Fluids made of light

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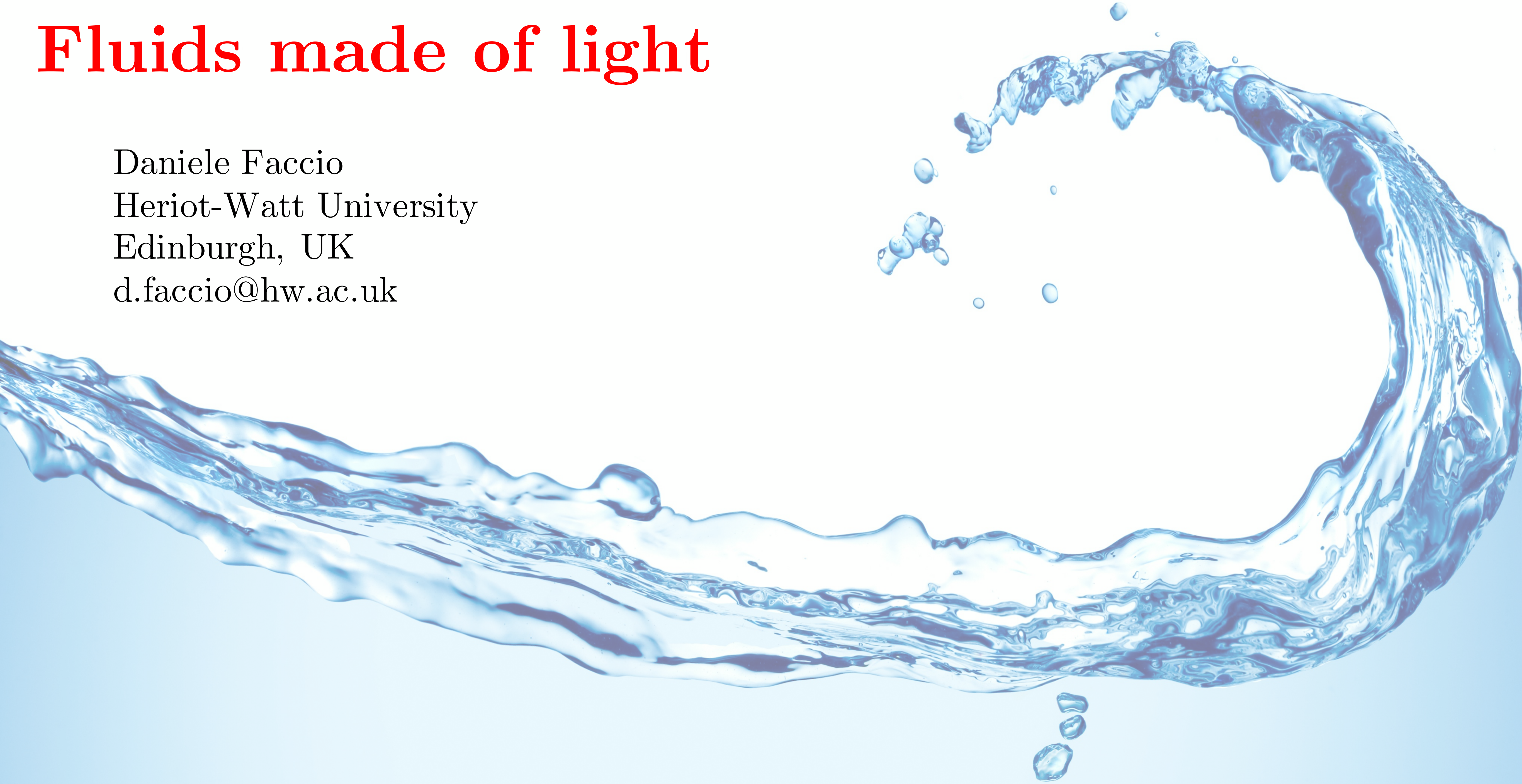


Propagating photon fluids

- Laser propagating through defocusing NL Medium

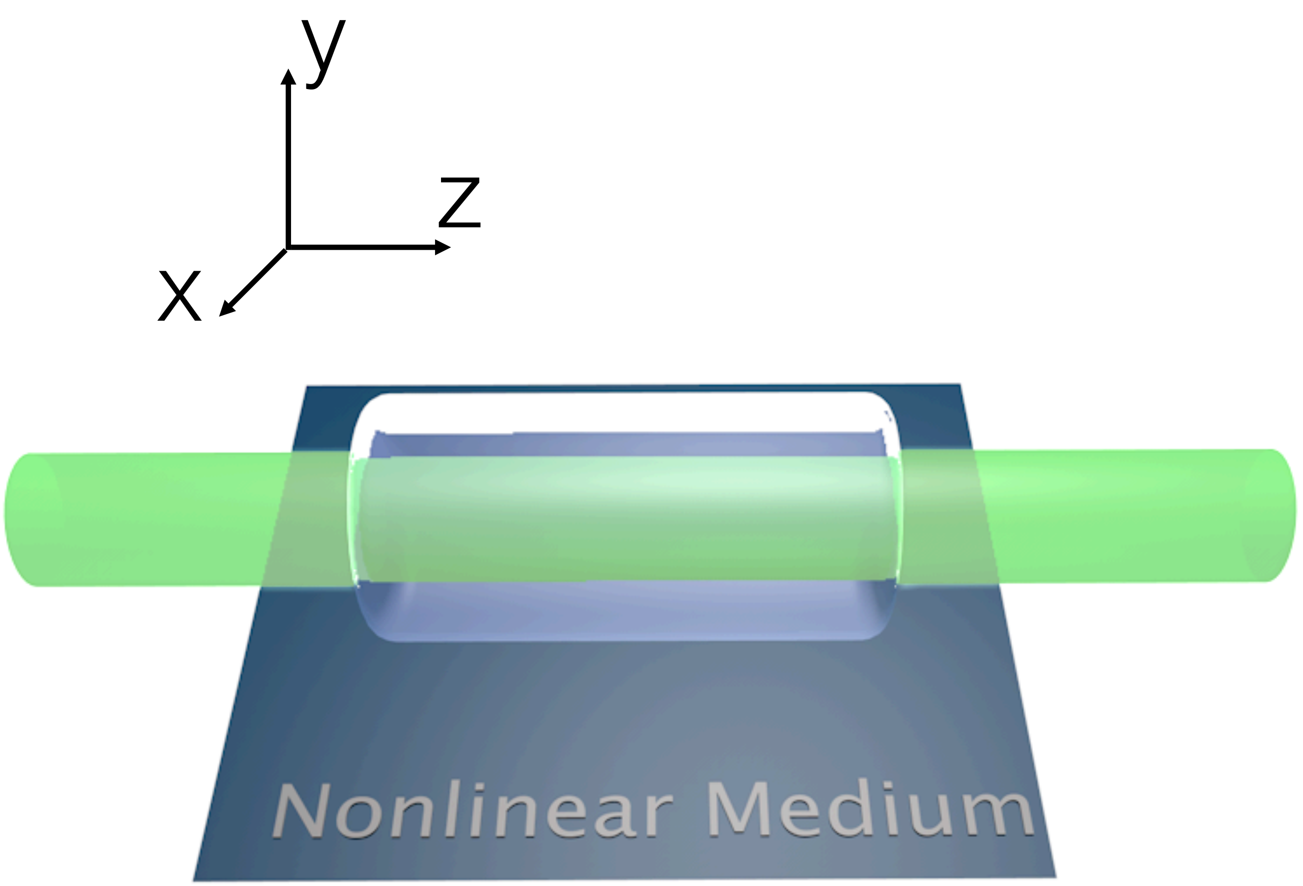
$$\partial_z E = \frac{i}{2k}\nabla^2 E - i\frac{kn_2}{n_0}|E|^2 E$$

- Transformation:

$$E = \sqrt{\rho}e^{i\phi}$$
$$v = (c/kn_0)\nabla\phi = \nabla\psi$$
$$t = z/n_0 c$$

- Hydrodynamical equations:

$$\partial_t \rho + \nabla(\rho v) = 0$$
$$\partial_t \psi + \frac{1}{2}v^2 + \frac{c^2 n_2}{n_0^3}\rho = 0$$



Pomeau, Rica, C.R. Acad. Sci. Paris (1993)
Chiao et al., PRA (1999)
Fleischer et al., Nat. Phys. (2007,2012), NJP (2012)
Picozzi et al. PRL (2005)
Review – Carusotto, Ciuti (2012)

Propagating photon fluids

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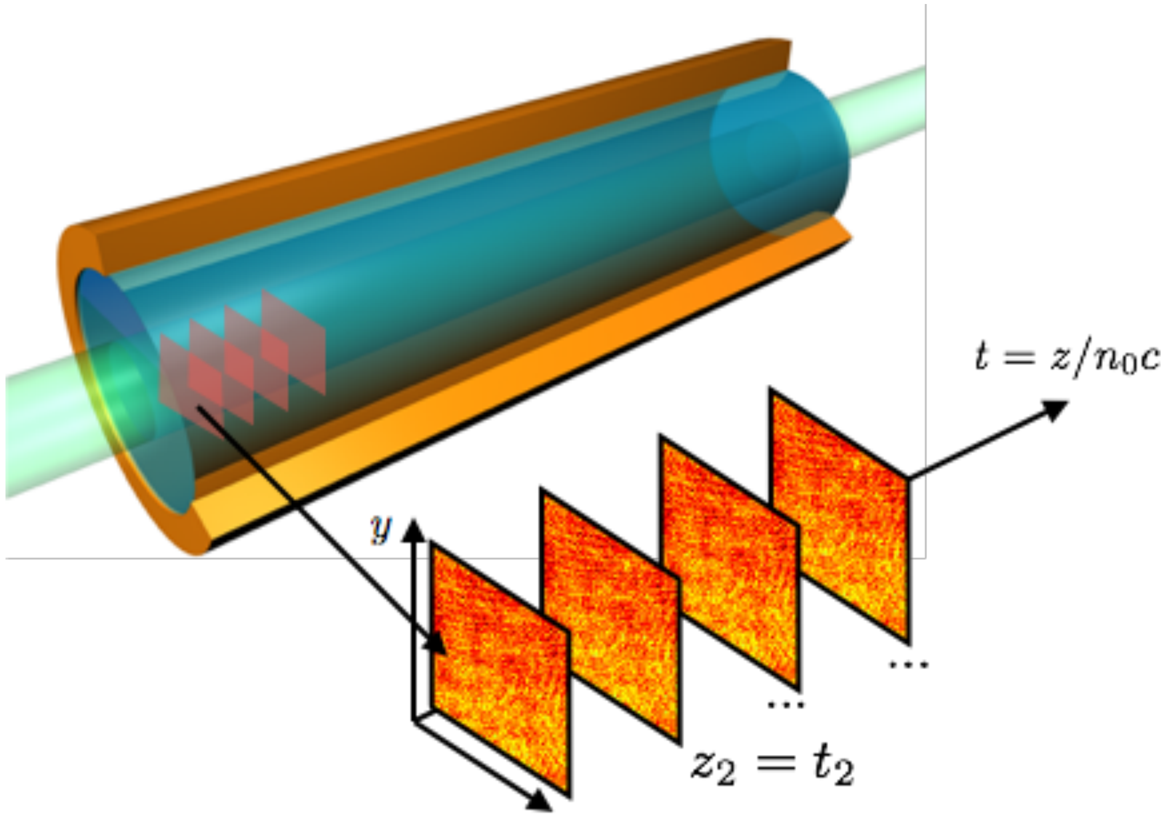


speed of sound

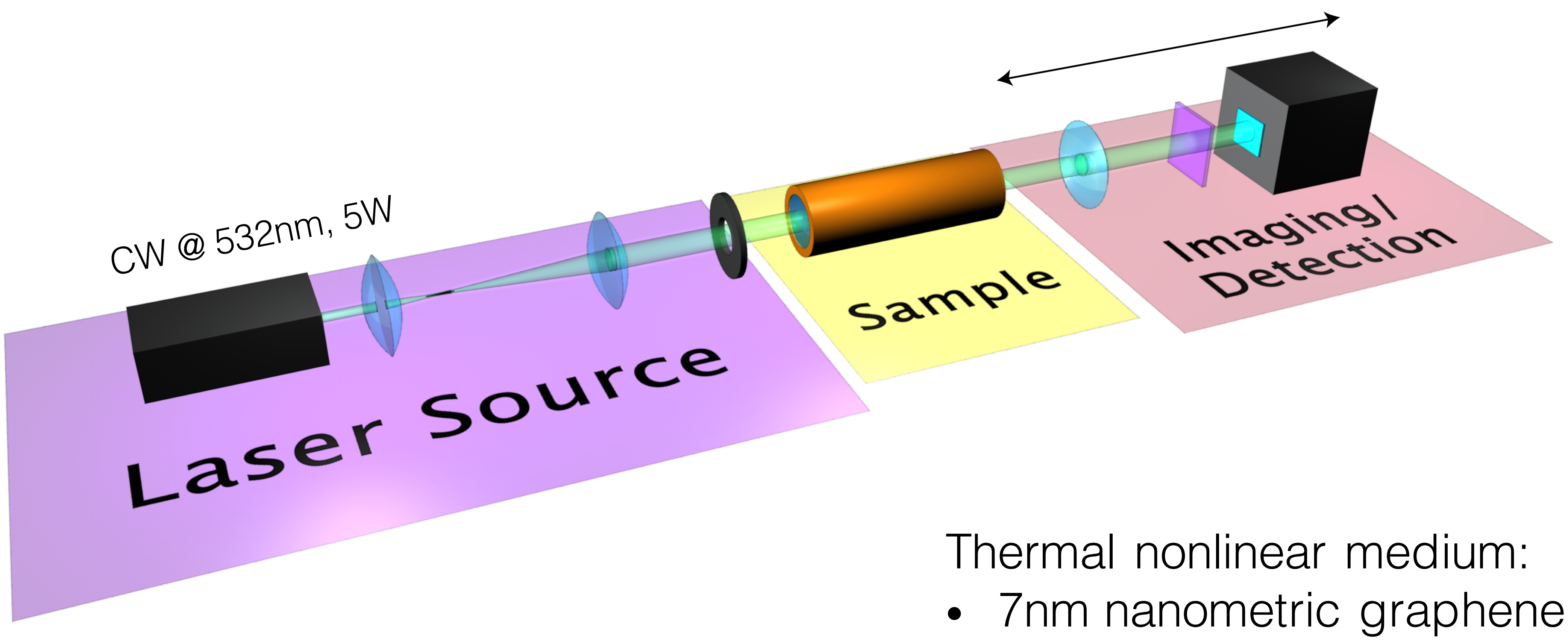
$$c_s \propto \sqrt{\Delta n}$$

Flow

$$v \propto \nabla\phi$$

Propagating photon fluids

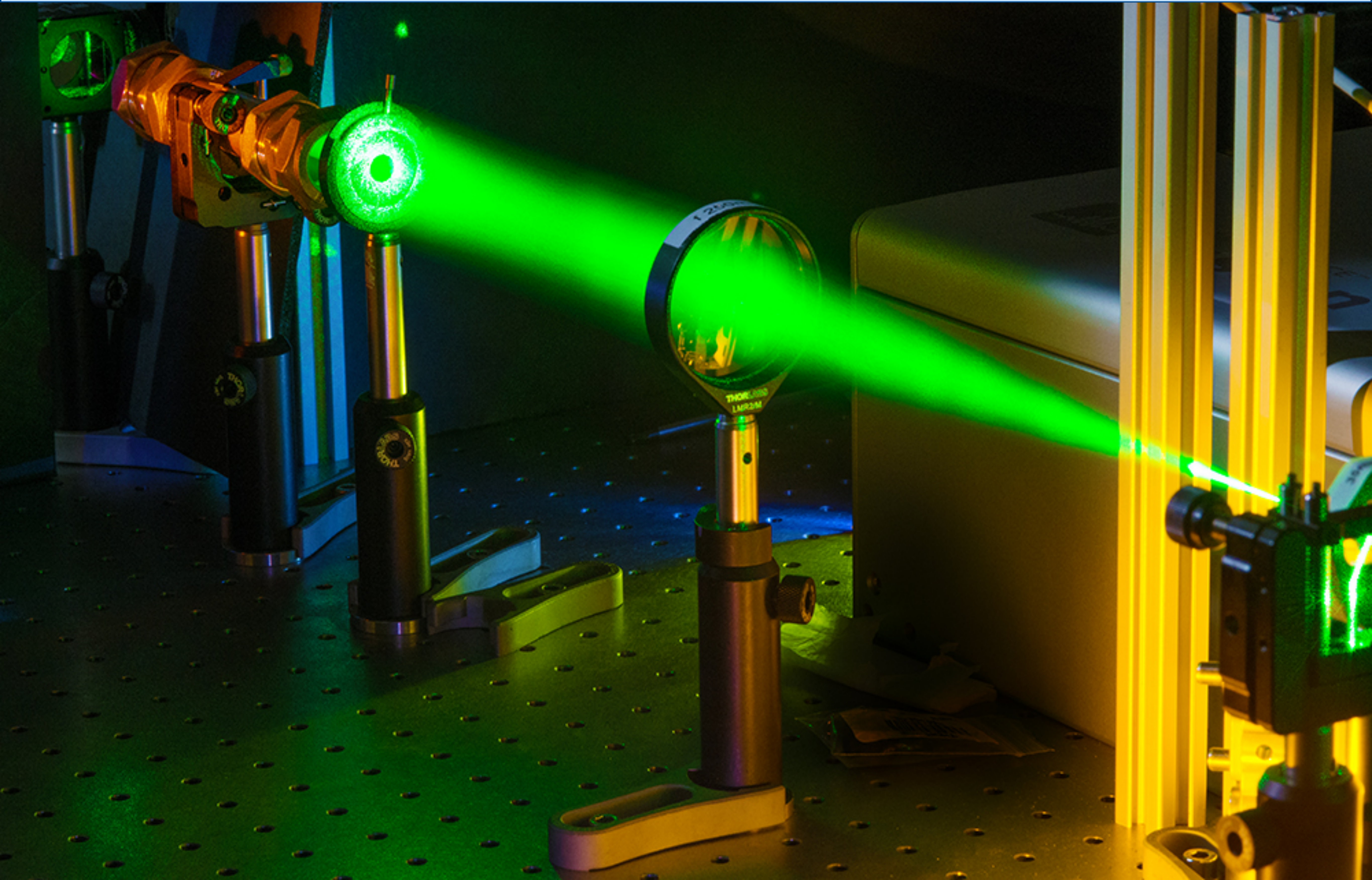
Experiments: example of layout



Thermal nonlinear medium:

- 7nm nanometric graphene flakes dissolved in Methanol
- Graphene provides efficient conversion of absorbed laser power into heat
- Methanol: $dn/dT = -4 \times 10^{-4} 1/K$

Experiments: example of layout

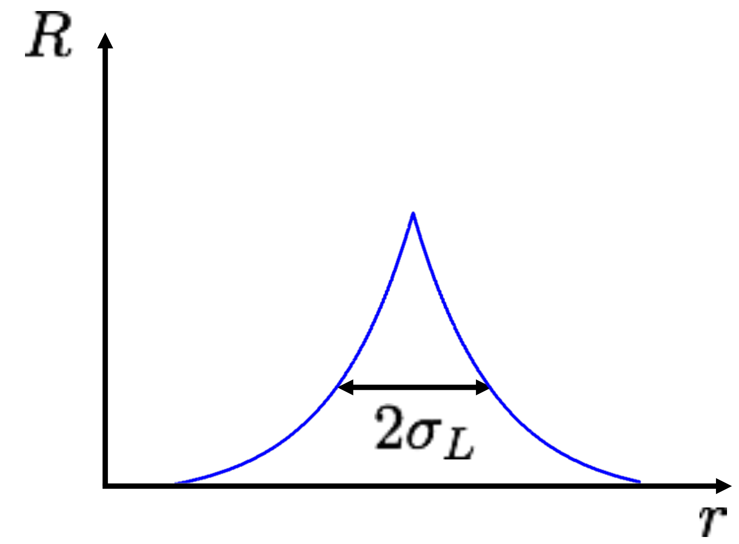


Nonlocal interaction

- Local nonlinearity $\Delta n = -n_2|E|^2 = -n_2 I$

- Thermal nonlinear media are highly nonlocal through heat transport through the medium

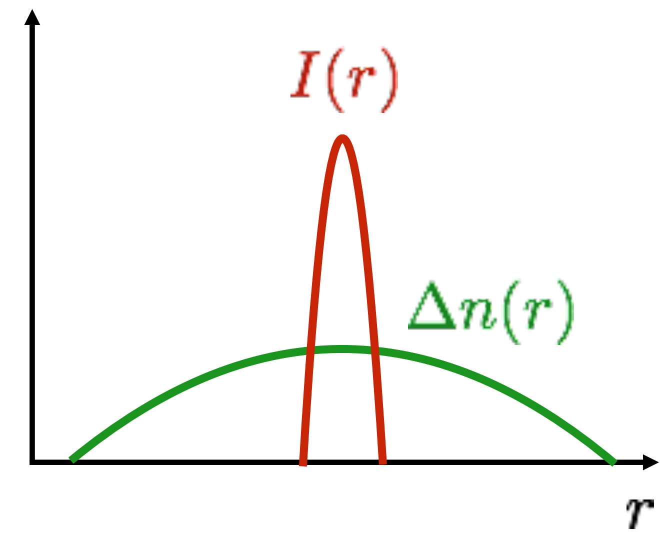
$$\Delta n(r) = \int \gamma R(r - r')I(r')dr'$$



- Response function

$$R(r) = 1/(2\sigma_L)e^{-\sqrt{r^2}/\sigma_L}$$

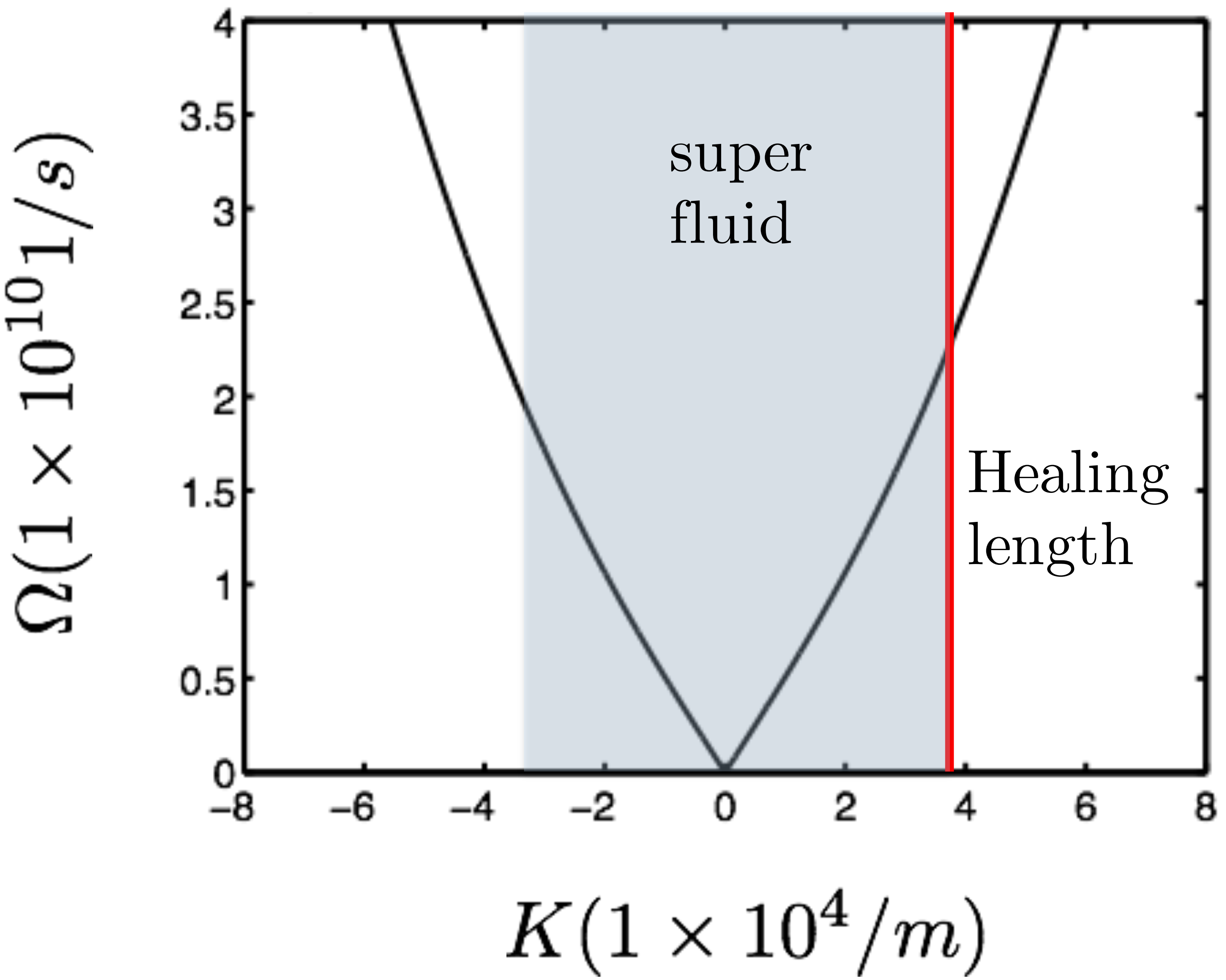
- Nonlocality results in a smoothing of the local nonlinearity



Bogoliubov dispersion

Hydrodynamical equations can be solved for small amplitude excitations:

$$(\Omega - vK)^2 = \frac{c^2 \Delta n}{n_0^3} \hat{R}(K) K^2 + \frac{c^2}{4k^2 n_0^2} K^4$$



$\Delta n > 0, \sigma = 0$

Bogoliubov dispersion

Hydrodynamical equations can be solved for small amplitude excitations:

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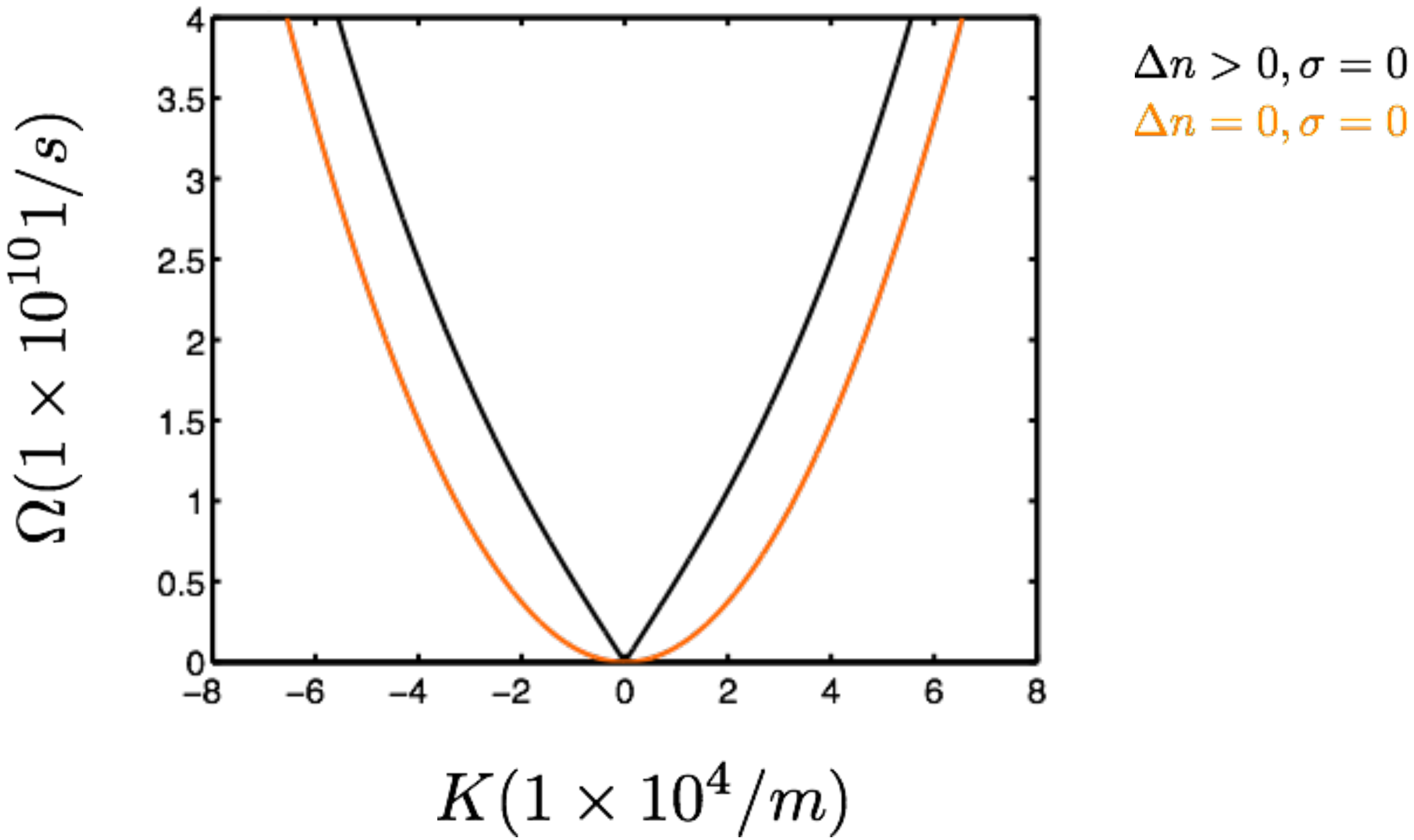

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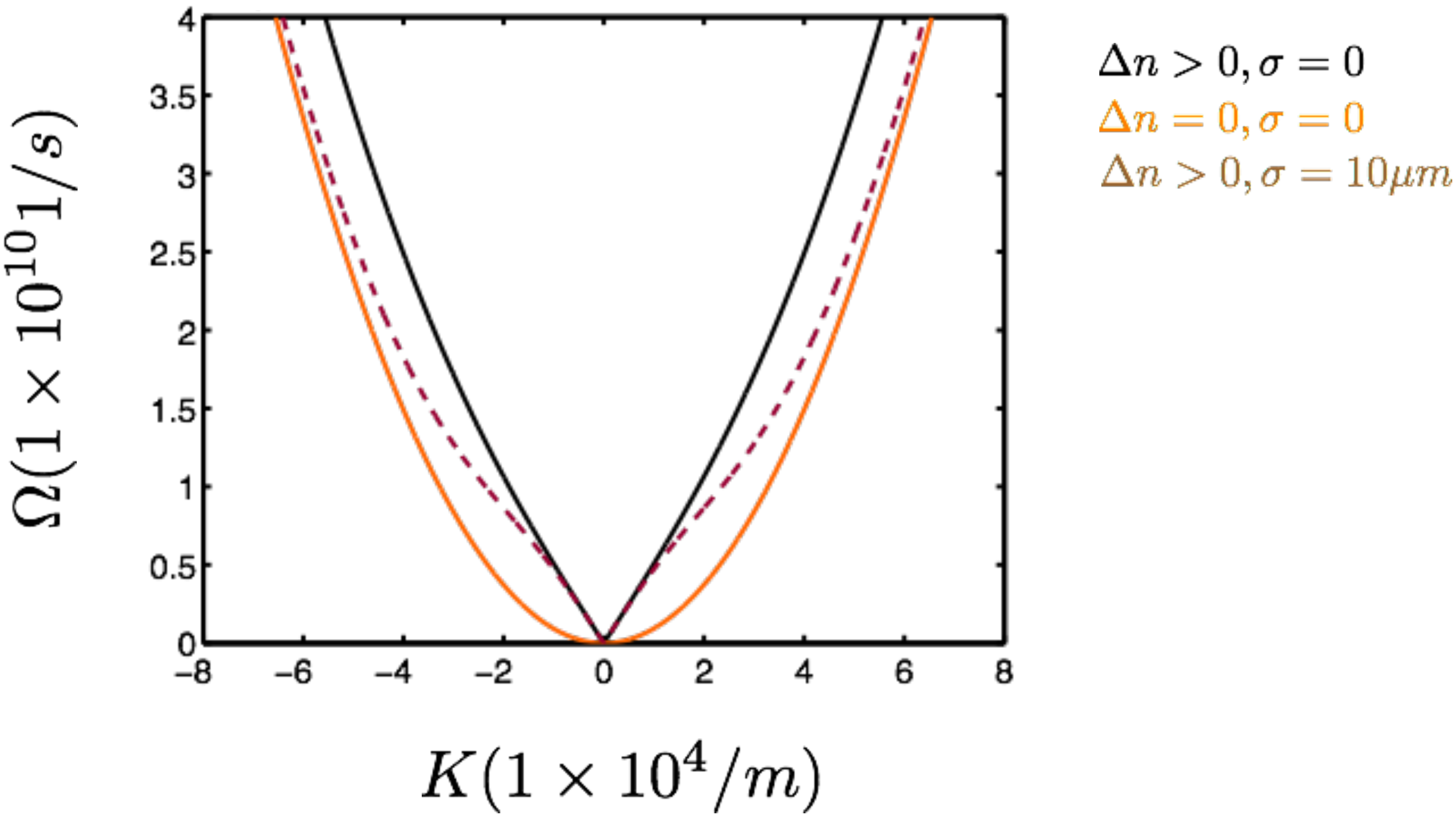

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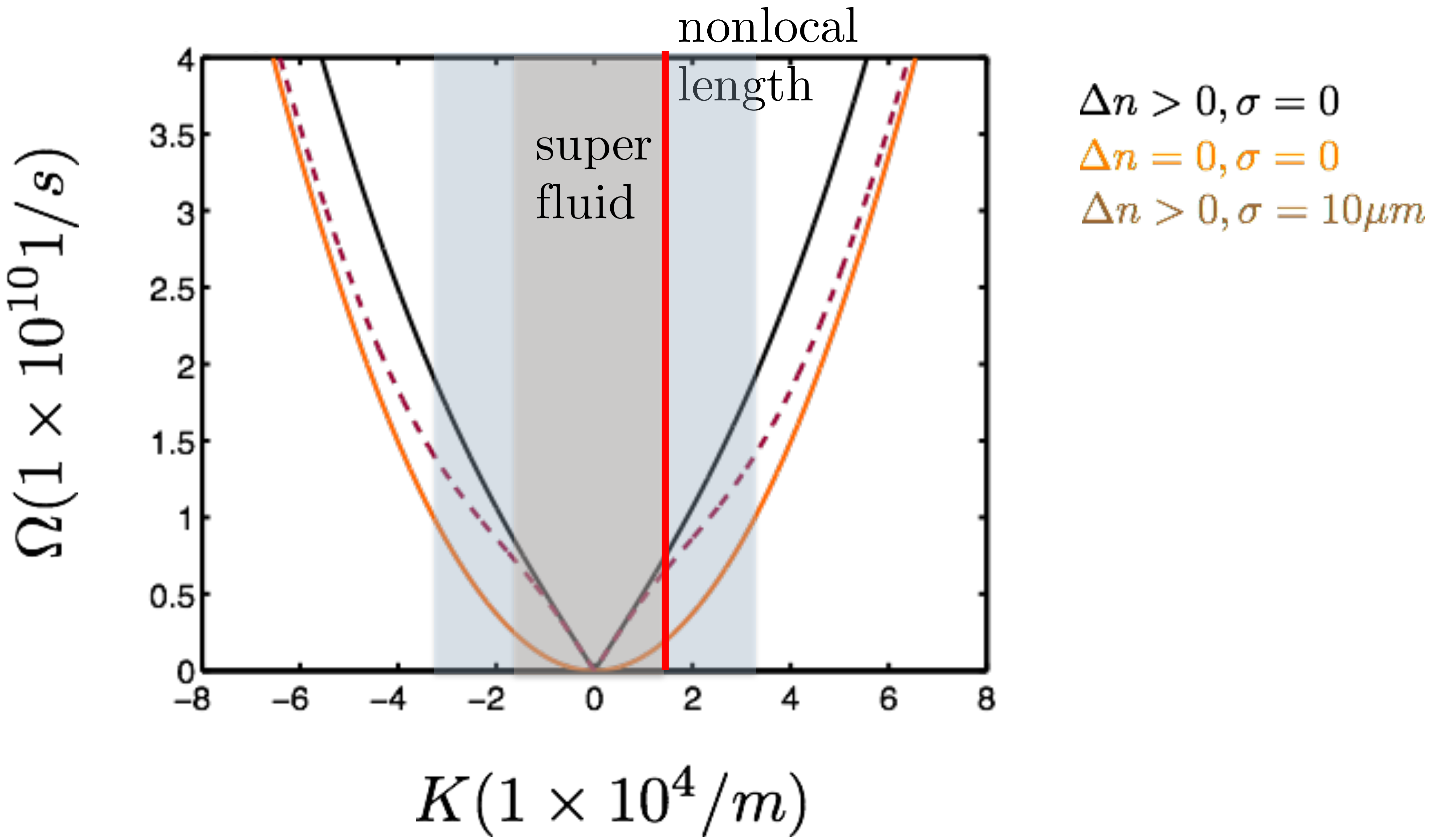
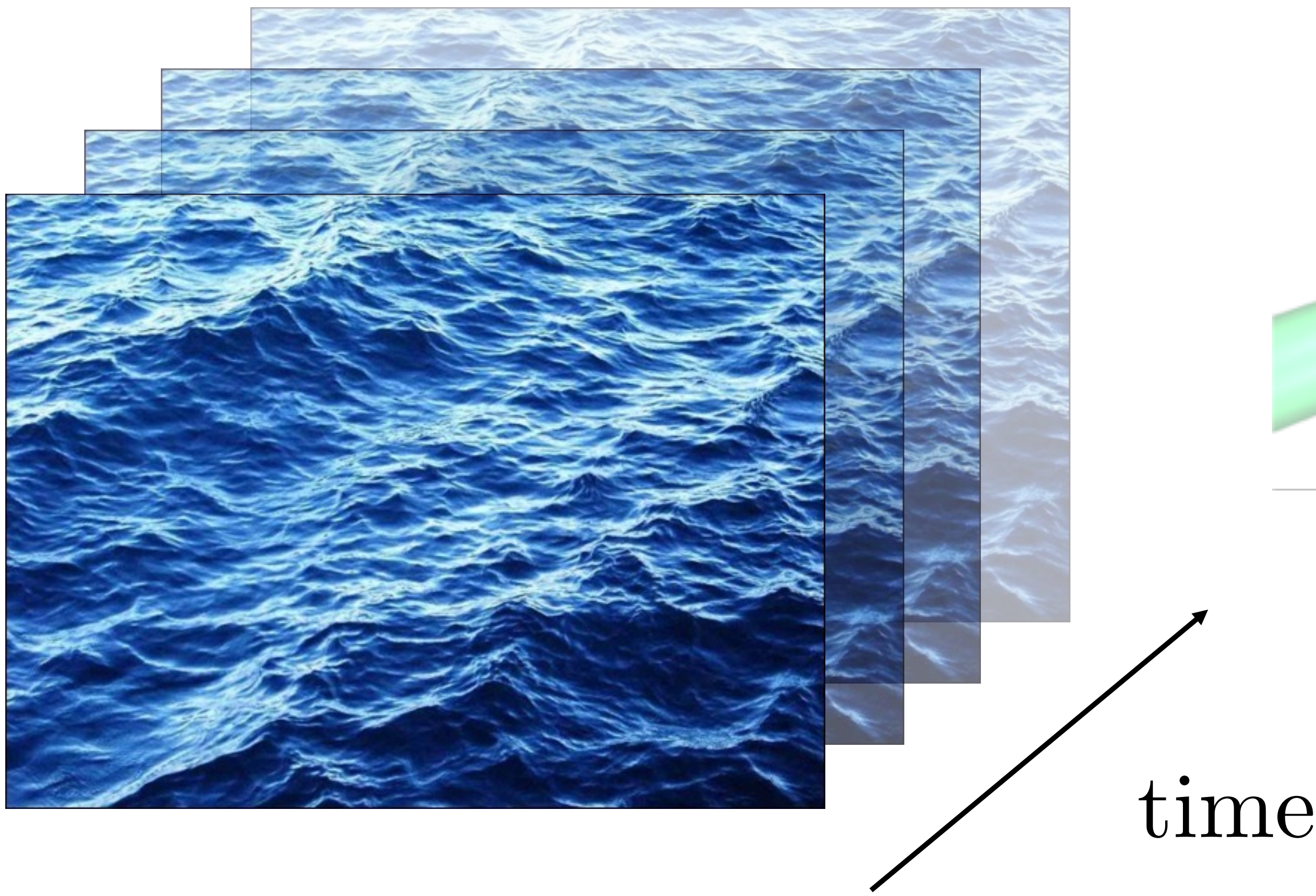
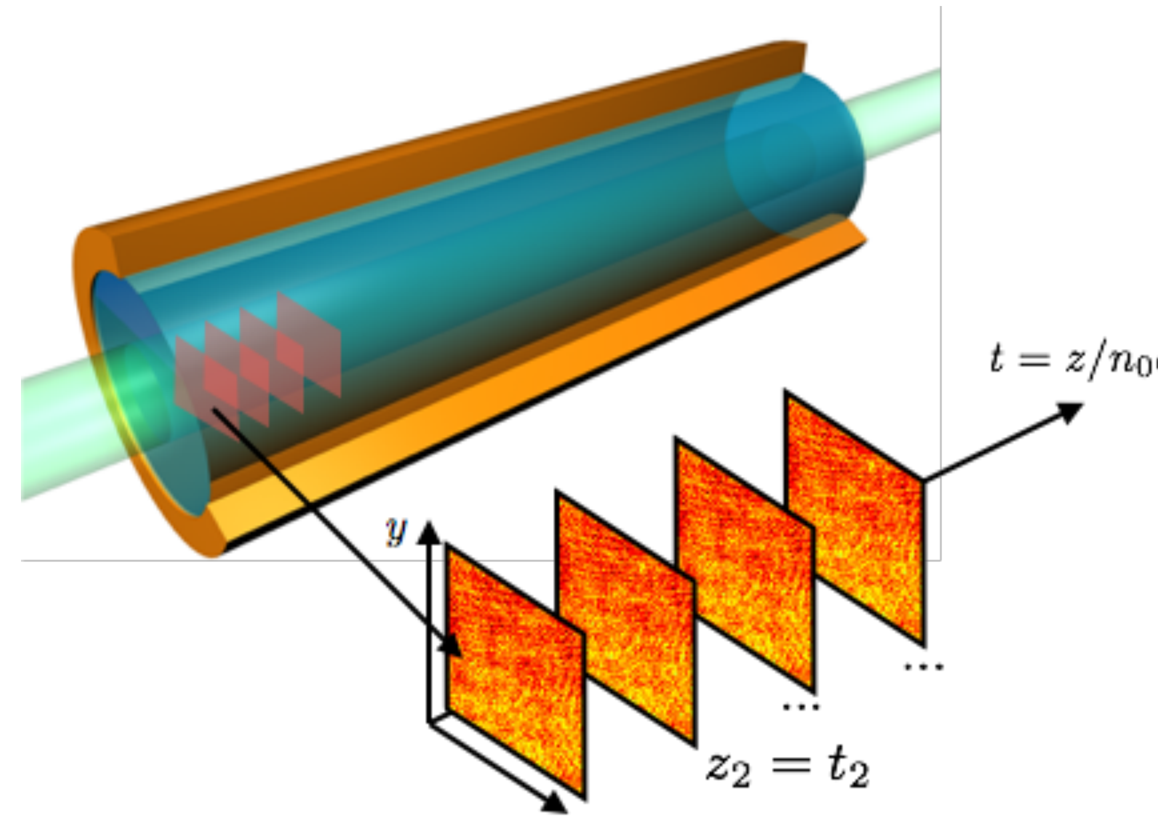


Oceanographic technique

Normal fluid



Photon fluid

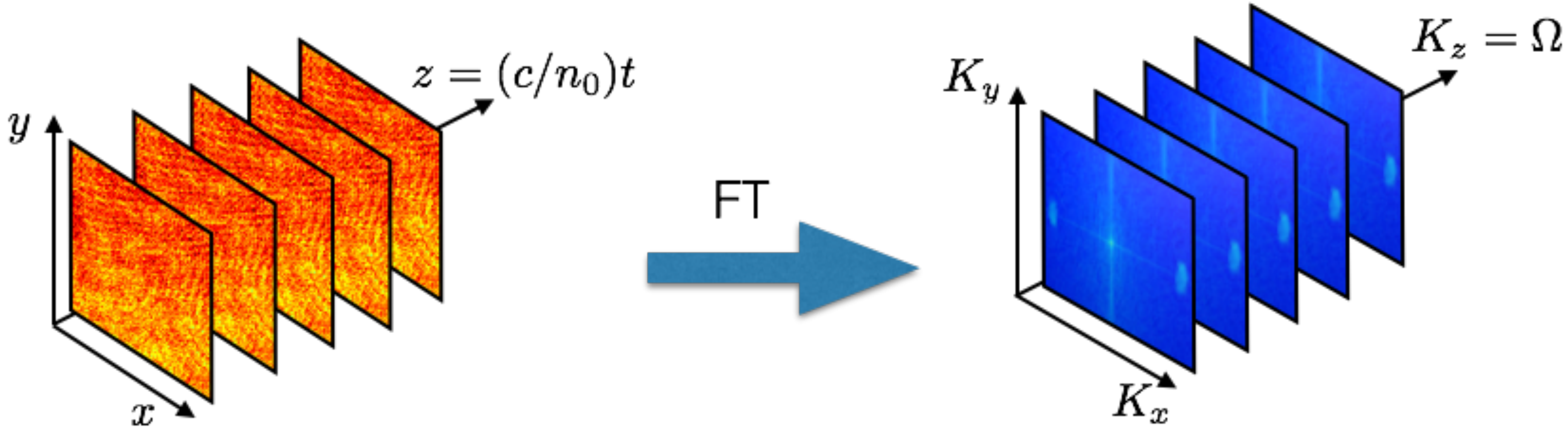


Amplitude noise that naturally lives on the beam behaves like waves on the ocean

Transient surface profile I(x,y,z) of the photon fluid obtained by imaging and scanning along z = scanning along “time”

Bogoliubov dispersion

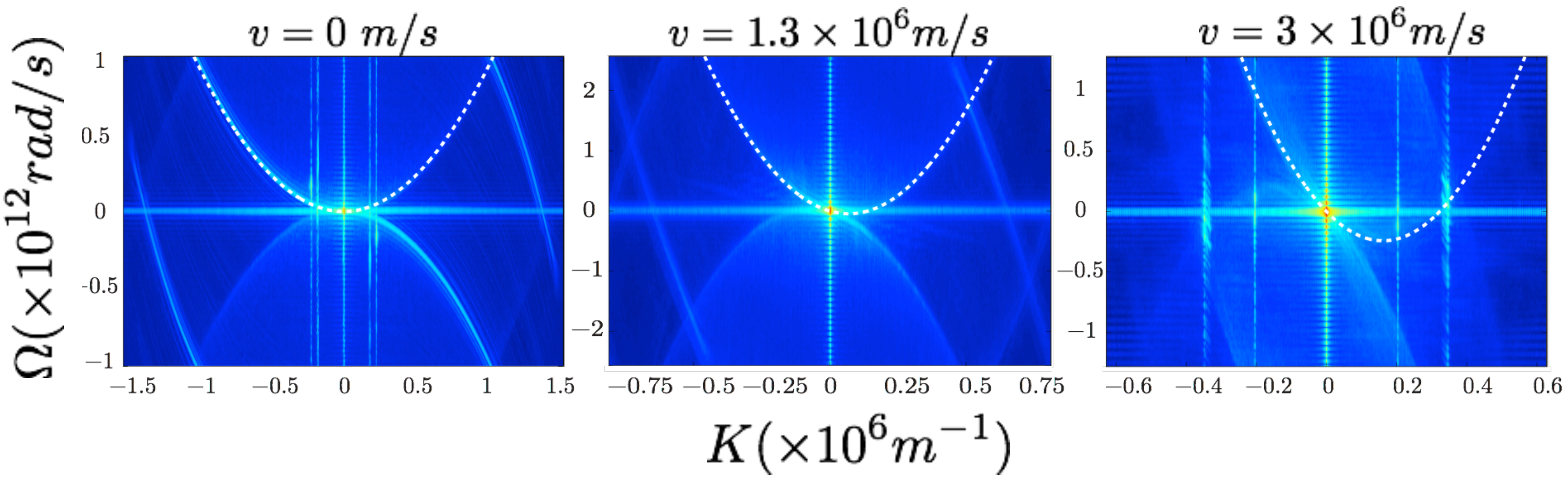
- Fourier transform:



- Dispersion of small amplitude noise on the laser beam

$$I(K_x, K_y, K_z = \Omega) = F[I(x, y, z)]$$

Bogoliubov dispersion



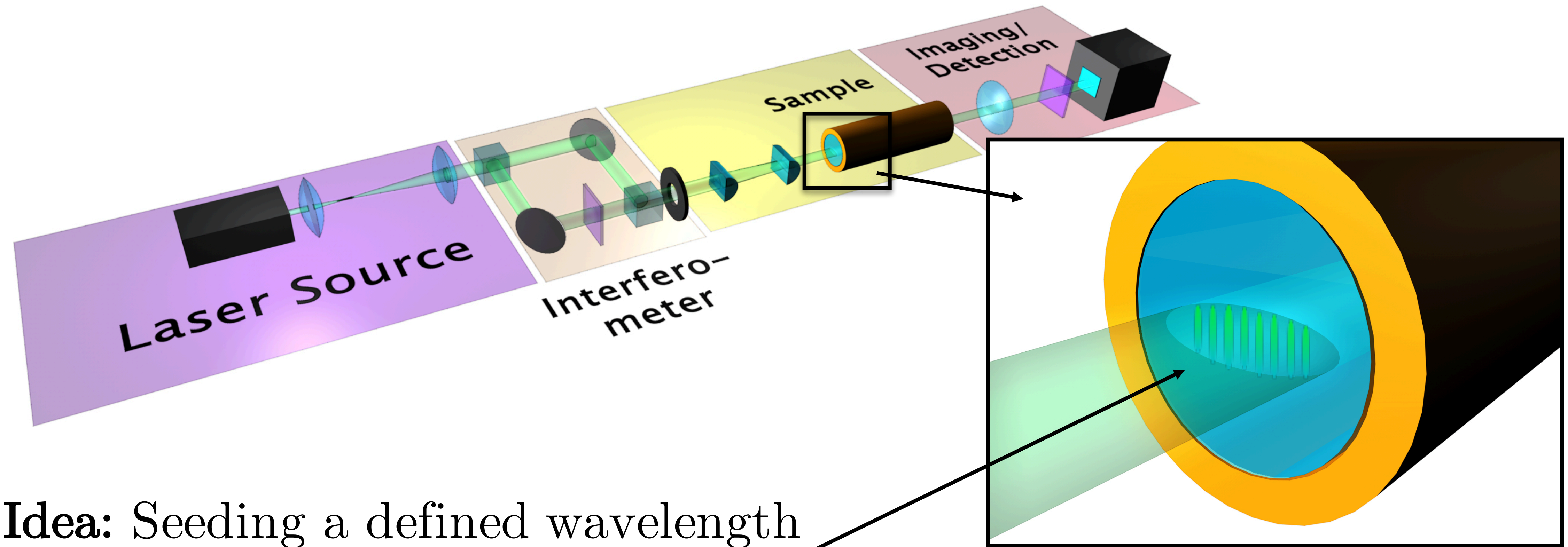
- Flow speed can be controlled by the phase gradient

$$(\Omega - vK)^2 = \frac{c^2 \Delta n}{n_0^3} \hat{R}(K) K^2 + \frac{c^2}{4k^2 n_0^2} K^4$$

- Nonlocality alters local dispersion relation for small wave vectors

D.Vocke et al., "Experimental characterization of nonlocal photon fluids,"
Optica 2, 484-490 (2015)

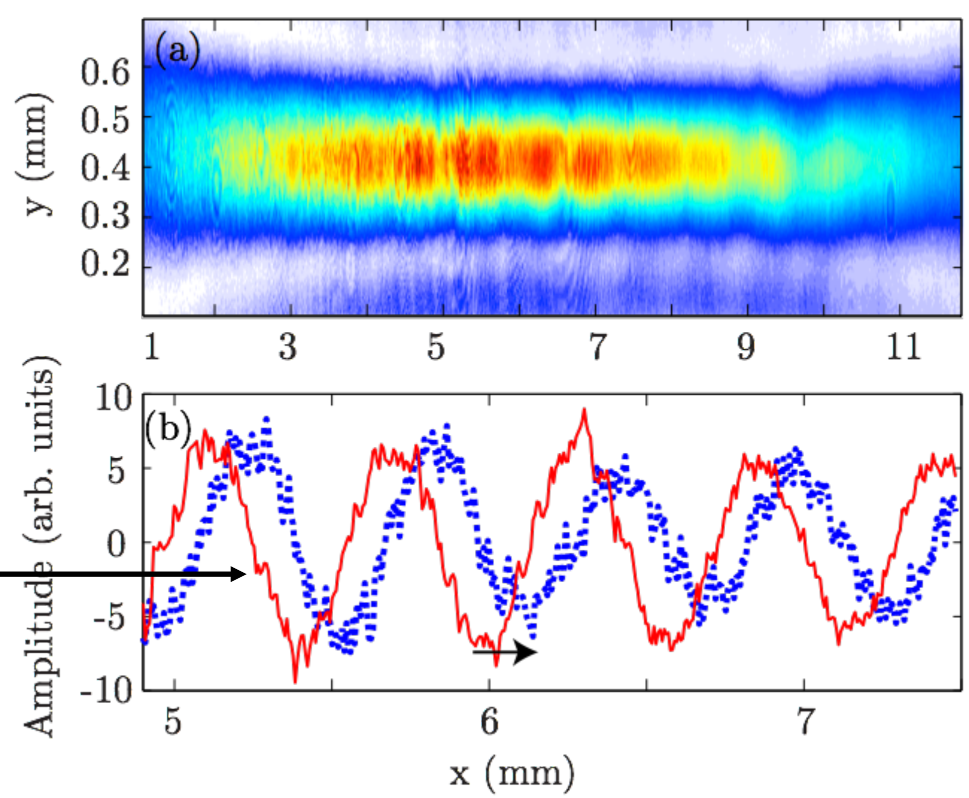
Pump probe technique



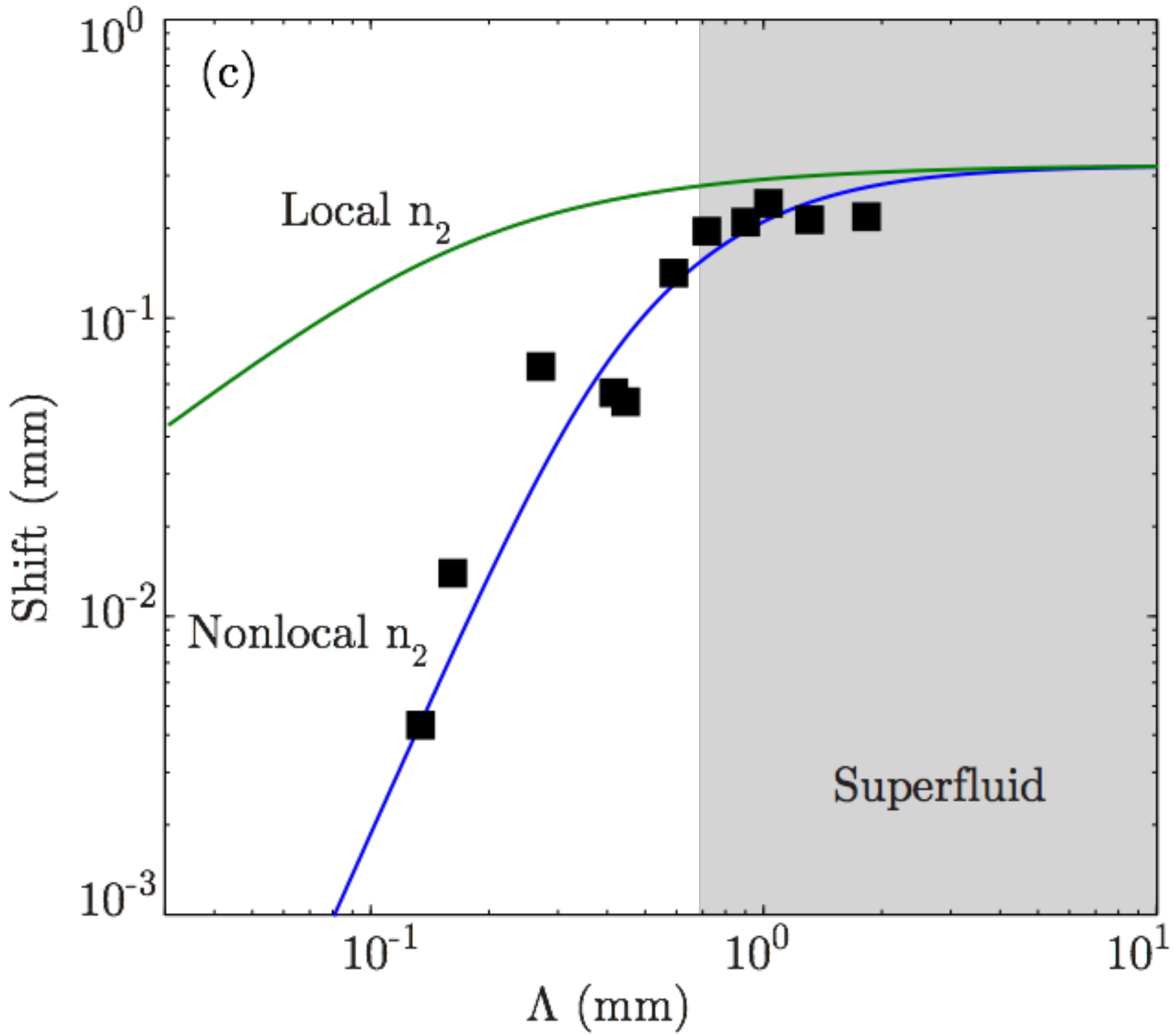
Idea: Seeding a defined wavelength and study its phase velocity

Input: Interference pattern
Output: shifted Interference pattern

$$\Delta S = c_s \tau \propto \sqrt{\Delta n}\; z$$



Pump probe techniques



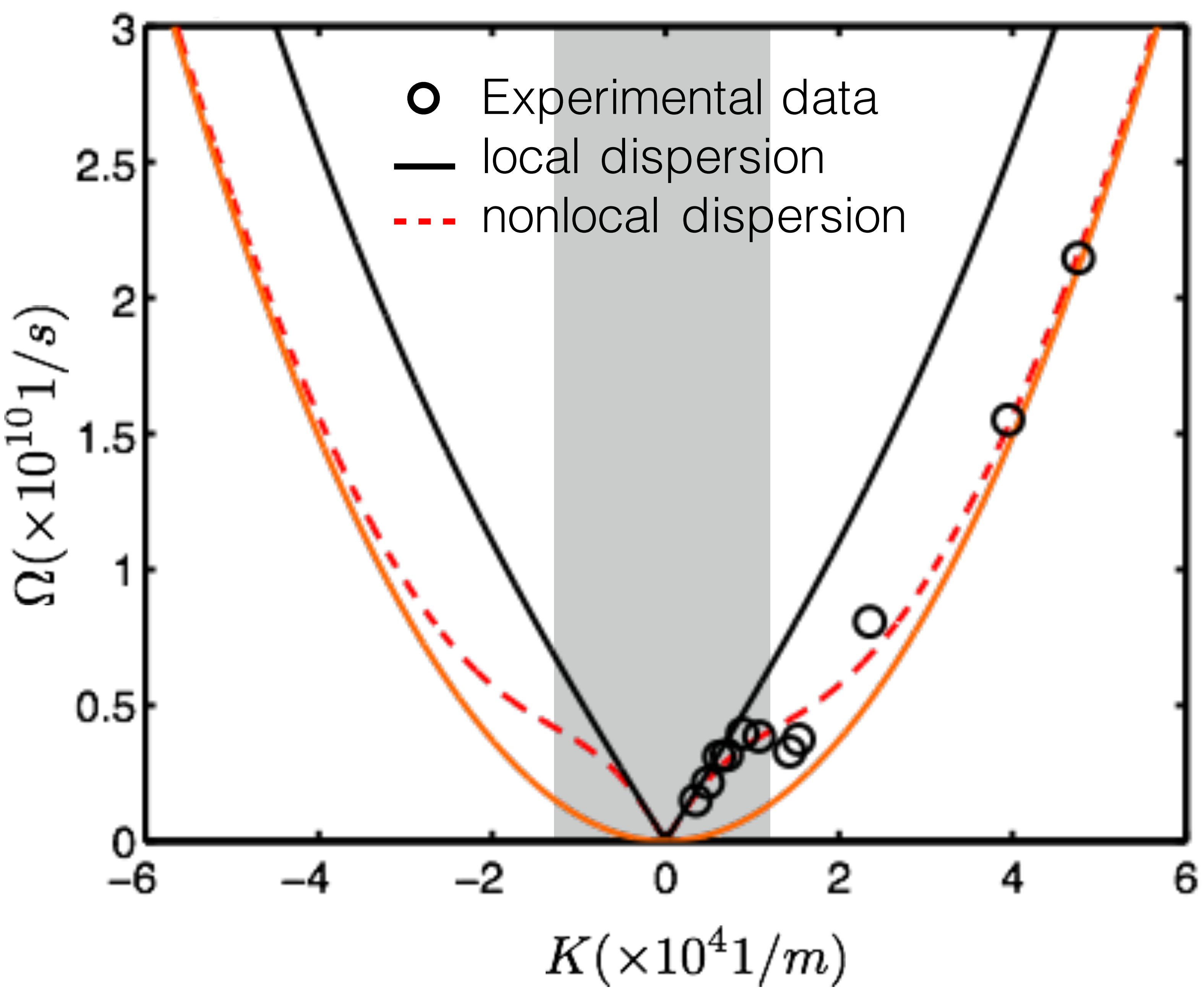
Best fit:

$$\sigma = 110\mu m$$
$$\Delta n = -7.6 \times 10^{-6}$$

$$\Delta S = c_s \tau = \frac{K}{2k}\left[\sqrt{1 + \frac{|\Delta n|}{n_0}\hat{R}(K)\left(\frac{2k}{K}\right)^2} - 1\right] z$$

D.Vocke et al., "Experimental characterization of nonlocal photon fluids," **Optica 2, 484-490 (2015)**

Bogoliubov dispersion



Excellent agreement with nonlocal Bogoliubov dispersion

D.Vocke et al., "Experimental characterization of nonlocal photon fluids," **Optica 2, 484-490 (2015)**

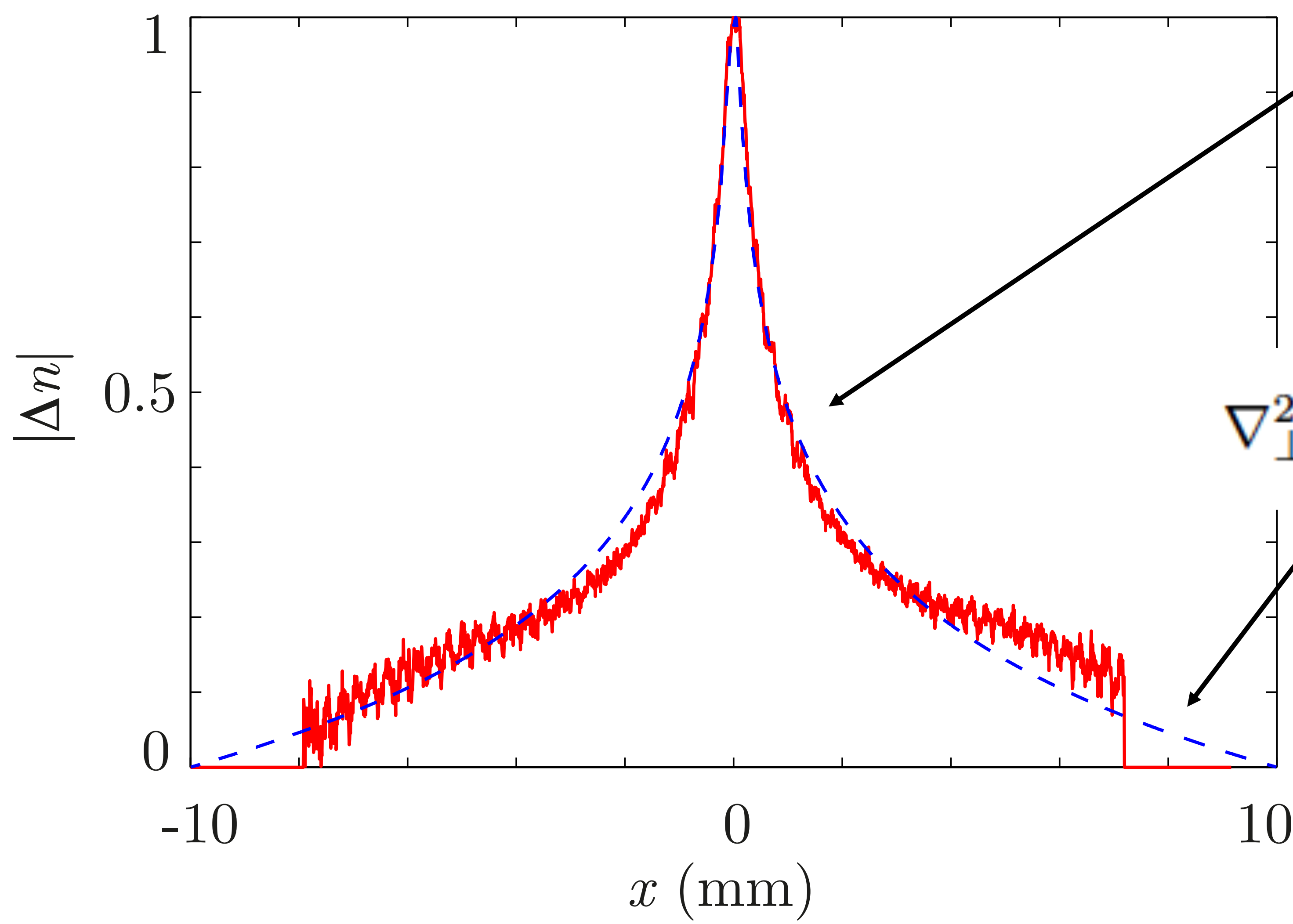
Geometry and nonlocality

Problem:

measured nonlocal length $\sigma = 120$ microns

But...

Measured response function has $\sigma = D/2 = 1$ cm



Measurement Based on Minovich et al., Opt. Lett. (2007)

$$\nabla_{\perp}^{2}(\Delta T) = -\frac{\alpha}{\kappa} I(x,y)$$

Geometry and nonlocality

Solution:

Beam/sample geometry modifies nonlocal interactions

Modify Poisson equation to account for boundaries through a loss term

$$\nabla_\perp^2(\Delta T) = -\frac{\alpha}{\kappa} I(x,y) \longrightarrow \nabla_\perp^2(\Delta T) - \frac{\Delta T}{\sigma^2} = -\frac{\alpha}{\kappa} I(x,y)$$

$$\downarrow$$

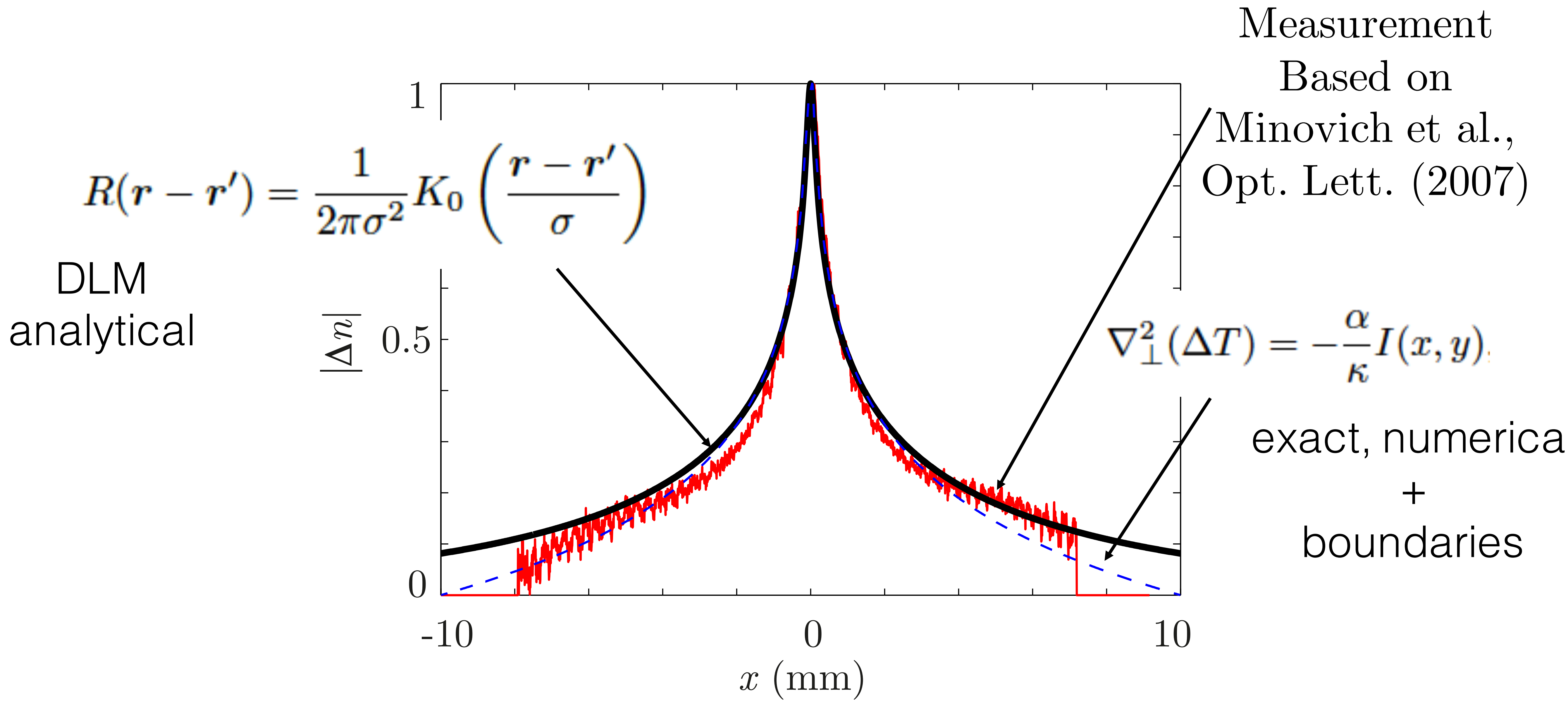
$$\hat{R}(K_x, K_y) = \frac{1}{1+\sigma^2(K_x^2 + K_y^2)}$$

$$\downarrow$$

$$R(\boldsymbol{r}-\boldsymbol{r}') = \frac{1}{2\pi\sigma^2} K_0\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{\sigma}\right)$$

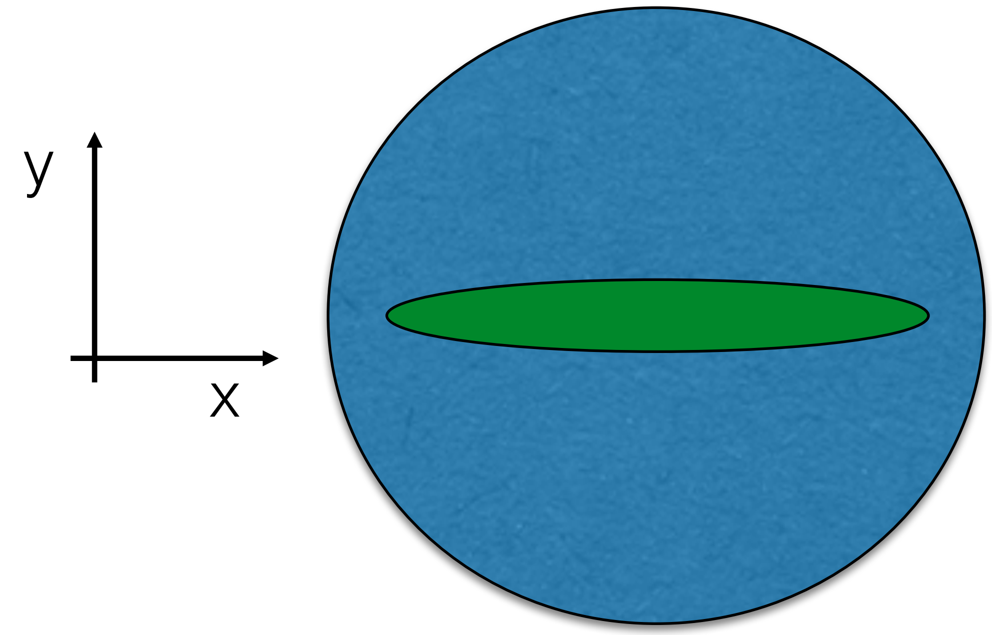
Geometry and nonlocality

"Distributed Loss Model" provides analytical solution to heat equation



Geometry and nonlocality

Assume a highly elliptical beam $w_y << w_x$



Restricting width -> modifies heat flow

$$\frac{\partial^2(\Delta T)}{\partial y^2} \approx -\frac{\Delta T}{w_y^2}$$

Alternatively -> geometry limits Response function to

$$K_y = 1/w_y = 2\pi/\Lambda_y$$

$$\hat{R}_{1D}(K_x) = \hat{R}(K_x, K_y = 1/w_y)$$

$$= 1/(1+\bar{\sigma}^2 K_x^2) \qquad \bar{\sigma} = (\sigma w_y)/\sqrt{w_y^2+\sigma^2}.$$

Effective nonlocal length

If one dimension is very small, this will dominate nonlocal behaviour

$$w_y << \sigma \longrightarrow \bar{\sigma} \sim w_y$$

Geometry and nonlocality

In the experiments reported in Optica, 2015, Vocke et al. :

Measured response function has
σ = D/2= 1 cm

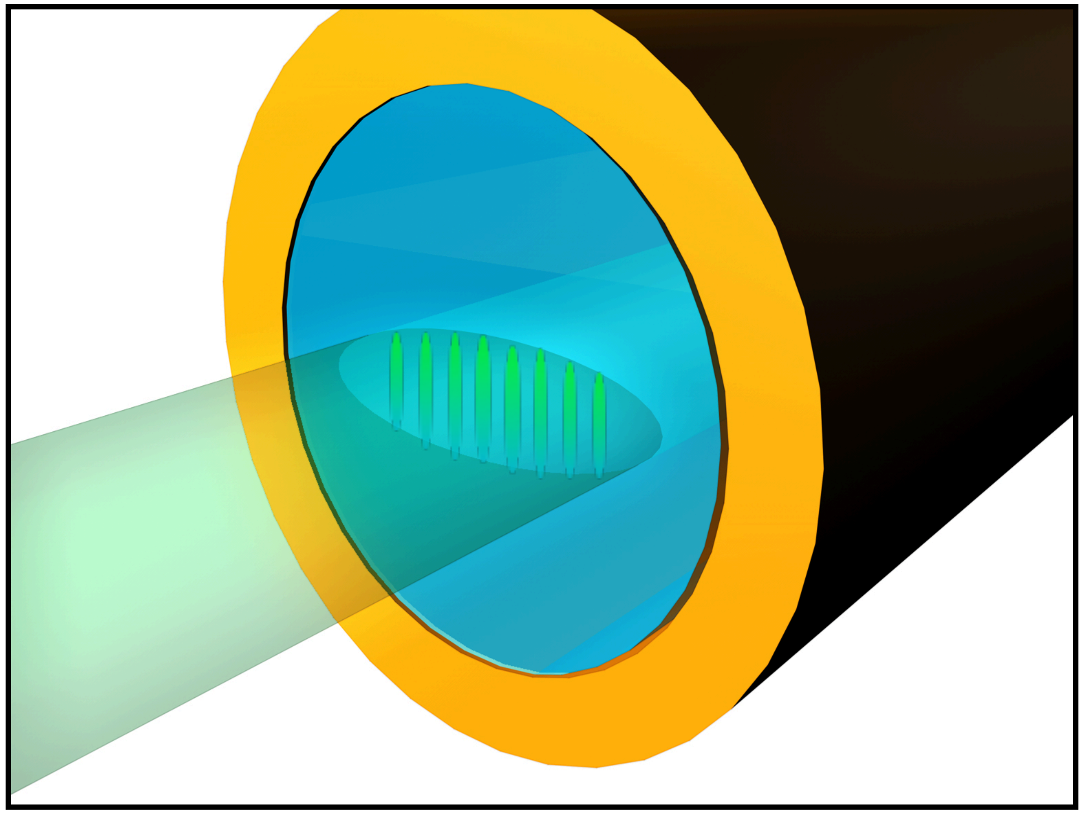
But...

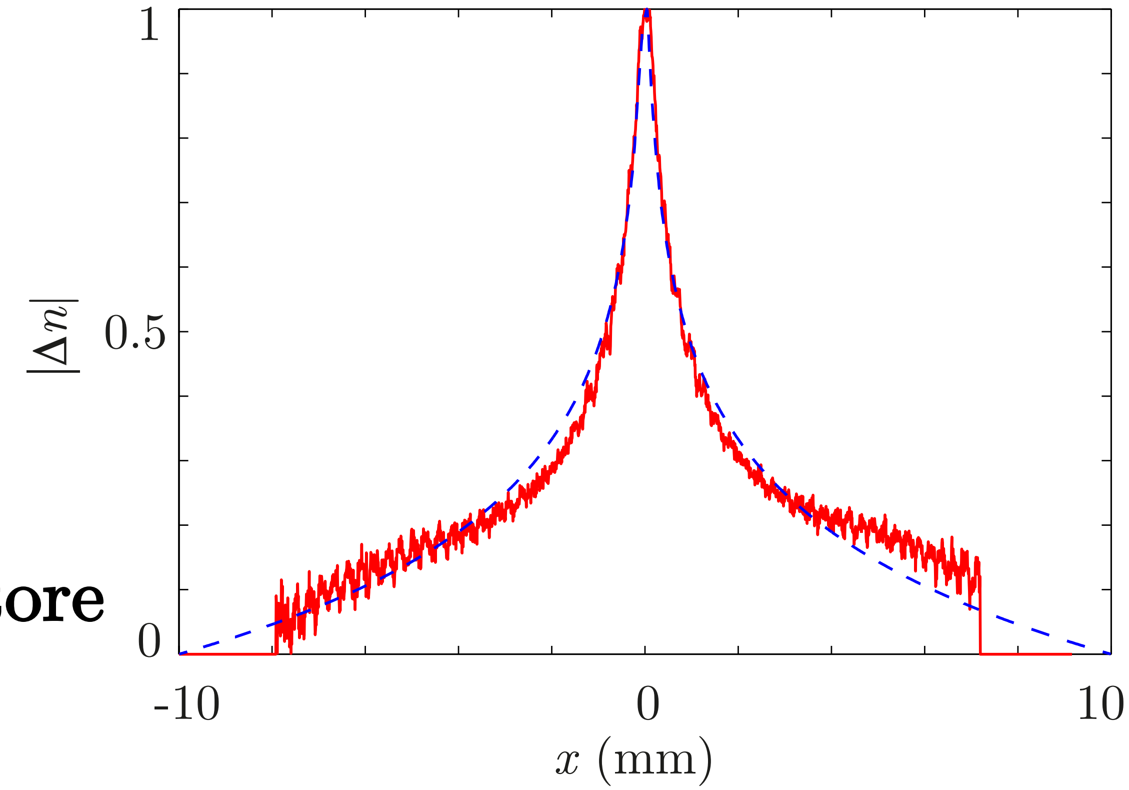
Highly elliptical beam wx = 1 cm,
wy = 130 microns

Measured nonlocal length σ = 120 microns

$$w_y << \sigma \longrightarrow \bar{\sigma} \sim w_y$$

By acting on the beam geometry we can restore superfluid behaviour over broad range





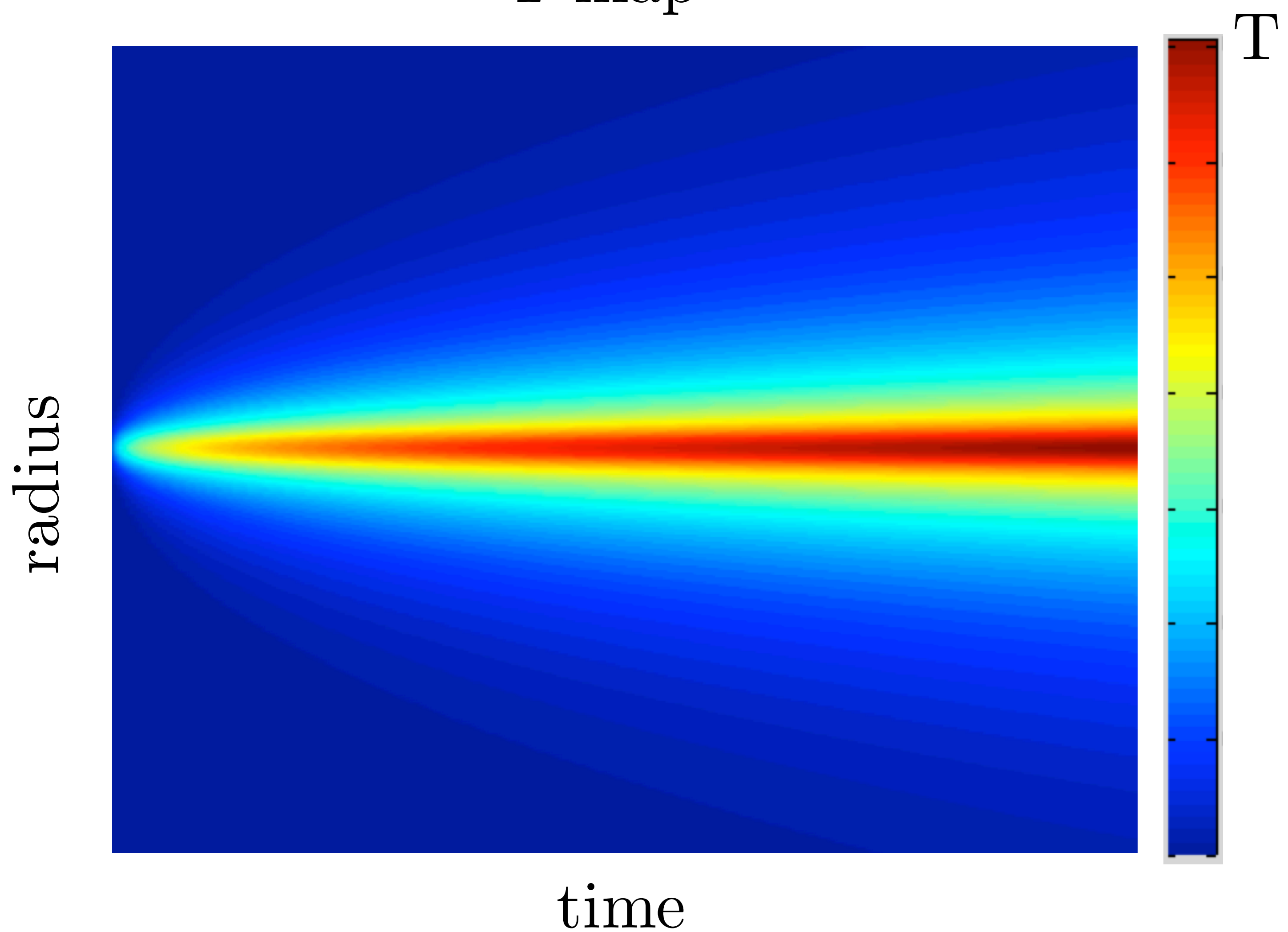
Transient nonlocality

Numerically Solve time-dependent heat diffusion eq.

Temperature rapidly reaches max (or close to max)
And slowly spreads out spatially

In other words, $\Delta n = \frac{\partial n}{\partial T}\Delta T$
can be controlled
Independently of
σ = spatial extent of heat profile

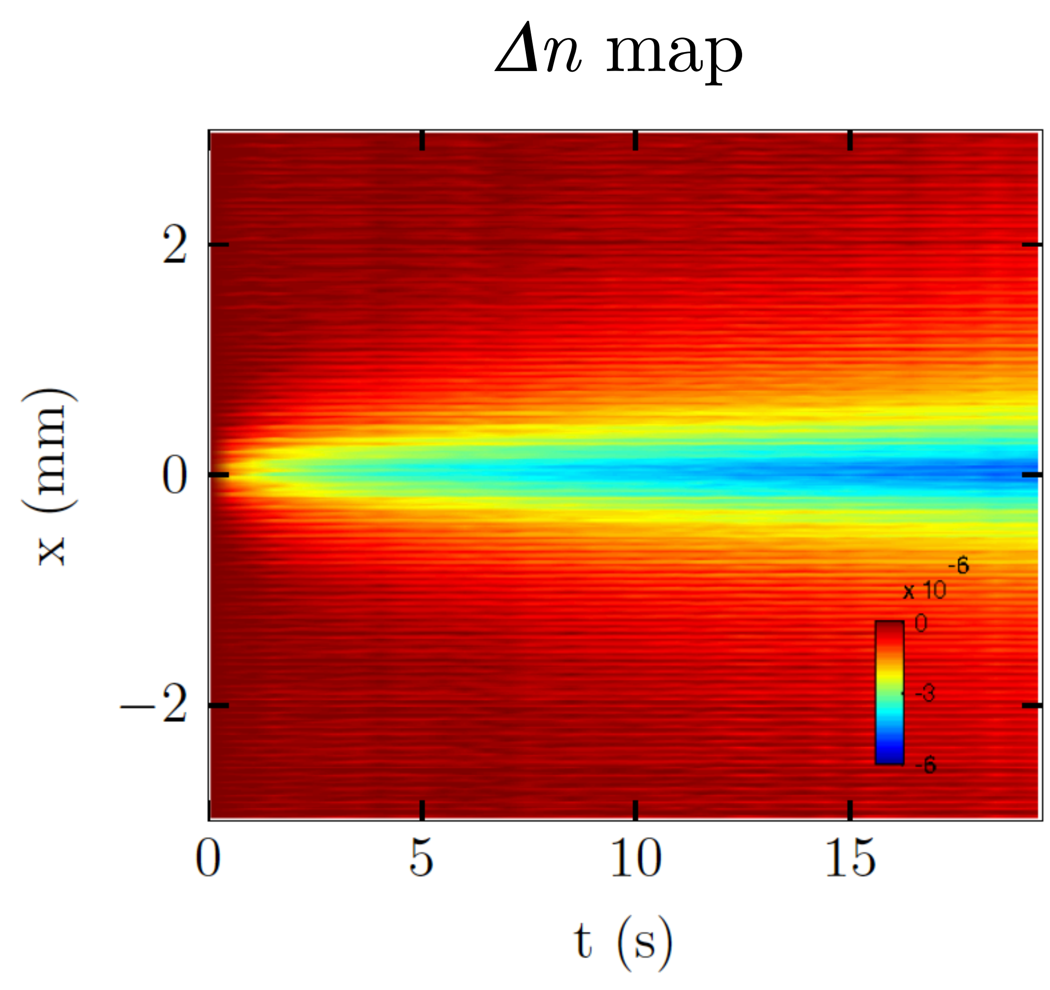
T map

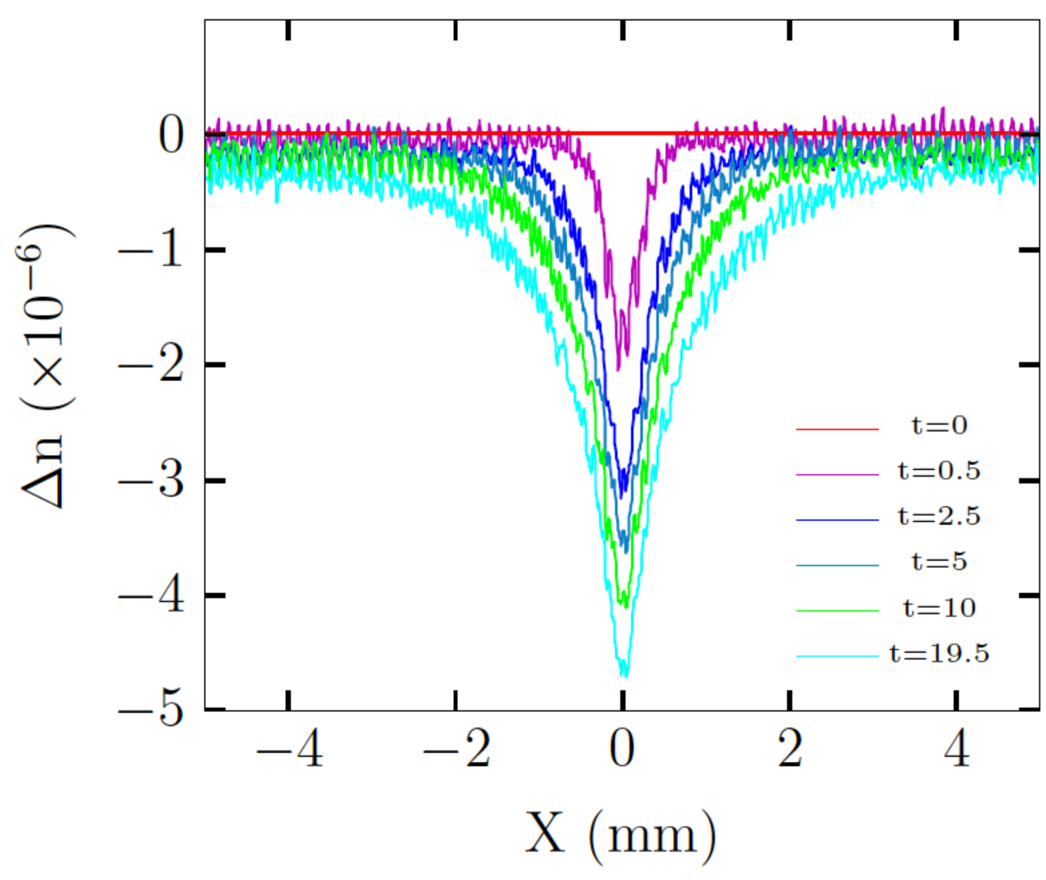


Transient nonlocality

Measurements of time-dependent heat diffusion:

Agree very well with numerics



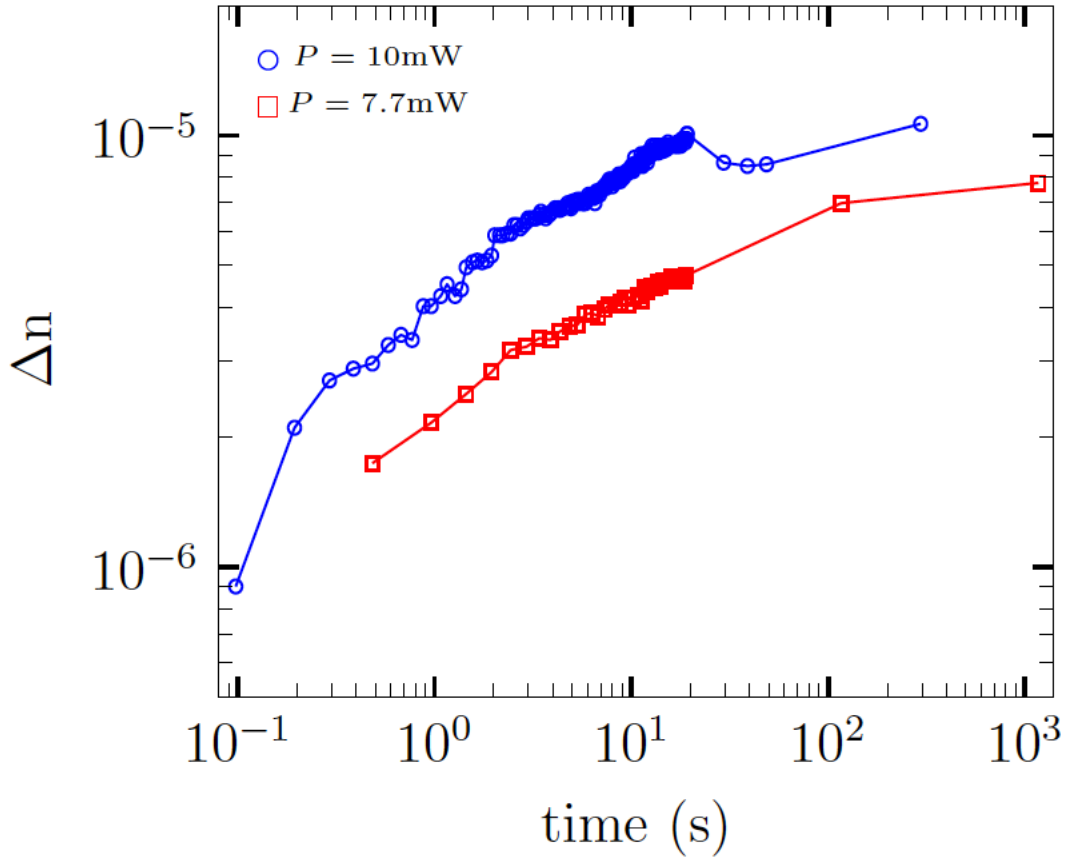


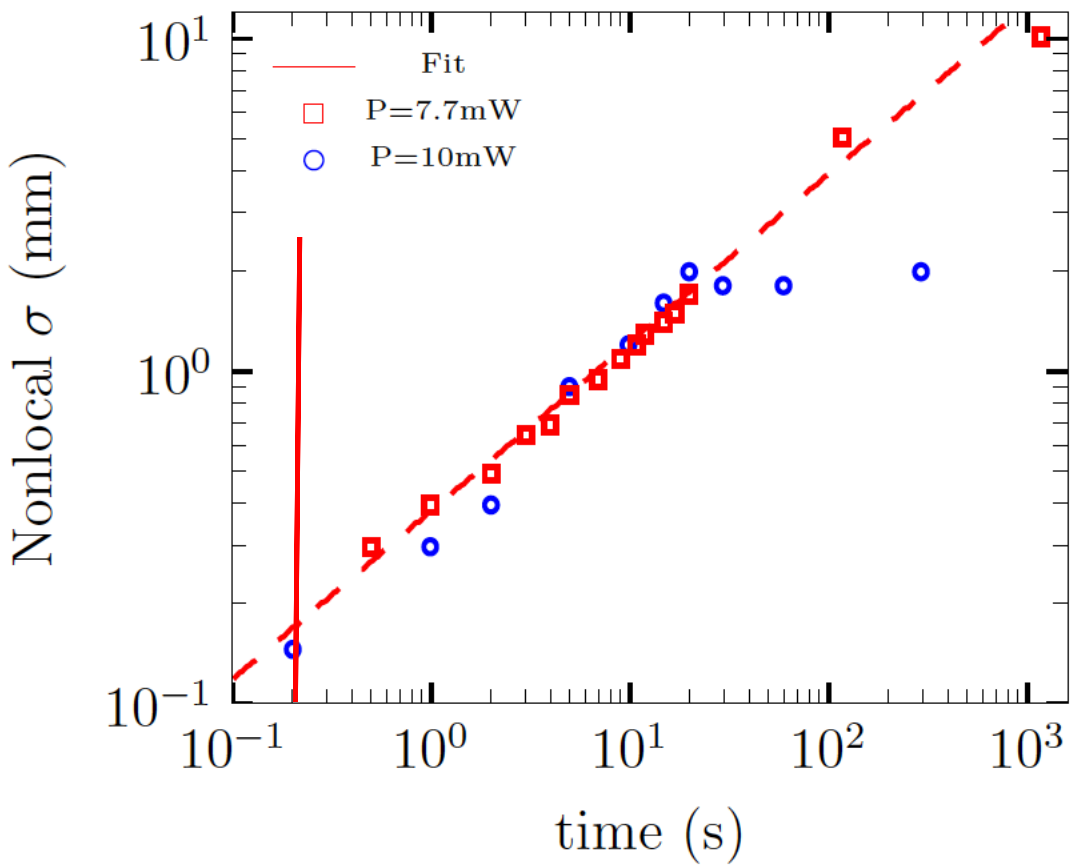
Transient nonlocality

Measurements of time-dependent heat diffusion:

Provides alternative route to controlling nonlocal length

Superfluidy is restored by measuring at short times (large Δn, small σ) (e.g. 200 ms after laser is switched on)





Superfluid flow around an obstacle

Idea: Angle knife edge/beam determines flow speed

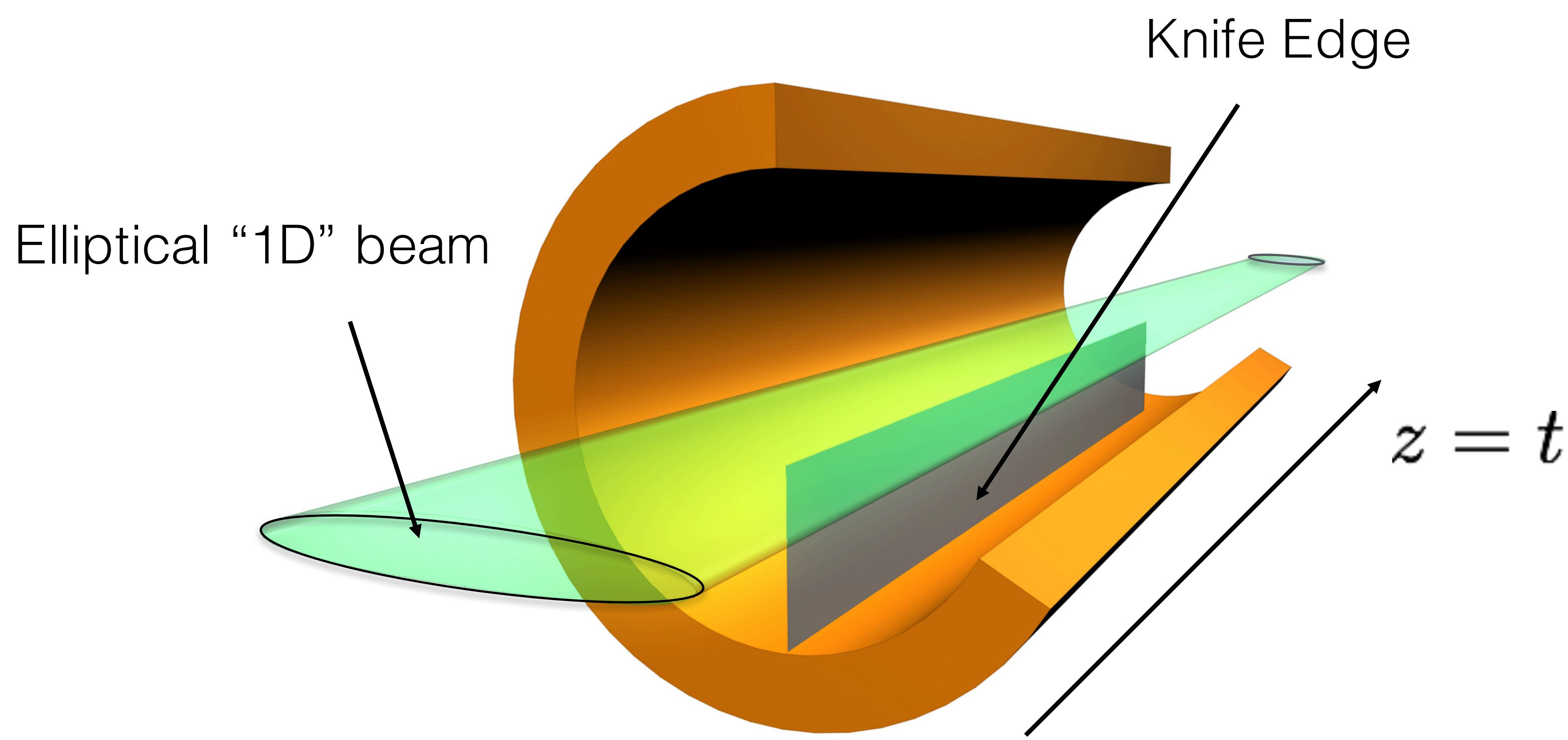
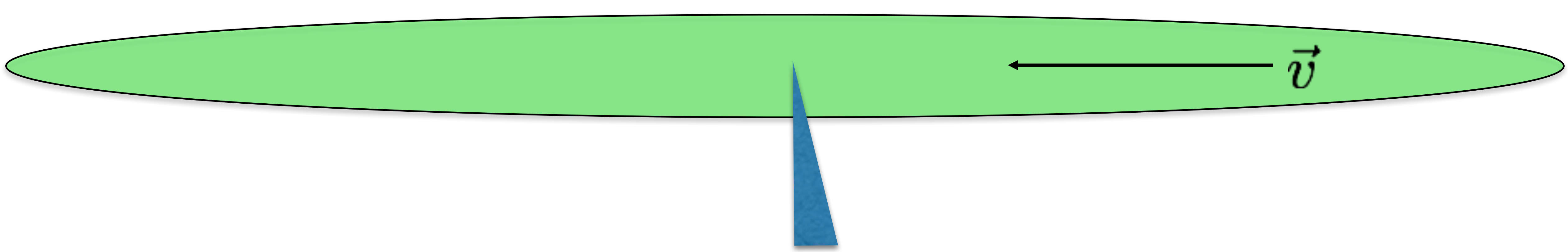
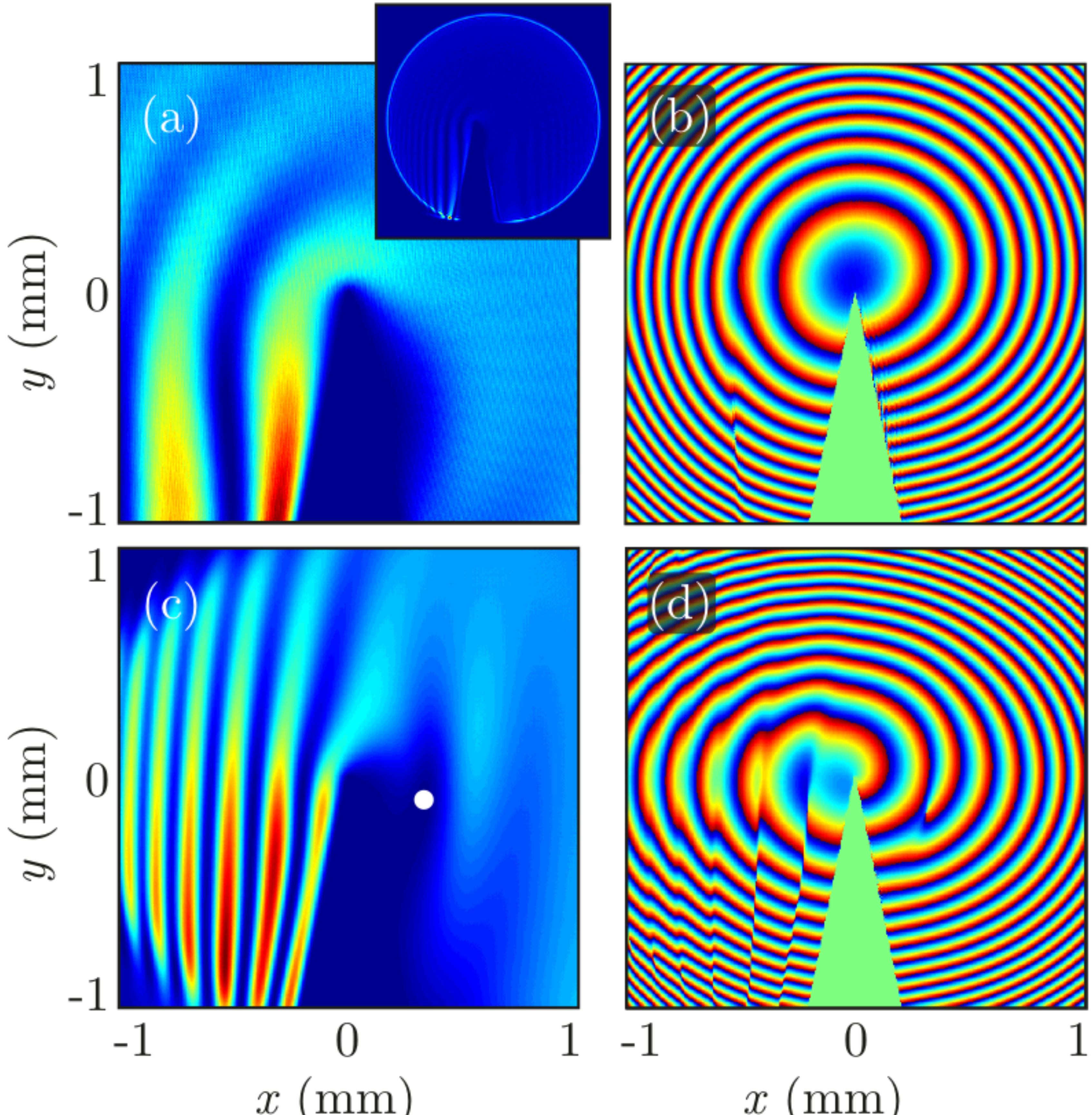


Image on camera:



Numerics



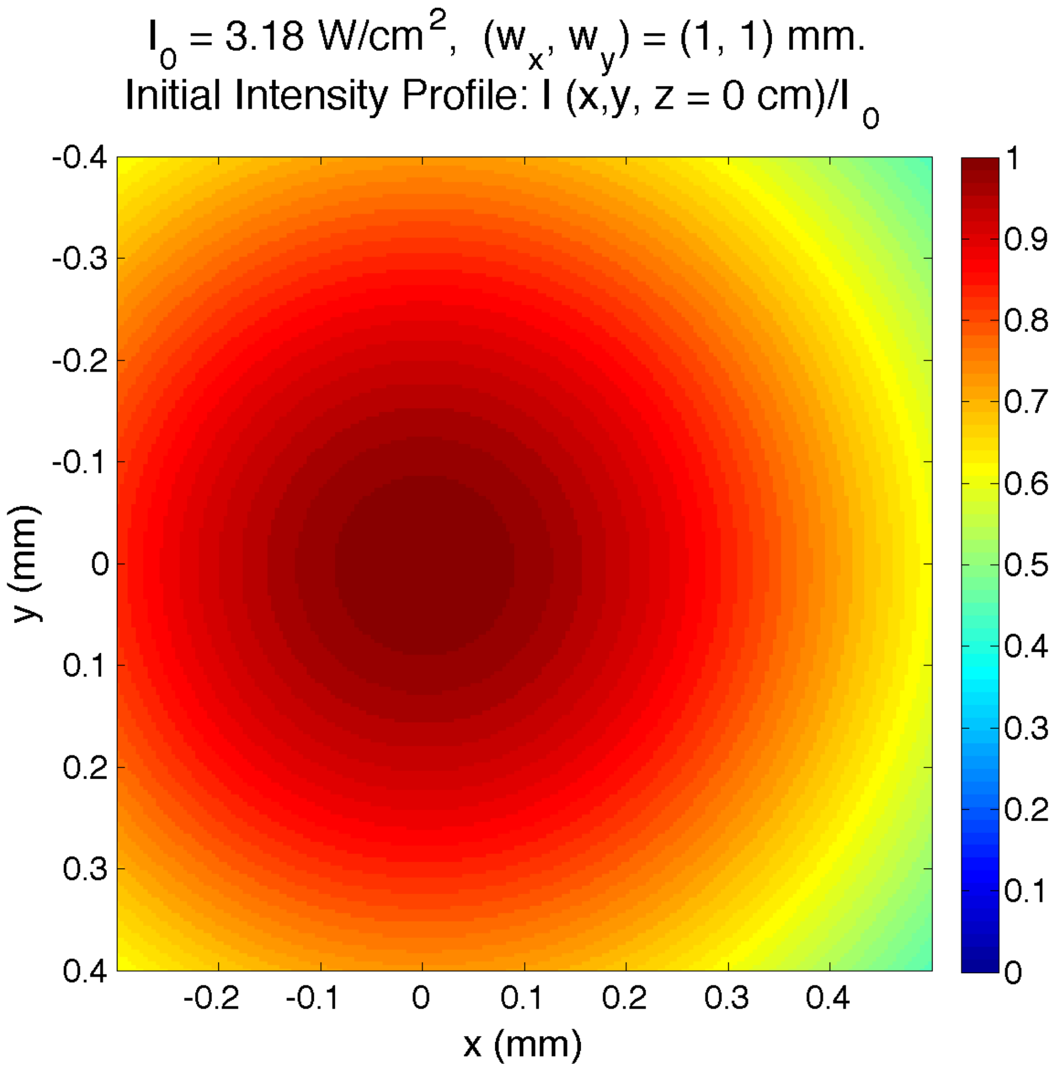
Round beam:
σ = D/2= 1 cm

Photon fluid flows around obstacle.

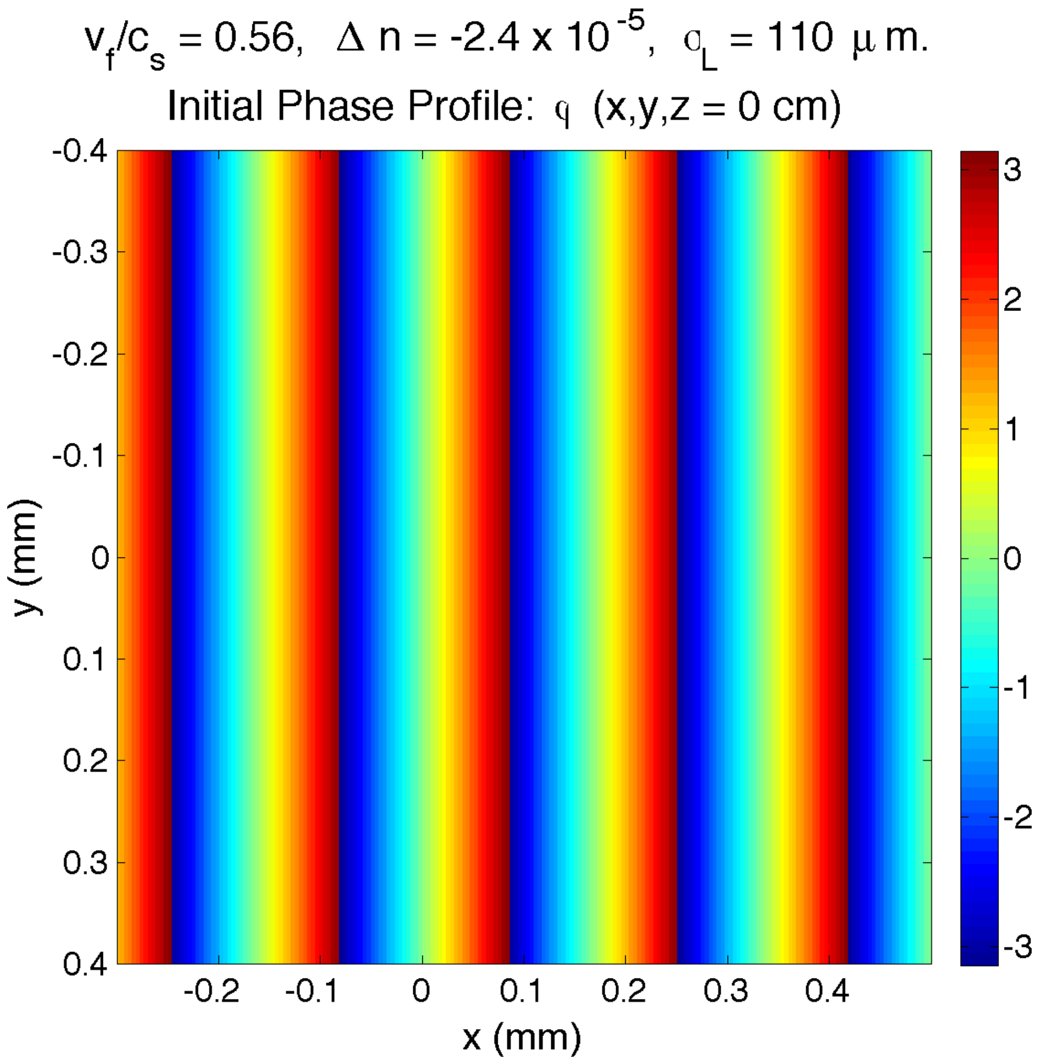
Elliptical beam:
σ = wy= 0.1 cm

Photon fluid flows around obstacle and sheds **quantised vortices**

Numerics

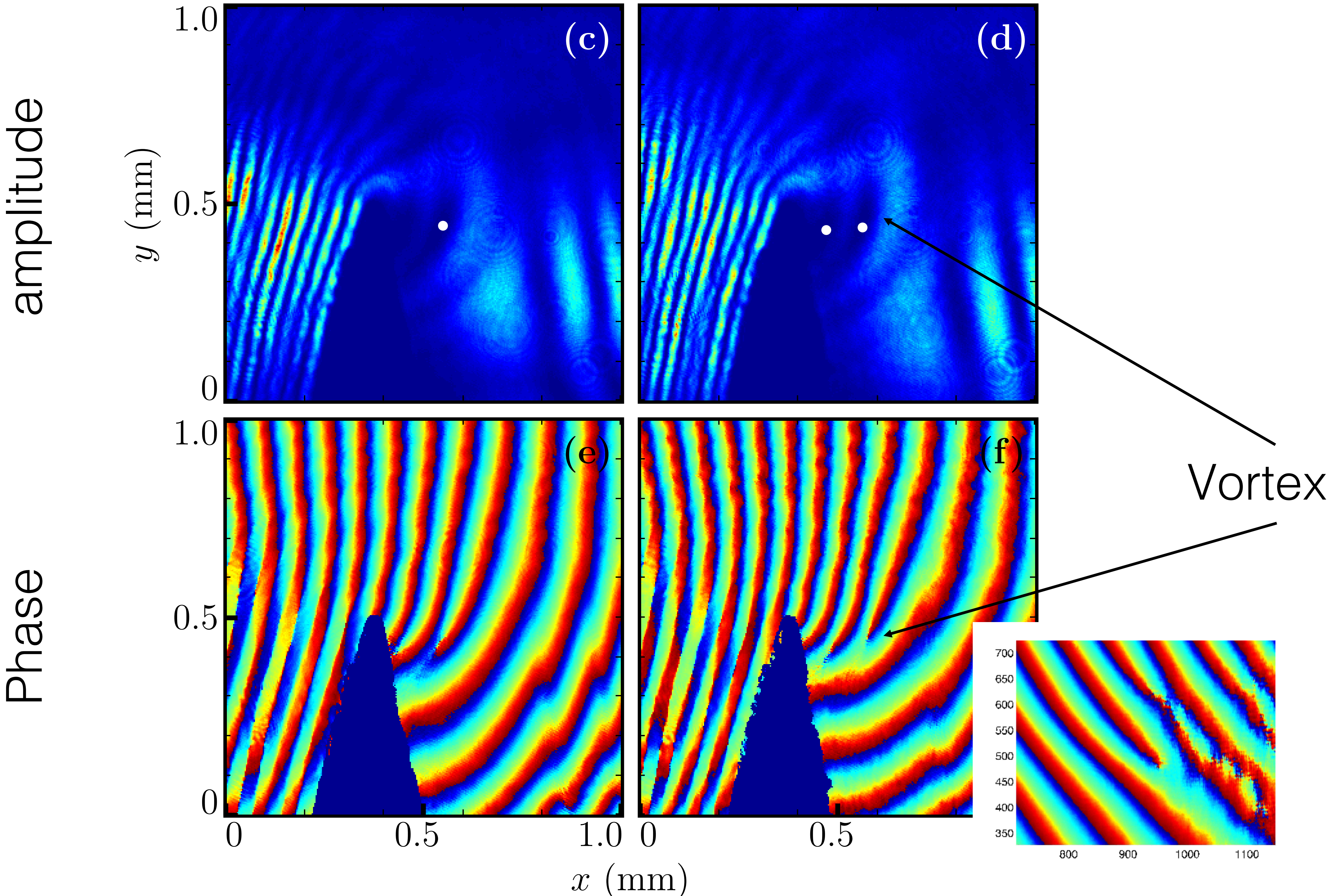


I_0 = 3.18 W/cm^2, (w_x, w_y) = (1, 1) mm.
Initial Intensity Profile: I (x,y, z = 0 cm)/I_0
y (mm)
x (mm)
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
-0.2
-0.1
0
0.1
0.2
0.3
0.4
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0



v_f/c_s = 0.56, Δ n = -2.4 x 10^{-5}, σ_L = 110 μm.
Initial Phase Profile: φ (x,y,z = 0 cm)
y (mm)
x (mm)
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
-0.2
-0.1
0
0.1
0.2
0.3
0.4
3
2
1
0
-1
-2
-3

Experiments



Analogue gravity in photon fluids

Phonons = small oscillations on fluid (beam) surface

Linearising equations leads to scalar field eq. in curved spacetime

$$\Delta\psi_1 \equiv (1/\sqrt{-g})\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi_1) = 0$$

$$g_{\mu\nu} = \left(\frac{\rho_0}{c_s}\right)^2 \begin{pmatrix} -(c_s^2 - v_0^2) & -\mathbf{v}_0^T \\ -\mathbf{v}_0 & \mathbf{I} \end{pmatrix}$$

(See papers by Francesco Marino, PRA, 2008-2009)

Analogue gravity in photon fluids

Event horizon in transonic flows:
flow speed v = phonon speed, c

$$v \propto \nabla \phi \qquad c_s = \sqrt{c^2 n_2 \rho_d / n_0^3}$$

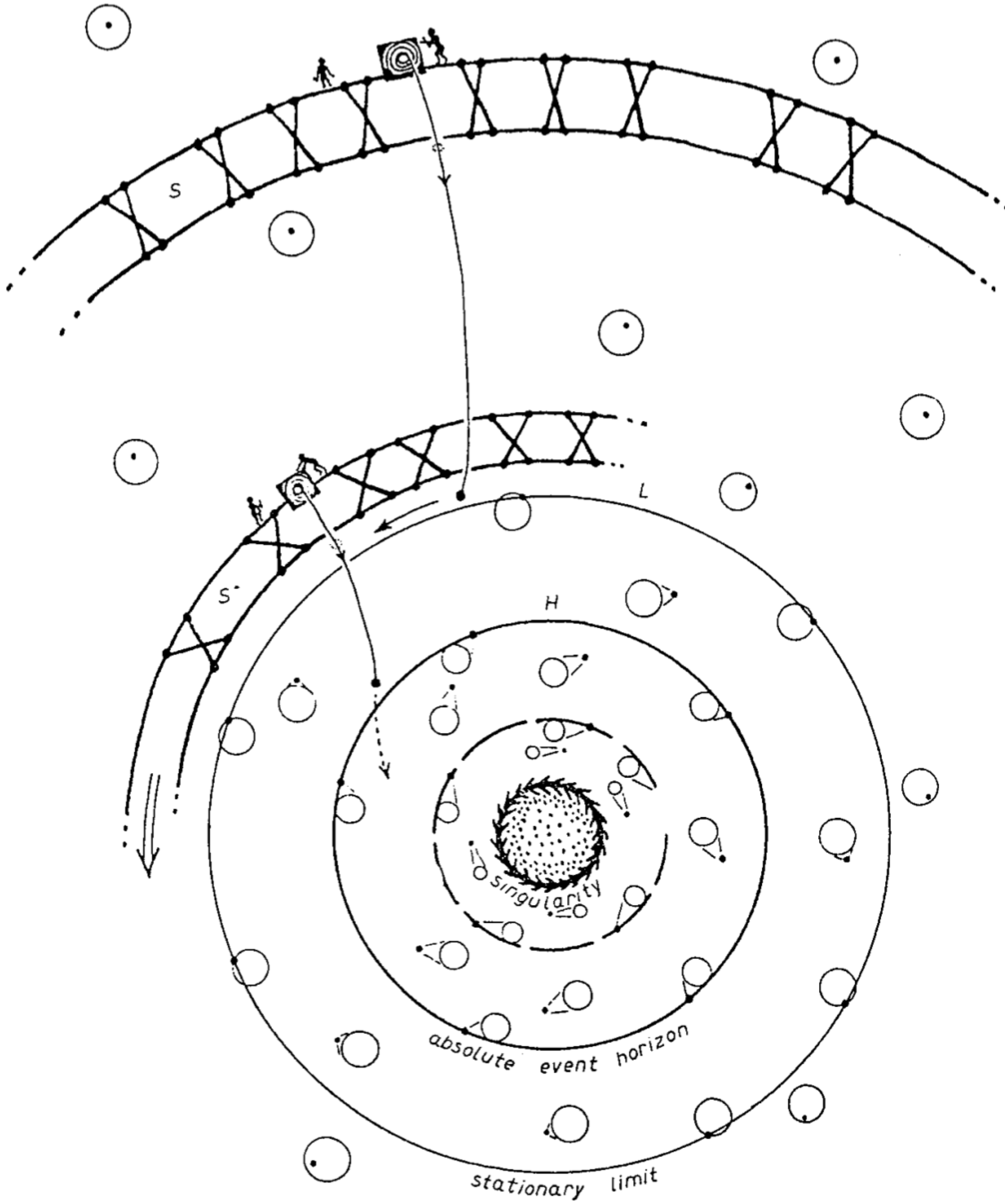
Two options:

1 – constant flow v, varying c (See papers by Carusotto, 2014)

2 – gradient in flow v, constant c

(See papers by Francesco Marino, PRA, 2008-2009)

Analogue gravity in photon fluids



General Relativity and Gravitation,
Vol. 34 (2002)
"Gravitational Collapse:
The Role of General Relativity"
R. Penrose

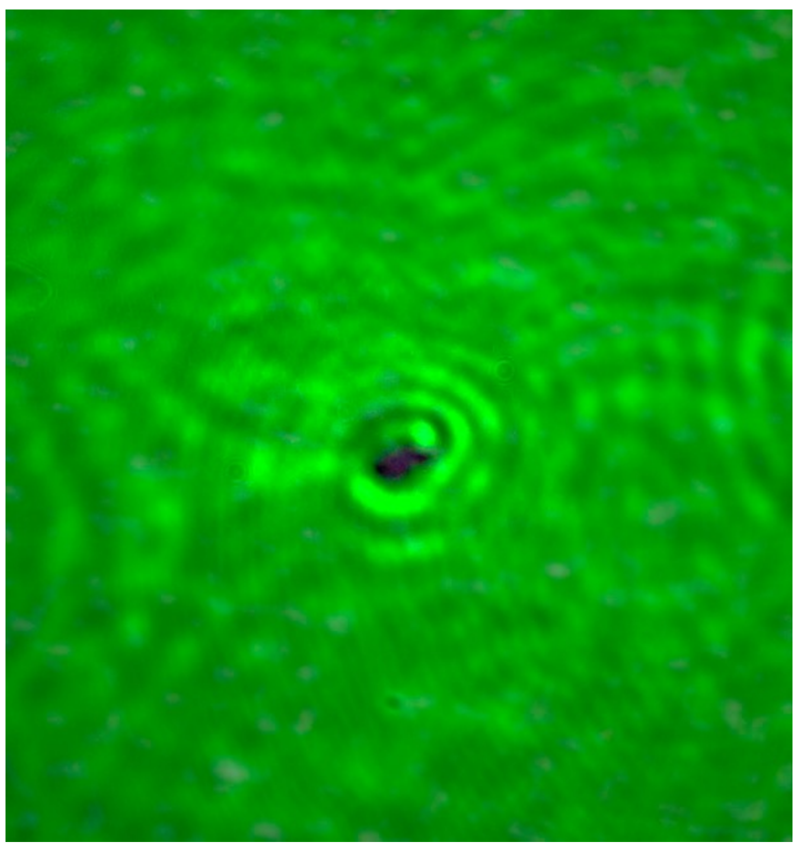
Analogue gravity in photon fluids

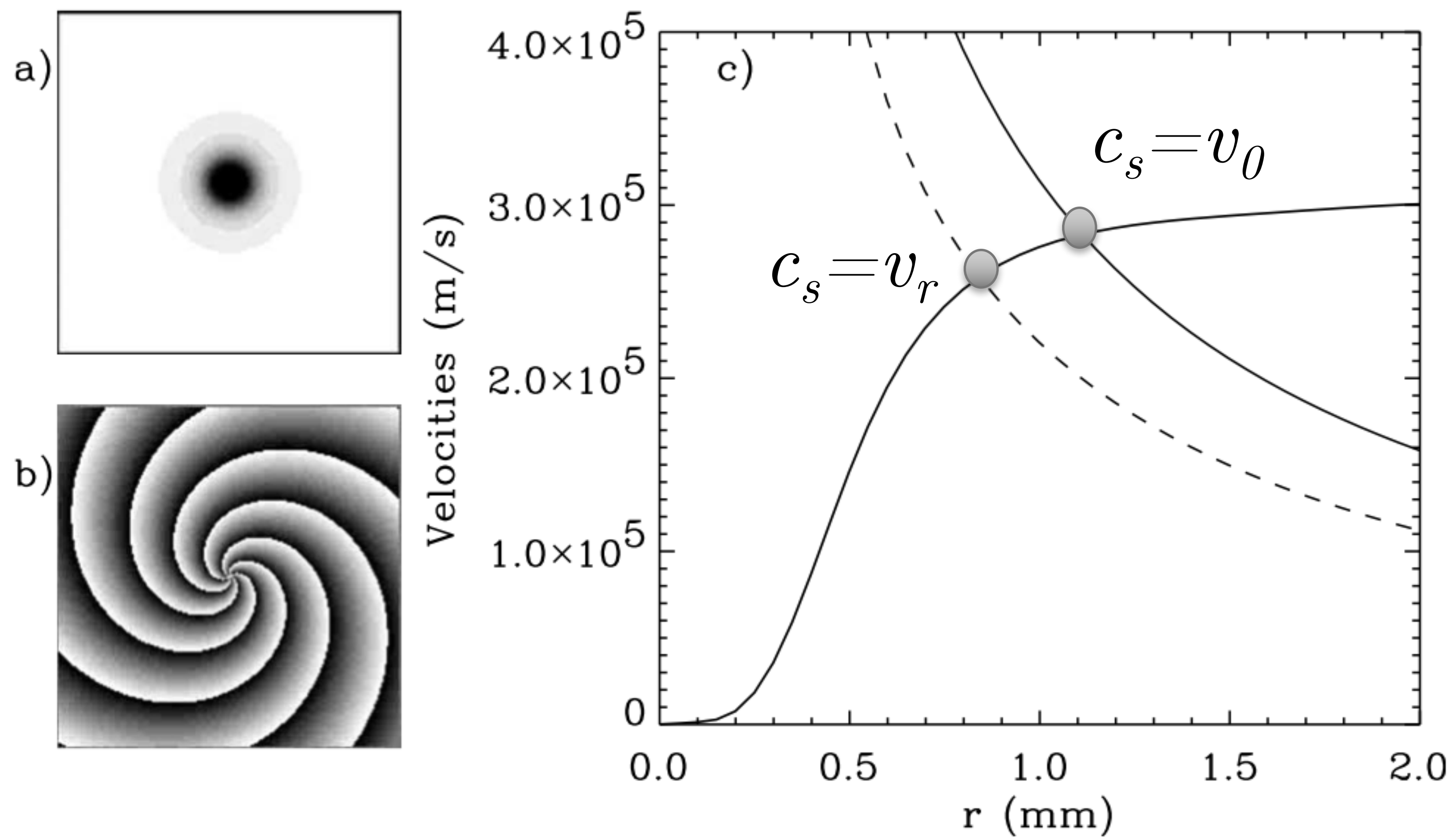
$$v_r = -c\pi / k n_0 \sqrt{r_0 r},$$

$$v_\theta = cm / k n_0 r,$$

$$c_s = \sqrt{c^2 n_2 \rho_d / n_0^3}$$

$$v_0 = \sqrt{v_\theta^2 + v_r^2}$$

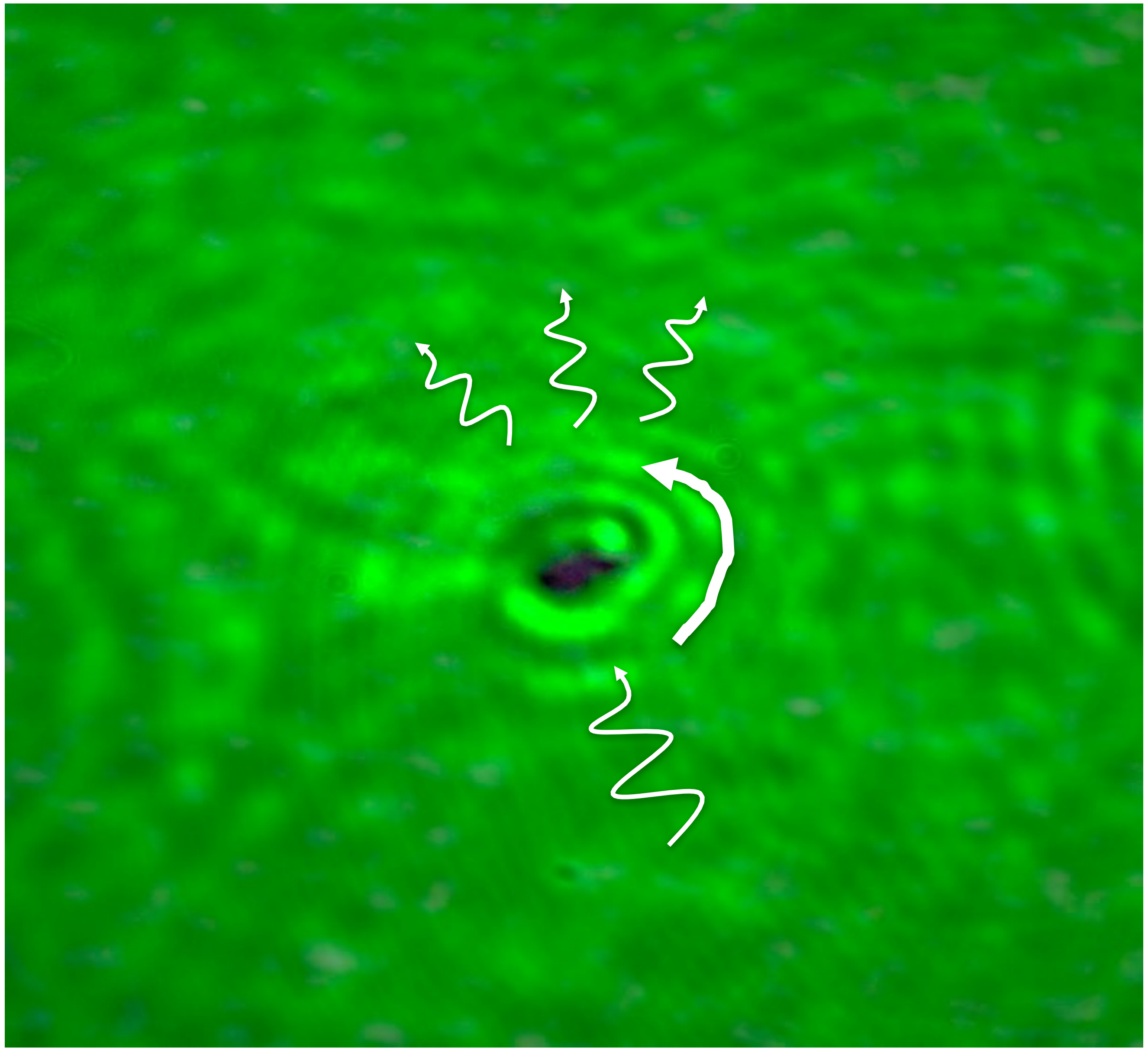




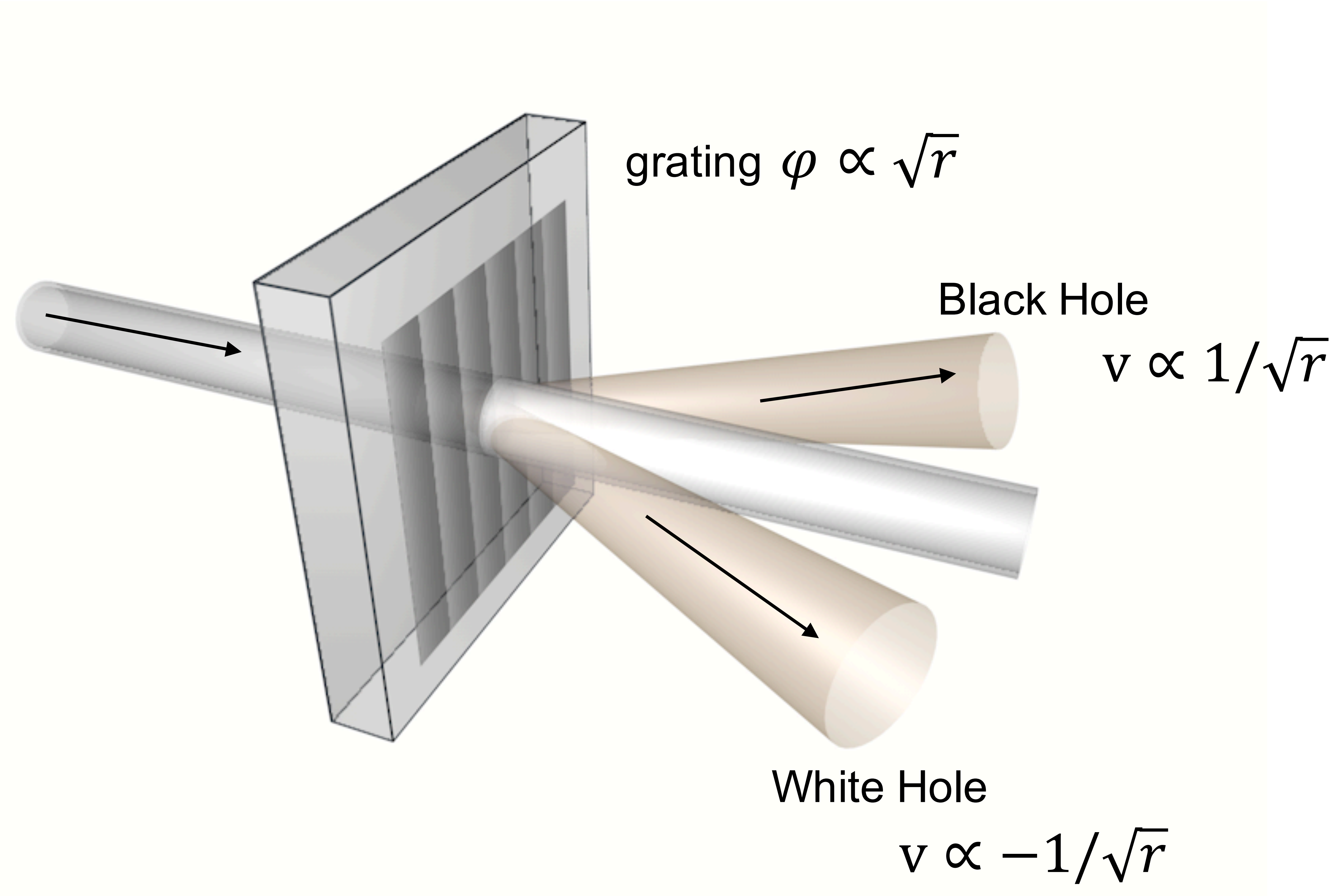
(See papers by Francesco Marino, PRA, 2008-2009)

Analogue gravity in photon fluids

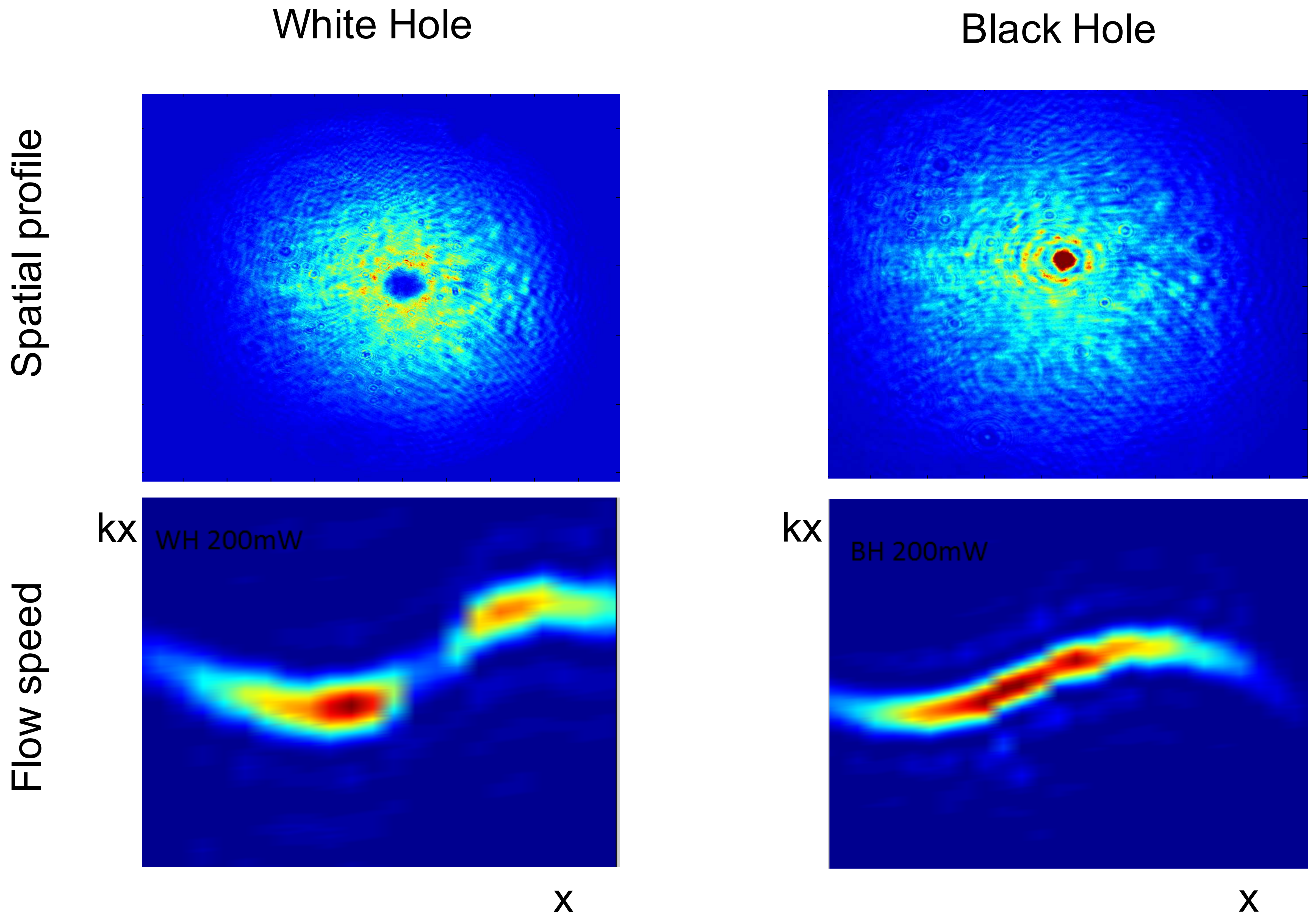
Amplification from rotational motion of an absorbing object (e.g. a black hole)



Analogue gravity in photon fluids

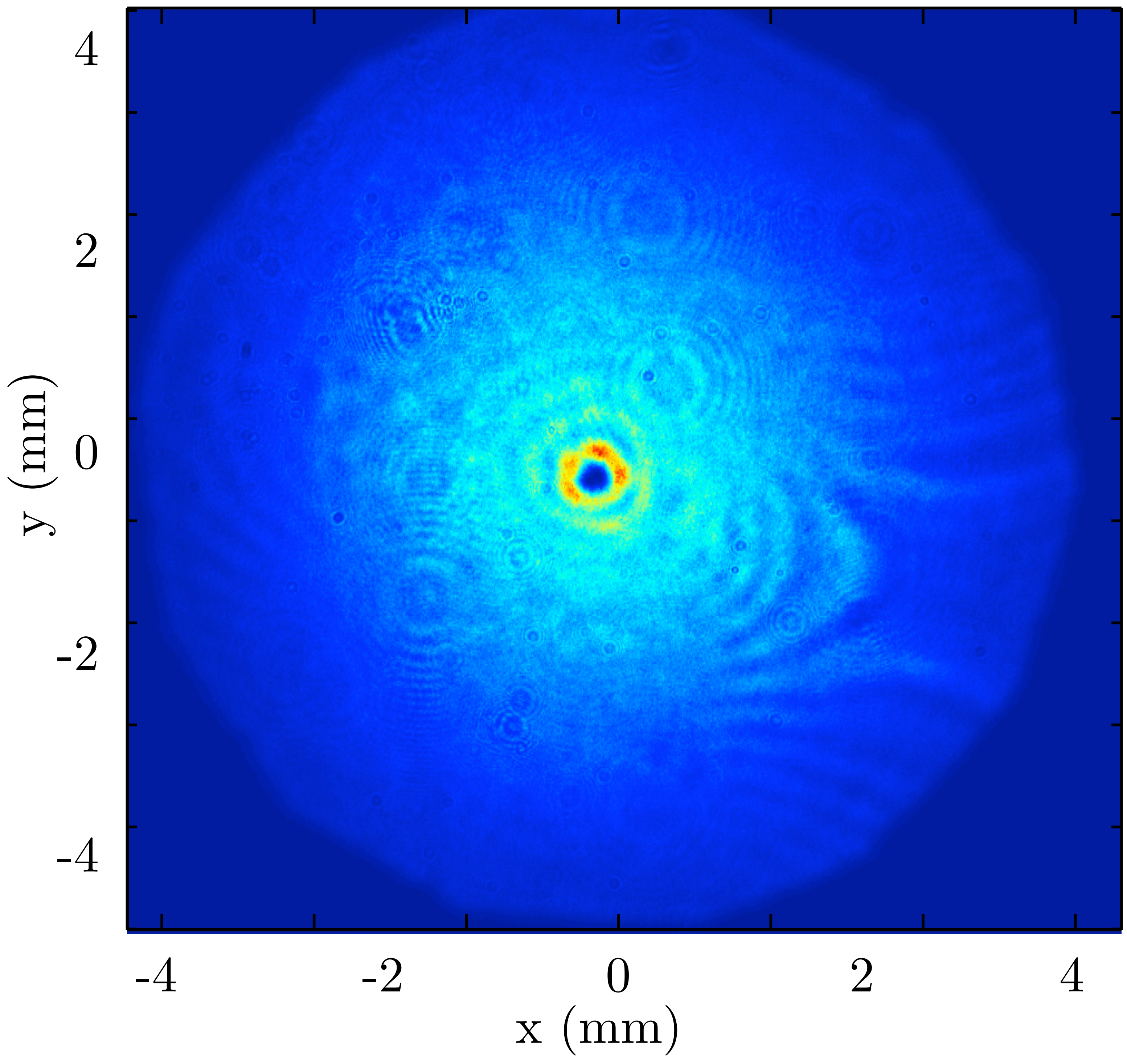


Analogue gravity in photon fluids



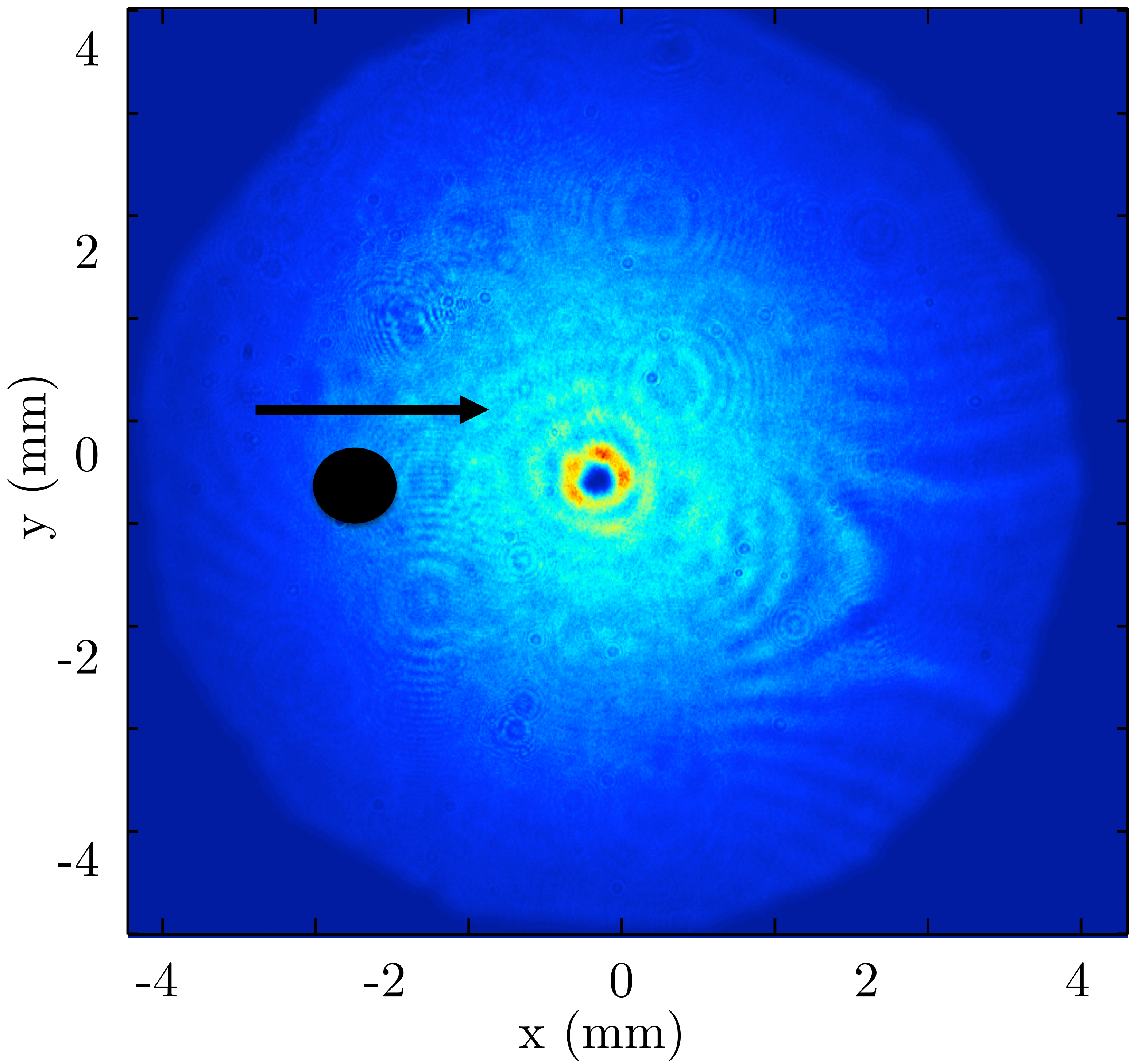
Analogue gravity in photon fluids

White Hole with rotation



Analogue gravity in photon fluids

Scan aperture and measure kx and ky

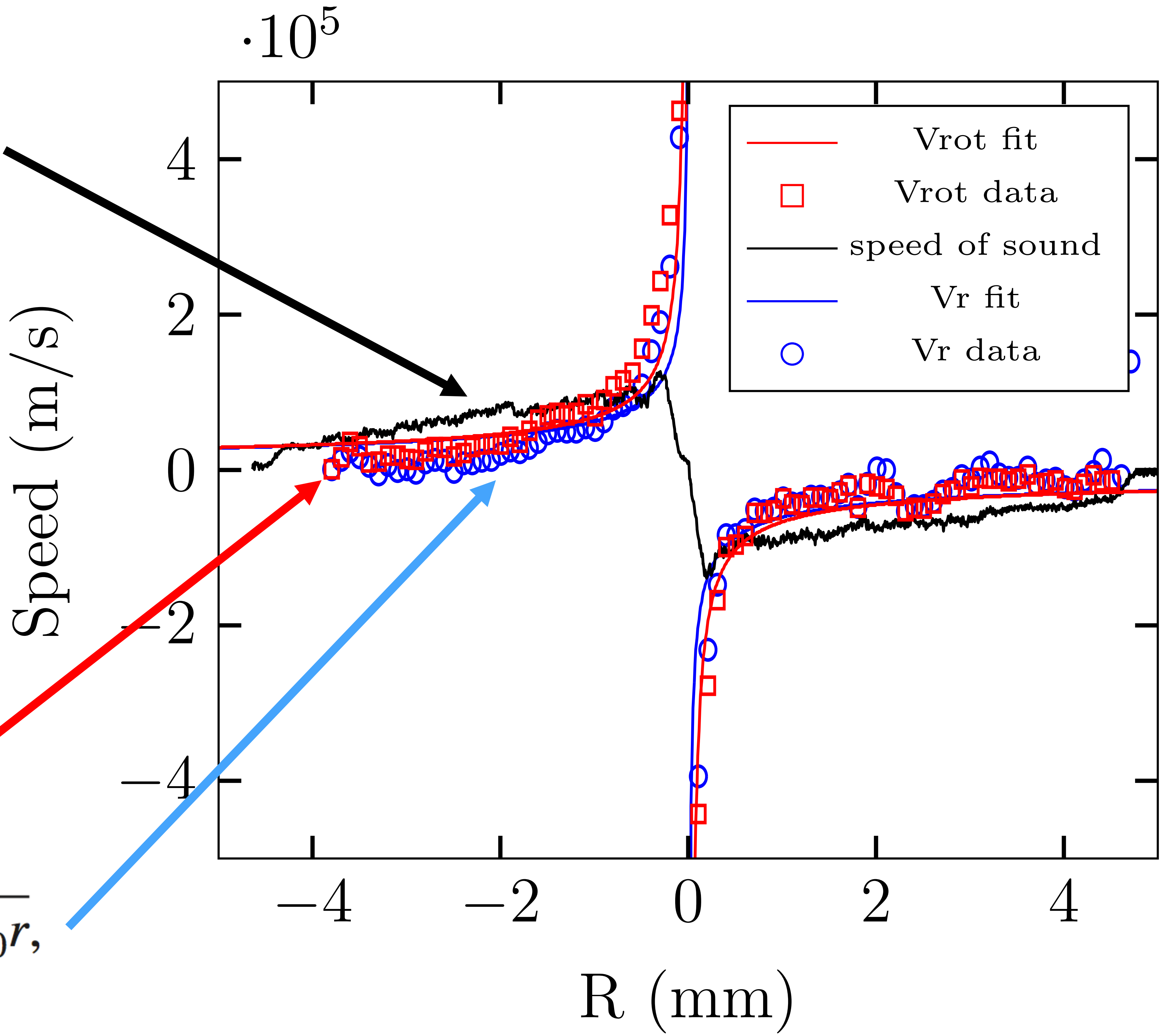


Analogue gravity in photon fluids

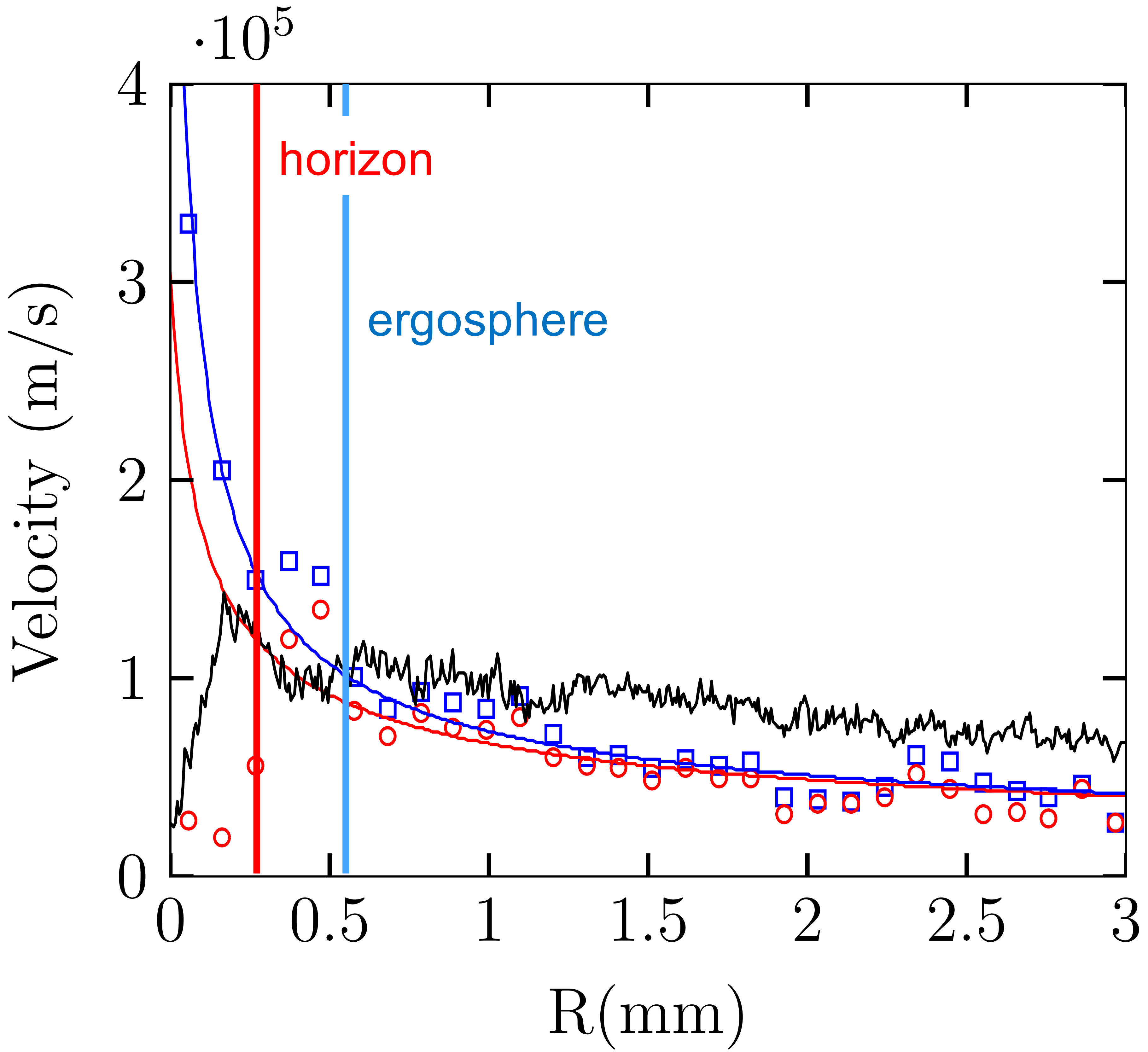
$c_s = \sqrt{c^2 n_2 \rho_d / n_0^3}$

$v_0 = \sqrt{v_\theta^2 + v_r^2}$

$v_r = -c\pi / k n_0 \sqrt{r_0 r},$

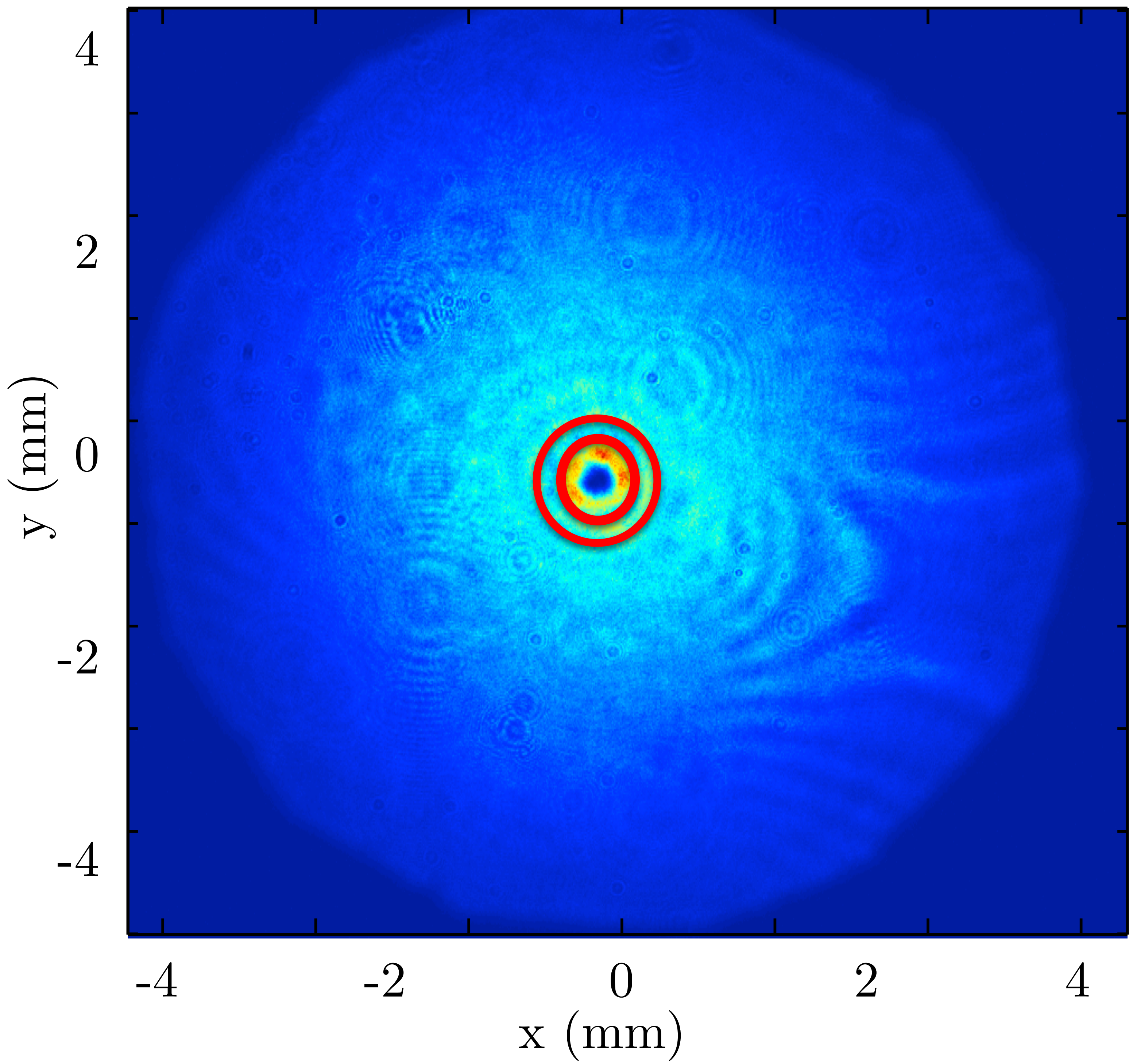


Analogue gravity in photon fluids



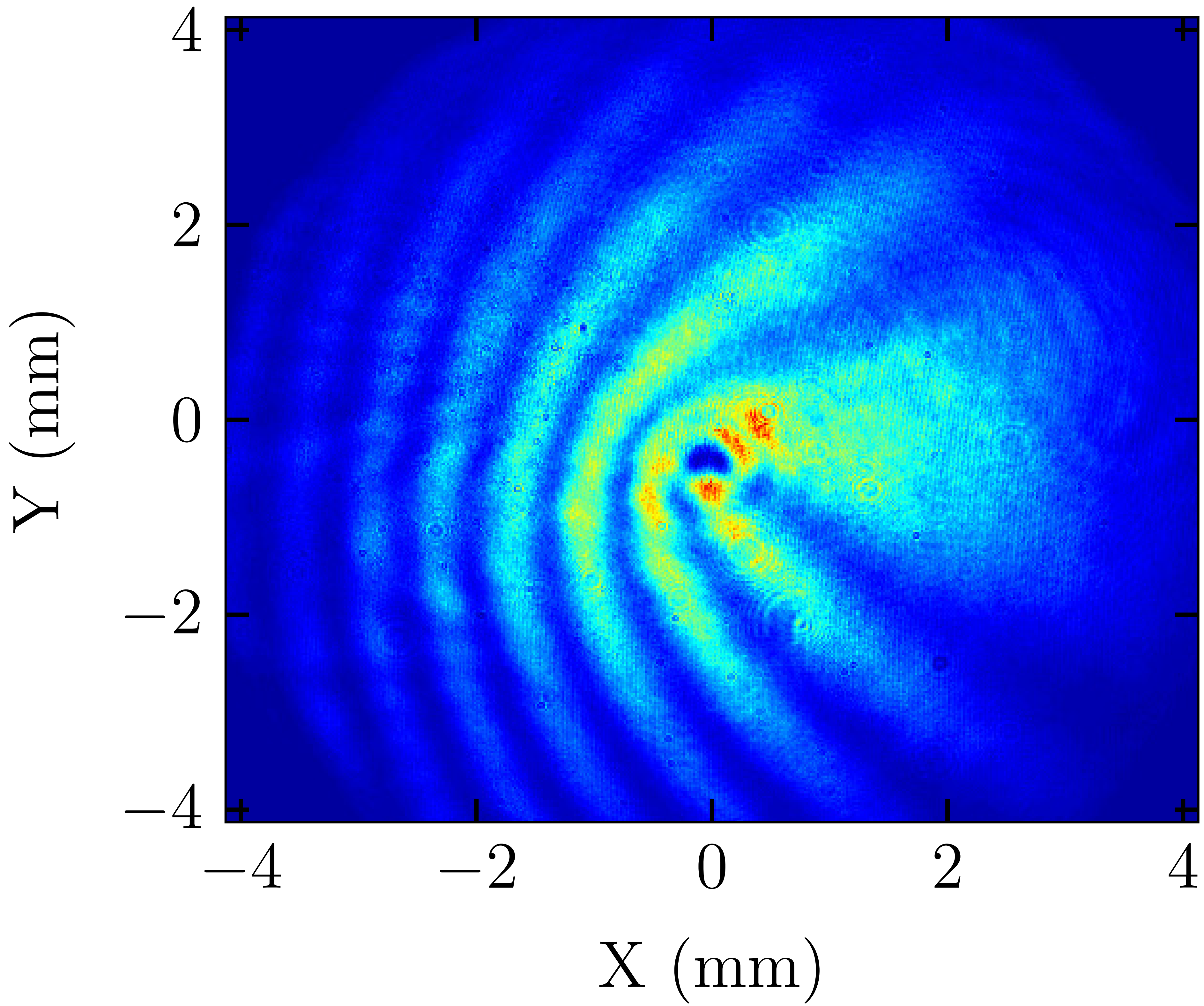
Analogue gravity in photon fluids

Scan aperture and measure kx and ky



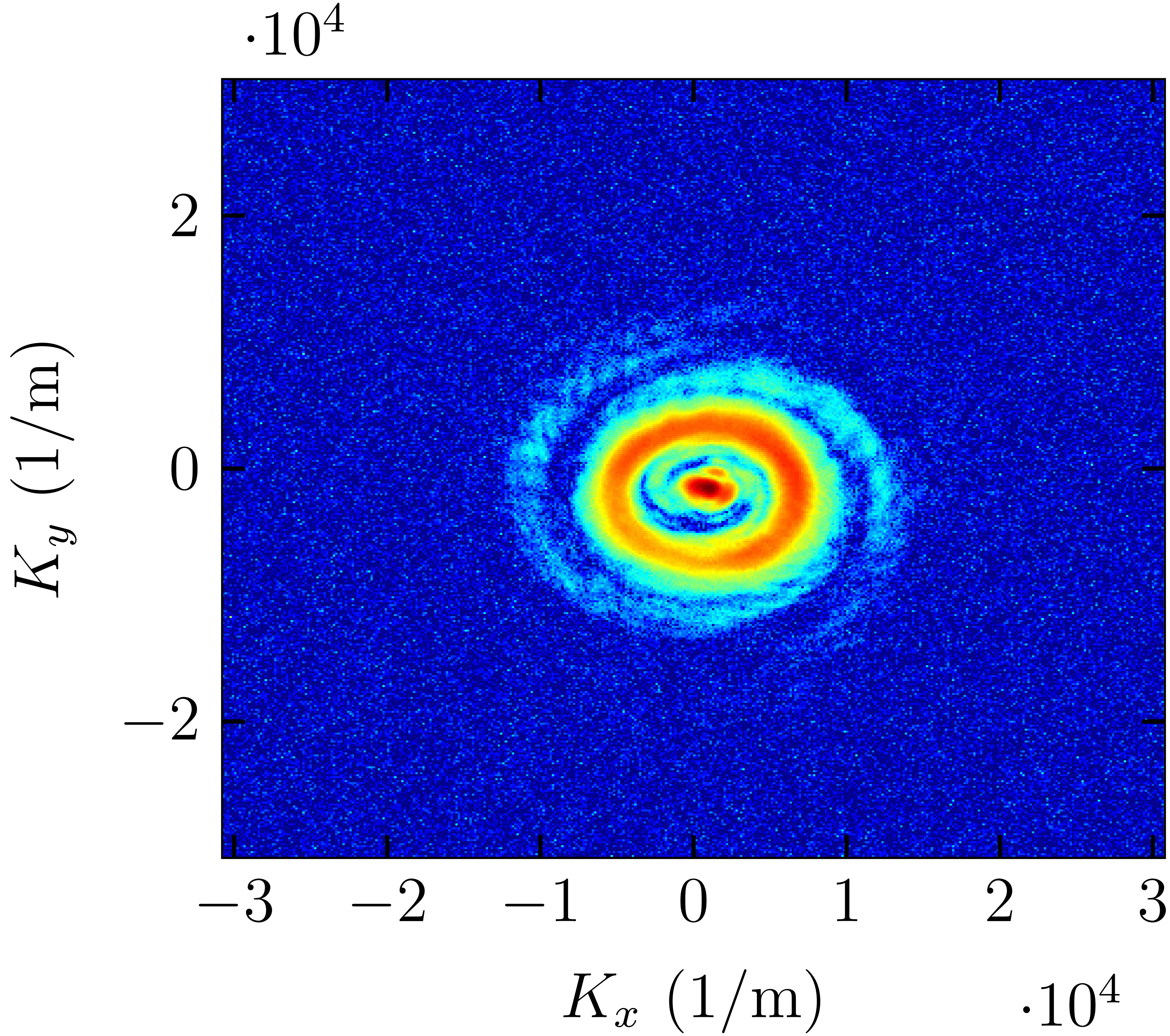
Scattering from rotating BHs

Near field image of BH + plane wave probe beam at an angle



Scattering from rotating BHs

Far field image of BH + plane wave probe beam at an angle



Newton Schrodinger equation

ARTICLES
PUBLISHED ONLINE: 31 AUGUST 2015 | DOI: 10.1038/NPHYS3451
nature physics

Optical simulations of gravitational effects in the Newton–Schrödinger system

Rivka Bekenstein*, Ran Schley, Maor Mutzafi, Carmel Rotschild and Mordechai Segev

$$i\hbar\psi_t + \frac{\hbar^2}{2m}\nabla^2\psi + m\phi\psi = 0$$

$$\nabla^2\phi = -4\pi G m|\psi|^2$$

NSE – Diosi, Penrose – study Gravity-induced quantum wavefunction collapse
Schrodinger equation +
Nonlocal, nonlinear gravitational interaction

$$i\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{2k}\nabla_\perp^2\mathcal{E} + k_0\Delta n\mathcal{E} = 0,$$

$$\nabla_\perp^2(\Delta n) = -\frac{\alpha\beta}{\kappa}|\mathcal{E}|^2$$

NLSE – standard **wave-equation in nonlinear optics +**
Nonlocal, nonlinear thermal interaction

Newton Schrodinger equation

nature COMMUNICATIONS

ARTICLE

Received 27 Feb 2016 | Accepted 7 Oct 2016 | Published 14 Nov 2016 DOI: 10.1038/ncomms13492 OPEN

Optical analogues of the Newton–Schrödinger equation and boson star evolution

Thomas Roger[1], Calum Maitland[1], Kali Wilson[1], Niclas Westerberg[1], David Vocke[1], Ewan M. Wright[1,2] & Daniele Faccio[1]

$$i\hbar\psi_t + \frac{\hbar^2}{2m}\nabla^2\psi + m\phi\psi = 0$$

$$\nabla^2\phi = -4\pi G m|\psi|^2$$

$$i\frac{\partial\mathcal{E}}{\partial z} + \frac{1}{2k}\nabla_\perp^2\mathcal{E} + k_0\Delta n\mathcal{E} = 0,$$

$$\nabla_\perp^2(\Delta n) = -\frac{\alpha\beta}{\kappa}|\mathcal{E}|^2$$

Newton Schrodinger equation

3D

$$i\hbar\psi_t + \frac{\hbar^2}{2m}\nabla^2\psi + m\phi\psi = 0$$

$$\nabla^2\phi = -4\pi G m|\psi|^2$$

2D

$$i\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{2k}\nabla_\perp^2 \mathcal{E} + k_0 \Delta n \mathcal{E} = 0,$$

$$\nabla_\perp^2(\Delta n) = -\frac{\alpha\beta}{\kappa}|\mathcal{E}|^2$$

Newton Schrodinger equation

$$\tilde{R}(K_\perp) = \left(\frac{\alpha\beta}{\kappa\gamma}\right)\frac{1}{K_\perp^2}$$

1/r Coulomb-like potential

$$R(\mathbf{r}_\perp - \mathbf{r}'_\perp) = -\left(\frac{\alpha\beta}{\kappa\gamma}\right)\frac{1}{2\pi}\ln\left(|\mathbf{r}_\perp - \mathbf{r}'_\perp|\right)$$

3D

$$i\hbar\psi_t + \frac{\hbar^2}{2m}\nabla^2\psi + m\phi\psi = 0$$

$$\nabla^2\phi = -4\pi G m|\psi|^2$$

2D

$$i\frac{\partial\mathcal{E}}{\partial z} + \frac{1}{2k}\nabla_\perp^2\mathcal{E} + k_0\Delta n\mathcal{E} = 0,$$

$$\nabla_\perp^2(\Delta n) = -\frac{\alpha\beta}{\kappa}|\mathcal{E}|^2$$

Newton Schrodinger Equation

Problem:
Analytical solution does not account for boundaries.
-> introduce boundaries using Distributed Loss Model

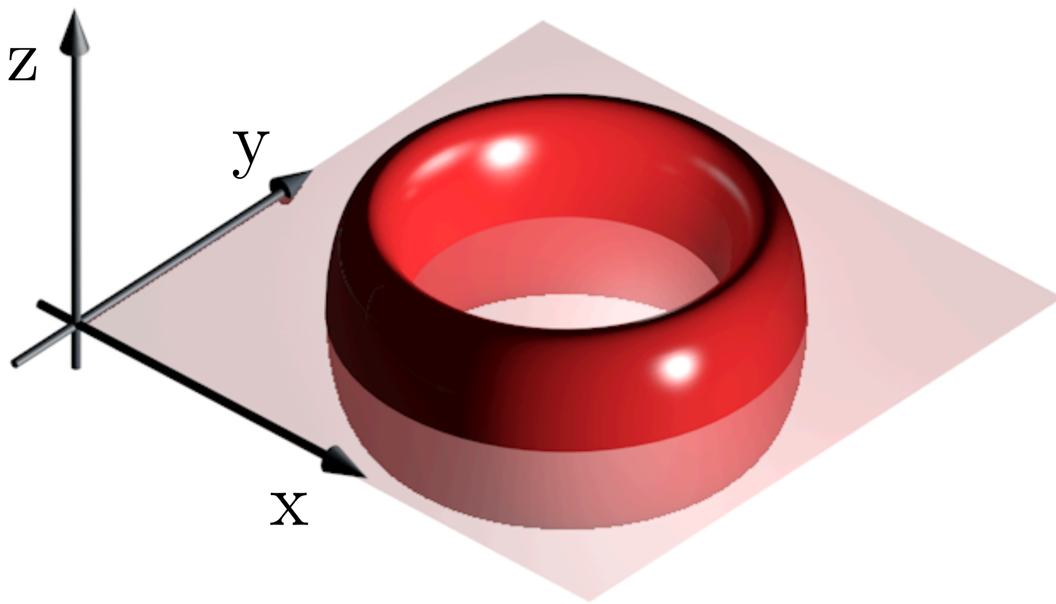
$$\tilde{R}(K_\perp) = \frac{1}{1+(\sigma K_\perp)^2}$$

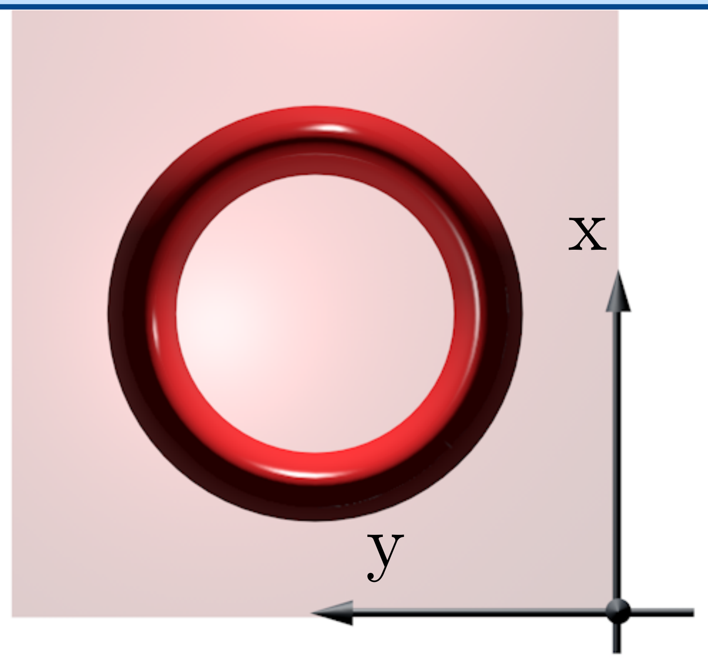
Well-behaved in direct space
(exp decaying behaviour)

$$R(\mathbf{r}_\perp - \mathbf{r'}_\perp) = \frac{1}{2\pi\sigma^2} K_0\left(\frac{|\mathbf{r}_\perp - \mathbf{r'}_\perp|}{\sigma}\right)$$

3D	**2D**
Gravitational (Coulomb-like) potential	DLM Thermal response function
$K_z \ll K_\perp$	$(\sigma K_\perp)^2 \gg 1$
$\tilde{R}(K_\perp) = 1/K_\perp^2$	$\tilde{R}(K_\perp) = 1/(\sigma K_\perp)^2$

Newton Schrodinger equation





Exact correspondence between NLSE+heat and SE+gravity when $(\sigma K_\perp)^2 \gg 1$

3D

Gravitational (Coulomb-like) potential

$$K_z \ll K_\perp$$

$$\tilde{R}(K_\perp) = 1/K_\perp^2$$

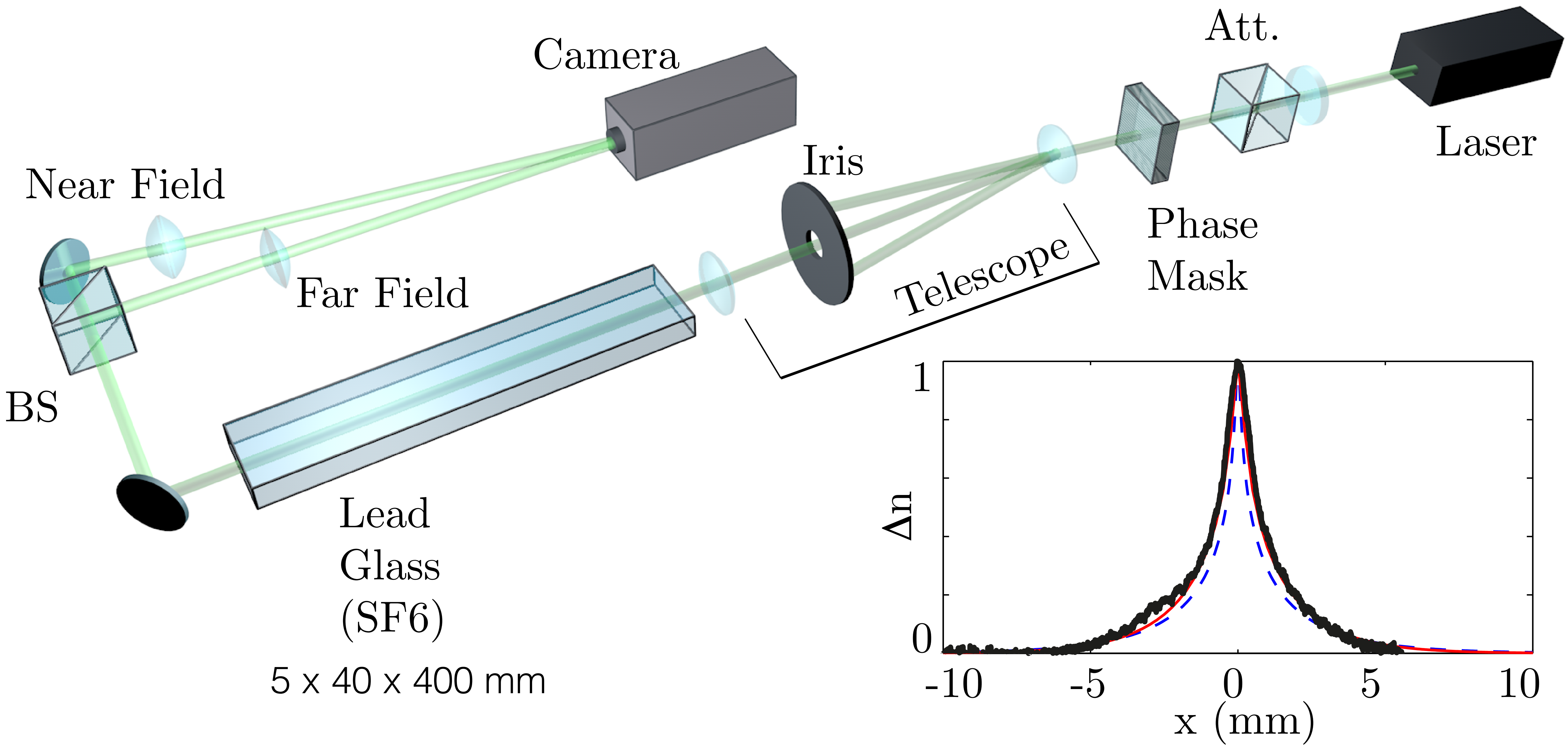
2D

DLM Thermal response function

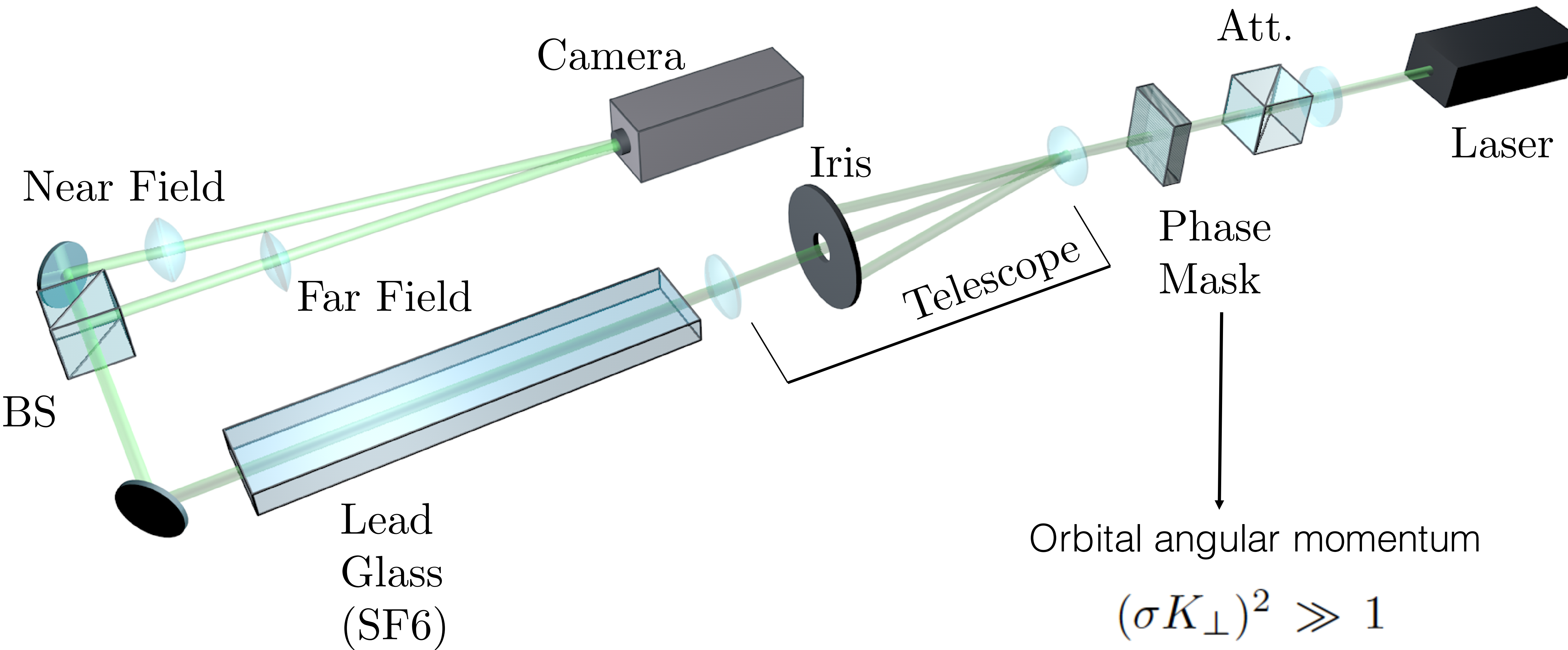
$$(\sigma K_\perp)^2 \gg 1$$

$$\tilde{R}(K_\perp) = 1/(\sigma K_\perp)^2$$

Experiments



Experiments



5 x 40 x 400 mm

Orbital angular momentum

$$(\sigma K_{\perp})^2 \gg 1$$

Rotating boson star

Boson stars

Boson cloud or BEC

Described by a wave function or order parameter

Einstein-Klein-Gordon equation predicts stable solution:
gravitational collapse is counter-balanced by "dispersion", i.e.
by Heisenberg uncertainty

In the weak field limit, EKG -> reduces to the NSE

Dark matter candidates, neutron star models, alternative to some BHs

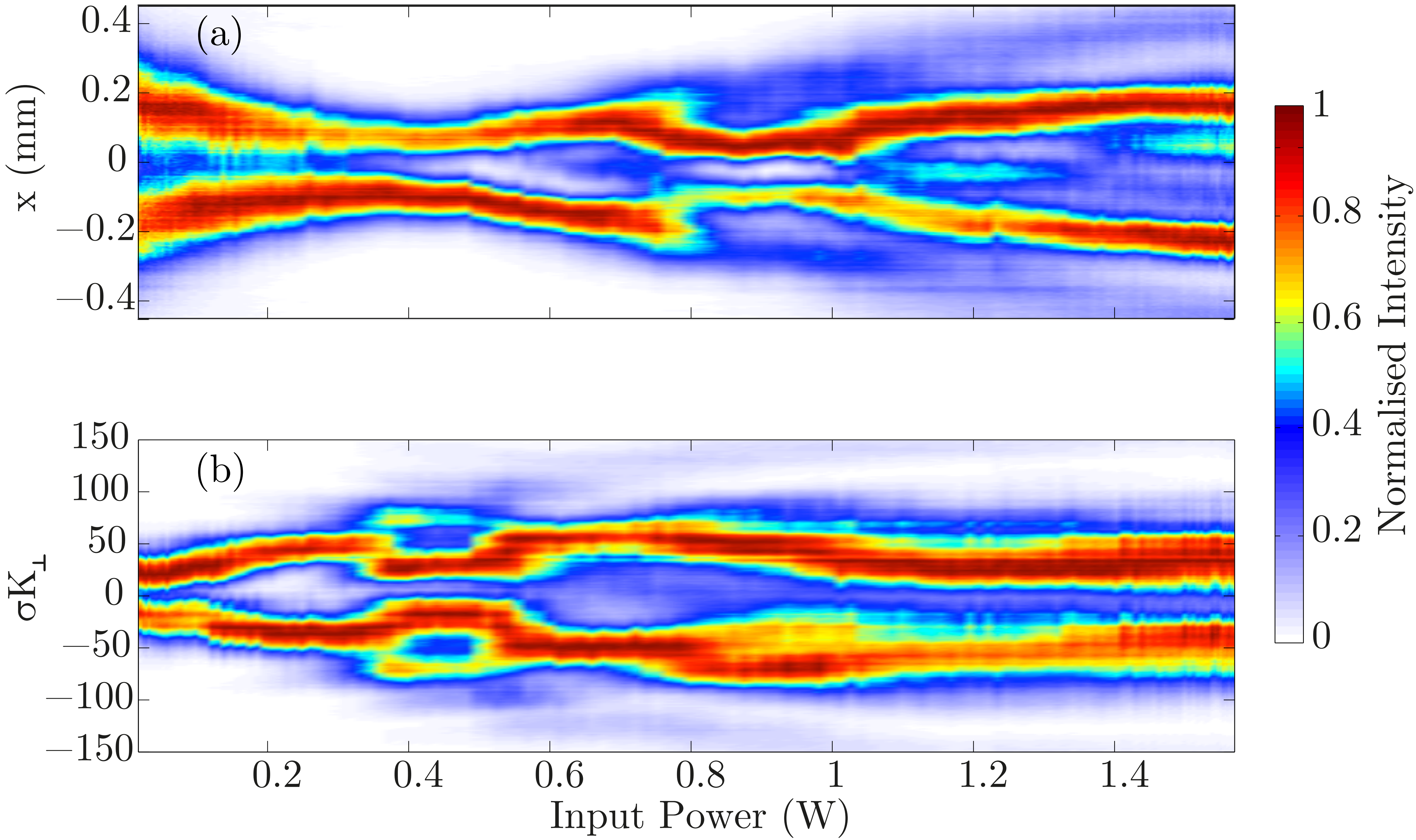
Living Rev. Relativity, **15**, (2012), 6
http://www.livingreviews.org/lrr-2012-6

LIVING REVIEWS *in relativity*

Dynamical Boson Stars

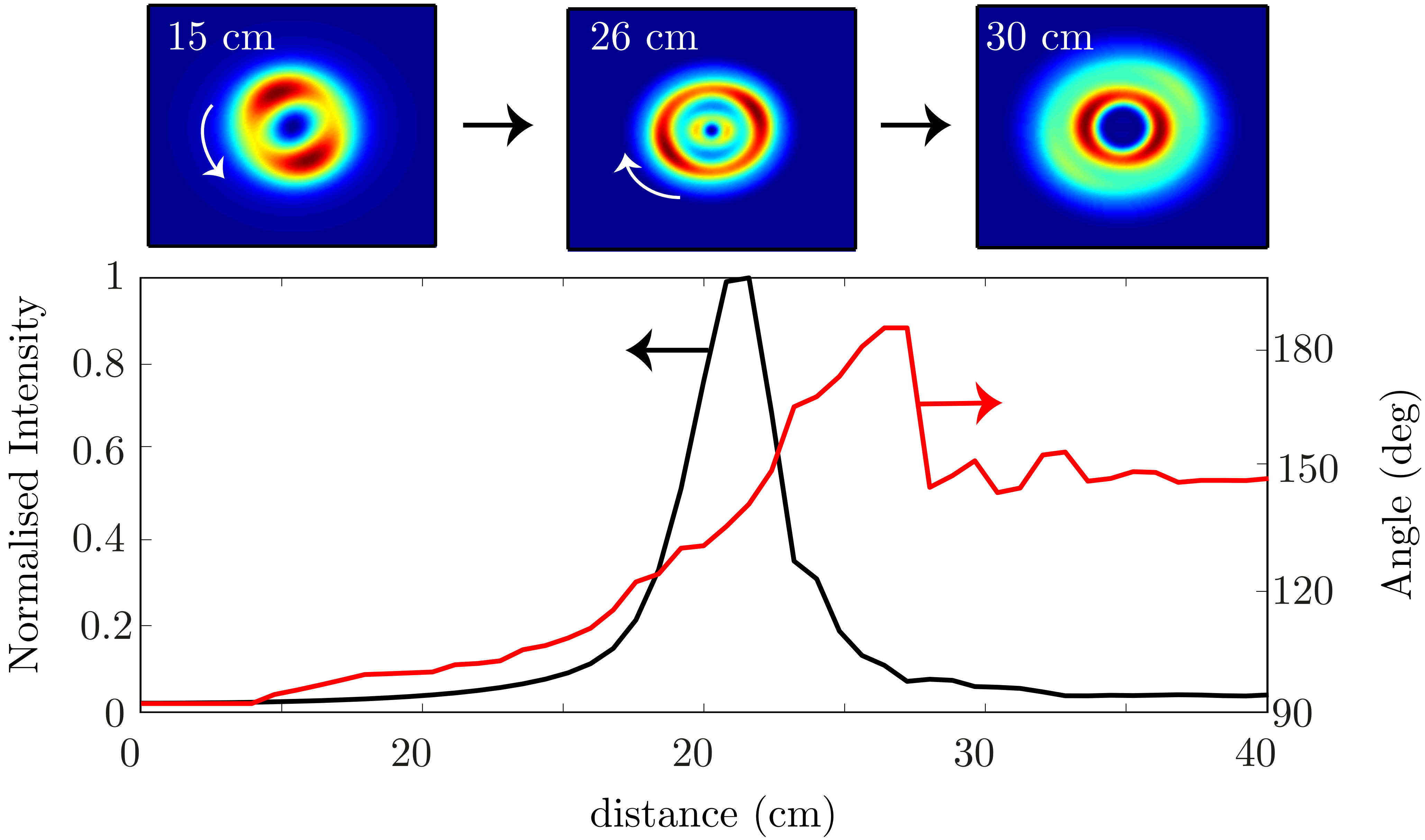
Steven L. Liebling

Experiments: Boson stars



Experiments: Boson stars

rotation dynamics

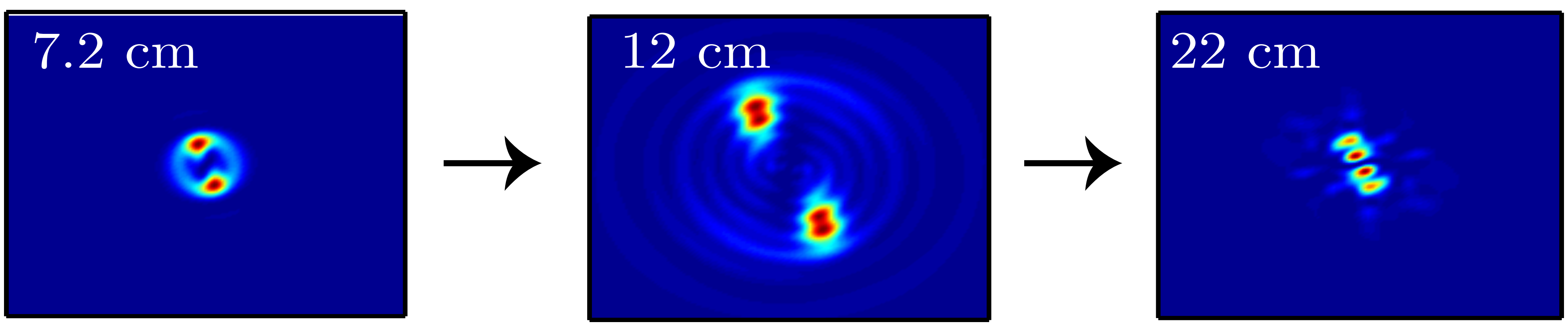


Experiments: Boson stars

Complex rotation dynamics

@ high power (star densities), collapse is arrested by OAM phase singularity

-> will stop the formation of a black hole !!



Summary

- Two dimensional photon fluids exhibit superfluid behaviour crucial for mimicking kinematics in Lorentz invariant systems
- -> black hole supperadiance from rotating BHs
- -> NSE turbulence and links with gravity
- -> Newton-Schrodinger equation in focusing regime
- -> Quantum effects ??
- -> go back to the Penrose idea: study wavefunction collapse