# Collapse models and spacetime symmetries

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# Outline

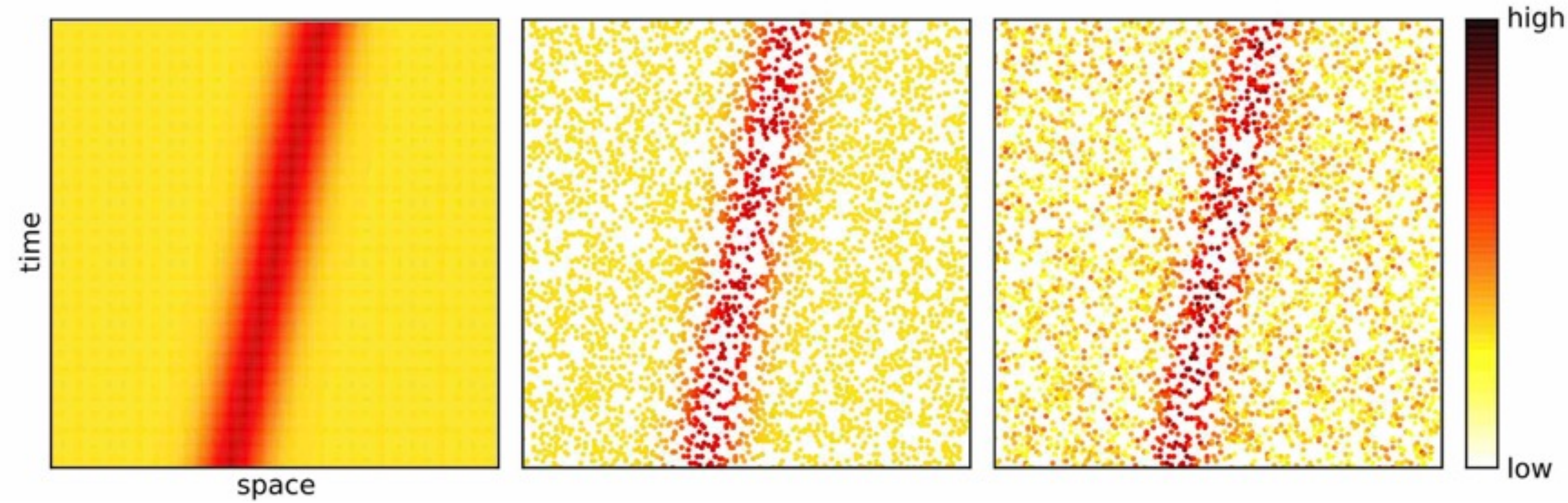
- What are collapse models?
- General structure of collapse models
- Collapse as inference
- Time symmetry
- Relativity
- Gravity

# What are collapse models?

- Collapse models describe the distribution of matter in space and time. **It is legitimate to regard this as the starting point**: there is matter in spacetime, how best to describe it.

- The role of the wavefunction is to determine the chance of a given matter distribution in the future, conditional on the matter distribution in the past. The physical collapse of the wave function can be viewed as a Bayesian updating based on new matter density information.

# Matter density



# Structure of collapse models

If no collapse between $s$ and $t$

$$|\psi_t\rangle = \hat{U}(t-s)|\psi_s\rangle$$

If collapse occurs at time $t$ (assumed to occur spontaneously)

$$|\psi_t\rangle \rightarrow |\psi_{t+}\rangle = \hat{J}(z_t)|\psi_t\rangle$$

$\hat{J}$ is the collapse operator, $z_t$ is a random variable with distribution

$$P_t(z) = \frac{\langle\psi_t|\hat{J}^2(z)|\psi_t\rangle}{\langle\psi_t|\psi_t\rangle}$$

Must have $\int dz \hat{J}^2(z) = \hat{1}$ (assumed $\hat{J} = \hat{J}^\dagger$ )

Generic scheme: equivalent to a random sequence of generalised quantum measurements

# Examples

The GRW model: Collapse events sprinkled uniformly in time

$$\hat{J}(\mathbf{z}_t) = \left(\frac{\alpha}{\pi}\right)^{3/4} \exp\left\{-\frac{\alpha}{2}(\hat{\mathbf{x}} - \mathbf{z}_t)^2\right\}$$

The CSL model: Collapse events sprinkled uniformly in spacetime

$$\hat{J}(z_x) = \left(\frac{\beta}{\pi}\right)^{1/4} \exp\left\{-\frac{\beta}{2}\left[\hat{N}(\mathbf{x}) - z_x\right]^2\right\}$$

$$x = (\mathbf{x}, t)$$

# Collapse as inference

Define discrete noisy information $$z_t = A(t) + B_t$$

underlying c-number variable

Gaussian noise

Assume that probability distribution for $A(t) = A$ is $$P_t(A) = \frac{|\langle A|\psi_t\rangle|^2}{\langle\psi_t|\psi_t\rangle}$$

Then $$P_t(A|z_t) = \frac{P_t(z_t|A)P_t(A)}{P_t(z_t)}$$

It turns out that this can be written as $$P_t(A|z_t) = \frac{|\langle A|\psi_{t+}\rangle|^2}{\langle\psi_{t+}|\psi_{t+}\rangle}$$

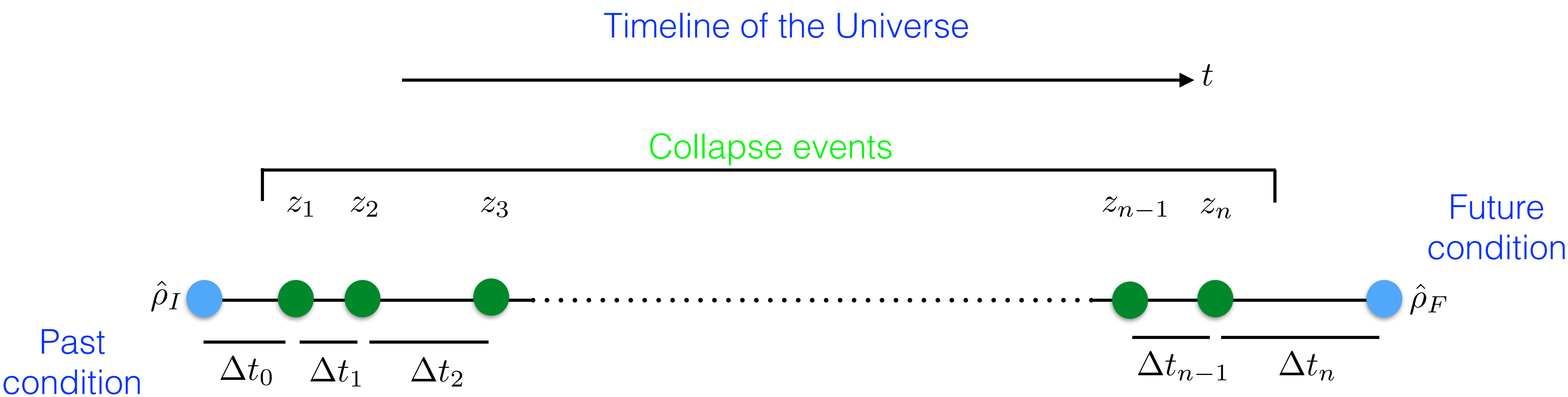
where $$|\psi_{t+}\rangle = \frac{1}{(2\pi\sigma^2)^{1/4}} \exp\left\{-\frac{1}{4\sigma^2}(\hat{A} - z_t)^2\right\} |\psi_t\rangle$$

and that $$P_t(z_t) = \frac{\langle\psi_{t+}|\psi_{t+}\rangle}{\langle\psi_t|\psi_t\rangle}$$

# Collapse as inference

- Collapse process is mathematically equivalent to a Bayesian updating on basis of noisy information
- The role of the wavefunction is in determining the probability of the variable $A$ along with its dynamics

# Time symmetry




Now if there exists a basis $\{|\phi_i\rangle\}$ such that

$$\langle\phi_i|J(z)|\phi_j\rangle^* = \langle\phi_i|J(z)|\phi_j\rangle$$
$$\langle\phi_i|U(t)|\phi_j\rangle^* = \langle\phi_i|U(-t)|\phi_j\rangle$$

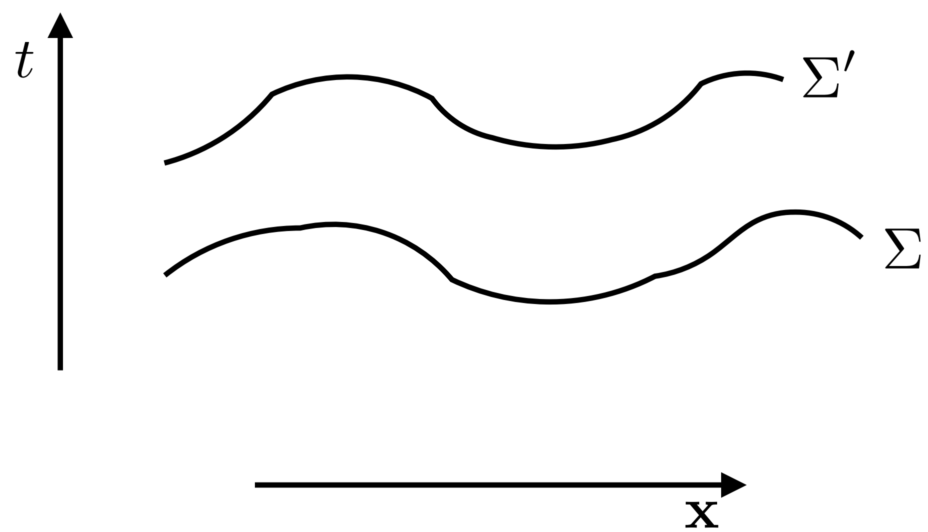
then

$$P(\{z_1, z_2, \ldots, z_n\}|\{\Delta t_0, \Delta t_1, \ldots, \Delta t_n\}; \hat{\rho}_I; \hat{\rho}_F) = P(\{z_n, \ldots, z_2, z_1\}|\{\Delta t_n, \ldots, \Delta t_1, \Delta t_0\}; \hat{\rho}_F^*; \hat{\rho}_I^*)$$

# Time symmetry

- This means that the collapse dynamics can be applied in either time direction and we arrive at the same probability for a given complete set of collapse outcomes
- At the level of the collapse outcomes there is structural time symmetry
- In terms of the wavefunction the description is time asymmetric (this is because the wavefunction is a means of conditioning on past collapse outcomes)

# Relativity

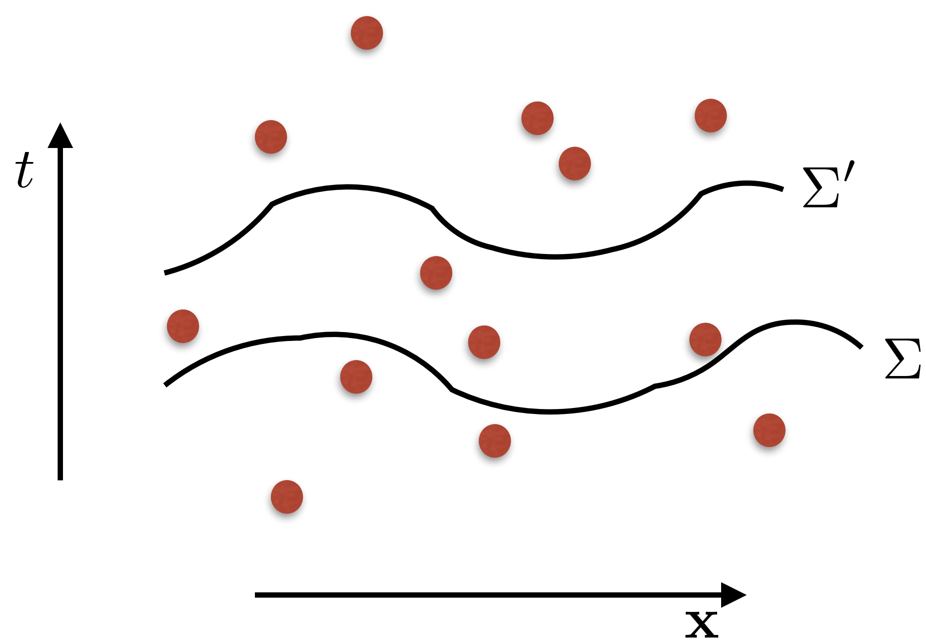



Interaction picture:

$$|\Psi_{\Sigma'}\rangle = \hat{U}(\Sigma', \Sigma)|\Psi_{\Sigma}\rangle$$

$$\hat{U}(\Sigma', \Sigma) = T \exp\left\{ -i \int_{\Sigma}^{\Sigma'} dx \hat{\mathcal{H}}_{\text{int}}(x) \right\}$$

# Relativity




Interaction picture:

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$$\hat{U}(\Sigma', \Sigma) = T\exp\left\{-i\int_{\Sigma}^{\Sigma'} dx\hat{\mathcal{H}}_{\rm int}(x)\right\}$$

Include collapse events:

$$|\Psi_\Sigma\rangle \to |\Psi_{\Sigma+}\rangle = \hat{J}_x(z_x)|\Psi_\Sigma\rangle$$

$$P(z_x) = \frac{\langle\Psi_{\Sigma+}|\Psi_{\Sigma+}\rangle}{\langle\Psi_\Sigma|\Psi_\Sigma\rangle}$$

$$\int dz\hat{J}_x^2(z) = \hat{1}$$

# Relativity

Given that these constraints are satisfied for space-like separated $x$ and $y$:

$$\left[\hat{\mathcal{H}}_{\text{int}}(x), \hat{\mathcal{H}}_{\text{int}}(y)\right] = 0$$

$$\left[\hat{J}_x(z_x), \hat{J}_y(z_y)\right] = 0$$

$$\left[\hat{J}_x(z_x), \hat{\mathcal{H}}_{\text{int}}(y)\right] = 0$$

then

A. for a given set of collapse outcomes $\{z_x\}$ for all collapse events at locations $\{x|\Sigma \prec x \prec \Sigma'\}$, the state $|\Psi_{\Sigma'}\rangle$ is unambiguously determined from $|\Psi_{\Sigma}\rangle$ independently of foliation used to get from $\Sigma$ to $\Sigma'$.
B. the probability of the set of collapse outcomes $\{z_x\}$ is independent of foliation used to get from $\Sigma$ to $\Sigma'$.

Furthermore, provided that $\hat{\mathcal{H}}_{\text{int}}$ and $\hat{J}_x$ are scalar operators then the probability is Lorentz invariant.

Again it makes more sense to treat the collapse outcomes as primitive and the quantum state as derivative.

- Assuming that collapse outcomes provide an empirically adequate picture, this results in Lorentz invariance at the phenomenal level in which underlying processes do not use a preferred frame.

- Contrast with a state history which is by necessity foliation dependent. But this is a natural consequence of its interpretation as a state of information about past collapses and the need to demarcate a past-future boundary for this purpose.

# Example

Complex scalar field

$$\hat{J}_x(z_x) = \left(\frac{\beta}{\pi}\right)^{1/4} \exp\left\{-\frac{\beta}{2}(|\hat{\phi}(x)|^2 - z_x)^2\right\}$$

This can be shown to reduce to the CSL model (without smearing of particle number density) in the non relativistic limit.

The lack of smearing over the field operator is a problem since it leads to infinite energy density increase.
Possible resolution: regulation by discrete spacetime—this would provide an effective smearing.

# Gravity

How to make sense of

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

quantum matter
stress energy tensor

Use the distribution of matter described by the collapse outcomes as the source of gravity.
[For Newtonian version see Tilloy & Diosi, PRD93 024026 (2015)]

This idea undermines some no-go theorems of semiclassical gravity.
For example:

- Eppley & Hannah I :  faster than light signalling

- Eppley & Hannah II:  uncertainty principle violation

But Einstein equation with collapsing quantum matter is inconsistent since

$$\nabla^{\mu} G_{\mu\nu} = 0$$

$$\nabla^{\mu} T_{\mu\nu} \neq 0$$

Possible resolution: discrete spacetime geometry

# Conclusions

- Collapse models describe definite-valued matter distributed in space and time.
- The wave function can be understood as representing a probability distribution for how matter in the future is distributed given the distribution of matter in the past.
- Collapse can be understood as Bayesian inference.
- Distribution of matter density is structurally time symmetric.
- Probability of a given distribution of matter is Lorentz invariant and independent of spacetime foliation.
- Hints at discrete spacetime structure (to control energy increases and enable consistent semiclassical gravity).