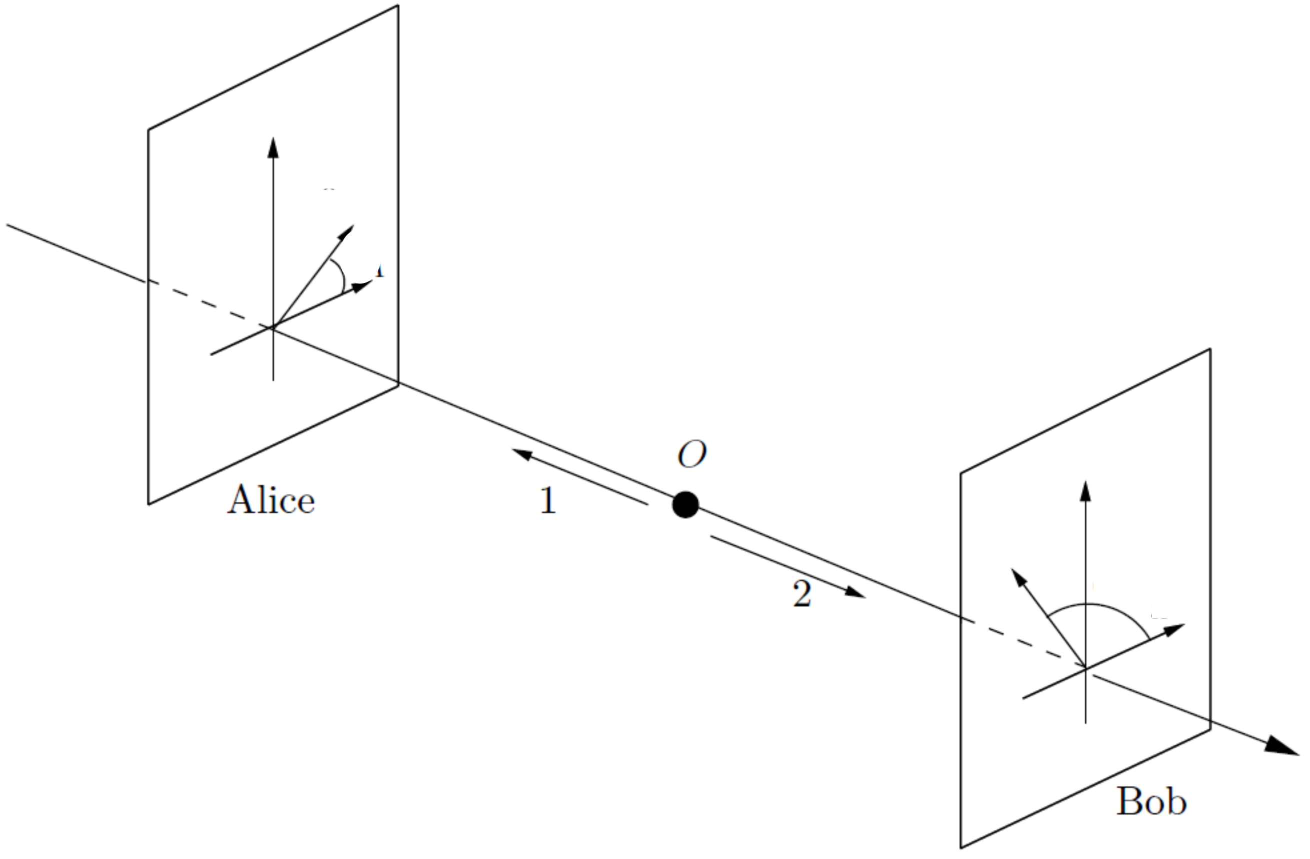


For the other correlation function

$$\left.\begin{aligned} \langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}'_{\mathbf{k}} \rangle_{\rm cl} &= e^{-4N_*} \\ \langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}'_{\mathbf{k}} \rangle &= e^{-2N_*} \end{aligned}\right\} \quad \frac{\langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}'_{\mathbf{k}} \rangle}{\langle \hat{\zeta}'_{\mathbf{k}} \hat{\zeta}'_{\mathbf{k}} \rangle_{\rm cl}} = e^{2N_*} \gg 1$$

The difference is hidden in the decaying mode …

Bell's experiment



A two-mode squeezed state is also an entangled state

$$|\Psi_{\mathbf{k}}\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{+\infty} e^{2in\varphi_k} (-1)^n \tanh^n r_k |n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle$$

We have quantum correlations between modes with wave-numbers k and -k