The Particle-Field Theory and Its Relativistic Generalization

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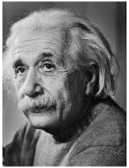

”Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us closer to the secret of the Old One. I, at any rate, am convinced that He is not playing at dice” [1]

[1] L. E. Ballentine, Am. J. Phys. **40**, (1972)

The PF Theory

A new foundation for describing micro-events from a deterministic causal standpoint is formulated, in which a micro-entity is supposed to be an allied particle-field system, instead of composing of a particle and (or) a field (wave). In this new approach, the principles of realism and causality based on the classic-like equations of motion are presumed.

Contents

Basic Elements of the PF theory

For a one-dimensional, one-particle microsystem, three physical entities are introduced:

Particle mass: $\mathbf{m}$ position $\mathbf{x(t)}$

$$\mathbf{m}\frac{\mathbf{d^2x(t)}}{\mathbf{dt^2}} = \mathbf{f_P}$$

For the conservative forces, the particle possesses a conserved energy

$$E_P = V_P + K_P$$

$\mathbf{K_P = \frac{p_P^2}{2m}}$ kinetic energy

$\mathbf{p_P}$: linear momentum of the particle

Field $X(x(t), t)$

Velocity of the field $\mathbf{v_F} = |\frac{\mathbf{dX}}{\mathbf{dt}}| = |\dot{\mathbf{X}}|$

$$\mathbf{m}\frac{\mathbf{d\dot{X}}}{\mathbf{dt}} = \mathbf{f_F}$$

For a conservative forcef_P $X = \chi(x(t))$

The field X merely enfolds the particle. Although the presence of the particle is essential for defining the force of the field. If there is no particle, there will not be any associated field too. The existence of the field depends on the existence of the particle, but the opposite is not true.

$E = E_P + E_F$ is an observable property. So:

$$\begin{aligned} E &= V_P + (E_F + \frac{p_P^2}{2m}) \\ &= V_P + \frac{p^2}{2m} \end{aligned}$$

Double feature of p
We postulate that p should satisfy the de Broglie relation:

$$p = \frac{h}{\lambda} = \hbar k$$

For stationary states, the form of $\chi(x(t))$ could be obtained from:

$$\chi'' = -k^2\chi; \qquad k^2 = \frac{p^2}{\hbar^2}; \qquad \chi'' = \frac{d^2\chi}{d^2x}$$

PF system

Neither the particle, nor the field representation alone is adequate for explaining the physical behavior of a microsystem. What really gives us a thorough understanding of the nature of a quantum system is a holistic depiction of both particle and its associated field which we call here a PF system.

The dynamics of the PF system can be described according to:

$$m\frac{d^2\dot{q}}{dt^2} = f_{PF}$$

q : Position of the PF system, $\dot{q}$: Velocity

$$V_{PF} = V_P + V_F; \quad K_{PF} = K_P + K_F; \quad E = E_P + E_F$$

$$K_{PF} = \frac{1}{2} m\dot{q}^2$$

$$\dot{q}^2 = \dot{x}^2 + |\dot{X}|^2; \quad dq^2 = dx^2 + |dX|^2$$

Trajectories of the PF system:

$$q(x,t) = \int dx \sqrt{\left(1 + |\frac{dX(x,t)}{dx}|^2\right)}$$

while we expect the particle to move along the infinitesimal displacement dx in the x direction, the displacement of the whole system is equal to dq, not dx. one can obtain the finite displacement q of a PF system in terms of the particle's location and time, when the field $X(x,t)$ is known.

Lorentz Invariance of the PF theory

Using the relativistic kinetic energy of the particle and stationary field

$$K_{rF} = K_{rP}\chi'^2 \qquad K_{rP} = m_0 c^2(\gamma_p - 1)$$

The kinetic energy of the PF system is defined as

$$\begin{aligned} K_{rPF} &= K_{rP} + K_{rF} \\ &= m_0 c^2(\gamma_{PF} - 1) \end{aligned}$$

$$\gamma_{PF} = (1 - \frac{\dot{q}^2}{c^2})^{-\frac{1}{2}}$$

$$\dot{q} = c\left(1 - \frac{1}{[(\gamma_p - 1)(1 + \chi'^2) + 1]^2}\right)^{-\frac{1}{2}}$$

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$$ds^2 = c^2 dt^2 - dq^2$$

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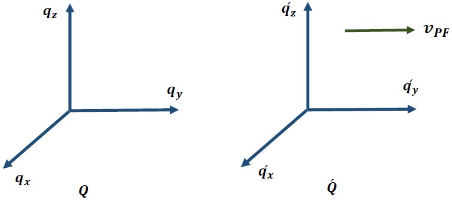

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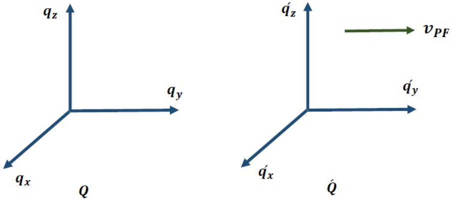

$$ds^2 = c^2 dt^2 - dq^2 = ds'^2 = c^2 dt'^2 - dq'^2$$

Relativistic Generalization of Time Independent Schrodinger Equation

The dynamics of a stationary real field in one dimension in the relativistic regime :

$$\frac{d(m_p \dot{\chi})}{dt} = f_{rF}$$

$$m_p = \gamma_p m_0; \qquad \gamma_p = \left(1 - \frac{v_p^2}{c^2}\right)^{-\frac{1}{2}}$$

For stationary real fields, we postulate the following equality as a general rule[2]:

$$- m_P \bar{\omega}^2 \chi = \gamma_P m_0 v_P^2 \chi''$$

[2] A. Shafiee, Pramana journal of physics, **76**, No. 6, 843-873, (2011).

$$\bar{\omega}^2 = k^2 v_P^2; \quad k = \frac{p}{\hbar};$$

P : relativistic de Broglie momentum
So

$$-\hbar^2 \chi'' = p^2 \chi,$$

Using the total energy of the PF system and

$$p = \gamma m_0 v$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \sqrt{1 + \frac{p^2}{m_0^2 c^2}}$$

Relativistc Schrodinger equation:

$$-\frac{\hbar^2}{2m_0}\chi'' + \frac{1}{2}m_0c^2\chi = \frac{E^2}{2m_0c^2}\left(1+\frac{V_{nrP}}{m_0c^2}\right)^{-2}\chi$$

$$-\frac{\hbar^2}{2m_0}\chi'' + \frac{1}{2}m_0c^2\chi = \frac{1}{2m_0c^2}(E - V_{nrP})^2\chi$$

Relativistic Hydrogen Atom

From Coulomb's law the potential energy is:

$$V(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\begin{aligned}
- & \frac{\hbar^2}{2m_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right)\right]\chi(\mathbf{r}) \\
+ & \frac{1}{2}m_0c^2\chi(\mathbf{r}) = \frac{1}{2m_0c^2}(E - V_{nrp}(r))^2\chi(\mathbf{r})
\end{aligned}$$

$$\chi(\mathbf{r}) = R(r)Y(\theta,\phi)$$

The energy of Hydrogen atom:

$$E = m_0 c^2 + E_n - \frac{E_n^2}{2m_0 c^2}[\frac{4n}{(l+\frac{1}{2})} - 3] - ...$$

$$E_n = -\frac{m_0}{2\hbar^2}(\frac{e^2}{4\pi\varepsilon_0})^2\frac{1}{n^2}$$

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The relativistic correction to E_n in the first-order perturbation theory:

$$E_r = -\frac{E_n^2}{2m_0 c^2}[\frac{4n}{(l+\frac{1}{2})} - 3]$$

Thank you