The Zeno Effect in Quantum Mechanics

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Time Evolution Operator

- Classical system decays exponentially…
- Quantum Systems:

Schrödinger equation gives us the time evolution operator (unitary) which tells us the dynamics of any quantum mechanical system

For the intermediate time, quantum system decays exponentially

For very short or very long time, the decay is NOT exponential

In the short time regime, some very interesting results can be found.

Here we will be dealing only with the short time regime

Suppose **H** is the Hamiltonian of a given quantum system which is in the state $|\phi\rangle$

H : Time independent

Time evolution of $|\phi\rangle$ is given by

$$U(t)|\phi\rangle = e^{-iHt}|\phi\rangle$$

$\hbar$ = 1 unit (numerically)

The survival probability is,

$$p(t) = \left|\langle \phi | U(t) | \phi \rangle\right|^2$$

which is actually,

$$p(t) = \langle \phi | e^{iHt} | \phi \rangle \langle \phi | e^{-iHt} | \phi \rangle$$

By expanding further we get, (for small times)

$$p(t)_{short} \approx 1 - (\Delta H)^2 t^2$$

where,

$$\Delta H = \sqrt{\langle \phi | H^2 | \phi \rangle - \langle \phi | H | \phi \rangle^2}$$

So the survival probability has a quadratic time dependence, for small time,

$$1-\left(\frac{(\Delta H)^2 t^2}{\hbar^2}\right)$$

Now, if the measurement is performed n times, at regular intervals of t/n, over a time span t then the survival probability will become

$$\left(1-\frac{(\Delta H)^2 (t/n)^2}{\hbar^2}\right)^n$$

for the limit $n \rightarrow \infty$

$$\left(1-\frac{(\Delta H)^2 (t/n)^2}{\hbar^2}\right)^n \rightarrow 1$$

Under continual "observation" the initial state has not decayed at all !

This is the Quantum Zeno Effect!

Why the name ‘Zeno’ ???

Zeno was the Greek philosopher and famous for his ”Arrow paradox”

At every moment of time, a moving arrow occupies a definitive position so at every moment, the arrow is at rest

(Same as “not decaying” of a system)

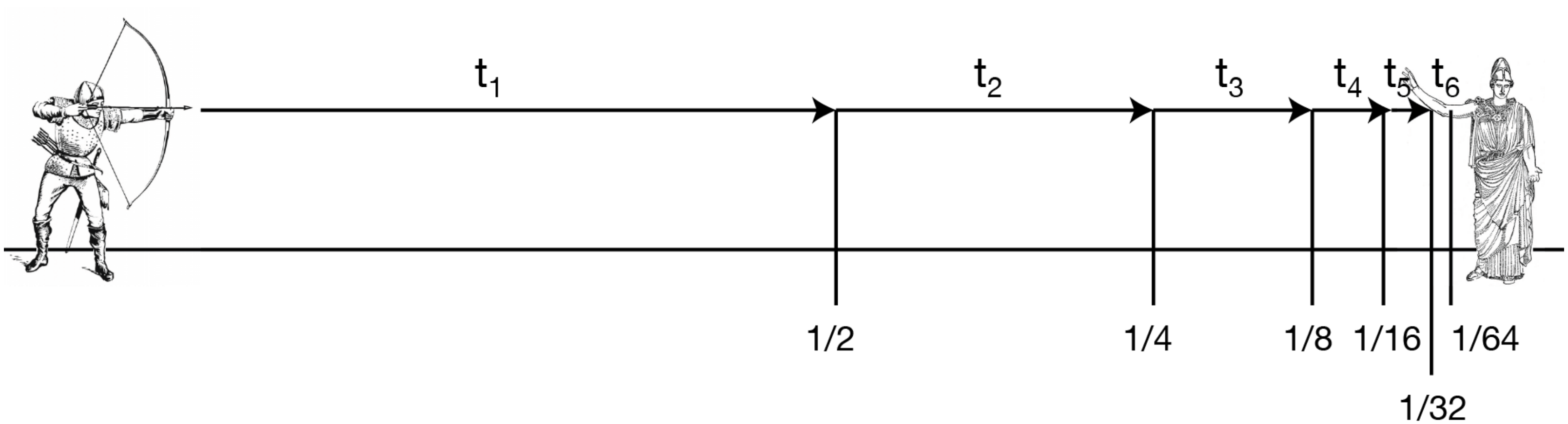


Image source: wikipedia

Turing paradox (1954)

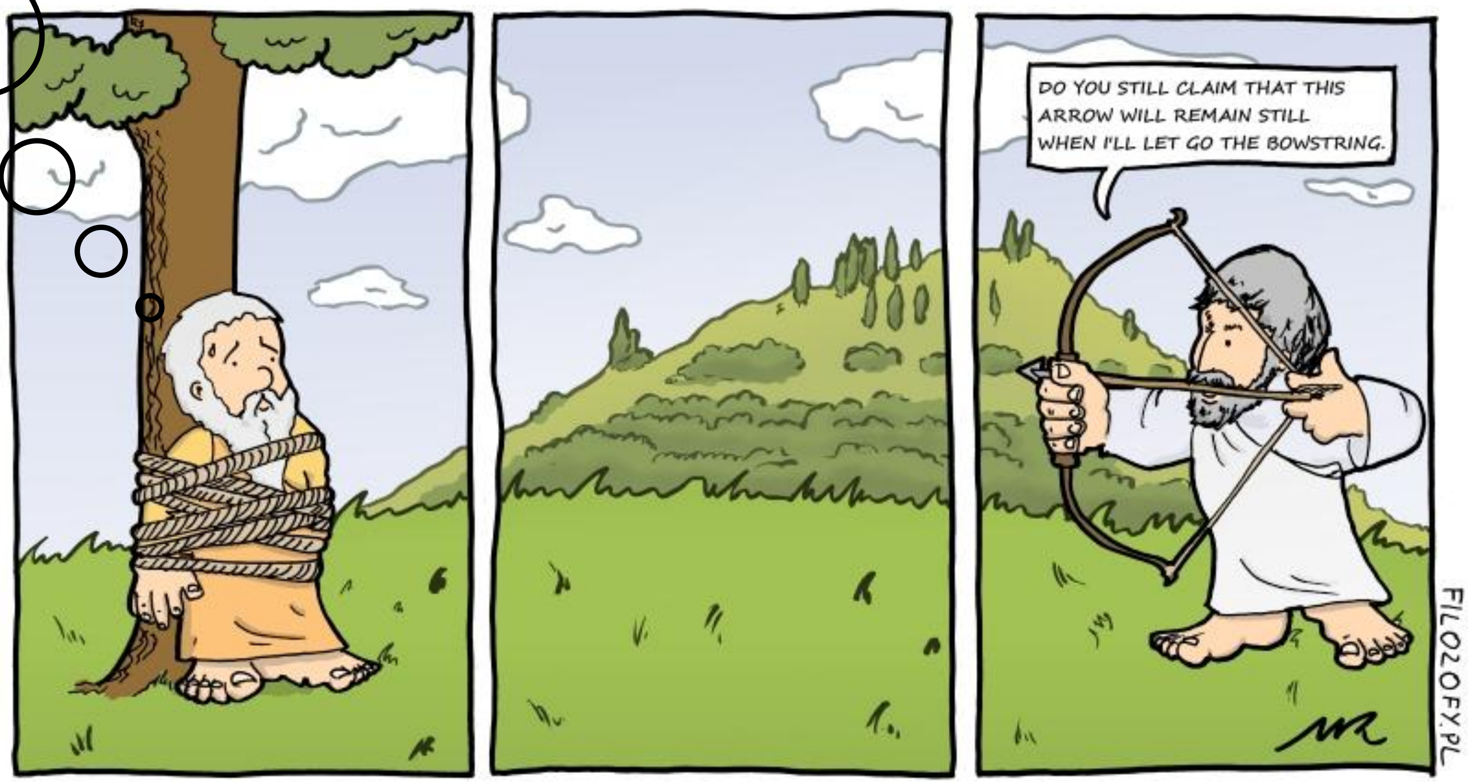
It is easy to show using standard theory that if a system starts in an eigenstate of some observable, and measurements are made of that observable *N* times a second, then, even if the state is not a stationary one, the probability that the system will be in the same state after, say, one second, tends to one as *N* tends to infinity; that is, that continual observations will prevent motion …

— *Alan Turing as quoted by A. Hodges in 'Alan Turing: Life and Legacy of a Great Thinker' p. 54*

Also famous as "watched pot phenomenon".

i.e watching pot never boils!

Whether all motion is just illusory??



http://filozofy.blox.pl/resource/93_Strzala_blog_e.jpg

Quantum Zeno Effect

B. Misra and E. C. G. Sudarshan were the first to provide a rigorous mathematical definition and introduced the notion of "quantum Zeno paradox".

"The Zeno's paradox in quantum theory", Journal of Mathematical Physics, Vol. 18, No. 4, pp. 756-763

Example:

We may describe the polarized photons as a system with two states,

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{“original”}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{“decayed”}$$

The effective Hamiltonian H is,

$$H = \begin{pmatrix} 0 & -i\omega \\ i\omega & 0 \end{pmatrix} \quad ; \quad H \begin{pmatrix} 1 \\ 0 \end{pmatrix} = i\omega \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H = \omega\sigma^{(2)} \quad ; \quad HH = \omega^2 \mathbf{I}$$

Time evolution operator will be:

$$e^{-itH} = \mathbf{I} - itH + (1/2!)(-it)^2 HH + (1/3!)(-it)^3 HHH + \ldots \text{upto } \infty$$

$$= \begin{pmatrix} \{1-((\omega t)^2/2!)+((\omega t)^4/4!)+\ldots\} & -\{\omega t-((\omega t)3/3!)+\ldots\ldots\} \\ \{\omega\, t-((\omega t)^3/3!)+\ldots\ldots\ldots\} & \{1-((\omega t)^2/2!)+((\omega t)^4/4!)+\ldots\ldots\} \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{pmatrix}$$

, a unitary matrix

The time-evolved state is

$$\Psi = e^{-itH} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \end{pmatrix}$$

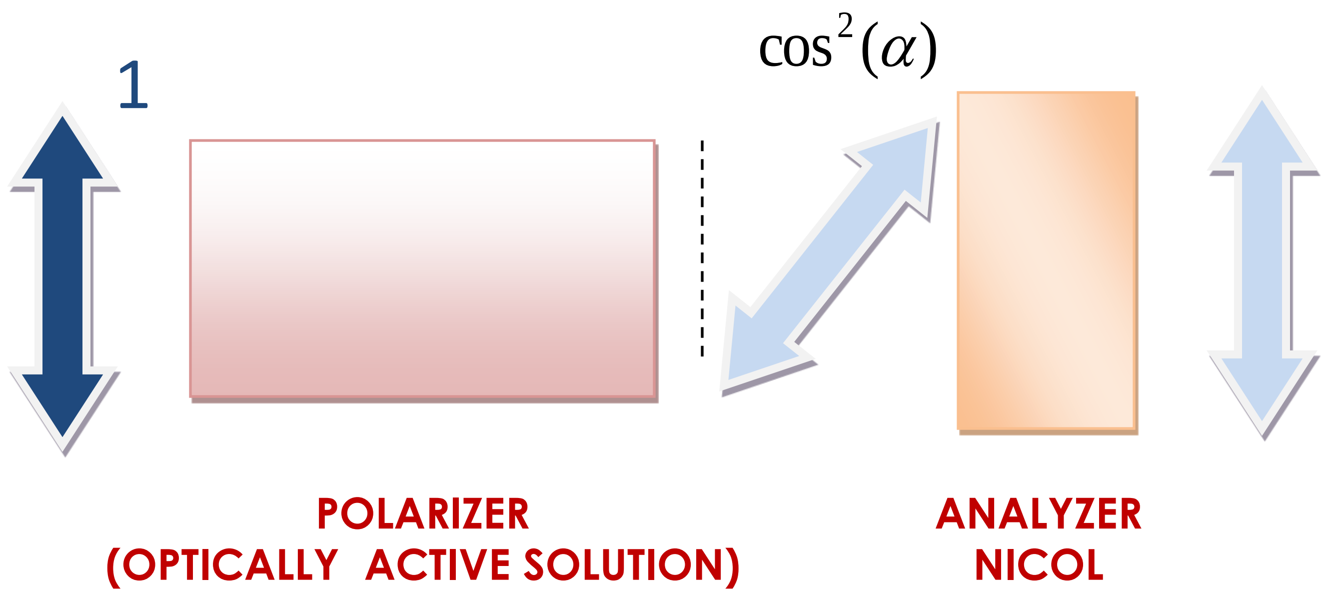
a_0

$$= \cos(\omega t) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin(\omega t) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

For small times t, we have quadratic decay law

$$|a_0|^2 = \cos^2(\omega t) \sim 1 - \omega^2 t^2$$

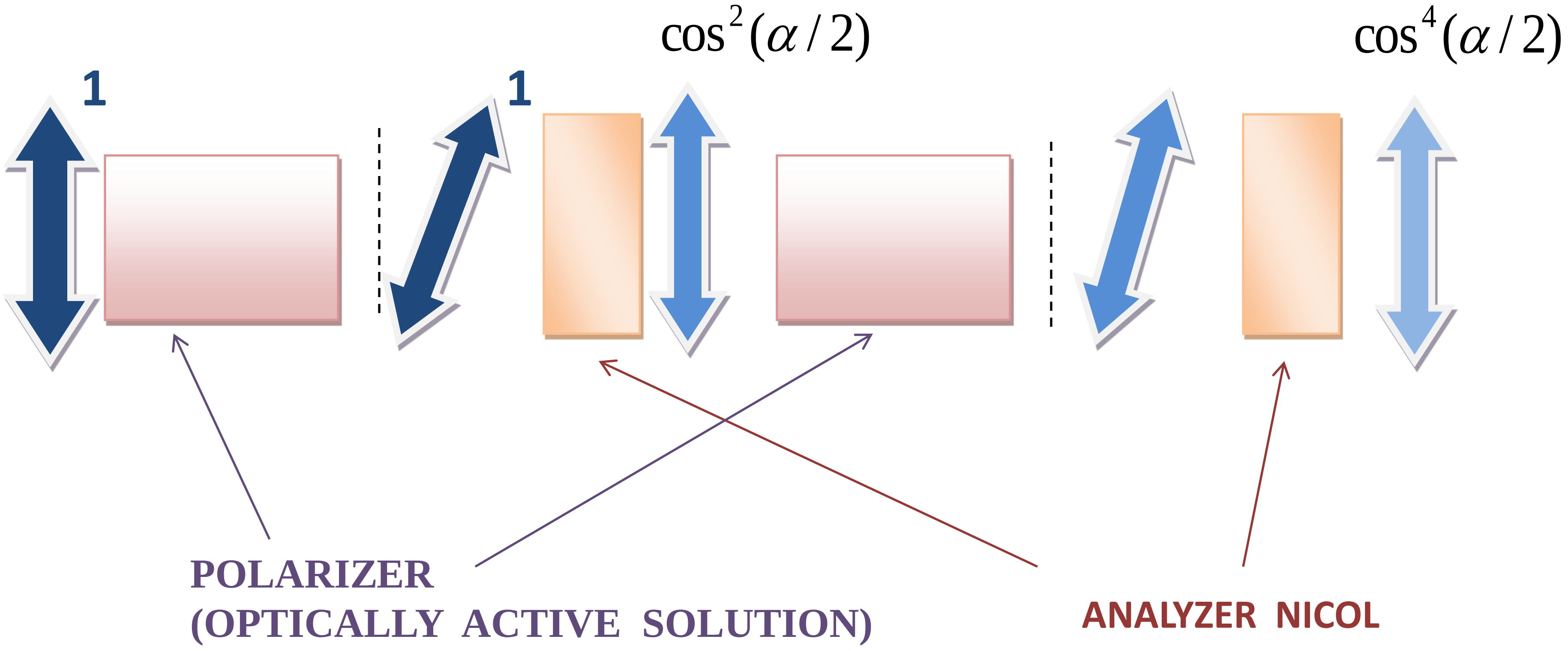
Example:



I "Project back" to the original polarization state

INTENSITY,

$$I = I_0 \cos^2(\alpha)$$


1
1
$\cos^2(\alpha/2)$
$\cos^4(\alpha/2)$
POLARIZER
(OPTICALLY ACTIVE SOLUTION)
ANALYZER NICOL

For small a,

$$\cos^2(\alpha) \sim (1-(\alpha^2/2))^2 \sim 1-\alpha^2$$

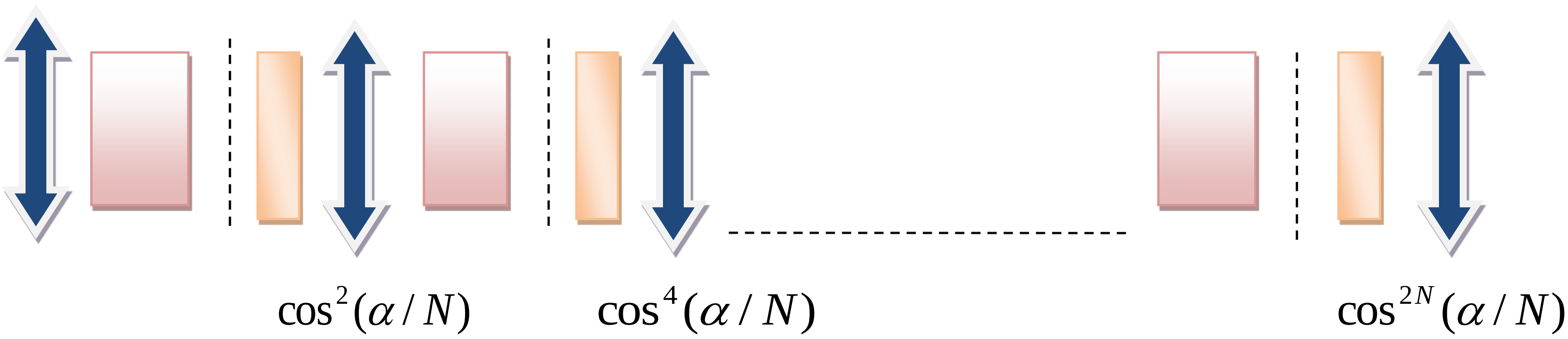
and

$$\cos^4(\alpha/2) \sim (1-(\alpha^2/8))^4 \sim 1-(\alpha^2/2)$$

Hence,

$$\cos^4(\alpha/2) > \cos^2(\alpha)$$

… Evolution slackened!



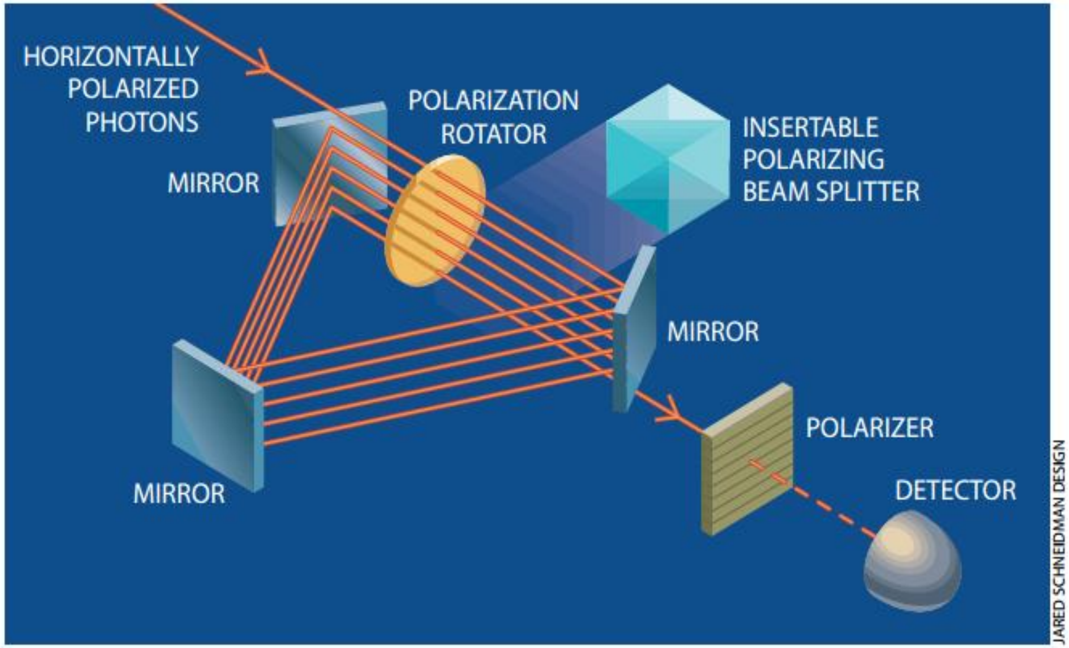
"Observation" (measurement of polarization)
is carried out continuously ($N \to \infty$)

$$\cos^{2N}\left(\frac{\alpha}{N}\right) = \left(1 - \left(\frac{1}{2}\right)\left(\frac{\alpha}{N}\right)^2 + O\left(\frac{1}{N^4}\right)\right)^{2N}$$

$$\cos^{2N}\left(\frac{\alpha}{N}\right) \rightarrow 1$$ **!!** $N \longrightarrow \infty$

ALL THE LIGHT, WITH ***CERTAINTY*** PASSES THROUGH, UNALTERED IN INTENSITY

.................QZE!!



Quantum Seeing in the Dark

Quantum optics demonstrates the existence of interaction-free measurements: the detection of objects without light—or anything else—ever hitting them

by Paul Kwiat, Harald Weinfurter and Anton Zeilinger

Probability for N stages can be given as, $(\cos^2(\Pi/2N))^N$

"EXPERIMENTAL REALIZATION

of the quantum Zeno effect was accomplished by making the photon follow a spiral-staircase path, so that it traversed the polarization rotator six times. Inserting a polarizer next to the rotator suppressed the rotation of the photon's polarization."

No. of stages	Probability
6	~ 66 %
20	~ 90 %
2500	~ 99.9 %
∞	100 % (Exactly!)

Kwiat et al., Scientific American, November 1996

Applications

PRL **115,** 140402 (2015) PHYSICAL REVIEW LETTERS week ending 2 OCTOBER 2015

Measurement-Induced Localization of an Ultracold Lattice Gas

Y. S. Patil, S. Chakram, and M. Vengalattore*
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(Received 23 April 2015; published 2 October 2015)

The process of measurement can modify the state of a quantum system and its subsequent evolution. Here, we demonstrate the control of quantum tunneling in an ultracold lattice gas by the measurement backaction imposed by the act of imaging the atoms, i.e., light scattering. By varying the rate of light scattering from the atomic ensemble, we show the crossover from the weak measurement regime, where position measurements have little influence on tunneling dynamics, to the strong measurement regime, where measurement-induced localization causes a large suppression of tunneling—a manifestation of the quantum Zeno effect. Our study realizes an experimental demonstration of the paradigmatic Heisenberg microscope and sheds light on the implications of measurement on the coherent evolution of a quantum system.

DOI: 10.1103/PhysRevLett.115.140402 PACS numbers: 03.65.Xp, 03.65.Ta, 03.67.-a, 37.10.Jk

PRL **100,** 090503 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Protecting Entanglement via the Quantum Zeno Effect

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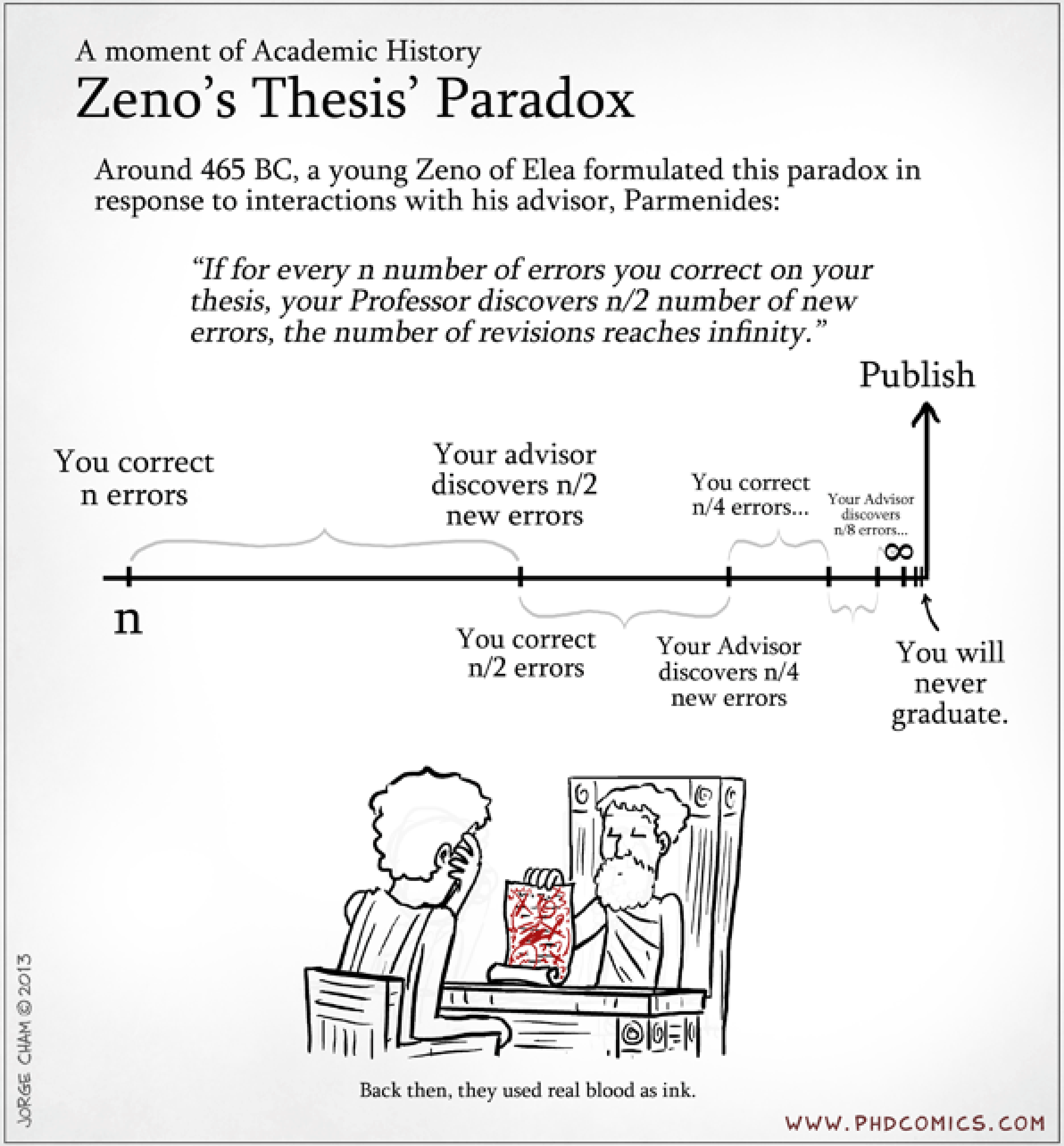
(Received 20 October 2007; published 7 March 2008)

We study the exact entanglement dynamics of two atoms in a lossy resonator. Besides discussing the steady-state entanglement, we show that in the strong coupling regime the system-reservoir correlations induce entanglement revivals and oscillations and propose a strategy to fight against the deterioration of the entanglement using the quantum Zeno effect.

DOI: 10.1103/PhysRevLett.100.090503 PACS numbers: 03.67.Mn, 03.65.Ta, 03.65.Ud, 03.65.Yz


A moment of Academic History
Zeno's Thesis' Paradox
Around 465 BC, a young Zeno of Elea formulated this paradox in response to interactions with his advisor, Parmenides:
"If for every n number of errors you correct on your thesis, your Professor discovers n/2 number of new errors, the number of revisions reaches infinity."
Publish
You correct n errors
Your advisor discovers n/2 new errors
You correct n/4 errors...
Your Advisor discovers n/8 errors...
∞
n
You correct n/2 errors
Your Advisor discovers n/4 new errors
You will never graduate.
Back then, they used real blood as ink.
JORGE CHAM © 2013
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Department of Physics,
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Thank you!